On-line Multi-Objective Optimization of Dynamic Systems: Pareto seeking control

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Abstract: Considering a Multi-Objective Optimization problem for a Dynamic System, we propose a controller that drives this system into a point on the Pareto front.At such a point, no change in any control variable can be made without deteriorating one of the objectives. Gradient Estimators are the core of this controller. A sinusoidal Perturbation based gradient estimation approach is adopted.

Keywords: Pareto Seeking Control, Multi Objective Optimization, Pareto Front

1. INTRODUCTION

The multi-objective optimization problem can be defined as the task of maximizing (or minimizing) a number of functions (also called objectives) subject to a number of constraints (or limitations) by adjusting a number of design (decision) variables. This idea presents a concept more than being a definition due to the absence of a unique single solution for this problem(Marler and Arora (2004)). Instead of the optimal solution (point), the term Pareto optimal is used in the multi-objective optimization problem. In simple words a Pareto optimal point is a point such that there is no other point that does not deteriorate at least one objective and at the same time it represent an optimal solution for one of the objectives. By Relaxing the condition of optimality of one of the objectives (Marler and Arora (2004)), the set of all weakly Pareto optimal points is obtained which is also termed the Pareto Frontier (or Front or Surface) and represents a trade off between the different objectives (Lampinen (2000); Caramia and Dell'Olmo (2008)).

2. PROBLEM DEFINITION

Consider a non linear system with input $\mathbf{u}(t) \in \mathbb{R}^m$ and a number of objectives to be optimized $\mathbf{y}(t) \in \mathbb{R}^l$. The problem of optimizing the objectives can be stated in a manner similar to the Extremum seeking control Problem (Zhang and Ordóñez (2007)) as:

$$\begin{aligned} \underset{\mathbf{u}}{\text{maximize}} \quad & \mathbf{y}(t) = [h_1(\mathbf{x}(t)), h_2(\mathbf{x}(t)), \dots h_l(\mathbf{x}(t))] \\ \text{subject to} \quad & g_i(\mathbf{x}(t), \mathbf{u}(t)) \leq 0,\ j = 1, \dots p, \\ & \frac{\mathrm{d}x_i(t)}{\mathrm{d}t} - f_i(\mathbf{x}(t), \mathbf{u}(t)) = 0,\ i = 1, \dots n, \\ & \mathbf{x}(0) = \mathbf{x}_0 \end{aligned} \tag{1}$$

where $\mathbf{x} = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n$ is a vector representing the state variables with $\mathbf{x}_0$ representing the initial condition of the states variable. Also, $\mathbf{f} = [f_1, f_2, \dots, f_n]^T$, $\mathbf{h} = [h_1, h_2, \dots, h_l]^T$, and $\mathbf{g} = [g_1, g_2, \dots, g_p]^T$ are vector-valued functions. Both $\mathbf{f}$, $\mathbf{h}$ and $\mathbf{g}$ are assumed to be sufficiently smooth.

Our goal is to find a controller that drives the system into a point within the Pareto front. This controller will be termed the Pareto Seeking Controller (PSC).

3. PROPOSED CONTROLLER

The idea of the proposed PSC is to adjust $\mathbf{u}(t)$ in a direction that improves all the objectives, and stops when reaching a point such that any small change in $\mathbf{u}$ will cause a reduction in one of the objectives. The algorithm requires knowledge of the gradient of all the objectives at the current operation point. Gradient estimators are discussed in the Extremum seeking problem, and one of the proposed estimators can be used in our scenario (Gelbert et al. (2012); Krstić and Wang (2000))

For the case of two objectives, the system is at the Pareto front if the gradient of the two objectives are in opposite direction. Then, the suitable direction is found by adding the normalized gradients:

$$\frac{du_i^0}{dt} = k \left(\frac{\partial \bar{y}_1}{\partial u_i} + \frac{\partial \bar{y}_2}{\partial u_i} \right),\ i = 1, 2 \tag{2}$$

In this way, the direction will be zero for all the points on the Pareto front, and no further changes are made.

Fig. 1 shows the case of a plant with two inputs and two objectives.

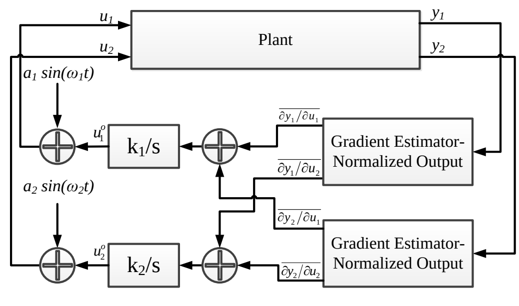

Fig. 1. A PSC for 2 × 2 Case

It should be noted that, the point that the system will converge is not unique, and will depend mainly on the initial condition.

4. SIMULATIONS

A plant with two inputs and two objective outputs were considered. The plant is represented as a cascade combination of linear dynamics and a static nonlinearity (Krstić (2000)), similar to Fig 2, with the following dynamics:

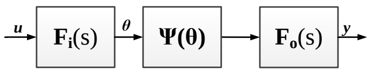

Fig. 2. An approximated non Linear System

$$\mathbf{F_i}(s) = \begin{bmatrix} \frac{1}{0.33s+1} & 0 \\ 0 & \frac{1}{0.5s+1} \end{bmatrix},$$

$$\mathbf{F_o}(s) = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{0.5s+1} \end{bmatrix}, \mathbf{\Psi}(\theta) = \begin{bmatrix} \psi_1(\theta) \\ \psi_2(\theta) \end{bmatrix},$$

A perturbation based gradient estimator were used to find the gradient. Two cases were simulated, the first case is a maximization problem with the non-linear functions as:

$$\psi_1(\theta) = (\theta - \theta_{\mathbf{1}}^*)^T \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} (\theta - \theta_{\mathbf{1}}^*)$$

$$\psi_2(\theta) = (\theta - \theta_{\mathbf{2}}^*)^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (\theta - \theta_{\mathbf{2}}^*)$$

where $\theta_1^* = [-2, +2]^T$ and $\theta_2^* = [+2, -2]^T$. Different starting points were simulated. The left part of Fig. 3 shows how the controlled variables are converging to the Pareto front, while the right part shows the objective functions increasing until the Pareto front is reached.

The second case is a minimization problem with non linear function (Lampinen (2000)):

$$\psi_1(\theta) = \theta(1)^2 + \theta(2),\ \psi_2(\theta) = \theta(1) + \theta(2)^2$$

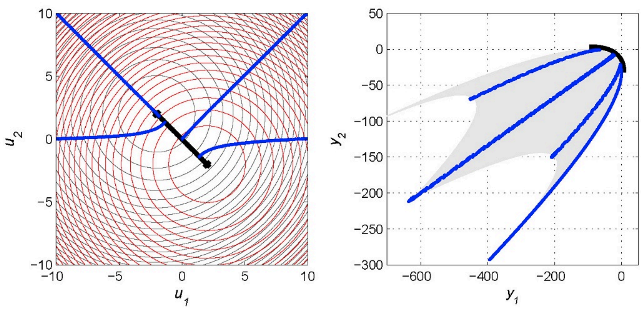

Fig. 3. First case, Left: u_1 and u_2 and the convergence to the Pareto Frontier of the input variables, Right: y_1 and y_2 and the convergence to the Pareto Frontier of the Objectives

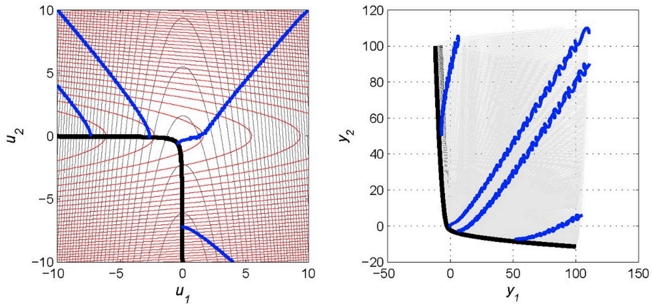

Fig. 4. Second case, Left: u_1 and u_2 and the convergence to the Pareto Frontier of the input variables, Right: y_1 and y_2 and the convergence to the Pareto Frontier of the Objectives

Fig. 4 shows the convergence to the Pareto Front for different cases.

5. CONCLUSION

This proposed controller is shown to drive the system to the Pareto front for two examples. Similar to the Extremum Seeking Control, a detailed knowledge of the system dynamics is not necessary. Only an estimate of the gradient of the output is required.

REFERENCES

Caramia, M. and Dell'Olmo, P. (2008). *Multi-objective Management in Freight Logistics*. Springer.

Gelbert, G., Moeck, J., Paschereit, C., and King, R. (2012). Advanced algorithms for gradient estimation in one-and two-parameter extremum seeking controllers. *Journal of Process Control*.

Krstić, M. (2000). Performance improvement and limitations in extremum seeking control. *Systems & Control Letters*, 39(5), 313–326.

Krstić, M. and Wang, H. (2000). Stability of extremum seeking feedback for general nonlinear dynamic systems. *Automatica*, 36(4), 595–601.

Lampinen, J. (2000). Multiobjective nonlinear pareto-optimization. *Pre-investigation Report, Lappeenranta University of Technology.*

Marler, R.T. and Arora, J.S. (2004). Survey of multi-objective optimization methods for engineering. *Structural and multidisciplinary optimization*, 26(6), 369–395.

Zhang, C. and Ordóñez, R. (2007). Numerical optimization-based extremum seeking control with application to abs design. *Automatic Control, IEEE Transactions on*, 52(3), 454–467.