Modelling Thermoelasticity and Hygroelasticity for Orthotropic Materials

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ABSTRACT

A definition of an ideally elastic orthotropic material was made assuming that the stress, strain, temperature and MC (moisture content) defined the state of the material. From this definition, an orthotropic linear elastic stress-strain relation was derived in an exact differential (i.e., incremental) form as

$$d\varepsilon = \frac{d\sigma}{E} + \alpha_0 dT + \beta_0 du - \frac{\sigma}{E^2}\left(\frac{\partial E}{\partial T}dT + \frac{\partial E}{\partial u}du\right) \quad (1)$$

where $\alpha_0(T,u)$ is the free thermal expansion coefficient (the thermal expansion coefficient at zero stress) and $\beta_0(T,u)$ is the free moisture-expansion coefficient (the moisture-expansion coefficient at zero stress) . The dependence of strain on stress, temperature and MC led to the last two extra terms in the incremental stress-strain equations compared to the same equations for a material where the elastic modulus does not depend on temperature and MC.

There have been earlier descriptions of the first extra term $-\frac{\sigma}{E^2}\frac{\partial E}{\partial T}dT$ when it comes to temperature expansion for metals, and the temperature expansion due to this term has been denoted the "stress dependency of the thermal expansion", [1]. For wood, the temperature expansion is often neglected in comparison to the MC expansion.

For expansion due to MC changes for hygroscopic materials such as wood, the last extra term $-\frac{\sigma}{E^2}\frac{\partial E}{\partial u}du$ have previously in many cases been disregarded. Instead a non-elastic mechano-sorptive strain increment $m\sigma du$ has been added to the strain increment $\frac{d\sigma}{E} + \beta_0 du$ and the constant m has been experimentally evaluated, [2], [3], [4] . This implies that the extra term $-\frac{\sigma}{E^2}\frac{\partial E}{\partial u}du$ did get included in the added mechanosorption term in those papers. Reported m-values are from about -0.1 to -0.3. Fig. 1 shows that the size of the extra term for MC expansion is significant in comparison to reported m-values, especially at high temperatures and high MC. This implies that a significant contribution to reported m parameters may be not due to mechanosorption defined as a nonlinear phenomenon, but instead due to the extra terms derived in this paper.

To summarize, there are different ways to mathematically describe phenomena such as mechanosorption in wood. An important issue is to separate the elastic contribution from the nonelastic contribution of the strains that appear. Therefore the following proposition is made: nonelastic behaviour in the form of added strain increment terms in the incremental stress-strain relations are the totally observed strain increment minus the linear elastic strain increment as it is defined in this paper.

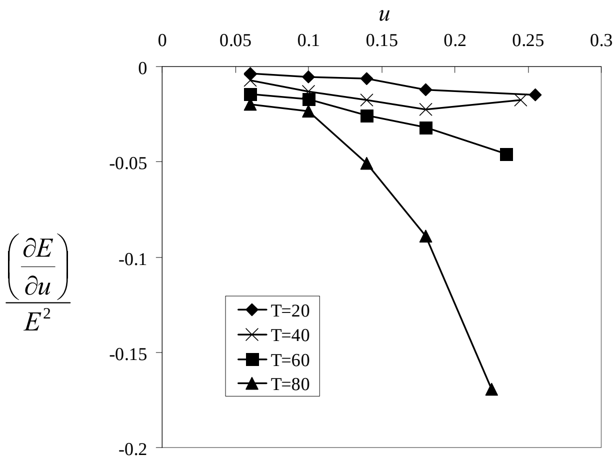

Fig. 1. $\frac{\left(\frac{\partial E}{\partial u}\right)}{E^2}$ as a Function of Moisture Content u at Temperatures 20°C, 40°C, 60°C and 80°C (MPa^{-1}).

Based on Data from [5].

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