Robust time-varying thresholds for supervision of valves in a flotation process

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Abstract—Robust thresholds for observer-based residuals are developed for the purpose of detecting clogging in the valves of a flotation process. The observer in the residual generator is a linear model of the flotation process extended with integrators and corrected with a linear feedback term. The integral states in the observer constitute the residual. Contribution to the residual comes not only from the faults but also from uncertainties in measurements, estimates and working point. To avoid false alarms generated by these uncertainties, robust time-varying thresholds depending on the uncertainties are derived.

Experiments on a froth flotation process with four cascade coupled tanks are carried out successfully. The data was provided by Boliden Area Concentrator in Boliden, Sweden.

I. INTRODUCTION

Froth flotation is an important and versatile mineral-processing technique in which the differences in physico-chemical surface properties of particles from different minerals is used. By using flotation reagents it is possible to separate the valuable minerals from the rocky material called *gangue*. In *reverse flotation* the gangue is separated into the froth but in *direct flotation* the minerals are transferred to the froth. An introduction to flotation can be found in [1].

In both reversed and direct flotation it is important to control the level of the flotation tank, see [2] for an account on a multivariable LQ-controller for a flotation plant. The levels are controlled with valves on the outflow of each tank and to ensure the controller is able to fulfill its performance requirements, a fault detection algorithm is needed to detect clogging of the valves at an early stage.

A. Fault detection

Fault detection and isolation (FDI) algorithms can be divided into two steps, residual generation and residual evaluation. Design and analysis of model-based fault detection and isolation algorithms has received significant attention in the literature over the years (see survey articles [3], [4], [5] and books [6], [7], [8]).

The residual generator produces a signal, the residual, which is, ideally, nonzero in the presence of fault and zero otherwise. Three different methods to generate the residual is by parameter estimation algorithms, see [9] and references therein, parity space methods [10], and state observers, *i.e.* Kalman filters [11] and Luenberger observers [12]. Different observer structures can be found in the fault detection literature, *e.g.* the dedicated observer scheme (DOS), see [13] or the generalized observer scheme (GOS), see [3], [6]. The dedicated observer is constructed such that each residual is only sensitive to one fault compared to the

generalized observer where each residual is sensitive to all faults except one.

In some rare cases it is possible to achieve perfect decoupling between uncertainties and faults, *i.e.* making the residual independent of the uncertainties, see [14] and references therein.

In the case where perfect decoupling is not possible, a robust detection threshold is needed to avoid false alarms while maintaining high sensitivity to faults.

Some work in this direction can be found in the literature. In [15], robustness against unstructured additive uncertainty in the frequency domain is considered, [16] presents a robust threshold for detection of clogging in the coal injection lines of a blast furnace and [17] for sensor fault detection for a turbofan jet engine. Other recent publications of robust thresholds are [18] and [19].

In [20] a deterministic robust threshold is presented and a fault detection algorithm is derived for a nonlinear system with full state measurement. An adaptive GOS structure observer is used and the threshold is derived to be the output of a linear system. The algorithm in [20] is complicated in the sense that it has many degrees of freedom, but has been shown to work in simulation.

B. Fault detection in the flotation process

Two papers that concern fault detection of valves in flotation processes are [21], [22]. In [21] the authors use a parameter estimation algorithm to estimate clogging in valves and in [22] a nonlinear observer is used. Experiments with these two algorithms, see [22], shows that the residuals are not decoupled from the uncertainties and that constant thresholds, which are used in [22], can consequently not be made both robust and sensitive to faults. Therefore, more robust thresholds which are sensitive to faults are needed.

C. Outline of the paper

The article is composed as follows. First, a description of the flotation process model is presented in Section II followed by a presentation of the residual generation algorithm, Section III. Section IV treats the residual evaluation step and is divided into two parts. In the first part, a sensitivity analysis is carried out to assess the impact on the residual by uncertainties and disturbances. A method for threshold calculation is suggested in the second part. Section V shows experiments on data from a real flotation process and some conclusions are drawn in Section VI.

II. FLOTATION PROCESS MODEL

The flotation process consists of cascade coupled tanks with control valves after each tank for the purpose of controlling the levels in the tanks. The measured level in the tanks are denoted $h_i(t)$ where $i = 1..n_h$ and n_h is the number of tanks. The input signals to the process are the valve control signals $u_i(t)$ and the external inflow to the tanks is denoted $q(t)$. The levels are measured in meter and the inflow in m^3/h .

The continuous time model of the levels $h(t)$ in the tanks can be described as a system of first order differential equations.

$$\dot{h}(t) = F(h(t), u(t), v(t)) + Dq(t) \quad (1)$$

where $D \in \mathbb{R}^{n_h \times n_q}$, $h(t) \in \mathbb{R}^{n_h}$, $v(t) \in \mathbb{R}^{n_h}$, $u(t) \in \mathbb{R}^{n_h}$ and $q(t) \in \mathbb{R}^{n_q}$ are, respectively, the input matrix, state, fault, control signal and inflow. The number of external inflows are denoted n_q . In the sequel the time argument t is dropped to enhance readability. The vector of nonlinear functions $F(h, u, v)$ is

$$F(h, u, v) = \begin{bmatrix} F_1(h, u, v) \\ \vdots \\ F_{n_h}(h, u, v) \end{bmatrix}$$

where

$$F_1(h, u, v) = -\frac{C_{v,1}(u_1)}{\alpha_{\Omega_1}} \sqrt{H_1(h)} - \frac{1}{\alpha_{\Omega_1}} v_1$$

where $v_1 = \gamma_{C_{v,1}} \sqrt{H_1(h)}$ is the volume flow loss due to clogging in Valve 1 and $\gamma_{C_{v,1}}$ is the clogging area. The function $C_{v,1}$ and the constant α_{Ω_1} are, respectively, the opening area of Valve 1 and liquid surface area of Tank 1 which is assumed to be a constant independent of the level in the tank. For $i \geq 2$

$$F_i(h, u, v) = \frac{C_{v,i-1}(u_{i-1})}{\alpha_{\Omega_i}} \sqrt{H_{i-1}(h)} + \frac{1}{\alpha_{\Omega_i}} v_{i-1} - \frac{C_{v,i-1}(u_i)}{\alpha_{\Omega_i}} \sqrt{H_i(h)} - \frac{1}{\alpha_{\Omega_i}} v_i$$

where $C_{v,i}$ is the valve opening area of valve i and α_{Ω_i} is the liquid surface area of tank i . The functions $H_i(h)$ are defined as

$$H_i(h) = 2g(h_i - h_{i+1} + \alpha_{h_i})$$

where g is the acceleration of gravity and α_{h_i} is the physical level difference between the zero-levels of tank i and $i+1$. For the last tank $H_n(h) = 2g(h_n + \alpha_{h_n})$ where α_{h_n} is the level difference between the zero-level and the outflow valve.

A. Linearization

The derivation of a robust threshold, see subsection IV-B, is simplified if the estimation error system is linear. A linear error system can be obtained by linearizing the process model which, since the process is almost linear, will introduce errors that are small compared to the uncertainties. The flotation process model is possible to linearize around the working point h_0, u_0, q_0 . The parameters h_0, q_0 are chosen and u_0 calculated by setting the derivative in equation (1) equal to zero, *i.e.* solving $0 = F(h_0, u_0, 0) + Dq_0$. A Taylor

expansion of the right hand side of equation (1) with respect to h, u, v and q gives the linearized model

$$\begin{aligned} \dot{\Delta}h &= A\Delta_h + D\Delta_q + B\Delta_u + E_v\Delta_v + \Gamma \\ y &= \Delta_h + \delta_w \end{aligned} \quad (2)$$

where δ_w is a measurement disturbance and $\Delta_h, \Delta_u, \Delta_q$ and Δ_v are defined by

$$\begin{aligned} h &= h_0 + \Delta_h & u &= u_0 + \Delta_u \\ q &= q_0 + \Delta_q & v &= v_0 + \Delta_v \end{aligned}$$

and the matrices A, B, E_v, Γ by

$$\begin{aligned} A &= F'_h(h_0, u_0, 0) & B &= F'_u(h_0, u_0, 0) \\ E_v &= F'_v(h_0, u_0, 0) & \Gamma &= F'(h_0, u_0, 0) + Dq_0 \end{aligned}$$

where prime indicates derivative with respect to the index, *i.e.*

$$f'_a \triangleq \frac{\partial f}{\partial a}$$

The matrix Γ will be equal to zero if neglecting uncertainties but nonzero otherwise and thus affect the process, see Section IV-A, and is therefore retained.

The derivative of the fault signal $v(t)$ can be expressed as

$$\begin{aligned} \dot{v}_i(t) &= \frac{\partial}{\partial t} (\gamma_{C_{v,i}}(t) \sqrt{H_i(h)}) \\ &= \dot{\gamma}_{C_{v,i}}(t) \sqrt{H_i(h)} + \gamma_{C_{v,i}}(t) \frac{\partial}{\partial t} \sqrt{H_i(h)} \end{aligned}$$

In the working point, the clogging $\gamma_{C_{v,i}}$ is defined to be zero, $\gamma_{C_{v,i}}(t) = 0$. Furthermore, a reasonable assumption is that the clogging develops slowly, *i.e.* $\dot{\gamma}_{C_{v,i}}(t) \approx 0$, which gives

$$\dot{v}(t) = 0$$

An augmented state vector can thus be defined as $x = [\Delta_h \ v]^T$ for which the dynamics are

$$\begin{aligned} \dot{x} &= \bar{A}x + \bar{D}\Delta_q + \bar{B}\Delta_u + \bar{\Gamma} \\ y &= \bar{C}x + \delta_w \end{aligned} \quad (3)$$

where

$$\begin{aligned} \bar{A} &= \begin{bmatrix} A & E_v \\ 0 & 0 \end{bmatrix}, & \bar{D} &= \begin{bmatrix} D \\ 0 \end{bmatrix}, & \bar{B} &= \begin{bmatrix} B \\ 0 \end{bmatrix} \\ \bar{C} &= \begin{bmatrix} I & 0 \end{bmatrix}, & \bar{\Gamma} &= \begin{bmatrix} \Gamma \\ 0 \end{bmatrix} \end{aligned}$$

III. RESIDUAL GENERATION

In order to estimate the state x in the linear system, (3), a Luenberger observer can be used, see [12]. The feedback will be the difference between the measured level Δ_h and the estimated level $\hat{\Delta}_h$ and therefore,

$$\begin{aligned} \dot{\hat{x}} &= \bar{A}\hat{x} + \bar{D}\Delta_q + \bar{B}\Delta_u + \bar{\Gamma} + L(y - \hat{y}) \\ \hat{y} &= \bar{C}\hat{x} \end{aligned} \quad (4)$$

where $L = [L_h^T \ L_v^T]^T$ and $L_h \in \mathbb{R}^{n_h \times n_h}$, $L_v \in \mathbb{R}^{n_h \times n_h}$.

In the design of the feedback gain it should be noted that the matrices A and B depend on the working point and therefore, changes in the working point result in a time-varying process model, (2). To simplify the threshold calculation a time-invariant system is needed. This can be

achieved by designing a feedback matrix which cancels the time dependence in the matrix $\bar{A}(t)$. It can be shown that this is possible to achieve while maintaining the possibility to place the observer poles arbitrarily.

IV. RESIDUAL EVALUATION

In Section III an observer was presented that will produce an estimate of the clogging. This estimate will be different from zero even when no clogging is present since there are uncertainties in the model which are neglected in the observer. These uncertainties come from *e.g.* valve parameter identification errors, linearization errors and measurement errors.

In [22], the residual is compared to a constant threshold. As can be seen in [22] there is an imminent risk of false alarms and therefore more robust thresholds are needed. The thresholds must exceed the residuals as long as no clogging has occurred and yet be low enough to be sensitive to the faults. A possible solution is to design the thresholds to depend on upper bounds of the above mentioned uncertainties.

A. Sensitivity analysis

There are uncertainties in the measurements, $y(t)$ and $\Delta_q(t)$, the control signal, $u_\Delta(t)$ and the valve model. The valve model is

$$C_v(t) = Ku(t) + c \quad (5)$$

which is commonly used in the process industry and also in [21], [22].

The actual valve parameters, K, c in (5) are not known exactly and estimated parameters $\hat{K}, \hat{c}$ are used instead.

The uncertainties in the valve model constants K and c causes uncertainties in the working point h_0, q_0 and u_0 . The true parameters, Δ_q and Δ_u , are measured and are assumed to be uncertain. Uncertainties in the measurement, y can be described as an uncertainty in $\bar{C}$ and some additive measurement noise, δ_w . In summary

$$\begin{aligned} h_0 &= \hat{h}_0 \circ (\mathbf{1}_{n_h} + \delta_{h_0}) & q_0 &= \hat{q}_0 \circ (\mathbf{1}_{n_q} + \delta_{q_0}) \\ u_0 &= \hat{u}_0 \circ (\mathbf{1}_{n_h} + \delta_{u_0}) & \Delta_q &= \Delta_{\hat{q}} \circ (\mathbf{1}_{n_q} + \delta_{\Delta_q}) \\ \Delta_u &= \Delta_{\hat{u}} \circ (\mathbf{1}_{n_h} + \delta_{\Delta_u}) & K &= \hat{K} \circ (\mathbf{1}_{n_h} + \delta_K) \\ c &= \hat{c} \circ (\mathbf{1}_{n_h} + \delta_c) & \bar{C}_i &= \hat{C}_i \circ (\mathbf{1}_{n_h} + \delta_y) \end{aligned} \quad (6)$$

where $\circ$ denotes the Hadamard product, *i.e.* entry-wise product of two matrices of the same size. The notation $\mathbf{1}_n$ represents a column vector of dimension n with each element equal to 1 while $\bar{C}_i$ and $\hat{C}_i$ are the i :th column of $\bar{C}$ and $\hat{C}$.

The matrices $\bar{A}, \bar{B}, \bar{D}, \bar{C}, \bar{\Gamma}$ in equation (3) depend on the uncertainties in (6),

$$\delta = [\delta_{h_0}^T \ \delta_{q_0}^T \ \delta_{u_0}^T \ \delta_{\Delta_q}^T \ \delta_{\Delta_u}^T \ \delta_K^T \ \delta_c^T \ \delta_y^T \ \delta_w^T]^T \in \mathbb{R}^{n_\delta}$$

The dependence is nonlinear but can be approximated by

first order Taylor expansions as

$$\begin{aligned} \bar{A}(\delta) &\approx \bar{A}(0) + \sum_{i=1}^{n_\delta} \bar{A}'_{\delta_i}(0) \delta_i \\ &\triangleq \hat{A} + \sum_{i=1}^{n_\delta} A_i \delta_i \triangleq \hat{A} + \delta_A \\ \bar{B}(\delta) &\approx \bar{B}(0) + \sum_{i=1}^{n_\delta} \bar{B}'_{\delta_i}(0) \delta_i \\ &\triangleq \hat{B} + \sum_{i=1}^{n_\delta} B_i \delta_i \triangleq \hat{B} + \delta_B \\ \bar{D}(\delta) &\approx \bar{D}(0) + \sum_{i=1}^{n_\delta} \bar{D}'_{\delta_i}(0) \delta_i \\ &\triangleq \hat{D} + \sum_{i=1}^{n_\delta} D_i \delta_i \triangleq \hat{D} + \delta_D \\ \bar{C}(\delta) &\approx \bar{C}(0) + \sum_{i=1}^{n_\delta} \bar{C}'_{\delta_i}(0) \delta_i \\ &\triangleq \hat{C} + \sum_{i=1}^{n_\delta} C_i \delta_i \triangleq \hat{C} + \delta_C \\ \bar{\Gamma}(\delta) &\approx \bar{\Gamma}(0) + \sum_{i=1}^{n_\delta} \bar{\Gamma}'_{\delta_i}(0) \delta_i \\ &\triangleq \hat{\Gamma} + \sum_{i=1}^{n_\delta} \Gamma_i \delta_i \triangleq \hat{\Gamma} + \delta_\Gamma \end{aligned} \quad (7)$$

where δ_i represents the uncertainty at the i :th position in the vector δ . The detailed description of the matrices in (7) are omitted due to lack of space. The process description in (3) can be rewritten by using the matrices from the equations in (7)

$$\begin{aligned} \dot{x} &= (\hat{A} + \delta_A)x + (\hat{D} + \delta_D)(\Delta_{\hat{q}} + \Delta_{\hat{q}} \circ \delta_{\Delta_q}) \\ &\quad + (\hat{B} + \delta_B)(\Delta_{\hat{u}} + \Delta_{\hat{u}} \circ \delta_{\Delta_u}) + (\hat{\Gamma} + \delta_\Gamma) \\ y &= \hat{C}x + \delta_C x + \delta_w \end{aligned} \quad (8)$$

and the observer in (4) is, by using matrices and vectors $\hat{A}, \hat{B}, \hat{C}, \hat{D}, \hat{\Gamma}, \Delta_{\hat{q}}$ and $\Delta_{\hat{u}}$ which only contains known parameters, modified to

$$\begin{aligned} \dot{\hat{x}} &= \hat{A}\hat{x} + \hat{D}\Delta_{\hat{q}} + \hat{B}\Delta_{\hat{u}} + \hat{\Gamma} + L(y - \hat{y}) \\ \hat{y} &= \hat{C}\hat{x} \end{aligned} \quad (9)$$

The dynamics of the error, $e = x - \hat{x}$ can be calculated by combining equations (8) and (9) into

$$\begin{aligned} \dot{e} &= (\hat{A} - L\hat{C})e + \delta_A x + \hat{D}(\Delta_{\hat{q}} \circ \delta_{\Delta_q}) \\ &\quad + \delta_D \Delta_{\hat{q}} + \delta_D(\Delta_{\hat{q}} \circ \delta_{\Delta_q}) + \hat{B}(\Delta_{\hat{u}} \circ \delta_{\Delta_u}) + \delta_B \Delta_{\hat{u}} \\ &\quad + \delta_B(\Delta_{\hat{u}} \circ \delta_{\Delta_u}) + \delta_\Gamma - L\delta_C x - L\delta_w \end{aligned} \quad (10)$$

where $\delta_D, \delta_{\Delta_q}, \delta_B, \delta_{\Delta_u}$ are assumed to be small and therefore $\delta_D(\Delta_{\hat{q}} \circ \delta_{\Delta_q})$ and $\delta_B(\Delta_{\hat{u}} \circ \delta_{\Delta_u})$ can be neglected. The inputs of (10) all depend linearly on δ as

$$\begin{aligned} \delta_A x &= (\sum_{i=1}^{n_\delta} A_i \delta_i) x = \sum_{i=1}^{n_\delta} A_i x \delta_i \\ &= \begin{bmatrix} A_1 x & \cdots & A_{n_\delta} x \end{bmatrix} \delta \\ \delta_D \Delta_{\hat{q}} &= (\sum_{i=1}^{n_\delta} D_i \delta_i) \Delta_{\hat{q}} = \sum_{i=1}^{n_\delta} D_i \Delta_{\hat{q}} \delta_i \\ &= \begin{bmatrix} D_1 \Delta_{\hat{q}} & \cdots & D_{n_\delta} \Delta_{\hat{q}} \end{bmatrix} \delta \\ \delta_B \Delta_{\hat{u}} &= (\sum_{i=1}^{n_\delta} B_i \delta_i) \Delta_{\hat{u}} = \sum_{i=1}^{n_\delta} B_i \Delta_{\hat{u}} \delta_i \\ &= \begin{bmatrix} B_1 \Delta_{\hat{u}} & \cdots & B_{n_\delta} \Delta_{\hat{u}} \end{bmatrix} \delta \\ \delta_C x &= (\sum_{i=1}^{n_\delta} C_i \delta_i) x = \sum_{i=1}^{n_\delta} C_i x \delta_i \\ &= \begin{bmatrix} C_1 x & \cdots & C_{n_\delta} x \end{bmatrix} \delta \\ \delta_\Gamma &= (\sum_{i=1}^{n_\delta} \Gamma_i \delta_i) \\ &= \begin{bmatrix} \Gamma_1 & \cdots & \Gamma_{n_\delta} \end{bmatrix} \delta \\ \hat{D}(\Delta_{\hat{q}} \circ \delta_{\Delta_q}) &= \begin{bmatrix} 0_{n_1 \times n_2} & \hat{D} \text{diag}(\Delta_{\hat{q}}) & 0_{n_1 \times n_3} \end{bmatrix} \delta \\ &\triangleq \begin{bmatrix} \hat{D}_1 & \cdots & \hat{D}_{n_\delta} \end{bmatrix} \delta \\ \hat{B}(\Delta_{\hat{u}} \circ \delta_{\Delta_u}) &= \begin{bmatrix} 0_{n_1 \times n_4} & \hat{B} \text{diag}(\Delta_{\hat{u}}) & 0_{n_1 \times n_5} \end{bmatrix} \delta \\ &\triangleq \begin{bmatrix} \hat{B}_1 & \cdots & \hat{B}_{n_\delta} \end{bmatrix} \delta \\ L\delta_w &= \begin{bmatrix} 0_{n_1 \times n_6} & L & 0_{n_1 \times n_h} \end{bmatrix} \delta \\ &\triangleq \begin{bmatrix} L_1 & \cdots & L_{n_\delta} \end{bmatrix} \delta \end{aligned}$$

where $n_1 = 2n_h$, $n_2 = 2n_h + n_q$, $n_3 = 5n_h$, $n_4 = 2n_h + 2n_q$, $n_5 = 4n_h$, $n_6 = 5n_h + 2n_q$. In the above, the property $x \circ y = \text{diag}(x)y$ is used where $\text{diag}(x)$ is a diagonal matrix of same size as the vector x and with the elements of x on the diagonal.

The error system (10) can thus be expressed as

$$\begin{aligned} \dot{e} &= (\hat{A} - L\hat{C})e + \delta_A x + \hat{D}\delta_{\Delta_q} + \delta_D \hat{\Delta}_q \\ &\quad + \hat{B}\delta_{\Delta_u} + \delta_B \hat{\Delta}_u + \delta_\Gamma - L\delta_C x - L\delta_w \\ &= (\hat{A} - L\hat{C})e + E_\delta \delta \end{aligned} \quad (11)$$

where the columns of E_δ are given by

$$\begin{aligned} E_{\delta,i} &= A_i x + B_{q_i} \hat{\Delta}_q + B_{u_i} \hat{\Delta}_u + \Gamma_i \\ &\quad + \hat{D}_i + \hat{B}_i - LC_i x - L_i \end{aligned}$$

Note that the matrix E_δ is time-varying but depends only on measured signals and known parameters.

Limits on the uncertainties are needed to be able to use equation (11) to construct robust thresholds. The uncertainties are assumed to be bounded by

$$\begin{aligned} |\delta_{h_0}| &< \delta_{\beta_1} & |\delta_{q_0}| &< \delta_{\beta_2} & |\delta_{u_0}| &< \delta_{\beta_3} \\ |\delta_{\Delta_q}| &< \delta_{\beta_4} & |\delta_{\Delta_u}| &< \delta_{\beta_5} & |\delta_K| &< \delta_{\beta_6} \\ |\delta_c| &< \delta_{\beta_7} & |\delta_y| &< \delta_{\beta_8} & |\delta_w| &< \delta_{\beta_9} \end{aligned} \quad (12)$$

When all the uncertainties are within their bound then $|\delta| < \delta_\beta$ where δ_β is a vector of constants as $\delta_\beta = \begin{bmatrix} \delta_{\beta_1}^T & \cdots & \delta_{\beta_{n_\delta}}^T \end{bmatrix}^T$.

B. Threshold calculation

To be able to infer the existence of a clogging, an evaluation signal $s(t)$ and a threshold $\Phi(t)$ must be defined. The threshold $\Phi(t)$ shall be derived such that $s(t) < \Phi(t)$ while no clogging occurs and the uncertainties are inside the bounds, specified in (12), to avoid false alarms. A common choice of evaluation signal is the 2-norm, *i.e.*

$$s(t) \equiv \|e\|_2 \triangleq \sqrt{\int_0^\infty |e(\tau)|^2 d\tau}$$

The benefit of using the 2-norm is that it is then straightforward to optimize the residual generator to minimize the influence of disturbances on $s(t)$ [14]. Unfortunately, the above evaluation function can not be realized exactly, since the value of $\|e\|_2$ is not known until $t = \infty$ and thus $s(t)$ must be modified to *e.g.*

$$s(t) = \sqrt{\int_0^t |e(\tau)|^2 d\tau} \quad (13)$$

However, as shown in [23], the optimality of the residual generator is then lost and therefore the authors instead suggest the evaluation signal

$$s(t) \triangleq |e(t)| \quad (14)$$

which also is the choice in [16]. In general, the evaluation signal is related to the residual by $s = Se$ where the evaluation operator S is necessarily causal *i.e.*

$$e_1(\tau) = e_2(\tau), \forall \tau \leq t \text{ implies that } (Se_1)(t) = (Se_2)(t)$$

In [17], the following class of evaluation operators is proposed

$$(\mathcal{S}_w e)(t) \triangleq \int_0^t w(t-\tau) |e(\tau)| d\tau \quad (15)$$

In general, the purpose of the weighting function w is to increase the influence from the most recent data, *e.g.* by windowing or exponential forgetting, in [17] the exponential forgetting weighting function $\mu e^{-\mu t}$ is used. Note also that the evaluation signal (14) proposed in [23] can be expressed as $s = \mathcal{S}_{\Theta_d} e$, where Θ_d is the dirac delta function. Furthermore, by generalizing the above evaluation operator to

$$(\mathcal{S}_w^p e)(t) \triangleq \left(\int_0^t w(t-\tau) |e(\tau)|^p d\tau \right)^{\frac{1}{p}}$$

then the 2-norm based evaluation signal defined by (13) can be represented as $s = \mathcal{S}_{\Theta_H}^2 e$, where Θ_H is the heaviside step function.

In this case the residual is the estimation of the clogging, $\hat{v}$. The weighting function is chosen to be the dirac delta function and thus the threshold function $\Phi(t)$ should satisfy

$$\Phi(t) \geq \sup_{\substack{|\delta| < \delta_\beta \\ v(t) \equiv 0}} (\mathcal{S}_{\Theta_d} \hat{v})(t) \quad (16)$$

Assuming that $v(t) \equiv 0$ yields

$$\hat{v}(t) = \begin{bmatrix} 0 & -I \end{bmatrix} e(t) \quad (17)$$

The error system in equation (11) is a linear and time-invariant differential equation which has the solution

$$e(t) = e^{(\bar{A}-L\bar{C})t} \left(\int_0^t e^{-(\bar{A}-L\bar{C})\tau} \tilde{u}(\tau) d\tau + e(0) \right) \quad (18)$$

where $\tilde{u}(t) = E_\delta(t)\delta$. By putting the initial conditions of the observer to zero, *i.e.* $\hat{v}(0) = 0$, and neglecting $h(0) - \hat{h}(0)$ the initial condition on the error is zero, $e(0) = 0$.

Using (17) and (18), an expression for the estimated clogging can be obtained, which is used to derive a robust detection threshold.

The input signal $\tilde{u}(t)$ in (18) can be written as

$$\tilde{u}(t) = \sum_{i=1}^{n_\delta} E_{\delta,i}(t) \delta_i$$

where $E_{\delta,i}$ is the i :th column vector of the matrix E_δ and δ_i is the i :th uncertainty. This gives the solution to the error system in equation (11) as

$$e(t) = e^{(\hat{A}-L\hat{C})t} \int_0^t e^{-(\hat{A}-L\hat{C})\tau} \sum_{i=1}^{n_\delta} E_{\delta,i} \delta_i d\tau$$

The thresholds are calculated by applying the operator in equation (16) to the error signal $\hat{v}$, *i.e.*

$$|\hat{v}(t)| = \begin{bmatrix} 0 & I \end{bmatrix} |e(t)| \quad (19)$$

where

$$\begin{aligned} |e(t)| &= |e^{(\hat{A}-L\hat{C})t} \int_0^t e^{-(\hat{A}-L\hat{C})\tau} \sum_{i=1}^{n_\delta} E_{\delta,i} \delta_i d\tau| \\ &= \left| \sum_{i=1}^{n_\delta} \delta_i e^{(\hat{A}-L\hat{C})t} \int_0^t e^{-(\hat{A}-L\hat{C})\tau} E_{\delta,i} d\tau \right| \\ &\leq \sum_{i=1}^{n_\delta} |\delta_i| |e^{(\hat{A}-L\hat{C})t} \int_0^t e^{-(\hat{A}-L\hat{C})\tau} E_{\delta,i} d\tau| \\ &\leq \sum_{i=1}^{n_\delta} \delta_{\beta_i} |e^{(\hat{A}-L\hat{C})t} \int_0^t e^{-(\hat{A}-L\hat{C})\tau} E_{\delta,i} d\tau| \end{aligned}$$

The threshold, $\Phi(t)$ is defined as

$$\Phi(t) \triangleq \begin{bmatrix} 0 & I \end{bmatrix} \sum_{i=1}^{n_\delta} \delta_{\beta_i} |\varphi_i(t)| \quad (20)$$

To simplify the calculation of the threshold the expression for $\varphi_i(t)$ is constructed as a system of differential equations. The expression for $\varphi_i(t)$ above is the solution to the differential equations

$$\dot{\varphi}_i(t) = (\hat{A} - L\hat{C})\varphi_i(t) + E_{\delta_i} \quad (21)$$

A special case is when all bounds on the uncertainties can be chosen equally. The advantage of this is that the heavy calculations of (20) and (21) then can be alleviated.

1) *The uncertainties are equal:* Assume all the uncertainties to be equal, $\delta_1 = \delta_2 = \dots = \delta_{n_\delta} \triangleq \delta_0$ and bounded by $|\delta_0| \leq \delta_{\beta_0}$. The solution to (11) is

$$e(t) = \delta_0 e^{(\hat{A} - L\hat{C})t} \int_0^t e^{(\hat{A} - L\hat{C})\tau} \tilde{u}_s(\tau) d\tau$$

where the input signal is $\tilde{u}_s(t) = E_{\delta} \mathbf{1}_{n_\delta}$. In this special case, when the operator in (16) is applied to the error signal $\hat{v}$, the result can be expressed as in equation (19) but where $|e(t)|$ is

$$\begin{aligned} |e(t)| &= |\delta_0 e^{(\hat{A} - L\hat{C})t} \int_0^t e^{(\hat{A} - L\hat{C})\tau} \tilde{u}_s(\tau) d\tau| \\ &\leq |\delta_{\beta_0} e^{(\hat{A} - L\hat{C})t} \int_0^t e^{(\hat{A} - L\hat{C})\tau} \tilde{u}_s(\tau) d\tau| \\ &\leq |\delta_{\beta_0}| |e^{(\hat{A} - L\hat{C})t} \int_0^t e^{(\hat{A} - L\hat{C})\tau} \tilde{u}_s(\tau) d\tau| \\ &\triangleq |\delta_{\beta_0}| |\varphi(t)| \end{aligned} \quad (22)$$

The function $\varphi(t)$ is the solution to the linear system

$$\dot{\varphi}(t) = (\hat{A} - L\hat{C})\varphi(t) + \tilde{u}_s(t)$$

which can easily be implemented. The threshold is defined as

$$\Phi(t) \triangleq \begin{bmatrix} 0 & I \end{bmatrix} \delta_{\beta_0} |\varphi(t)| \quad (23)$$

V. EXPERIMENTS WITH FLOTATION PROCESS

Experiments have been carried out on data from Boliden's flotation series at the Boliden Area Concentrator, Sweden. This is a direct flotation process and consists of four tanks *i.e.* $n_h = 4$ and the only inflow is to the first tank, $n_q = 1$. Levels for all four tanks and the inflow are measured.

Two different working conditions exists, either the flotation process is controlled by a multivariable LQ-controller, described in [2], or by four separate PI-controllers. When the PI-controllers are used, the levels and the control signals vary significantly, [22] compared to the LQ-controller. Therefore, the critical case, regarding false alarms, will be when the PI-controllers are used. The flow q is varying severely between $50m^3/h - 350m^3/h$ with a mean of $170m^3/h$. The chosen equilibrium point and some physical parameters are $h_0 = \begin{bmatrix} 0.42 & 0.40 & 0.40 & 0.38 \end{bmatrix}^T$, $\alpha_\Omega = \begin{bmatrix} 17 & 25 & 17 & 17 \end{bmatrix}^T$, $\alpha_h = \begin{bmatrix} 0.4 & 0.4 & 0.8 & 1.5 \end{bmatrix}^T$. The control signal working point is $\hat{u}_0 = \begin{bmatrix} 24.1 & 25.7 & 13.8 & 15.6 \end{bmatrix}^T$ and by using these working condition the matrices $\hat{A}$, $\hat{B}_u$, $\hat{B}_q$, $\hat{C}$ from equation (4) can

TABLE I
LIMITS ON UNCERTAINTIES.

Uncertainty	Tank 1	Tank 2	Tank 3	Tank 4
δ_{h_0}	0	0	0	0
δ_{q_0}	0	--	--	--
δ_{u_0}	0.01	0.01	0.07	0.1
δ_{Δ_q}	0.07	--	--	--
δ_{Δ_u}	0	0	0.05	0.05
δ_K	0.01	0.01	0.02	0.03
δ_c	0	0	0	0
δ_y	0	0	0	0
δ_w	0	0	0	0

be determined. The working point is time invariant so the feedback gain L from equation (4) can be calculated according to Kalman theory or pole placement theory. In this experiment the feedback gain is calculated using Kalman theory.

The uncertainties in (6) are expected to be different and therefore their limits, (12), are chosen differently so equation (20), (21) are used to calculate the threshold. The limits of the uncertainties have been determined by hand and are shown in Table I. The criterion used is that the threshold should be as close as possible to the residual but still larger at all times and have similar behavior as the residual. It shall be noted that it is difficult to distinguish between the effects of all of the uncertainties although they, in principle, all have different impact on the process. An automatic way to determine the limits of the uncertainties should be developed in the future.

The fault detection method is tested on measured data collected from the flotation process at Boliden Area Concentrator in Boliden. The data has been manipulated in order to simulate clogging. A clogging means that the actual valve opening is less than expected and can thus be simulated by adding a positive quantity to the logged control signal which gives total control of the fault. A reasonable assumption is that when clogging occurs it is a relatively slow process and then the valve control signal will increase as almost the only evidence of a clogging. A ramp signal is added to the control signal, to simulate the clogging on Valve 2, starting at time $t = 4000s$, stopping at $t = 8000s$ and increasing 10 units during these 4000s.

Figure 1 shows the result when an experiment, without clogging, is carried out. As can be noted, the residual is strictly smaller than the threshold and no alarm is raised.

The experiments, with clogging, are shown in Figure 2. It can be seen that the residual for Valve 2 is larger than the threshold and thus the clogging has been detected. None of the other residuals are indicating any clogging so there is no false alarm.

VI. CONCLUSION

A nonlinear model of the froth flotation process has been linearized and the resulting linear process model extended with integrals and corrected by a linear feedback term constitutes the observer. The estimation of the clogging is given by a part of the observer state vector.

An explicit expression for how the residual depends on the process uncertainties has been derived. Robust thresh-

Fig. 1. The clogging estimate of valve 1,2,3,4 are shown in the picture a,b,c,d respectively. The solid line shows the estimate and the dashed the threshold. No false alarm is raised.

Fig. 2. The clogging estimate of valve 1,2,3,4 are shown in the picture a,b,c,d respectively. The solid line shows the estimate and the dashed the threshold. The simulated clogging in Valve 2 is detected at $t = 5644s$.

olds have been derived depending on assumptions about the limits of the process uncertainties.

Experiments on data collected at Boliden Area Concentrators in Boliden, Sweden have been carried out successfully. During the experiments no false alarm occurred and simulated faults were detected quickly.

VII. ACKNOWLEDGEMENT

Funding of the project AIS33 from Vinnova is gratefully acknowledged.

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