Robust Colour and Range Sensing for Robotic Applications Using a Stereoscopic Light Stripe Scanner

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Abstract

This paper presents an integrated, low-level approach to removing sensor noise, cross talk, spurious specular reflections, and solving the association problem in a light stripe scanner. Most single-camera scanners rely on the laser brightness exceeding that of the entire image. Our system uses two cameras to measure the stripe and combines knowledge of the light plane orientation to produce useful validation properties. The key result is the development of a condition relating image plane measurements and camera intrinsic parameters, which allows validation/association to be performed independently of 3D reconstruction. The same equations are used to improve ranging accuracy compared to single-camera systems. We also show how the system may be self-calibrated using measurements of an arbitrary non-planar target. As validation allows operation in ambient light, registered colour and range are captured in the same sensor. An experimental scanner demonstrates the effectiveness of the proposed techniques.

1. Introduction

Light stripe range cameras are used in many robotic systems for purposes such as navigation [1][2], obstacle detection [5], object recognition [3], and visual servoing [4]. In most cases, active sensing is preferred over passive stereo because it provides a computationally efficient and robust solution to the correspondence problem. However, these and most other systems also assume favourable lighting conditions and surface reflectance properties, allowing easy detection of the light stripe as the brightest feature in the environment.

The range sensor presented in this paper is intended for use on a domestic/office robot for object recognition and manipulation. Thus, operation must be in ambient light without any special preparation of the environment, and both colour and range are desired. In these conditions, a number of mechanisms for measurement noise exist: smooth surfaces can cause multiple specular reflections ('ghost' stripes), arbitrary environmental

Figure 1: Stereoscopic light stripe ranging system.

features may have a stripe-like appearance and cross talk between sensors may arise if multiple robots operate in the same location.

A number of researchers have already proposed techniques for eliminating spurious measurements. In [5], the light stripe is pulsed to distinguish between the laser and ambient noise. In [6], specular reflections are eliminated by moving the scanner relative to the scene and analysing the motion of the reconstructed range data. Both techniques have the obvious disadvantage that a number of range images are required for reliable noise rejection. Trucco *et al* [7] use stereo cameras to observe the laser stripe, but treat the system as two independent sensors. Range measurements are validated by comparing reconstructed 3D points between sensors for consistency. Since validation occurs after reconstruction, a mechanism for associating multiple measurements is not proposed. Instead, scan lines with multiple stripes (such as reflections) are simply discarded. Magee *et al* [8] also use stereo cameras. In this case, the stereo fields are combined using a minimum intensity operation to eliminate spurious reflections. This technique relies heavily on *a priori* knowledge of the scanned target.

The system we propose here provides a more rigorous treatment of stereo light stripe scanning. Validation is

performed on raw image plane measurements, allowing uniform rejection of sensor noise, spurious reflections and cross talk. Unlike previous systems, these techniques are extended to solve the association problem for multiple measurements and improve the ranging accuracy over a single camera system. The ability to eliminate cross-talk allow these techniques to be applied to multiple sensor systems. For example, Hofman [9] describes a scanner using six light stripe projectors arranged on the faces of a cube, which are activated *sequentially* to avoid cross talk. Stereoscopic light stripe ranging would allow more than one scanner to be operated *simultaneously* with minimal cross talk.

Light stripe scanners acquiring colour and range have also been presented in previous work. Haverinen and Rönning [10] describe a system using one camera for range and a second for colour. The colour and range data are registered using a calibrated mapping. Conversely, our work allows the light stripe to be extracted from images exposed in ambient lighting so range and colour from each camera are registered implicitly.

The following Section briefly describes the experimental hardware used in this work. Section 3 develops the underlying theory of stereoscopic light stripe ranging, followed with some image processing implementation details in Section 4. Section 5 presents experimental results verifying of robustness the proposed system.

2. System Configuration

Figure 1 shows the components in the stereoscopic light stripe scanner. A cylindrical lens mounted on a laser diode module generates the vertical light plane. Rotating the laser about a vertical axis scans the stripe across a scene, and the rotation angle is measuring by an optical encoder connected to the output shaft via a toothed belt. PAL colour cameras capture stereo images of the stripe at 320x240 pixel resolution. The relative positions of the laser and cameras are initially unknown, and are determined using the image-based calibration technique described later. Images are captured and processed at PAL frame rate (25 Hz) on a 450 MHz Pentium III PC. Motor control and optical encoder measurements are implemented with a PIC microcontroller, which communicates with the PC via a serial link.

3. Stereoscopic Light Stripe Ranging

3.1 General Principles

Just as single-camera laser scanning is analogous to binocular stereo, a stereo scanner is equivalent to a trinocular system in which the 3D reconstruction problem is over-constrained, and the additional information is used for validation. The key observation here is that

all valid measurements of a scanning laser must lie on the light plane. We now show how a condition for checking light plane occupancy can be developed in terms of stereo image plane measurements.

Let L and R represent the coordinate frames at the optical centre of two cameras separated by an arbitrary transform. Let $\Omega_L = (A_L, B_L, C_L, D_L)^T$, represent the plane equation parameters of the light plane expressed in camera L , and similarly $\Omega_R = (A_R, B_R, C_R, D_R)^T$ for the same plane viewed in camera R . Then a given point on the light plane, at $\mathbf{x}_L = (x_L, y_L, z_L)^T$ and $\mathbf{x}_R = (x_R, y_R, z_R)^T$ in L and R respectively, must satisfy

$$(\Omega_i)^T \cdot \mathbf{x}_i = A_i x_i + B_i y_i + C_i z_i + D_i = 0 \quad (1)$$

for $i=L, R$. Using a pin hole model with focal length f to describe both cameras, coordinates $\mathbf{x} = (x, y, z)^T$ project to the image plane at $\mathbf{u} = (u, v)^T$ via

$$(u, v)^T = (x, y)^T \cdot f / z \quad (2)$$

Projecting condition (1) via (2) gives

$$A_i u_i + B_i v_i + C_i f = -D_i f / z_i, i = L, R \quad (3)$$

Usually the image plane coordinate u_i of a point on the light plane is the measured value, and the actual point $\mathbf{x}$ in space is unknown. We now eliminate the unknown z_i from (3) by dividing the plane equations in L and R :

$$\frac{A_L u_L + B_L v_L + C_L f}{A_R u_R + B_R v_R + C_R f} = \frac{D_L z_R}{D_R z_L} \quad (4)$$

If the optical axes of the two cameras are parallel and the images planes are coplanar, we have the special condition $z_L = z_R \equiv z$. In this case, (4) reduces to:

$$A_L u_L + B_L v_L + C_L f - k(A_R u_R + B_R v_R + C_R f) = 0 \quad (5)$$

where $k=D_L/D_R$. The above condition linearly relates image plane measurements and is satisfied only by points on the light plane. In addition to providing a simple validation mechanism, (5) can be used to solve the association problem for multiple measurements.

It is important to note that the condition of parallel optical axes and coplanar images planes, also called *rectilinear stereo*, does not necessarily constrain the camera arrangement. Even when this condition is not met mechanically, a variety of techniques in *projective rectification* [11] allow the effective image coordinates for rectilinear stereo to be recovered analytically.

3.2 System Model

We now apply the above principles to derive a system model for our light stripe camera. Figure 2 shows the system configuration, where L and R denote the left and right camera frames and P is the frame of the light plane

scanner. The camera frames have been mechanically arranged to produce rectilinear stereo, which allows us to use the notation $y_L = y_R \equiv y$ and $z_L = z_R \equiv z$. The cameras focal lengths f are assumed to be equal and known. The light plane axis of rotation intersects the camera xz -plane at $\mathbf{x}_{0L} = (x_{0L}, 0, z_0)^T$ and $\mathbf{x}_{0R} = (x_{0R}, 0, z_0)^T$ in L and R respectively. During a scan, the light plane frame P rotates about the y_P -axis with angle θ_y , with $\theta_y = 0$ when the light plane is parallel to the camera optical axes. The rotation axis may also be slightly offset from the vertical, as characterized by the small fixed angles θ_x and θ_z .

Let Ω_P represent the plane equation parameters for the light plane in P . We can express Ω_P as

$$\Omega_P = (1, B, 0, D)^T \quad (6)$$

where B and D are typically small quantities indicating the non-vertical component of the plane and distance of the plane from the axis of rotation. Now let $\mathbf{H}_i^P$, $i = L, R$, represent the homogeneous transformation matrices from P to the camera frames. Using the matrices $\mathbf{R}_x(\theta)$, $\mathbf{R}_y(\theta)$ and $\mathbf{R}_z(\theta)$ to represent rotations about the x , y and z -axes by angle θ , and $\mathbf{T}(\mathbf{x})$ to represent translation by $\mathbf{x}$, these transformations may be expressed as

$$\mathbf{H}_i^P = \mathbf{T}(x_{0i}, 0, z_0) \cdot \mathbf{R}_z(\theta_z) \cdot \mathbf{R}_x(\theta_x) \cdot \mathbf{R}_y(\theta_y) \quad (7)$$

for $i = L, R$. Then, the plane equation parameters transform from P to L and R as

$$\Omega_i = (\mathbf{H}_i^P)^{-T} \cdot \Omega_P, \quad i = L, R \quad (8)$$

Thus the plane equations for the light plane in the two camera frames can be written as

$$\Omega_i^T \cdot (\mathbf{H}_i^P)^{-1} \cdot \mathbf{x}_i = 0, \quad i = L, R \quad (9)$$

Following the procedure outlined in the previous section, the plane equations are projected onto the image planes using the pin hole camera model (2), and combined to eliminate the unknown z . This results in

$$k_1 u_L + k_2 v + k_3 f - k_4 (k_1 u_R + k_2 v + k_3 f) = 0 \quad (10)$$

where $v_L = v_R \equiv v$ (since $y_L = y_R$) and

$$\begin{aligned} k_1 &= c_y c_z - s_x s_y s_z - B c_x s_z \\ k_2 &= c_y s_z + s_x s_y c_z + B c_x c_z \\ k_3 &= -c_x s_y + B s_x \\ k_4 &= \frac{z_0 c_x s_y - x_{0L} (c_y c_z - s_x s_y s_z) + B (x_{0L} c_x s_z - z_0 s_x) + D}{z_0 c_x s_y - x_{0R} (c_y c_z - s_x s_y s_z) + B (x_{0R} c_x s_z - z_0 s_x) + D} \end{aligned}$$

where $c_i = \cos(\theta_i)$ and $s_i = \sin(\theta_i)$. By making the simplifying assumptions $B, D, \theta_x, \theta_z \ll 1$, we can eliminate many of the insignificant terms in (10) and rearrange the equation to give an approximate model:

Figure 2: Light stripe camera coordinate frames.

$$u_L + \alpha u_R + \beta v + \gamma f = 0 \quad (11)$$

where

$$\alpha = -\frac{a \cos \theta_y + b \sin \theta_y + c}{\cos \theta_y + b \sin \theta_y + c} \quad (12)$$

$$\beta = \frac{(1-a)(\theta_z \cos \theta_y + \theta_x \sin \theta_y + B)}{\cos \theta_y + b \sin \theta_y + c} \quad (13)$$

$$\gamma = -\frac{(1-a) \sin \theta_y}{\cos \theta_y + b \sin \theta_y + c} \quad (14)$$

and

$$a = x_{0L}/x_{0R}, \quad b = -z_0/x_{0R}, \quad c = -D/x_{0R} \quad (15)$$

Equation (11) provides a condition for validating raw laser stripe measurements given knowledge of the light plane angle θ_y , and is central to the techniques proposed in this work. Section 4.1 describes how (11) is used to form correct associations between multiple measurements while simultaneously rejecting spurious data.

3.3 Active Calibration of Light Plane Parameters

System calibration involves determining the unknown parameters α , β and γ in equation (11). If the mechanical properties of the system are known, these values may be calculated directly via (12)-(14). Alternatively, we now present a strategy for calibrating the system using only image-based measurements.

The calibration procedure begins by measuring the position of the light stripe on each of n scan lines in a stereo image pair to produce a set of coordinates $\mathbf{u}_{L,i}$ and $\mathbf{u}_{R,i}$, $i=1 \dots n$. To sufficiently constrain the solution of (11), the measured points must not be co-linear. Hence, any non-planar but otherwise arbitrary object can be used as the calibration target. The calibration parameters may then be estimated via linear regression by forming the residue

$$r_i = u_{L,i} + \alpha u_{R,i} + \beta v_i + \gamma f, \quad i = 1 \dots n \quad (16)$$

Figure 3: Image based calibration; estimated parameters α , β and γ (points) and fitted models (solid lines).

and solving the set of linear equations

$$\frac{\partial}{\partial j} \left(\sum_{i=1}^n r_i^2 \right) = 0, j = \alpha, \beta, \gamma \quad (17)$$

The solution of (17) gives the calibration parameters at a particular angle of the light plane θ_y . To determine the variation of α , β , and γ with θ_y , the laser is scanned across the calibration target and the regression is performed for each captured frame. The laser shaft encoder count e is also measured, and is assumed to vary linearly with θ_y according to:

$$\theta_y = m_1 e + m_2 \quad (18)$$

Figure 3 plots typical measurements of the calibration parameters against encoder count for our experimental system. The calibration target in this case was a corner formed by two boxes (chosen with favourable reflectance properties), and the light stripe was detected using the processing techniques described in Section 4.

The final step in the calibration process is to fit the model equations (12)-(14) and (18) to the measured data, providing estimates for the model parameters (a , b , c , B , θ_x , θ_z , m_1 , m_2). Since we cannot find a closed form solution, the models were fitted using the iterative numerical *downhill simplex* method [12]. A suitable initial estimate of the model parameters was chosen manually. The final estimates are plotted on Figure 3 along with the measured data. In all cases, the residues of the fitted models are less than 2% of the full-scale values.

Finally, given the estimated model parameters (a , b , c , B , θ_x , θ_z , m_1 , m_2), we can calculate α , β and γ for any laser plane angle and use (11) to validate/associate our light stripe measurements. The calibration technique presented here is simple, fast and accurate, requiring only a few scans of a non-planar object. Furthermore, since it is independent of intrinsic camera parameter estimation, image-based calibration allows for robust validation independent of ranging accuracy.

3.4 Stereo Reconstruction

In most stereo systems, three-dimensional information is recovered by triangulation of the image plane meas-

urements via the inverse camera model. In this system we can also use knowledge of the light plane angle to further constrain and improve the position estimation. Let b_0 be the baseline separation of the stereo cameras, noting $b_0 = x_{OL} - x_{OR}$. Then, using (12)-(14) we can write the pin hole projection of the plane equations (9) as:

$$u_L + \frac{\beta}{\alpha+1}v + \frac{\gamma}{\alpha+1}f = \frac{fb_0\alpha}{z(\alpha+1)} \quad (19)$$

$$u_R + \frac{\beta}{\alpha+1}v + \frac{\gamma}{\alpha+1}f = -\frac{fb_0}{z(\alpha+1)} \quad (20)$$

For light plane measurements u_L and u_R , we first form the residues r_L and r_R of the projected plane equations:

$$r_L = u_L + \frac{\beta}{\alpha+1}v + \frac{\gamma}{\alpha+1}f - \frac{fb_0\alpha}{z(\alpha+1)} \quad (21)$$

$$r_R = u_R + \frac{\beta}{\alpha+1}v + \frac{\gamma}{\alpha+1}f + \frac{fb_0}{z(\alpha+1)} \quad (22)$$

The optimal three-dimensional reconstruction is taken as the point $\mathbf{x}$ which minimizes the sum of the squared residues:

$$\frac{\partial}{\partial z} r_L^2 \Big|_{u_L} + \frac{\partial}{\partial z} r_R^2 \Big|_{u_R} = 0 \quad (23)$$

Solving the (23) for z gives the optimal recovered depth:

$$z = \frac{fb_0(\alpha^2+1)}{(\alpha u_L - u_R)(\alpha+1) + v\beta(\alpha-1) + f\gamma(\alpha-1)} \quad (24)$$

The x and y coordinates of the reconstructed point are recovered via the plane equations and camera model:

$$x_L = \frac{\alpha b_0 - \beta y - \gamma z}{\alpha+1}, \quad x_R = -\frac{b_0 + \beta y + \gamma z}{\alpha+1}, \quad y = \frac{vz}{f} \quad (25)$$

Note that accurate knowledge of b_0 is required only for metric reconstruction, as it affects only isotropic scale.

4. Implementation

4.1 Light Plane Detection and Association

The purpose of light plane detection is to determine the position of the reflected stripe on each scan line of the

captured stereo images. Stripe extraction is performed using intensity data only. Since (11) is a necessary but not sufficient condition for validation, we provide additional robustness by requiring the light stripe to be a moving feature. Thus, the first step is to subtract successive frames and remove the stationary background. Processing only non-background regions also reduces the computational expense of the following steps.

Candidate locations for the light stripe are found by searching scan lines for *local* maxima in the intensity profile and applying pre-defined thresholds on the pulse shape. However, these thresholds can be chosen to select candidates optimistically, since the following steps will reject false measurements.

For scan line i at height v_i , the result is a set of m candidates for the stripe in the left camera at $u_{L,j}$, $j = 1 \dots m$, and n candidates in the right camera at $u_{R,k}$, $k = 1 \dots n$. Only one pair of these candidates can describe the actual stripe, and the association problem is solved using (11). First, for each candidate $u_{R,k}$ in R , the corresponding measurement $\hat{u}_{L,k}$ in L is predicted as

$$\hat{u}_{L,k} = -(\alpha u_{R,k} + \beta v_i + \gamma) \quad (26)$$

The optimal correspondence is then chosen as the candidates l in L and r in R that minimize the position error

$$(l, r) = \min_{(j,k)} |u_{L,j} - \hat{u}_{L,k}|, j = 1 \dots m, k = 1 \dots n \quad (27)$$

Finally the minimum position error is compared to an error threshold to validate the correspondence. This procedure simultaneously solves the association problem for multiple candidates while eliminating noise and cross talk. Given the stripe location on each scan line, the range profile is then recovered with (24)-(25).

4.2 Sub-Region Processing

For a given plane angle, equations (19) and (20) reveal that possible light plane measurements can only occur in a sub-region of the images. Taking the limit for points far from the cameras gives one region boundary at:

$$\lim_{z \rightarrow \infty} u_i = -\frac{\beta v + \gamma}{1 + \alpha}, i = L, R \quad (28)$$

Furthermore, taking the limit $z \rightarrow 0$ for points close to the cameras, the other boundary is at $u_L \rightarrow +\infty$ and $u_R \rightarrow -\infty$. Now if w is the captured image width, laser stripe measurements of the scan line at v must be constrained in L and R to the u -coordinate ranges:

$$u_L \in \left[-\frac{\beta v + \gamma}{1 + \alpha}, +\frac{w}{2} \right], u_R \in \left[-\frac{w}{2}, -\frac{\beta v + \gamma}{1 + \alpha} \right] \quad (29)$$

Processing pixels only within these ranges therefore recovers all available stereo data with a total pixel throughput equivalent to a single image. Using this

Figure 4: Multiple light stripe experiment.

Figure 5: Colour-range image without validation

Figure 6: Colour-range image with validation.

technique effectively halves the computational expense of image processing compared to a naive implementation, and allows the experimental system presented here to operate a full PAL frame rate.

4.3 Dynamic Effects

Measurements involving multiple sensors and dynamic targets usually require synchronization. In this system the cameras are synchronized in hardware and exposed simultaneously. Measurement validation also requires accurate knowledge of the light plane angle during camera exposure. However, encoder count acquisition can-

not be synchronized with camera exposure in hardware. The solution adopted here was to accurately measure the delay between exposure and corresponding encoder count acquisition. This involved sweeping the light plane in a known pattern while time stamping the encoder data and the measured position of the light plane. The encoder data and image-based measurements of the light plane trajectory were then matched to determine the acquisition delay. During normal operation, the shaft position for a given image is determined by offsetting the encoder data by the acquisition delay, and linearly interpolating between measured encoder values.

5. Experimental Results

To demonstrate the robustness of stereoscopic laser scanning, the proposed techniques were implemented on the experimental system described in Section 2. A typical office object was scanned in ambient lighting, and a mirror was positioned to create an image of the object and scanning laser (Figure 4). The arrangement is equivalent to the object and its image being scanned simultaneously by two robots, synthesizing the effects of spurious reflections and cross talk.

Figure 5 shows a 3D rendering of the colour/range data captured by the system without the validation described in Section 3. The stripe is simply detected as the brightest pulse on each scan line in the stereo pair, and depth is recovered using rectilinear stereopsis. Clearly, gross association errors result in noisy range data, as the system cannot distinguish between the true and reflected laser stripe. Figure 6 shows the object captured under the same conditions, but using equation (11) to correctly identify the real stripe and reject the 'ghost' stripe. Comparison with Figure 5 highlights the robustness of the system against spurious sample points that may arise from reflections and cross talk. Also noteworthy is the implicitly accurate registration of colour and range. In fact, the raw data captured here is sufficiently clean for higher level processing without additional filtering.

6. Conclusions and Future Work

In this paper, we develop an condition relating camera intrinsic parameters, light plane orientation and image plane measurements for a stereoscopic light stripe scanner. Unlike previous methods for noise rejection, we show how this condition provides a uniform, low-level mechanism for validating light stripe measurements to reject sensor noise, spurious reflections and cross talk, solve the association problem and provide improved accuracy and efficiency. We also present an image-based method for calibrating light plane parameters, allowing for robust validation even when accurate ranging is not required. An experimental colour-range scan-

ner is calibrated and used to scan an object in the presence of noise to verify the proposed methodology.

We intend to extend this work to a multi-stripe system to increase data acquisition rate, using these techniques to solve the association problem that arises when multiple stripes illuminate the target. We also intend to explore the idea that strongly validated measurements may be used to find adjacent weak measurements, or those masked by sensor noise, assuming properties such as smoothness of the scanned surface. Finally, an overarching goal is to perform some object modelling and recognition based on the acquired data.

7. References

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