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Experimental Determination of the Single Point Echo of an Ultrasonic Transducer Using a Tomographic Approach

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Experimental determination of the single point echo of an ultrasonic transducer using a tomographic approach

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Abstract

The impulse response method is often used to study the ultrasonic field radiated by planar or gently curved transducers. This, however, requires one transmitting and one receiving transducer which has to be placed with great accuracy in the ultrasonic field. Here we propose a measuring method with only one transducer which acts both as transmitter and receiver. Echoes from sliding halfplanes are registered and by using tomographic methods, the echo from a single point in the same plane as the halfplanes can then be calculated. Beside the ultrasonic field, these calculations will yield us the transducer's characteristics. These can be used to extract further information from an echo of an arbitrary surface, such as the gradient of the surface.

Figure 1: Echoes from plane surfaces. Perpendicular (above) and 5° angle of inclination (below)

I INTRODUCTION

A very common way of imaging with ultrasonics is to use the pulse-echo method [6]. However, there are more information in the echoes besides the arrival time, such as the gradient of the surface at the measured point. To be able to extract this information, a detailed knowledge of the transducer is required, such as the single point echo. In our experiments we have used a focused airborne ultrasonic transducer with a radius of curvature of 45 mm. Two echoes from a plane surface with a sloping angle of 0° and 5° respectively are shown in Fig. 1. The shape of the echo is dependent on the angle of inclination [2]. The focal point has the same distance from the transducer in both measurements.

II THEORY

In our experiments we have assumed an axisymmetric focused transducer. The reason for this is simplicity, but the method can be extended to unsymmetric transducers. Suppose that we have a line which lies in a plane parallel to the focalplane and is located at a height z from the transducer. The line has a perpendicular distance p to

the axis of the transducer, see Fig. 2. The transducer works both as a transmitter and a receiver. The echo $f_\lambda(p, z, t)$ from the line will be a function of distance p , height z and, of course, time t . Since the height z was constant in our experiments and t is only a parameter in our calculations, we will drop them from now on.

If $f(r)$ equals the echo from a single point at a distance r as indicated in Fig. 2, we can write the line-echo, by using superposition, as

Figure 2: Reflecting line

$$f_\lambda(p) = \int_L f(r) dr = 2 \int_p^\infty f(r) \frac{r}{\sqrt{r^2 - p^2}} dr \quad (1)$$

Since the transducer is axisymmetric, we can assume the line to be parallel to the y -axis for the sake of simplicity. The expression in (1) is known as the *Abel transform*. The inversion of (1) can be found in [1] and is calculated in the following manner:

$$f(r) = -\frac{1}{\pi} \int_r^\infty \frac{f'_\lambda(p)}{\sqrt{p^2 - r^2}} dp \quad (2)$$

However, there are practical problems in measuring the echo from a line. The line has to be very narrow to be considered a line, which will yield weak echoes with a low SNR. We will also get diffraction effects from the edges if the line isn't infinitesimal thin. These problems can be solved if we instead measure the echoes from halfplanes. If we denote the echo from a perpendicular halfplane with $f_\pi(p)$, where p is the same distance as before, we get the following relationship between the line-echo and the halfplane-echo

$$\begin{aligned} f_\pi(p) &= \int_p^\infty f_\lambda(x) dx \\ f_\lambda(p, t) &= -f'_\pi(p) \end{aligned} \quad (3)$$

Inserting (3) in (2), we obtain the final expression

$$f(r) = \frac{1}{\pi} \int_r^\infty \frac{f''_\pi(p)}{\sqrt{p^2 - r^2}} dp \quad (4)$$

III NUMERICAL EVALUATION

Inversion algorithms are known for (1), but since we have to differentiate our measurements, we will experience numerical problems. We can also evaluate (4) directly. Since the integrand is singular at its lower bound, this can't be done in a straightforward manner. Our solution to this problem was to use a regularization of the singular integral as shown in [4]. The kernel $1/\sqrt{p^2 - r^2}$ is replaced with an approximated kernel that has no singularities. We formed the approximated kernel by convolving the original kernel with $\Psi_\epsilon(p)$, where $\lim_{\epsilon \rightarrow 0} \Psi_\epsilon(p) = \delta(p)$, and $\delta(p)$ is the Dirac function. This will cause the singularity to "smooth" out. Numerous functions $\Psi_\epsilon(p)$ are possible to apply and we have tried the most common ones used in tomography.

IV RESULTS AND DISCUSSION

The single point echoes are calculated and agrees to the theoretically determined ones. The echo from a point placed at the focal point is shown in Fig. 3. This echo

Figure 3: Calculated echo from point reflector at the focal point

is especially interesting, since the transducer's spatial impulse response is equal to a Dirac pulse at the focal point [5]. This means that the echo will be the transducer's acoustoelectric transfer function convolved with itself. The echo from a sloping plane surface is estimated and reasonable agreement is found with the measured echo. The theory can be expanded by using the Radon transform instead of the Abel transform, which allows non-axisymmetrical transducers [3].

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