ECHO-CANCELLATION IN A SINGLE-TRANSDUCER ULTRASONIC IMAGING SYSTEM

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ABSTRACT

During the last ten years, time-reversal of acoustic fields have been shown to be a very useful technique in ultrasonic imaging and testing. With the use of transducer arrays, it is possible to steer the sound beam to an arbitrary position within the medium, even if the medium is inhomogeneous or contains scatterers.

The major drawback with traditional beamforming and time-reversal techniques is that they require the use of transducer arrays. Recently a new technique was presented that, with the use of a waveguide, makes it possible to focus sound arbitrarily, with only one transducer elements. A problem with the setup is that the signal-to-noise ratio (SNR) is degraded because of interfering echoes from the waveguide.

In this paper, we present an echo-cancellation scheme that results in an SNR gain of approximately 33 dB. This enables the new technique to be used in pulse-echo mode, where this was not previously possible.

1. INTRODUCTION

In medical imaging applications, arrays of ultrasound transducers have been used for a long time. The use of arrays makes it possible to focus the sound beam in either transmit or receive mode. The focusing is done electronically, without any mechanical displacement of the transducers. There are also a number of potential non-destructive testing applications in the industry, but because of the expensive hardware required by conventional array techniques, the use has so far been limited. Ing *et al.* [1] recently presented a new focusing technique that only requires one transducer. This paper extends the technique with a noise cancellation method, which enables the technique to be used in pulse-echo mode. The focusing is based on the principle of acoustic time-reversal, which was first introduced by Fink, *et al.* [2].

The next sections give a background to acoustic time-reversal, and readers already familiar with this can skip directly to section 2.4.

2. BACKGROUND

2.1. Acoustic Time-Reversal

In a lossless fluid medium with a spatially dependent compressibility $\kappa(\mathbf{r})$ and density $\rho(\mathbf{r})$, the speed of sound, $c(\mathbf{r})$, is given by $c(\mathbf{r}) = \sqrt{\rho(\mathbf{r})\kappa(\mathbf{r})}$, where $\mathbf{r}$ is the location in space. The propagation equation of an acoustic pressure field $p(\mathbf{r}, t)$ is then given in [3] as:

$$\vec{\nabla} \cdot \left(\frac{\vec{\nabla} p(\mathbf{r}, t)}{\rho(\mathbf{r})} \right) - \frac{1}{\rho(\mathbf{r}) c^2(\mathbf{r})} \frac{\partial^2 p(\mathbf{r}, t)}{\partial t^2} = 0. \quad (1)$$

Now, if the pressure field $p(\mathbf{r}, t)$ is a solution to Eq. (1), then $p(\mathbf{r}, -t)$ is also a solution, *i.e.,* the wave equation is invariant to time-reversal. This property holds as long as the medium has a frequency independent attenuation. If not, the wave equation will contain odd-order derivatives of t and thus, the invariance to time-reversal is lost. Another requirement for the invariance to hold is conservation of energy [2].

An interesting consequence of the invariance to time-reversal of the propagation equation is that, if the complete three-dimensional (3D) sound pressure field $p(\mathbf{r}, t)$ from a point-like source is recorded, with an infinite number of point-like transducers and then time-reversed and re-emitted, the time-reversed pressure field will propagate back to the point source. Because the causality requirement has to be met in any practical realization of this experiment, the re-emitted pressure wave will instead be $p(\mathbf{r}, T - t)$, where T is the duration of the original sound wave.
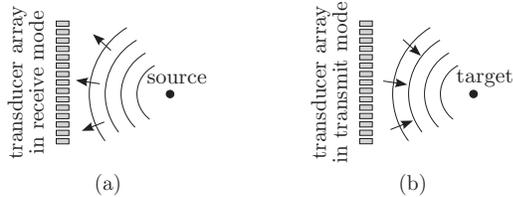
Figure 1: The principle of time-reversal of ultrasonic fields, using a linear transducer array. (a) Recording the field transmitted by a source. (b) Re-emitting time-reversed version of the recorded pulses. The sound field will back-propagate to the original location of the source.

It was shown in [2] that the time-reversal procedure can be interpreted as a spatio-temporal matched filter to the medium. This is true even if the medium contains multiple scatterers.

2.2. Time Reversal With Linear Arrays

Of course, the full 3D time-reversal cavity consisting of an infinite number of point-like transducers, required to capture the entire sound field, is a purely theoretical construction. In practice, this system has to be replaced by a finite number of transducers which all have a certain, non-zero, area. This can be 1D or 2D arrays, either planar or pre-focused. A 1D linear array is probably the most commonly used.

Time-reversal focusing with a linear array, often called a time-reversal mirror (TRM) consists of three steps:

1. Illuminating the target with a plane wave.

2. Recording the backscattered sound pressure wave. $p(\mathbf{r}, t)$.

3. Re-transmitting $p(\mathbf{r}, T - t)$.

Fig. 1 illustrates the last two steps, where the source marked in the Fig. 1a) could be either an active source, or a passive scatterer. If the entire sound field is captured, the performance of the focusing is very good, *i.e.,* the focal spot will be almost point-like. In practice, this is never the case, and the focal spot will have a certain spread, because of diffraction losses stemming from the limited aperture of the array.

2.3. Iterative Time-Reversal

The TRM described in the previous section enables us to focus sound at the location of a scatterer, even if the medium consists of layers with different sound velocities, *i.e.,* the TRM corrects for phase aberrations in the medium.

If the medium contains multiple scatterers, the time-reversal procedure will still be a realization of the spatio-temporal matched filter to the medium. However, focusing on a specific target becomes more complicated. The solution is to iterate the steps of the time-reversal. It was shown by Prada *et al.* [4] that the iterations will cause the sound beam to focus on the strongest scatterer. By a method known as D.O.R.T. [5] it is also possible to select other targets than the strongest scatterer.

2.4. Single-Transducer Time-Reversal

The principle of time-reversal has been shown to be useful in both medical and non-destructive testing applications. The major drawback with the technique is the need of large and expensive hardware. This is because the sound field has to be sampled at each array element individually, and the transmitter has to be fully programmable with separate D/A converters for each array element. In practice this requires in the range 64–128 A/D and D/A converters.

The performance of the focusing is partly determined by how much of the sound field that can be captured, *i.e.,* the array aperture. It was shown in [6] that a waveguide can be used to increase the effective aperture of the array. Because of reflections in the waveguide, most of the sound field will reach the elements of the array. This means that if the waveguide is designed properly, the number of transducer elements can be reduced. In [1] this idea was applied to the extreme case, with only one single transducer. Experimental results show that this technique can be used to focus the sound field at an arbitrary point in the medium, without any mechanical displacement of the transducer, at the cost of only a one-channel time-reversal system. The new method can be used either to locate sources in the medium, or to focus the sound at a desired point. Fig. 2 shows the setup.

In many cases, however, there are no active sources in the medium, but defects or other inhomogeneities. If a sound wave is transmitted into the medium, these inhomogeneities give rise to reflected sound waves. These waves can then be used to locate the inhomogeneities. This technique is widely used in traditional ultrasonic echographic imaging systems, where a transducer array is used.

When the single-element method is used in echographic mode, a series of problems arises. In the echographic configuration depicted in Fig. 2, the transducer is first used to transmit a short pulse. The sound will then propagate through the waveguide and out in the medium. The same transducer is then used as receiver to record the backscattered sound field. The problem in this setup is that the multipath propagation inside the waveguide is present for a long time.
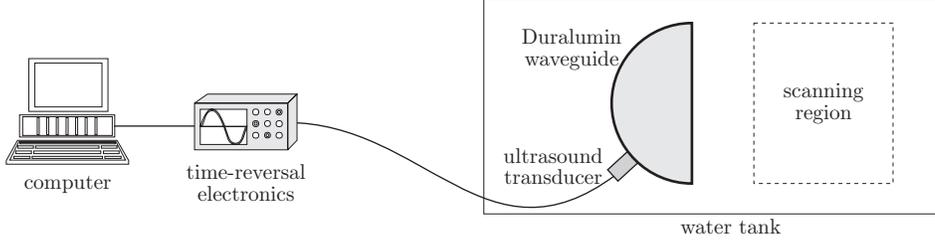
Figure 2: Experimental setup for the single-transducer imaging system.

These *interfering echoes* will overlap with the *desired echo*, coming back from the medium. The interfering echo is much stronger than the backscattered signal. To be able to locate the target, or to steer a transmitted beam to the point of the target, the interfering echo must be suppressed.

In this paper we present a technique for cancelling the interfering echo from the waveguide. The cancellation results in approximately 33 dB gain in SNR compared to the original signal. The echo-cancellation is based on a low-rank parametrization of the interfering echo, estimated from calibration data.

3. ECHO-CANCELLATION PRINCIPLE

The interfering echo is almost deterministic, but exhibits small variations due to sampling jitter, temperature variations, etc. For a deterministic signal the cancellation problem can easily be solved by subtracting it away. However, in our case the small random fluctuations are large enough for this not to work. Our approach is to first determine a parametric model of the interfering echo. The first part of the received signal contains only the interference, and can thus be used to estimate the model parameters. This is then used to predict the remaining part of the interfering echo, which can then be cancelled efficiently. The linear model is derived by performing a singular value decomposition (SVD) of a large set of measured calibration signals that only contains the interfering echo, *i.e.*, the scanning region in Fig. 2 contains no scatterers.

For the calibration measurements, the received signal is:

$$r_c(t) = e(t) + n(t), \qquad (2)$$

where $e(t)$ denotes the undesired interfering echo and $n(t)$ is additive white Gaussian noise. For the normal case, when we have a desired echo from reflectors in the medium, the received signal can be written as

$$r(t) = s(t) + e(t) + n(t), \qquad (3)$$

where $s(t)$ denotes the desired echo signal. The signal is sampled, with an 8 bit A/D converter, at 30 MHz,

during 200 μs, resulting in the corresponding discrete time signal model

$$r_k = s_k + e_k + n_k, \qquad k = 0, 1, ...5999. \qquad (4)$$

With vector notation this can be written as

$$\mathbf{r} = \mathbf{s} + \mathbf{e} + \mathbf{n}, \qquad (5)$$

where the signal vectors are 6000×1. Due to the propagation delay of the desired echo signal, the first part of the signal will only contain the interfering echo plus noise. In our case, approximately the first 2000 samples will never contain any part of the desired signal $s(t)$, thus we can write

$$r_k = e_k + n_k, \qquad k = 0, 1, ...1999. \qquad (6)$$

We use a top-bar to denote the truncated vectors containing only the first 2000 elements, $\bar{\mathbf{r}} = [r_0, r_1, \ldots, r_{1999}]$,

$$\bar{\mathbf{r}} = \bar{\mathbf{e}} + \bar{\mathbf{n}}. \qquad (7)$$

The interfering echo can be represented with a linear combination of a small set of basis vectors

$$\mathbf{e} = \sum_{i=1}^{n} \mathbf{b}_i a_i = \mathbf{Ba}, \qquad (8)$$

where $\mathbf{a} = [a_1, a_2, ...a_n]^T$ is the coefficient vector and $\mathbf{B}$ is a matrix containing the basis vectors $\mathbf{b}_i$. The matrix $\mathbf{R}_c$, which contains many calibration measurements $\mathbf{r}_c$ of the interfering echo $\mathbf{e}$, is factored using the SVD, as

$$\mathbf{R}_c = \mathbf{USV}^T. \qquad (9)$$

The first n columns of $\mathbf{U}$, corresponding to the n largest singular values (denoted by $\mathbf{U}_n$) form the optimal low-rank approximation (in the least-squares sense) to the column space of $\mathbf{R_c}$. Let these be the basis vectors of our model, that is $\mathbf{B} = \mathbf{U}_n$. The first part of the received signal contains only the interfering echo, and can thus be used to estimate the unknown parameter vector $\mathbf{a}$, expressed as

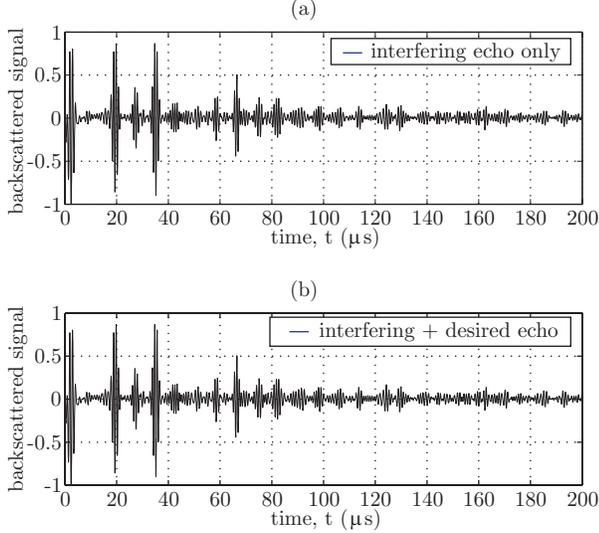$$\bar{\mathbf{e}} = \bar{\mathbf{B}}\mathbf{a}, \qquad (10)$$

Figure 3: Original pulses. (a) Echo from the waveguide only. (b) Echo from the waveguide superimposed on the desired signal.
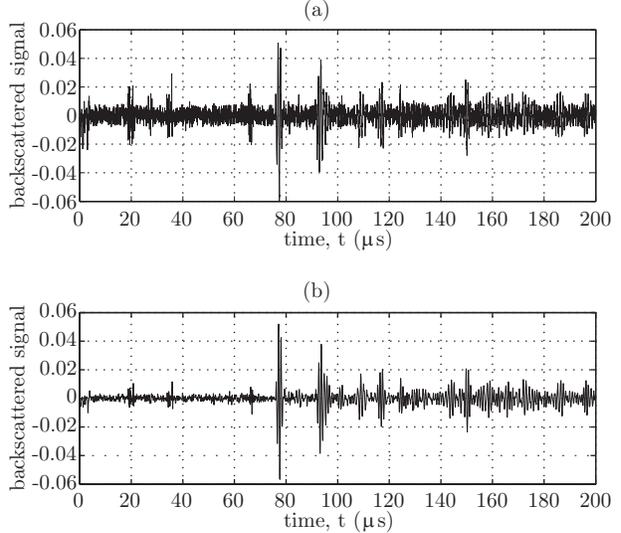

Figure 4: Filtered pulses. (a) Desired echo, extracted from the received signal. (b) Same as (a) after applying a 5:th order Butterworth low-pass filter with a cut-off frequency of 3 MHz.

where $\bar{\mathbf{B}}$ has dimensions $(2000 \times n)$ and consists of the upper part of $\mathbf{B}$. Using the first part, $\bar{\mathbf{r}}$, of the received signal $\mathbf{r}$, a least-squares estimate of the parameter vector is given by:

$$\hat{\mathbf{a}} = \left(\bar{\mathbf{B}}^T \bar{\mathbf{B}}\right)^{-1} \bar{\mathbf{B}}^T \bar{\mathbf{r}}. \tag{11}$$

The estimate of the entire interfering echo then becomes:

$$\hat{\mathbf{e}} = \mathbf{B}\hat{\mathbf{a}} = \mathbf{B} \left(\bar{\mathbf{B}}^T \bar{\mathbf{B}}\right)^{-1} \bar{\mathbf{B}}^T \bar{\mathbf{r}}. \tag{12}$$

Finally, our after subtracting off the estimated interfering echo, the echo-cancelled signal is given by $\hat{\mathbf{s}} = \mathbf{r} - \hat{\mathbf{e}}$.

4. EXPERIMENTAL RESULTS

All experiments were made using the setup described in Fig. 2. For each measurement, the transducer was first used to transmit a short pulse with a center frequency of 1 MHz. The same transducer was then used to record the backscattered signal, coming from both the Duralumin waveguide and any scatterers present in the region in front of the waveguide. The entire setup was immersed in water at room temperature.

First, 100 calibration measurements were made without any scatterers in the medium. These measurements contain only the interfering echo, and were used to estimate the linear model described in the previous section. The measured signals are represented by the matrix $\mathbf{R}_c$ (6000×100) in Eq. (9) above. Fig. 3a) shows an example of such a measured signal.

The second set of measurements was performed with the same setup, but with a thin copper wire

present in the scanning region, acting as a reflector. Again, 100 signals were recorded. Fig. 3b) show a typical example of those signals. Comparing with the first plot, we can not distinguish any desired echo signal. That signal is completely drowned by the much stronger interfering echo.

The SVD of the matrix $\mathbf{R}_c$, reveals one very large singular value and the other 99 are slowly decreasing in magnitude. Thus we conclude that one basis vector and parameter could be sufficient in our model, but to be on the safe side we chose to use the first two vectors. Fig. 4a) shows the signal after echo-cancellation with our second order model. Fig. 4b) shows the same signal after low-pass filtering. Because the desired signal is an echo of a signal generated by a 1 MHz source, the filter was designed as a 5:th order Butterworth low-pass filter with a cut-off frequency of 3 MHz. The desired echo signal that starts at about 76 μs is now clearly distinguishable. Note the difference in scale on the Y-axis between Fig. 3 and 4. The interfering echo is almost entirely removed.

Fig. 5 shows the estimated power spectral density (PSD) of the signal before and after echo cancellation, averaged over all 100 recorded pulses. Fig. 5a) shows the PSD of the part of the signal in Fig. 4a) that contains the desired signal, $i.e.$, 70-200 μs. Fig. 5b) shows the PSD of the first part of the signal in Fig. 4a), $i.e.$, 0-70 μs. Finally, Fig. 5c) shows the PSD of latter part (70-200 μs) of the interfering signal in Fig 3a) that overlaps with the desired signal. To estimate the SNR gain, we calculate the signal and noise
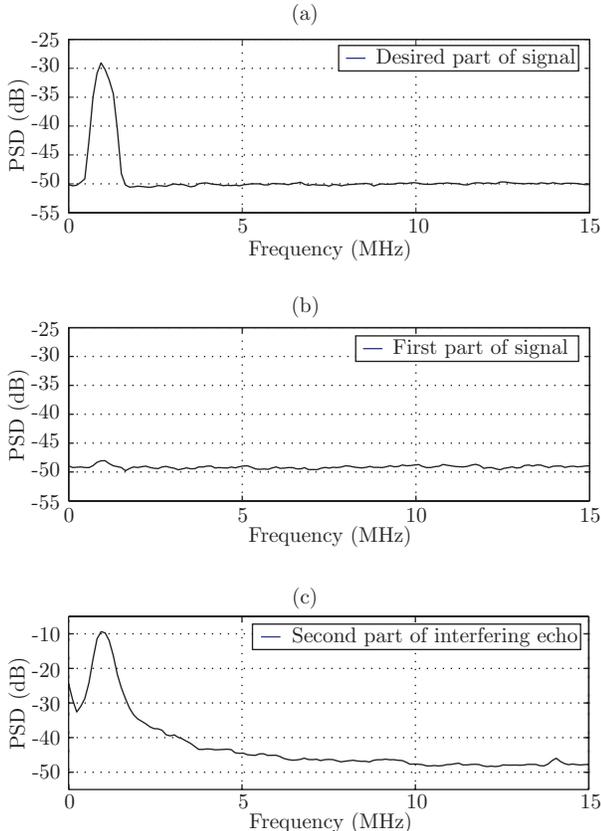
Figure 5: Spectra of the desired echo, (a), and the remaining noise after filtering, (b). The PSD of the interfering echo in the interval 70-200 μs, (c).

energy in the interval between 0 and 2 MHz. The resulting SNR gain (between Fig. 5c) and Fig. 5b)) then becomes approximately 33 dB.

We see that the spectrum is essentially flat for this part, which shows that no interfering echo remains after the echo cancellation, only white noise.

5. DISCUSSION

Traditional delay-line beamforming, using a transducer array, works well for focusing sound through homogeneous media. There are also techniques for correction of phase aberrations, as long as the inhomogeneity is located close to the array [7, 8]. If not, *i.e.* the medium contains scatterers or layers with different sound velocities, the delay-line beamforming will not work.

Using time-reversal with an array (TRM), a single scatterer can be located even if the sound velocity changes through the medium, as long as there is no multiple scattering. If different target locations can be probed, a TRM can be used to focus even through multiple scattering media. This is because the time-

reversal process realizes the spatio-temporal matched filter to the medium. If there are several targets, the iterative TRM [4] can be used in order to select target.

A common drawback with all the techniques mentioned above, is the need of a transducer array. With the single-transducer setup proposed by Ing *et al.* (see Fig. 2), a performance similar to the delay-line beamforming can be expected, in terms of focal spot spread. The main difference is the loss of sound energy at the interface between the water and the waveguide. As opposed to a normal transducer, the waveguide is not at all matched to the medium. This causes signal energy to be lost each time the sound passes the interface. For the same reason, an iterative time-reversal process is difficult with the current setup, but could be used if the loss in SNR due to the reflections is not important.

One example of this kind of application would be a non-destructive testing (NDT) setup, where the *detection* of faults (*e.g.* cracks in a solid material) is more important than to actually *imaging* the medium. In medical imaging applications, the SNR is much more critical, and therefore the current setup is more useful in NDT applications.

6. CONCLUSIONS

In this paper we present an echo-cancellation scheme based on a low-rank parametrization of the interfering echo generated within the waveguide. The filtering results in a gain of approximately 33 dB in SNR compared to the unprocessed signal. The time-reversal procedure can then be used to focus an ultrasound beam at the location of the target, using only one single transducer.

7. REFERENCES

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