Experimental Validation of Idealized Two-Axis Models of Faulty Induction Machines

Mikael Stocks[†], Fedor Rodyukov[‡] and Alexander Medvedev^{††}

Abstract—In this paper, two dynamical models of the Induction Machine (IM) subject to electrical faults in the stator and rotor respectively are validated against experimental data. The considered stator faults are inter-turn short circuit in one of the phases and increased phase winding resistance. The rotor model is validated against an IM with one broken rotor bar, but otherwise healthy. The model parameters are estimated by the Gauss-Newton method and the obtained estimates are compared and discussed. The validation results show, for each of the considered faults, very good agreement between simulations and experiments. This makes the models promising in various applications involving asymmetric IM such as fault tolerant motor drives and motor condition monitoring.

NOMENCLATURE

Variables		Parameters	
ψ	- magnetic flux	R	- electrical resistance
i	- electrical current	L	- electrical lumped inductance
u	- feeding voltage	M	- electrical mutual inductance
γ	- rotor angle	J	- rotor moment of inertia
M_t	- mechanical load	D	- rotor damping
		p	- number of pole-pairs

For both the *variables* and *parameters*, a capital letter subscript refers to the corresponding stator axis in a conventional three-axis system. In the same manner, a small letter subscript refers to the corresponding rotor axis. Similarly, for the *variables*, a subscript α, β refers to the corresponding α or β -axis in a two-axis description of the IM. A bar, $\bar{\cdot}$, denotes a scaled (dimensionless) *variable*. Where appropriate, SI-units are used throughout the paper. A prime superscript above a *parameter*, $\prime$, means that the parameter is evaluated in a two-axis framework. Bold capital letters stand for matrices, whereas small/capital letters denote scalars and vectors. Finally, differentiation with respect to time is expressed by the dot, i.e. $\dot{x} = \frac{dx}{dt}$.

I. INTRODUCTION

It is well known [1] that under certain idealizing assumptions, healthy Induction Machines (IMs) can be well described by a few ordinary differential equations and algebraic relationships, replacing the original sets of partial differential equations stemmed from the physics of the process. This is usually termed as space phasor approach, [2]. One generally starts by defining three axes for the stator where the consecutive axes are shifted $\frac{2\pi}{3}$ ahead of each other, due to the fact that the three stator phases are normally equally spaced. Then either two (spaced $\frac{\pi}{2}$) or three axes (spaced $\frac{2\pi}{3}$) are defined as well for the rotor. The space phasor approach yields a description of the IM, resulting in six, seven or eight

differential equations, including the *mechanical* part. These models are in the sequel referred to as *three-axis* models.

From the three-axis description of the IM, one usually proceeds by utilizing physical relations between currents and/or voltages to transform the model into two sets of orthogonal axes ($\frac{\pi}{2}$ apart of each other), introducing one set for the stator and one for the rotor, hence resulting in very concise equations. These models are in sequel referred to as *two-axis* models. In fact, a complete two-axis dynamical description of the IM is given by only five or six real-valued ordinary differential equations, four for the electrical part and one or two for the mechanical part. These symmetrical models can be written as a set of three (or four) complex-valued differential equations. A classical approach to modelling of symmetrical IMs is given in [1] and another one, very often cited, can be found in [2].

Unfortunately, many of the idealizing assumptions introduced in the derivation of the two-axes models fail to apply to IMs with faults. However, in some cases, this approach can still be utilized with some modifications, hoping that the significant properties are still captured, although some assumptions are violated. Such models are mainly used to describe stator faults. In [3] for example, a model is developed in two axes for inter-turn short circuits in possibly all three stator windings simultaneously. However, such a general model probably is too complex to be useful in engineering practice. A similar approach is given in [4]. Simple models including the main effects of rotor faults are hard to find, however one is given in [5], where a model including the effects of a faulty rotor bar is derived.

More often, faulty IMs are modelled by introducing a separate electrical equation for each mesh in the rotor (winding functions approach) and sometimes extra equations are also added for the stator. This approach is particularly suitable for the case of squirrel cage rotor, but can be as well used for wound rotors. In some aspects, this approach leads to a better model when the IM is subject to a fault, producing a more realistic description of the motor dynamics. However, considering separate meshes results in a high differential order of the model and makes it prohibitively complex for control engineering ends. In [6], for example, a general model is developed for IMs. With this model, a broad variety of faults can be modelled both in stator and rotor, as well as different stator connections. Another model, valid for net-connected IMs and inter-turn short circuit in one stator phase, is given in [7]. The latter paper explains in detail the underlying physical principles of the IM in general and constitutes a valuable reference. Finally, an approach to high-order modelling of rotor faults can be found in [8].

In this paper, two mathematical models of asymmetric IMs in two axes are described and experimentally validated. A systematic and unabridged derivation of the models can be found in [9] and condensed derivations can be found in [10] and [11]. Utilizing standard idealizing assumptions, the developed models explain which motor parameters are

[†]Systems and Interaction, Luleå University of Technology, SE-971 87 Luleå, SWEDEN, e-mail: Stocks@ltu.se

[‡]Faculty of Mathematics and Mechanics, Saint-Petersburg State University, Peterhof, St. Petersburg, RUSSIA, e-mail: fr@fr11189.spb.edu

^{††}Information Technology, Uppsala University, SE-75105 Uppsala, SWEDEN, e-mail: Alexander.Medvedev@it.uu.se

influenced by electrical faults in one of the stator or rotor phases. The models do not though describe in what way a fault comes about. Therefore, one cannot, by means of these models, evaluate exactly in what way the motor dynamics are altered by a given fault. Instead, identification of the parameters influenced by the faults has to be performed from experimental data. The simulation results are shown in the sequel to agree well with experimental data despite relative simplicity of the models.

II. NORMALIZED MODELS OF ASYMMETRIC IM

Normalized (dimensionless) mathematical models are often employed in order to minimize the number of independent parameters involved in a system description. In this section, such models are presented for the case of an IM with an electrical asymmetry.

A. Faulty stator

For the IM with a fault in stator phase A , such a model for the *electrical* part is given by

$$\begin{aligned}\dot{\bar{\psi}}_{\alpha S} &= -\alpha_A \bar{\psi}_{\alpha S} + \alpha_A \frac{\mu}{\mu_A} (1 - \mu_A) \bar{x}_{\alpha S} + \bar{u}_{\alpha S} \\ \dot{\bar{\psi}}_{\beta S} &= -\alpha_S \bar{\psi}_{\beta S} + \alpha_S (1 - \mu) \bar{x}_{\beta S} + \bar{u}_{\beta S} \\ \dot{\bar{x}}_{\alpha S} &= -\alpha_r \frac{\mu}{\mu_A} \bar{x}_{\alpha S} - \bar{\omega} \frac{k_0}{k_1} \bar{x}_{\beta S} + \alpha_r \bar{\psi}_{\alpha S} \\ \dot{\bar{x}}_{\beta S} &= -\alpha_r \bar{x}_{\beta S} + \bar{\omega} \frac{k_1}{k_0} \bar{x}_{\alpha S} + \alpha_r \bar{\psi}_{\beta S}\end{aligned}\quad (1)$$

including the output map to normalized stator currents

$$\begin{aligned}\bar{i}_{\alpha S} &= \frac{1}{\mu_A} \bar{\psi}_{\alpha S} - \frac{\mu}{\mu_A} \frac{1 - \mu_A}{\mu_A} \bar{x}_{\alpha S} \\ \bar{i}_{\beta S} &= \frac{1}{\mu} \bar{\psi}_{\beta S} - \frac{1 - \mu}{\mu} \bar{x}_{\beta S}\end{aligned}\quad (2)$$

The parameters of the *electrical* part are as follows

$$\begin{aligned}\mu_A &= 1 - \frac{M_{Ar}^2}{L'_A L'_r}; \quad \mu = 1 - \frac{M^2}{L'_S L'_r}; \\ k_0 &= \frac{1 - \mu}{\mu}; \quad k_1 = \frac{M}{M'_{Ar}} \frac{1 - \mu_A}{\mu_A}; \\ \alpha_A &= \frac{2}{3} \frac{1}{\mu_A} \frac{R'_A}{L'_A}; \quad \alpha_S = \frac{2}{3} \frac{1}{\mu} \frac{R_S}{L'_S}; \quad \alpha_r = \frac{2}{3} \frac{1}{\mu} \frac{R_r}{L'_r};\end{aligned}\quad (3)$$

The corresponding description of the *mechanical* part is

$$\dot{\bar{\omega}} = -\alpha_m \bar{\omega} + \delta (\bar{M}_{em} - \bar{M}_l) \quad (4)$$

where $\bar{\omega} = p\omega = p\dot{\gamma} = \dot{\bar{\gamma}}$ and the electromechanical torque is

$$\bar{M}_{em} = k_1 \left[\bar{\psi}_{\beta S} \bar{x}_{\alpha S} - \bar{\psi}_{\alpha S} \bar{x}_{\beta S} - \left(1 - \frac{\mu}{\mu_A} \right) \bar{x}_{\alpha S} \bar{x}_{\beta S} \right]$$

The parameters for the *mechanical* subsystem, including the load $\bar{M}_l$, are given by

$$\alpha_m = \frac{D}{J}; \quad \delta = \frac{p^2}{J} \frac{u_m^2}{L'_S}; \quad \bar{M}_l = \frac{1}{p} \frac{L'_S}{u_m^2} M_l$$

Since (1) and (4) are connected in a bilinear way, the complete model of the IM is essentially nonlinear.

The normalized voltages and currents in the model are related to the measured on a real IM ones as

$$u_S = \frac{3}{2} u_m \bar{u}_S; \quad i_{\alpha S} = K_A \bar{i}_{\alpha S}; \quad i_{\beta S} = K_S \bar{i}_{\beta S} \quad (5)$$

$$K_A = \frac{2}{3} \frac{u_m}{L'_A}; \quad K_S = \frac{2}{3} \frac{u_m}{L'_S} \quad (6)$$

where it is optional to assume that the input is bounded by $|u_{\alpha S}| \leq \frac{3}{2} u_m, |u_{\beta S}| \leq \frac{3}{2} u_m$. Since the measured electrical quantities correspond to those in the three axis model, the necessary transformations from three axes to two axes are provided in Appendix.

B. Faulty rotor

Defining $\mu_a = 1 - \frac{M_{Sa}^2}{L'_S L'_a}$, $\alpha_a = \frac{2}{3} \frac{1}{\mu_a} \frac{R_a}{L'_a}$, a normalized description of the *electrical* part for an IM with a fault in rotor phase a is

$$\begin{aligned}\dot{\bar{\psi}}_{\alpha P} &= -\alpha_S \frac{\mu}{\mu_a} \bar{\psi}_{\alpha P} + \bar{\omega} \bar{\psi}_{\beta P} + \alpha_S \bar{z}_{\alpha P} + \bar{u}_{\alpha P} \\ \dot{\bar{\psi}}_{\beta P} &= -\alpha_S \bar{\psi}_{\beta P} - \bar{\omega} \bar{\psi}_{\alpha P} + \alpha_S \bar{z}_{\beta P} + \bar{u}_{\beta P} \\ \dot{\bar{z}}_{\alpha P} &= -\alpha_a \bar{z}_{\alpha P} + \alpha_a \frac{\mu}{\mu_a} (1 - \mu_a) \bar{\psi}_{\alpha P} \\ \dot{\bar{z}}_{\beta P} &= -\alpha_r \bar{z}_{\beta P} + \alpha_r (1 - \mu) \bar{\psi}_{\beta P}\end{aligned}\quad (7)$$

including the transformation to normalized projected currents

$$\bar{i}_{\alpha P} = \frac{1}{\mu_a} \bar{\psi}_{\alpha P} - \frac{1}{\mu} \bar{z}_{\alpha P}; \quad \bar{i}_{\beta P} = \frac{1}{\mu} \bar{\psi}_{\beta P} - \frac{1}{\mu} \bar{z}_{\beta P} \quad (8)$$

and the input/output map to normalized voltages/currents

$$\begin{aligned}\bar{u}_P &= \mathbf{T}_{\bar{\gamma}}^T \bar{u}_S; \quad \bar{i}_S = \mathbf{T}_{\bar{\gamma}} \bar{i}_P \\ \mathbf{T}_{\bar{\gamma}} &= \begin{bmatrix} \cos(\bar{\gamma}) & -\sin(\bar{\gamma}) \\ \sin(\bar{\gamma}) & \cos(\bar{\gamma}) \end{bmatrix}\end{aligned}\quad (9)$$

The *mechanical* subsystem is modelled as

$$\begin{aligned}\dot{\bar{\gamma}} &= \bar{\omega} \\ \dot{\bar{\omega}} &= -\alpha_m \bar{\omega} + \delta (\bar{M}_{em} - \bar{M}_l)\end{aligned}\quad (10)$$

where the electromagnetical torque is given by

$$\bar{M}_{em} = \frac{1}{\mu} \left[\bar{\psi}_{\beta P} \bar{z}_{\alpha P} - \bar{\psi}_{\alpha P} \bar{z}_{\beta P} + \left(1 - \frac{\mu}{\mu_a} \right) \bar{\psi}_{\alpha P} \bar{\psi}_{\beta P} \right]$$

and $\bar{M}_l$ is the load. Note that the differential equation for the electrical angle, $\bar{\gamma}$, is necessary in order to complete the description of the electrical part of the model. The electrical angle enters the model equations via the transformation $\mathbf{T}_{\bar{\gamma}}$. Being unitary, $\mathbf{T}_{\bar{\gamma}}$ does not alter any important system properties of the IM model, such as stability ([9]), or observability/controllability, see [12].

In addition to the earlier given relationships between the normalized electrical quantities and the measured ones, note that

$$i_S = K_S \bar{i}_S \quad (11)$$

In the case of faulty rotor, there are less parameters involved than in the faulty stator model. Though, one more differential equation is needed to properly describe the dynamics of the IM.

III. MODEL VALIDATION

Two different mass-produced models of industrial IMs were used for validation experiments, one for stator faults and one for rotor faults. The rotor angle γ , the three stator currents (i_A , i_B and i_C) and the three stator voltages (u_A , u_B and u_C) were measured. In both cases, the *electrical* part of the model in question is validated since γ is readily available.

During the experiments, the IMs were connected to a three-phase mains, with frequency $f_s = 50$ Hz, and the voltages adjusted by a transformer to $u_m = 75$ V, in order to reduce magnetic saturation of the cores in the stator and rotor.

To guarantee sufficient excitation, the model parameters were identified during a start-up sequence with the conventional Gauss-Newton algorithm, see [9]. To reduce the uncertainties in the experimental conditions, the biases as well as the amplification gains in the sensors were estimated from the measurements. However, neither numerical data nor the identification method are presented here to save space.

In Fig. 1, the positive envelope of the stator phase current under start-up of a healthy IM is depicted.

Fig. 1. Start-up of a healthy IM. Envelope of the stator currents, $i_{eS} = (i_{\alpha S}^2 + i_{\beta S}^2)^{\frac{1}{2}}$. The data sequence begins at 0.5 s.

A. Stator Fault

All experiments of this section were performed on two 1.1 kW, two pole-pair induction machines (ASEA:MT 90 S 24-4, 26 bars). In one of the motors, some of the windings in one phase were inter-turn short circuited, for approximately $\frac{1}{6}$ of the particular phase. Another one was used for experiments on symmetrical IM and with increased phase resistance. For the latter case, an extra resistance was simply connected in series with one of the stator phases. The resistance was chosen so that the ratio between the resistances in the faulty phase and a healthy phase was approximately $\frac{9}{4}$. For parameter identification, model (1) combined with (2, 5) was used. From the identified model parameters the motor parameters can be easily evaluated using (3, 6).

1) *Healthy IM*: For reference, the identified model parameters are expressed in terms of conventional IM parameters, completely defining the dynamics of the IM. Described in the two-axis system, the time constants of the stator phases and the rotor phases are

$$\tau_A = \frac{L'_A}{R'_A}, \quad \tau_S = \frac{L'_S}{R'_S}; \quad \tau_r = \frac{L'_r}{R_r} \quad (12)$$

From the parameters in Table I and (3), the resistances of the stator phases in the two-axis system can be derived; $R'_A = 9.899$, $R'_S = 9.393$.

The measured and simulated by the identified model stator currents are compared in Fig. 2. Since the IM dynamics are

Fault invariant parameters		Fault influenced parameters	
Parameter	Value	Parameter	Value
τ_S	0.03999	τ_A	0.03258
τ_r	0.08007	μ_A	0.1020
μ	0.08726	L'_A	0.3225
L'_S	0.3757	$\frac{M_{Ar}}{M}$	1.001

TABLE I

ESTIMATED VALUES OF MODEL PARAMETERS FOR HEALTHY IM

time-varying, three characteristic time intervals are considered to highlight the model quality. The first one is in the beginning of the data sequence. The second one is in the middle of the data set where the amplitudes of the currents drop most rapidly, see Fig.1. Finally, the third one is at the end of the data sequence when the motor has achieved stationary conditions. As can be seen, the identified model of the symmetrical IM explains the measurements quite well. Notably, even for the healthy IM, some imbalance is indicated by the estimated parameters. However, the quotient $\frac{M_{Ar}}{M} = 1.001$ proves the IM symmetrical from a magnetic point of view.

Fig. 2. Comparison of measured data and model output for the healthy IM at three different time intervals. Solid lines - measured data. Dash-dotted lines - simulations.

2) *Increased Phase Resistance*: In this experiment, nothing is altered internally in the IM, but an additional resistance of 10Ω is connected in series with phase A to simulate a poor connection of the winding. For cold IM, the nominal value of R_S is about 8.5Ω . The identified from logged data model parameters are given in Table II. Similar to the case of healthy IM one obtains $R'_A = 16.45$, $R'_S = 9.825$. In [9] (and [10]), it is shown that $\frac{3}{2}R'_A = R_A + \frac{1}{2}R_S$ which

Fault invariant parameters		Fault influenced parameters	
Parameter	Value	Parameter	Value
τ_S	0.03346	τ_A	0.02279
τ_r	0.07954	μ_A	0.08867
μ	0.09933	L'_A	0.3748
L'_S	0.3287	$\frac{M_{Ar}}{M}$	1.008

TABLE II

ESTIMATED MODEL PARAMETERS FOR INCREASED PHASE RESISTANCE

yields $R_A = 19.76 \Omega$, *i.e.* mainly R_A changes (*c.f.* Section III-A.1). Note that the estimated increase in R_A is about 9.5Ω and close to what is expected. Comparing Table I and Table II, it can be seen that almost all estimated parameters have changed. However $\frac{M_{Ar}}{M} = 1.008$, which indicates that magnetic balance is preserved, exactly as expected.

To conclude, a good indication for this kind of fault seems to be a significant difference between parameters R_S and R_A (or R'_A) while retaining magnetic symmetry, *i.e.* $\frac{M_{Ar}}{M} \approx 1$.

Fig. 3. Comparison of measured data and model output for the IM with increased resistance in one phase at three different time intervals. Solid lines - measured data. Dash-dotted lines - Simulations.

3) *Inter-turn Short Circuit*: In the same manner as for the healthy IM, one obtains $R'_A = 9.467$, $R_S = 8.165$.

For inter-turn short circuit, the estimated parameters behave in a manner completely different to what intuitively might be expected. For example, using the estimated values of R'_A and R_S one obtains $R_A = 10.12 \Omega$, *i.e.* $R_A > R_S$, despite of the short circuit. However, many of the assumptions for idealized IMs ([9]) are violated and the reasoning based on experiences with the symmetrical motor would be of limited use in this case.

Nevertheless, the estimated parameters clearly indicate a severe imbalance in the IM. For example, the time constants

Fault invariant parameters		Fault influenced parameters	
Parameter	Value	Parameter	Value
τ_S	0.05430	τ_A	0.01487
τ_r	0.07761	μ_A	0.2175
μ	0.08057	L'_A	0.1407
L'_S	0.4434	$\frac{M_{Ar}}{M}$	0.8471

TABLE III

ESTIMATED MODEL PARAMETERS FOR INTER-TURN SHORT CIRCUIT

Fig. 4. Comparison of measured data and model output for the IM with inter-turn short circuit in one phase at three different time intervals. Solid lines - measured data. Dash-dotted lines - simulations.

significantly differ between the healthy and the faulty phases, *i. e.* τ_S and τ_A . It is also the case for the goodness factors μ , μ_A and the inductances L'_S , L'_A . Moreover, a good indication that an inter-turn short circuit is present seems to be that the quotient $\frac{M_{Ar}}{M}$ significantly differs from one.

To summarize, the estimated model parameters confirm significant imbalance, but the parameters behave in a fairly unexpected manner. Further research is needed to understand whether the reason for this is that significant phenomena have been neglected in the model derivation, or it is due to problems with parameter identifiability, *i. e.* low sensitivity of the measurements to the motor parameters alternation.

B. Rotor Fault

The experiments for the case of rotor fault were performed on two 1.1 kW, two pole-pair induction machines (ABB:M3AA 90 L-4, 26 bars). This is another motor type compared to the one used in the experiments with the stator faults. In one of the motors, one rotor bar was completely broken at three equidistant points by drilling. The other one was used for experiments on a symmetrical IM. For parameter identification, model (7) combined with (8, 9, 11)

was used. Note that for the model of an IM with faulty rotor, it is also necessary to estimate the absolute angle of the rotor, or equivalently, the initial value of it, since the model is dependent on that information. Estimation of the rotor initial angle may be performed in the same way as estimation of any other parameter but falls outside of the scope of this paper.

1) *Healthy IM*: Since a different IM was used in the experimental setup, a new set of model parameters is provided in Table IV. In the table above, the time constants τ_S and

Fault invariant parameters		Fault influenced parameters	
Parameter	Value	Parameter	Value
τ_S	0.06490	τ_a	0.09439
τ_r	0.09402	μ_a	0.07491
μ	0.07486		
L'_S	0.4088		

TABLE IV
ESTIMATED MODEL PARAMETERS FOR HEALTHY IM

τ_r are evaluated according to (12) and $\tau_a = \frac{L_a}{R_a}$. From the parameters in Table IV and the parameter definitions, $R_S = 6.299$. The obtained model is compared to the experimental data in Fig. 5. Similarly to the case of stator fault, three characteristic time intervals are considered within the identification data set to examine the quality of the identified model. As it can be seen from the plots, the identified model describes the actual time-varying motor dynamics very well. Interestingly, as a comparison of Table I and Table IV reveals, the estimated parameters of the faulty stator model seems to indicate much more imbalance even for a healthy IM when used. It is fair to say that the model for rotor fault indicates no imbalance at all. This is likely due to the fact that the imbalance of any real-life IM is concentrated mainly to the stator and hardly captured by the faulty rotor model. The faulty stator model also has more degrees of freedom than the model for faulty rotor, which results in a lower parameter estimation accuracy. As pointed out before, these effects may as well be a parameter identifiability issue.

Fault invariant parameters		Fault influenced parameters	
Parameter	Value	Parameter	Value
τ_S	0.05855	τ_a	0.08286
τ_r	0.08474	μ_a	0.09505
μ	0.08108		
L'_S	0.3994		

TABLE V
ESTIMATED MODEL PARAMETERS BROKEN ROTOR BAR

2) *Broken Rotor bar*: In the same manner as for the healthy motor case, one obtains $R_S = 6.822$. For this case it is expected that both the time-constants and the goodness factors should differ between the healthy and the faulty rotor phases. A quick check reveals that there is no notable difference between the time-constants τ_r and τ_a , but there is a significant difference between the goodness factors μ and μ_a .

The case of a broken rotor bar is considered to be well modelled, which is also expected from the physics of the process. Comparing μ and μ_a indicates that a fault is present.

Fig. 5. Comparison of measured data and model output for the healthy IM at three different time intervals. Solid lines - measured data. Dash-dotted lines - simulations.

Fig. 6. Comparison of measured data and model output for the IM with one broken rotor bar at three different time intervals. Solid lines - measured data. Dash-dotted lines - simulations.

IV. DISCUSSION

It should be noted that some of the estimated imbalance in the IMs is due to inherent asymmetry in all real-life IMs as well as calibration errors of the sensors used to obtain the identification data. Hence, for an IM to be considered faulty, some kind of decision threshold has to be enforced to distinguish between nominal model uncertainty and fault. A good idea would be to concentrate on differences between fault independent parameters and parameters expected to be influenced by fault, instead of focusing on their absolute values. In this way, much of the effects of heating and other uncertainties are made less significant, since such effects are expected to influence all the motor phases in a similar manner.

By examining Fig. 2, Fig. 3, Fig. 4, Fig. 5 and Fig. 6, it can be concluded that the agreement between simulations and measured data is very good. In two of three fault types, the model parameters behave as it can be anticipated from the physics of the fault. In the case of stator phase inter-turn short circuit however, more explanatory work has to be done. Still, some systematic discrepancies between the models and experimental data can be noticed.

In particular, at the end of each validation sequence, the simulated data contain higher frequency oscillations. At this point it is not quite clear where these oscillations come from. One possible explanation is that these oscillations are mainly caused by inaccuracy of the rotor angle measurements. In Fig. 7 the normalized measured rotor speed is shown, *i.e.* $\bar{\omega}_n = \omega/\omega_{max}$. Note that ω is evaluated in a straightforward way by taking differences divided by the time elapsed between the two consecutive measured angles, γ . It can be seen that the speed signal is very noisy, especially at higher rates. Note that the angle measurements are exact in the sense that they contain no measurement noise and only "time resolution" error is present. Simulations show that such noise significantly affects the stator current, but this effect is not dealt with here. However, the phenomenon

Fig. 7. Unfiltered normalized measured rotor speed $\bar{\omega}_n = \omega/\omega_{max}$.

mentioned above is not the only possible explanation of the high frequency content in the simulated stator currents. In Fig. 2 through Fig. 6, it can be observed that the measured currents also contain higher frequency components. Hence, some of these measured oscillations should be present in the models as well. For the time being, it is not yet decided to what extent they should be modelled, because one effect is hard to distinguish from the other.

Furthermore, sometimes there are oscillations present in the measured currents which are not explained by the models, see Fig. 4 and Fig. 6. The cause of this is not clear. However, when modelling IMs in a conventional way, many idealization assumptions are made. As pointed out in [9] some of these assumptions are unlikely to hold for faulty IMs.

To complicate things even more, other phenomena in the measured currents seems to be present as well, *e. g.* saturation. However, the saturation effect should not be significant in the collected experimental data, because of the precautions taken in the course of the experiments, and might be neglected.

From the identification results it can be concluded that the main properties of the currents are very well explained by the derived models, *i. e.* the transient behavior and the dominating frequencies of the currents. In most cases, the estimated parameters behave exactly as expected when a fault occurs, but not always.

V. CONCLUSIONS

In this paper, results of experimental validation of two-axis models of IMs with electrical faults have been presented. In particular, one of the models describes the effects of a fault in a stator phase and another considers a fault in a rotor phase. The derived models explain experimental data very well. Possible applications of the models are fault detection and fault-tolerant motor drives.

APPENDIX

Transformations:

The transformations from three to two axes are given by

$$u_S = \mathbf{T}_{32}^T u_{3S}; \quad i_S = \mathbf{T}_{23} i_{3S}$$

for the forward transformations, and

$$i_{3S} = \mathbf{T}_{32} i_S$$

for the inverse current transformation, where

$$\mathbf{T}_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{bmatrix}; \quad \mathbf{T}_{32} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}^T$$

REFERENCES

- [1] D. C. White and H. H. Woodson, *Electromechanical Energy Conversion*. New York: John Wiley & Sons, 1959.
- [2] P. Vas, *Vector Control of AC Machines*. Oxford Science Publications, 1990.
- [3] M. Arkan, D. Kostic-Perovic, and P. J. Unsworth, "Modelling and simulation of induction motors for stator fault diagnosis," in *International Symposium on Diagnostics for Electrical Machines, Power Electronics and Drives SDEMPED*, 2001.
- [4] S. Bachir, S. Tnani, T. Poinot, and J. C. Trigeassou, "Stator fault diagnosis in induction machines by parameter estimation," in *International Symposium on Diagnostics for Electrical Machines, Power Electronics and Drives SDEMPED*, 2001.
- [5] C. Kral, "Derivation of the space phasor equations and the required parameters of a squirrel cage induction machine with a faulty rotor bar," in *International Symposium on Diagnostics for Electrical Machines, Power Electronics and Drives SDEMPED*, 1999.
- [6] X. Luo, Y. Liao, H. A. Toliyat, A. El-Antably, and T. A. Lipo, "Multiple coupled circuit modeling of induction machines," *IEEE Transactions on Industry Applications*, vol. 31, no. 2, May/June 1995.
- [7] G. M. Joksimovic and J. Penman, "The detection of inter-turn short circuits in the stator windings of operating motors," *IEEE Transactions on Industrial Electronics*, vol. 47, no. 5, October 2000.
- [8] R. Fiser, V. Ambrosic, and R. Leskovec, "Modelling of dynamic and steady-state performance of induction motor having rotor asymmetry," in *International Symposium on Diagnostics for Electrical Machines, Power Electronics and Drives SDEMPED*, 2001.
- [9] M. Stocks, F. Rodyukov, and A. Medvedev, "Idealized two-axis models of faulty induction machines," CSEE / Control Engineering Group, Tech. Rep. ISSN 1402-1528 / ISRN LTU-FR-05/05-SE / NR 2005:05, 2005.
- [10] —, "Idealized two-axis model of induction machines under stator fault," in *1st IEEE Conference on Industrial Electronics and Applications ICIEA*, Singapore, 2006.
- [11] —, "Idealized two-axis model of induction machines under rotor fault," in *1st IEEE Conference on Industrial Electronics and Applications ICIEA*, Singapore, 2006.
- [12] M. Stocks and A. Medvedev, "On observability and controllability of faulty induction machines," in *American Control Conference ACC*, Minneapolis, USA, 2006.