On Covariances for fusing Laser Rangers and Vision with Sensors Onboard a Moving Robot

Jonas Nygåards¹ and Åke Wernersson^{2,1}

¹) Robotics / Autonomous Mechanical Systems, IKP, Linköping University,
 S-58183 Linköping, Sweden, jonny@ikp.liu.se, robotics@ikp.liu.se

²) Robotics & Automation, Luleå University of Technology,
 S-97187 Luleå, Sweden, akewe@sm.luth.se, robotics@sm.luth.se

Abstract

Consider a robot to measure or operate on man made objects randomly located in the workspace. The optronic sensing onboard the robot are a scanning range measuring time-of-flight laser and a CCD camera. The goal of this paper is to give *explicit* covariance matrices for the extracted geometric primitives in the surrounding workspace. Emphasis is on correlation properties of the stochastic error models during motion. Topics studied includes;

- Covariance of Radon/Hough peaks for plane surfaces
- Covariance's for the intersection of two planes
- Equations for combining vision features, plane surfaces and range discontinuities.
- Explicit equations of how the covariance matrices are transformed during the robot motion.

Typical applications are; Models for verification and updating of CAD-models when navigating inside buildings and industrial plants. Accumulating sensor readings for a telecommanded robot.

Keywords: Optronic Sensors, Range Laser, Camera, Radon / Hough transform, Estimation of Planes, Estimation of Corners, Primitives in CAD-models, Navigation.

1. Introduction

Figures 2-4 are laser measurements and a CCD image from a fairly uncluttered indoor scene. The goal of this paper is not algorithms for extracting the geometric primitives but rather *explicit* covariance matrices for describing the uncertainty. The estimated primitives should both be used for controlling the robot, for sensor allocation and also allowing new measurements after motion of the robot to be fused in a consistent way.

Related work on navigation of mobile robots based on plane surfaces are found in, say; [Cro89], [HBC96], [LFW96], [MoC89] and [SSC90] while [KLO93], presents a non parametric solution for natural environments. Special references to covariances for estimating planes are found in [Kan94] and [Lar95].

The *explicit* equations for the covariances of the estimated features are approximations based on models for sensor resolution, number of measurements in a sector etc. The coordinate systems

are selected by geometry and the resulting covariances outlines the nature of the uncertainty in the estimates of planes and their intersections.

The result complement earlier results on stochastic maps [MoC89], [SSC90] and related work as [Tar92] towards an important class of applications. The motivation behind the work is the need for explicit models in design of feedback control and sensor planning as in [NNW97] and [NiN96].

2. Covariance Matrix for the Extracted Radon Parameters of a Plane

Consider N range measurements on a segment of a plane. In this section a covariance matrix for the

Figure 1 A vertical plane parametrized by the range ρ and the perpendicular orientation ϕ . On a segment of the plane N range measurements are measured. An approximation of the covariance matrix for (ρ, ϕ) is given below

extracted parameters describing the plane is derived. First consider a vertical plane described by the distance ρ to the sensor and the orientation ϕ of the perpendicular. On a segment of the vertical plane a set of ranges $r_1 \dots r_N$ are measured with a constant step δ_β in angle. These N measurements gives a Center of Gravity (COG) described in polar co-ordinates as (r_c, β_c) , compare this with Figure 1.

Figure 2 Three projections and a 3D-view of range data from a ladar scan of the corner in the CCD image of Figure 3. The data has been transformed to a co-ordinate frame where the line of intersection between the two walls is the z-axis. The x-axis is given as the normal of the plane corresponding to the trapezoid above the window. Note that the wall is not flat! - the planes above and under the window have a mutual difference of 2-5 cm. Several objects are easily identified like the radiator below the window, an I-beam at $x=0.5$ m, $y=1$ m and a pipe at $x=0.3$ m, $y=0.2$ m. At $x=1.2-2$ m there is a concrete slab and a couple of stones leaning against the wall.

Figure 3 Wide angle CCD-image of a corner in the laboratory. The heat radiators, the I-beam and pipe in the corner and some flowers in the window are clearly visible. To the left is a slab of concrete leaning towards the wall. The image is partially compensated for radial distortion.

Figure 4 A horizontal range slice from the range measuring laser. The pipe and I-beam in the corner are detected as well as the flowers. The window does not give any return to the laser. The bookshelf to the left is barely visible.

Next, drop a local Cartesian coordinate system (ξ, γ) at the COG. Each measurement can then be written

$$\gamma_i = k(\xi_i - \bar{\xi}) + \bar{\gamma} + \varepsilon_i \quad (1)$$

where ε_i is the range noise with variance σ_r^2 . In this equation $\xi_n \approx r_c \delta_\beta \left(n - \frac{N}{2}\right)$ and $\gamma_i \approx r_i - r_c$.

Using linear regression gives the estimate

$$\hat{k} = \frac{\sum_{i=1}^N (\gamma_i - \bar{\gamma})(\xi_i - \bar{\xi})}{\sum_{i=1}^N (\xi_i - \bar{\xi})^2} \quad (2)$$

$$Var(k) \approx \frac{\sigma_r^2}{\sum_{i=1}^N (\xi_i - \bar{\xi})^2} \approx \frac{\sigma_r^2}{N} \quad (3)$$

$$\sum_{i=1}^N \left(r_c^2 \delta_\beta^2 i - r_c^2 \delta_\beta^2 \frac{N+1}{2} \right)^2 \approx r_c^2 \delta_\beta^2 N^3$$

From Figure 1, estimates of the (ρ, ϕ) representation of the line follows as

$$\hat{\rho} = r_c \cos(\arctan(\hat{k})) \quad (4)$$

$$\hat{\phi} = \beta_c + \arctan(\hat{k}) \quad (5)$$

Ignoring the uncertainty in β gives

$$\sigma_\phi^2 = Var(\phi) \approx \frac{12\sigma_r^2}{(r\delta_\beta)^2 N^3 (1 + \hat{k}^2)^2} \approx \frac{12\sigma_r^2}{(r\delta_\beta)^2 N^3} \quad (6)$$

Introducing

$$\sigma_{\rho_c}^2 = \cos^2(\arctan(\hat{k})) \frac{\sigma_r^2}{N}$$

the error covariance can be written

$$Cov\left(\begin{bmatrix} \rho & \phi \end{bmatrix}^T\right) \approx \begin{pmatrix} \sigma_{\rho_c}^2 + \hat{\eta}^2 \sigma_\phi^2 & \hat{\eta} \sigma_\phi^2 \\ \hat{\eta} \sigma_\phi^2 & \sigma_\phi^2 \end{pmatrix} \quad (7)$$

where $\hat{\eta} = -r_c \sin(\arctan(\hat{k}))$ is the distance between the COG and the closest point on the plane. Now if the robot moves the orientation uncertainty should to still pivot around the COG when the old measurements are compared with the new.

Interpretation: *A robot travelling along the surface at a constant distance keeps $(\hat{\rho}, \hat{\phi})$ constant but the covariance matrix will change due to changing η*

In the regression analysis the uncertainty in the direction β of the range samples is not included.

3. Fisher Information Matrix for the Extracted Radon Parameters of a Plane

Below we are to describe the estimates using the information matrix in the sense of Fisher i. e. the

inverse of the covariance matrix. First consider a plane parametrized as before by $u = [\rho, \phi]$ and the distance function:

$$F(z, u) = x \cos(\phi) + y \sin(\phi) - \rho = 0 \quad (8)$$

A single measurement on this plane is

$$z_i = [x_i, y_i] + \varepsilon_i \quad (9)$$

where the noise is modelled as $E[\varepsilon_i] = 0$ $E[\varepsilon_i \varepsilon_i^T] = R$

The information matrix from a single measurement is then approximated to the first order by the matrix:

$$\begin{aligned} & \frac{\partial F(z, u)}{\partial u} \left(\frac{\partial F(z, u)}{\partial z} \text{cov}(z) \left(\frac{\partial F(z, u)}{\partial z} \right)^T \right)^{-1} \left(\frac{\partial F(z, u)}{\partial u} \right)^T = \\ & [-1 \quad -x \sin(\phi) + y \cos(\phi)] (\sigma_f^2)^{-1} [-1 \quad -x \sin(\phi) + y \cos(\phi)]^T \\ & = \frac{1}{\sigma_f^2} \begin{bmatrix} 1 & -\eta \\ -\eta & \eta^2 \end{bmatrix} \end{aligned} \quad (10)$$

where the identities

$$\eta = -x \sin(\phi) + y \cos(\phi)$$

$$\sigma_f^2 = [\cos(\phi) \quad \sin(\phi)] R [\cos(\phi) \quad \sin(\phi)]$$

are used.

The geometric interpretations of η and σ_f^2 are

η = distance from the COG to the perpendicular of the plane.

σ_f^2 = projection of the measurement noise covariance on the normal direction of the plane i. e. the direction where the noise "contributes".

The resulting information matrix of the N measurements is then:

$$\sum_{i=1}^N \frac{1}{\sigma_{\beta}^2} \begin{bmatrix} 1 & -\eta_i \\ -\eta_i & \eta_i^2 \end{bmatrix} \approx \frac{1}{\sigma_{\beta}^2} \begin{bmatrix} N & -N\bar{\eta} \\ -N\bar{\eta} & \sum \eta_i^2 \end{bmatrix} \quad (11)$$

Comparing with the inverse of (7) gives

$$\begin{aligned} & \left(\begin{pmatrix} \sigma_{\rho_c}^2 + \hat{\eta}^2 \sigma_\phi^2 & \hat{\eta} \sigma_\phi^2 \\ \hat{\eta} \sigma_\phi^2 & \sigma_\phi^2 \end{pmatrix} \right)^{-1} = \begin{pmatrix} \frac{1}{\sigma_{\rho_c}^2} & -\frac{\hat{\eta}}{\sigma_{\rho_c}^2 \sigma_\phi^2} \\ -\frac{\hat{\eta}}{\sigma_{\rho_c}^2} & \frac{\sigma_{\rho_c}^2 + \hat{\eta}^2 \sigma_\phi^2}{\sigma_{\rho_c}^2 \sigma_\phi^2} \end{pmatrix} \approx \\ & \frac{1}{\cos^2(\phi) \sigma_r^2} \begin{pmatrix} N & -N\hat{\eta} \\ -N\hat{\eta} & \left(\frac{(r_c \delta_\beta)^2 N(N-1)(N+1)}{12} + N\hat{\eta}^2 \right) \end{pmatrix} \end{aligned} \quad (12)$$

For 3D measurements $z_i = [x_i, y_i, z_i] + \varepsilon_i$ the plane is parametrised by $u = [\rho, \theta, \phi]$. Introduce the distance function:

$$F(z, u) = x \cos(\phi) \cos(\theta) + y \sin(\phi) \cos(\theta) - z \sin(\theta) - \rho = 0 \quad (13)$$

The corresponding information turn out to be

$$\sum_{i=1}^N \frac{1}{\sigma_{\beta}^2} \begin{bmatrix} 1 & \xi_i & -\eta_i \cos(\theta) \\ \xi_i & \xi_i^2 & -\xi_i \eta_i \cos(\theta) \\ -\eta_i \cos(\theta) & -\xi_i \eta_i \cos(\theta) & \eta_i^2 \cos^2(\theta) \end{bmatrix} \quad (14)$$

where the (η, ξ) parametrisation is

$$\eta = -x \sin(\phi) + y \cos(\phi)$$

$$\xi = x \cos(\phi) \sin(\theta) + y \sin(\phi) \sin(\theta) + z \cos(\theta)$$

and the projected noise is

$$\sigma_j^2 = \begin{bmatrix} \cos(\phi) \cos(\theta) & \sin(\phi) \cos(\theta) & -\sin(\theta) \end{bmatrix}^* R \begin{bmatrix} \cos(\phi) \cos(\theta) & \sin(\phi) \cos(\theta) & -\sin(\theta) \end{bmatrix}^T$$

4. The Covariance Matrix for an Intersecting 'Vision Plane'

For convenience consider the intersection of two planes in Figure 5, expressed in the co-ordinate system attached to plane 1 i. e. Plane 1 constrain the intersection to lie along the η -direction:

$$\mathbf{x}_{1i} = \begin{bmatrix} 0 & \eta_i & 0 \end{bmatrix}$$

$$C(\mathbf{x}_{1i}) = \begin{bmatrix} \sigma_{\rho_1}^2 + \eta_i^2 \sigma_{\phi_1}^2 & 0 & \eta_i \sigma_{\phi_1}^2 \\ 0 & s & 0 \\ \eta_i \sigma_{\phi_1}^2 & 0 & \sigma_{\phi_1}^2 \end{bmatrix} \quad (15)$$

Since η_i is unknown s should be considered as very large i.e. $1/s \rightarrow 0$. Now express the intersection as a point on plane 2 relative to the local co-ordinate system at plane 1:

$$\mathbf{x}_{2i} = R(\phi_1 - \phi_2) \begin{bmatrix} \rho_{2i} & \eta_{2i} & 0 \end{bmatrix}$$

$$C(\mathbf{x}_{2i}) = R(\phi_1 - \phi_2) \begin{bmatrix} \sigma_{\rho_2}^2 + \eta_{2i}^2 \sigma_{\phi_2}^2 & 0 & \eta_{2i} \sigma_{\phi_2}^2 \\ 0 & s & 0 \\ \eta_{2i} \sigma_{\phi_2}^2 & 0 & \sigma_{\phi_2}^2 \end{bmatrix} \times R(\phi_1 - \phi_2)^T + C(\mathbf{x}_{1i}) + M C(\mathbf{x}_{1i}) M^T \quad (16)$$

where $R(\phi)$ is the rotation matrix around the vertical and M is the magnification matrix due to the orientation uncertainty, [D-W88], given by:

$$R(\phi) = \begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 0 & -\eta_{2i} \\ 0 & 0 & \rho_{2i} \\ 0 & 0 & 0 \end{bmatrix}$$

The two expressions for the intersection can now be fused using a Kalman filter:

$$\mathbf{x}_i = (\mathbf{I} - \mathbf{K}) \mathbf{x}_{1i} + \mathbf{K} \mathbf{x}_{2i} \quad (17)$$

using the Kalman gain [BSL93 p.139]

$$\mathbf{K} = C(\mathbf{x}_i) C(\mathbf{x}_{12})^{-1}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ -\cot(\phi_1 - \phi_2) & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Figure 5 The intersection between two planes where plane 1 could be the plane of an edge detected in a camera and plane 2 estimated from range measurements.

$$C(\mathbf{x}_i) = (C(\mathbf{x}_{1i})^{-1} + C(\mathbf{x}_{2i})^{-1})^{-1} =$$

$$\begin{bmatrix} \sigma_a^2 & \sigma_a^2 \cot(\phi_1 - \phi_2) & 0 \\ \sigma_a^2 \cot(\phi_1 - \phi_2) & \frac{\sigma_b^2 + \sigma_a^2 \cos^2(\phi_1 - \phi_2)}{\sin^2(\phi_1 - \phi_2)} & 0 \\ 0 & 0 & \sigma_{\phi_1}^2 \end{bmatrix} \quad (18)$$

where σ_a^2 and σ_b^2 are the [1,1] elements of the covariance matrices $C(\mathbf{x}_{1i}), C(\mathbf{x}_{2i})$ respectively.

$$\sigma_a^2 = \cos^2(\arctan(\hat{k}_1)) \frac{\sigma_r^2}{N_1} + \hat{\eta}_{1i}^2 \frac{12\sigma_r^2}{r_1^2 \delta_\beta^2 N_1^3} \quad (19)$$

$$\sigma_b^2 = \cos^2(\arctan(\hat{k}_2)) \frac{\sigma_r^2}{N_2} + \hat{\eta}_{2i}^2 \frac{12\sigma_r^2}{r_2^2 \delta_\beta^2 N_2^3}$$

$$+ \sigma_a^2 \cos^2(\phi_1 - \phi_2) + \sigma_{\phi_1}^2 (2\eta_{1i} \cos(\phi_1 - \phi_2) \times (\rho_{2i} \sin(\phi_1 - \phi_2) + \eta_{2i} \cos(\phi_1 - \phi_2)) + (\eta_{2i} \cos(\phi_1 - \phi_2) + \rho_{2i} \sin(\phi_1 - \phi_2))^2)$$

$$(20)$$

Note that in the calculation of the inner inverses in (18) the limit for $1/s \rightarrow 0$ is used.

The transformation from the co-ordinates of plane 1 to world co-ordinates is performed using the formalism presented by Smith, Self and Cheeseman in [SSC90] (cp. the transformation in eq. (16)).

The generalisation for 3D is to transform the problem to a co-ordinate system with a z-coordinate direction parallel to the direction of the intersection. If the

direction of the two planes is represented by normal vectors the direction of the new z-axis is:

$$\mathbf{e}_z = \frac{\mathbf{n}_1 \times \mathbf{n}_2}{\|\mathbf{n}_1 \times \mathbf{n}_2\|} \quad (21)$$

In this coordinate system the solution for the intersection in eq. (15)-(19) can be used. Given perpendicular distance estimates $\hat{\rho}_1, \hat{\rho}_2$ and the estimated direction vectors $\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2$ for the two intersecting planes a more general formula for the point on the line with position vector perpendicular to the line is:

$$\begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \frac{(\hat{\rho}_1 - \hat{\rho}_2(\hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2))}{1 - (\hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2)^2} \hat{\mathbf{n}}_1 + \frac{(\hat{\rho}_2 - \hat{\rho}_1(\hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2))}{1 - (\hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2)^2} \hat{\mathbf{n}}_2 \quad (22)$$

As reference frame orientation one of the planes normals is used as x-direction and the normalised cross product of the normals as z. i.e. the reference frame of the corner is:

$$\mathbf{R}_3 = \begin{bmatrix} \hat{\mathbf{n}}_1^T \\ ((\hat{\mathbf{n}}_1 \times \hat{\mathbf{n}}_2) / \|\hat{\mathbf{n}}_1 \times \hat{\mathbf{n}}_2\| \times \hat{\mathbf{n}}_1)^T \\ (\hat{\mathbf{n}}_1 \times \hat{\mathbf{n}}_2)^T / \|\hat{\mathbf{n}}_1 \times \hat{\mathbf{n}}_2\| \end{bmatrix} = \begin{bmatrix} \mathbf{e}_x \\ \mathbf{e}_y \\ \mathbf{e}_z \end{bmatrix} \quad (23)$$

Following Kanatani [Kan94] a first order approximation of the covariance of a normalised vector is:

$$\text{If } \mathbf{n}_i = \frac{\mathbf{a}}{\|\mathbf{a}\|} \text{ then} \quad (24)$$

$$\mathbf{C}(\mathbf{n}_i) \approx \frac{1}{\|\mathbf{a}\|^2} (\mathbf{I} - \mathbf{n}_i \mathbf{n}_i^T) \mathbf{C}(\mathbf{a}) (\mathbf{I} - \mathbf{n}_i \mathbf{n}_i^T)$$

With resulting covariance for the direction along the intersection (assuming independence between plane estimates)

$$\mathbf{C}(\mathbf{e}_z) = \mathbf{C} \left(\frac{\hat{\mathbf{n}}_1 \times \hat{\mathbf{n}}_2}{\|\hat{\mathbf{n}}_1 \times \hat{\mathbf{n}}_2\|} \right) = \frac{\mathbf{P}_z (\tilde{\mathbf{n}}_2 \mathbf{C}(\hat{\mathbf{n}}_1) \tilde{\mathbf{n}}_2^T + \tilde{\mathbf{n}}_1 \mathbf{C}(\hat{\mathbf{n}}_2) \tilde{\mathbf{n}}_1^T) \mathbf{P}_z}{\|\hat{\mathbf{n}}_1 \times \hat{\mathbf{n}}_2\|^2} \quad (25)$$

where the projection matrix is

$$\mathbf{P}_z = (\mathbf{I} - \mathbf{e}_z \mathbf{e}_z^T)$$

and the tilde (~) denotes the skew symmetric matrix corresponding to a vector cross product i.e.

$$\tilde{\mathbf{n}}_1 \mathbf{n}_2 = \mathbf{n}_1 \times \mathbf{n}_2$$

$$\tilde{\mathbf{n}}_1 = \begin{bmatrix} 0 & -n_{z1} & n_{y1} \\ n_{z1} & 0 & -n_{x1} \\ -n_{y1} & n_{x1} & 0 \end{bmatrix} \quad (26)$$

Observe also that $\|\hat{\mathbf{n}}_1 \times \hat{\mathbf{n}}_2\|^2 = 1 - (\hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2)^2 \equiv s_\alpha^2$.

The covariances and cross covariances with the other direction vectors are:

$$\mathbf{C}(\mathbf{e}_x) = \mathbf{C}(\hat{\mathbf{n}}_1)$$

$$\mathbf{C}(\mathbf{e}_y) = \mathbf{P}_y \left(\tilde{\mathbf{e}}_z \mathbf{C}(\hat{\mathbf{n}}_1) \tilde{\mathbf{e}}_z^T + \tilde{\mathbf{n}}_1 \mathbf{C}(\mathbf{e}_z) \tilde{\mathbf{n}}_1^T - \frac{\tilde{\mathbf{n}}_1 \tilde{\mathbf{n}}_2 \mathbf{C}(\hat{\mathbf{n}}_1) \tilde{\mathbf{e}}_z^T + \tilde{\mathbf{e}}_z \mathbf{C}(\hat{\mathbf{n}}_1) (\tilde{\mathbf{n}}_1 \tilde{\mathbf{n}}_2)^T}{s_\alpha} \right) \mathbf{P}_y$$

$$\mathbf{C}(\mathbf{e}_x, \mathbf{e}_y) = \mathbf{P}_x \left(\mathbf{C}(\hat{\mathbf{n}}_1) \tilde{\mathbf{e}}_z^T - \mathbf{C}(\hat{\mathbf{n}}_1) (\tilde{\mathbf{n}}_1 \tilde{\mathbf{n}}_2)^T / s_\alpha \right) \mathbf{P}_y$$

$$\mathbf{C}(\mathbf{e}_x, \mathbf{e}_z) = \frac{\mathbf{P}_x \mathbf{C}(\hat{\mathbf{n}}_1) \tilde{\mathbf{n}}_2^T \mathbf{P}_z}{s_\alpha}$$

$$\mathbf{C}(\mathbf{e}_y, \mathbf{e}_z) = \mathbf{P}_y \left(\tilde{\mathbf{e}}_z \mathbf{C}(\hat{\mathbf{n}}_1) \tilde{\mathbf{n}}_2^T / s_\alpha - \tilde{\mathbf{n}}_1 \mathbf{C}(\mathbf{e}_z) \right) \mathbf{P}_z$$

5. An example of uncertainty for two intersecting planes

We conclude by an example illustrating the double coned shape of the uncertainty. For simplicity, the intersection of the two planes defines the z-axis, see Figure 6. Also, let plane 1 have its normal in the direction of the x-axis. The angle between the planes is ϕ_2 .

The laser is sampling in a rectangular grid with spacing Δ on each plane as illustrated in Figure 6. For plane 1 the $m_1 \times n_1$ range measurements are:

$$\mathbf{Z}_1 = \left\{ \begin{bmatrix} 0 \\ \Delta \left(i - \frac{m_1 + 1}{2} \right) + \eta_1 \\ \Delta \left(j - \frac{n_1 + 1}{2} \right) + \xi_1 \end{bmatrix} + \mathbf{v}_{ij}, i = 1..m_1, j = 1..n_1 \right\} \quad (26)$$

where the range noise is independent in the grid with

$$E[\mathbf{v}_{ij}] = \mathbf{0}, E[\mathbf{v}_{ij} \mathbf{v}_{ij}^T] = \sigma_r^2$$

Plane 2 has angle ϕ_2 from the x-axis. The $m_2 \times n_2$ measurements on plane 2 are (expressed in a co-ordinate frame oriented along the normal of plane 2):

$$\mathbf{Z}_2 = \left\{ \begin{bmatrix} 0 \\ \Delta \left(i - \frac{m_2 + 1}{2} \right) + \eta_2 \\ \Delta \left(j - \frac{n_2 + 1}{2} \right) + \xi_2 \end{bmatrix} + \mathbf{w}_{ij}, i = 1..m_2, j = 1..n_2 \right\} \quad (27)$$

where

$$E[\mathbf{w}_{ij}] = \mathbf{0}, E[\mathbf{w}_{ij} \mathbf{w}_{ij}^T] = \sigma_r^2$$

Figure 6 The intersection between measured planes where the grids represents the measurement site. The twisted tubular surface represents 3σ uncertainties for the line of intersection between the two planes. Finally the points represents the measurements with $\sigma_r = 0.033\text{ m}$, $\Delta = 0.1\text{ m}$.

For this selected case the information matrices for the two planes becomes

$$\Gamma_1 = \frac{m_1 n_1}{\sigma_r^2} \begin{bmatrix} 1 & \xi_1 & -\eta_1 \\ \xi_1 & \xi_1^2 + \frac{\Delta^2(n_1^2 - 1)}{12} & -\xi_1 \eta_1 \\ -\eta_1 & -\xi_1 \eta_1 & \eta_1^2 + \frac{\Delta^2(m_1^2 - 1)}{12} \end{bmatrix} \quad (28)$$

$$\Gamma_2 = \frac{m_2 n_2}{\sigma_r^2} \begin{bmatrix} 1 & \xi_2 & -\eta_2 \\ \xi_2 & \xi_2^2 + \frac{\Delta^2(n_2^2 - 1)}{12} & -\xi_2 \eta_2 \\ -\eta_2 & -\xi_2 \eta_2 & \eta_2^2 + \frac{\Delta^2(m_2^2 - 1)}{12} \end{bmatrix} \quad (29)$$

where the last matrix is expressed in a co-ordinate frame oriented as plane 2.

For each of the planes the corresponding covariance matrices are

$$\text{cov}([\rho, \theta, \phi]) = \begin{bmatrix} \sigma_c^2 + \sigma_\phi^2 \eta^2 + \sigma_\theta^2 \xi^2 & -\xi \sigma_\theta^2 & \eta \sigma_\phi^2 \\ -\xi \sigma_\theta^2 & \sigma_\theta^2 & 0 \\ \eta \sigma_\phi^2 & 0 & \sigma_\phi^2 \end{bmatrix} \quad (30)$$

where $\sigma_c^2 = \frac{\sigma_r^2}{mn}$

$$\sigma_\theta^2 = \frac{12\sigma_c^2}{\Delta^2(n^2 - 1)} \quad \sigma_\phi^2 = \frac{12\sigma_c^2}{\Delta^2(m^2 - 1)}$$

The estimate of the x-y coordinates for the intersection has, using (18), the covariance matrix:

$$C(x_i) = (C(x_{1i})^{-1} + C(x_{2i})^{-1})^{-1} = \begin{bmatrix} \sigma_a^2 & \sigma_a^2 \cot(\phi_2) \\ \sigma_a^2 \cot(\phi_2) & \frac{\sigma_a^2 \cos^2(\phi_2) + \sigma_b^2}{\sin^2(\phi_2)} \end{bmatrix} \quad (31)$$

where

$$\sigma_a^2 = \frac{\sigma_r^2}{m_1 n_1} \left(1 + \eta_1^2 \frac{12}{\Delta^2(m_1^2 - 1)} + \xi_1^2 \frac{12}{\Delta^2(n_1^2 - 1)} \right)$$

$$\sigma_b^2 = \frac{\sigma_r^2}{m_2 n_2} \left(1 + \eta_2^2 \frac{12}{\Delta^2(m_2^2 - 1)} + \xi_2^2 \frac{12}{\Delta^2(n_2^2 - 1)} \right)$$

The surface representing the uncertainty of the line of intersection in Figure 6 was calculated using equation (31) in a reference plane of varying height i.e. for varying ξ_1, ξ_2 .

The covariances for the direction estimates, as expressed in a coordinate system oriented as plane 1 are: Plane 1

$$\text{cov}(n_1) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_{\theta 1}^2 & 0 \\ 0 & 0 & \sigma_{\phi 1}^2 \end{bmatrix} \quad (32)$$

with $\sigma_{c1}^2 = \sigma_r^2 / m_1 n_1$ and

$$\sigma_{\theta 1}^2 = \frac{12\sigma_{c1}^2}{\Delta^2(n_1^2 - 1)} \quad \sigma_{\phi 1}^2 = \frac{12\sigma_{c1}^2}{\Delta^2(m_1^2 - 1)}$$

Plane 2

$$\text{cov}(n_2) = \begin{bmatrix} \sin(\phi_2)^2 \sigma_{\theta 2}^2 & -\sin(\phi_2) \cos(\phi_2) \sigma_{\theta 2}^2 & 0 \\ -\sin(\phi_2) \cos(\phi_2) \sigma_{\theta 2}^2 & \cos(\phi_2)^2 \sigma_{\theta 2}^2 & 0 \\ 0 & 0 & \sigma_{\phi 2}^2 \end{bmatrix} \quad (33)$$

with $\sigma_{c2}^2 = \sigma_r^2 / m_2 n_2$ and

$$\sigma_{\theta 2}^2 = \frac{12\sigma_{c2}^2}{\Delta^2(n_2^2 - 1)} \quad \sigma_{\phi 2}^2 = \frac{12\sigma_{c2}^2}{\Delta^2(m_2^2 - 1)}$$

Using equation (25) the covariance for the direction of the intersection then follows as:

$$C(e_z) = C((\hat{n}_1 \times \hat{n}_2) / \|\hat{n}_1 \times \hat{n}_2\|) = \frac{(I - e_z e_z^T)(\hat{n}_2 C(\hat{n}_1) \hat{n}_2 + \hat{n}_1 C(\hat{n}_2) \hat{n}_1)(I - e_z e_z^T)}{\sin(\phi_2)^2} \quad (34)$$

i.e.

$$C(e_z) = \begin{bmatrix} \sigma_{\theta 1}^2 & -\sigma_{\theta 1}^2 \cot(\phi_2) & 0 \\ -\sigma_{\theta 1}^2 \cot(\phi_2) & \frac{\sigma_{\theta 2}^2 + \sigma_{\theta 1}^2 \cos(\phi_2)^2}{\sin(\phi_2)^2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Using [SSC90], the example and the results above can be transformed to other co-ordinate systems.

6. Conclusions and future work

Navigation based on stochastic modelling and estimated line / surface segments from range data has proven successful in many cases. An explicit expression of the first order approximation of the covariance for the Hough / Radon line parameters $[\rho \ \phi]^T$ is

$$C([\rho \ \phi]^T) \approx \begin{bmatrix} \sigma_\rho^2 + \eta^2 \sigma_\phi^2 & \eta \sigma_\phi^2 \\ \eta \sigma_\phi^2 & \sigma_\phi^2 \end{bmatrix}$$

where ρ =perpendicular distance, η =distance from normal to COG along the line and ϕ =the orientation of the line normal. An important result concerning the representation of lines and planes measured from a moving platform is that the "centre of gravity" for old measurements "follow along with the robot as it moves". This is expressed by how η varies as the robot moves.

Using the covariance for separate lines the expression for the covariance of the intersection was also presented. The results are easily generalised to apply to planes and their intersections. Also, the results are directly applicable to range cameras, provided the range dependent resolution is introduced, i. e. $\sigma_r \approx \text{constant } R^2 \sigma_0$

With a good representation of the covariance for intersections of planes the fusion of laser range measurements and edges detected in a CCD image can be performed. The intersection of the infinite plane represented by the CCD edge and planes detected in laser data are combined. The same applies for a corner generated by two planes.

In the present work no attention has been paid to the problem of segmentation and association. However, a good representation of uncertainty is a prerequisite for good associations and one natural extension of the work is to consider the association problem using a Gaussian Mixture setting [NNW96],[BSL93 pp.461-465].

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