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Waves for design of wind-power plants in shallow seas

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ABSTRACT

Usually there is little knowledge of long-term wave conditions at prospective sites for wind-power plants, while the deep-water or open sea conditions may be more known and geographically less varying. However, most wind-energy plants are intended for water depths less than 20 m. A concept for assessing design waves at a near-shore site is to transform the off-shore wave spectra to the target site by a model for spectral wave-energy transfer over the actual bottom topography. The inshore spectra can be used for linear statistics of extreme waves and design wave loads can be produced. In this context it is important to know the realism of used spectral forms.

Based on 58 measured wave spectra at 6 m water depth at the near-shore wind farm Bockstigen in the Baltic the most realistic spectrum was found to be the TMA spectrum, which is a JONSWAP spectrum modified for shallow water. Some few examples are given.

INTRODUCTION

For the design of structures offshore or near-shore against wave loads it is necessary to have some realistic design waves. These could be regular design waves for extreme loads or irregular waves for dynamic load cases and fatigue problems. Based on measurements at Bockstigen the TMA shallow water spectrum is recommended here for assessing irregular waves. The TMA spectrum is a modified JONSWAP spectrum. (Hughes, 1984)

THE WAVE MEASUREMENTS

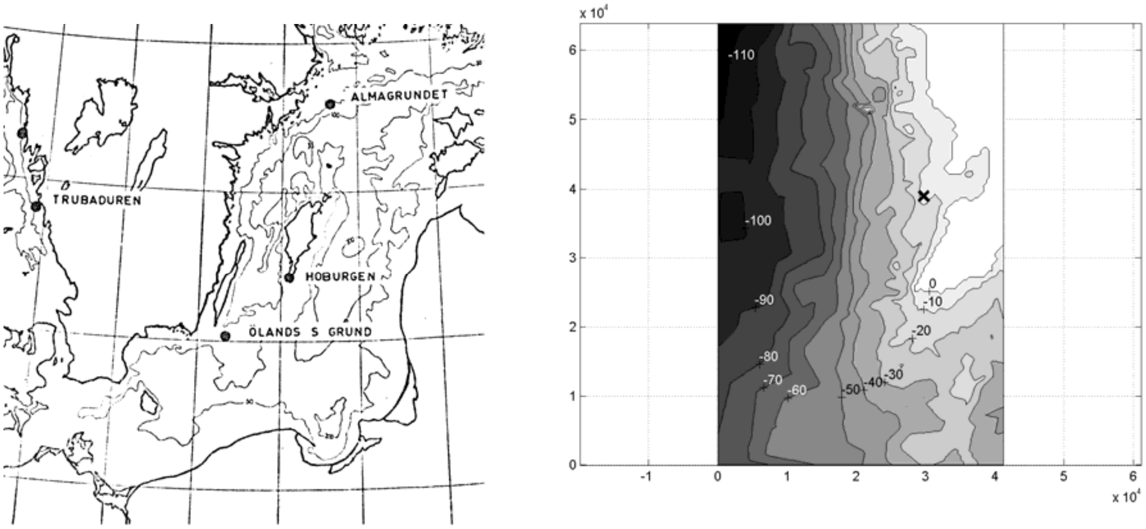

Figure 1. Left: The island of Gotland in the Baltic. Bockstigen is situated somewhat north of Hoburgen on the west coast. Right: The bathymetry at Bockstigen with the site of measuring marked by a cross.

Waves were measured at Bockstigen on with a pressure and directional bottom mounted equipment (Valeport) and with a wave-radar (SAAB), the latter measuring the water surface elevation directly. Bockstigen is situated on the southwest coast of the Island of Gotland in the Baltic. See Figure 1. Analysis of records from the same occasion shows that the pressure equipment mounted at 6 m water depth gives inaccurate information of the second-order double frequency waveform due to the attenuation of the dynamic pressure with depth, because, when amplifying the attenuated pressure signal by dividing with the attenuation factor, noise will also be amplified. The wave-radar gives the actual geometric waveform directly, however, with some inaccuracies at peaked, breaking waves. In both measurements it is difficult to differentiate between first-order and second-order waves at the same frequency. Measuring in one point the wave celerity cannot be evaluated. Examples of variance spectra of surface elevation from the two types of equipment at the same occasion are shown in Figure 2.

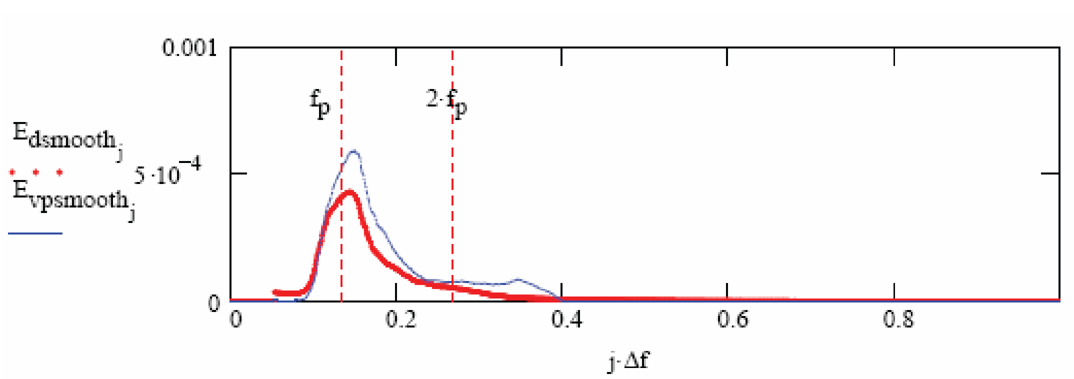

Figure 2. Smoothed amplitude spectra of Waveradar (thick graph) and Valeport (thin graph) measurements. An approximation for the peak frequency, f_p, is shown in the figure.

STANDARD SPECTRA

There are many wave spectra used for waves offshore in deep water, i.e. when the wavelength is smaller than twice the water depth. See e.g. Gran (1992). A fundamental spectrum is the Pierson-Moscowitz spectrum, which should describe wave spectra for fully developed sea, or fully arisen sea (FAS), when a constant wind blowing infinitely long cannot increase the energy in the waves, but the energy transfer is balanced by dissipation. This spectrum is a one-parameter spectrum completely described by the wind speed:

$$S_{PM}(\omega) = \alpha g^2 \omega^{-5} e^{-0.74(\omega_o/\omega)^4} \qquad \ldots(1)$$

where $\alpha = 0.0081$ is Phillip's constant,
g the earth acceleration,
$\omega_o = g/(U_{19.5})$ and
$U_{19.5}$ = the wind speed at the height 19.5 m above still water level.

See Figure 3 for examples of PM spectra for some wind speeds.

For the PM spectrum the mean period or zero-up-crossing period, T_z, can be evaluated from the spectrum as T_{01} or T_{02} ($T_{01} = 1.086\ T_{02}$) and the spectral peak period to $T_p = 1.30\ T_{01} = 1.408\ T_{02} = 1.14\ T_o$. $T_o = 2\pi/\omega_o$. T_p is the modal period or the period for the spectral peak.

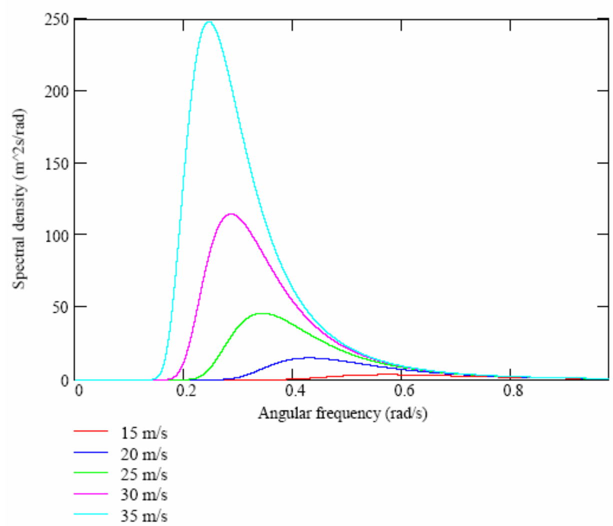

Figure 3. PM spectra for some wind speeds.

A variant of the one-parameter PM-spectrum is the ITTC spectrum (ITTC = International Towing Tank Conference).

$$S_{ITTC}(\omega) = \alpha g^2 \omega^{-5} e^{-\frac{3.11}{H_s^2}\omega^{-4}} \qquad \ldots(2)$$

In this spectrum the significant wave height, H_s, is used instead of the wind speed or mean period.

Mostly the sea state is, however, not fully developed as the wind speed and direction change, the fetch is too short, or the duration is not long enough, especially for strong winds and high waves. Then two-parameter spectra for developing seas can be used, e.g. some in which the wave height and frequency are the parameters. This was originally proposed by Bredtschneider and offers more flexibility, because the energy of the spectra can be placed at arbitrary locations on the frequency axis with the demanded significant wave height. Such spectra belonging to the PM-family are also, somewhat incorrectly, referred to as PM-spectra. Two spectra are the ISSC spectra for swell-dominated sea Equation (3) and for wind waves Equation (4) (ISSC = International Ship Structures Congress, 1964). The periods T_{01} and T_{02} are estimated by moments of measured spectra and are both estimates of the zero-crossing period.

$$S_{ISSCa}(\omega) = 0.11\left[\frac{H_s}{\left(\frac{T_{01}}{2\pi}\right)^2}\right]^2 \omega^{-5} e^{-0.44\left(\frac{T_{01}}{2\pi}\omega\right)^{-4}} \qquad \ldots(3)$$

$$S_{ISSCb}(\omega) = \frac{1}{4\pi}\left[\frac{H_s}{\left(\frac{T_{02}}{2\pi}\right)^2}\right]^2 \omega^{-5} e^{-\left(\frac{1}{\pi}\right)\left(\frac{T_{02}}{2\pi}\omega\right)^{-4}} \qquad \ldots(4)$$

Two-parameter spectra still give too little freedom to reproduce realistic spectra of developing sea. In 1973 Hasselman *et al.* published the JONSWAP spectrum, which was one of the results from the Joint North Sea Wave Project.

$$S_{JONSWAP}(\omega) = S_{PM}(\omega)\gamma^{e^{-\frac{1}{2}\left(\frac{\omega-\omega_p}{\sigma(\omega)\omega_p}\right)^2}}, \qquad \ldots(5)$$

where $\gamma \exp\left(\exp\left[-\frac{1}{2}\left(\frac{\omega-\omega_p}{\sigma(\omega)\omega_p}\right)^2\right]\right)$ is the peak enhancement factor,

ω the angular frequency,

ω_p the modal angular frequency (peak of spectrum)

$\sigma(\omega) = \sigma_a$ if $\omega < \omega_p$, "standard deviation" of the peak enhancement factor to the left and

$\sigma(\omega) = \sigma_a$ if $\omega > \omega_p$, "standard deviation" of the peak enhancement factor to the right.

By Hasselmann originally recommended values are, when the fetch, F, and the wind speed, U_{10}, at 10 m height is used:

γ = 3.30
σ_a = 0.07
σ_b = 0.09
$\alpha = 0.076 F_o^{-0.22}$
$\omega_p = 7\pi(g/U_{10})F_o^{-0.33}$ and
$F_o = gF/(U_{10})^2$.

For the above JONSWAP spectrum $T_{01} = 1.073\ T_{02} = 0.834\ T_p$.
The JONSWAP spectrum is in common use for design of drilling platforms in the offshore industry because it offers more flexibility with its five parameters, and can produce realistic spectra. The parameters are then chosen from wave statistics combined with systematic parameter fitting.

Note in Figure 4 the different characteristics of the two spectra with the sharp peak of the JONSWAP spectrum on top of the PM spectrum. In Figure 4 the JONSWAP spectrum has a larger variance and thus larger significant wave height. In Figure 5 the JONSWAP spectrum and the PM spectrum are given the same variance and significant wave height but then the PM spectrum is shifted to a higher peak period.

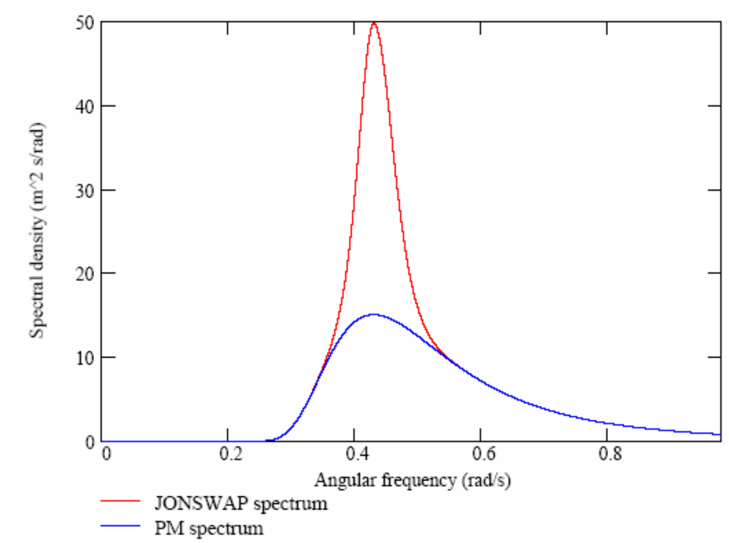

Figure 4. A JONSWAP spectrum and PM spectrum with the peak frequency, $\omega_p = 0.878\ \omega_o = 0.431$ rad/s.

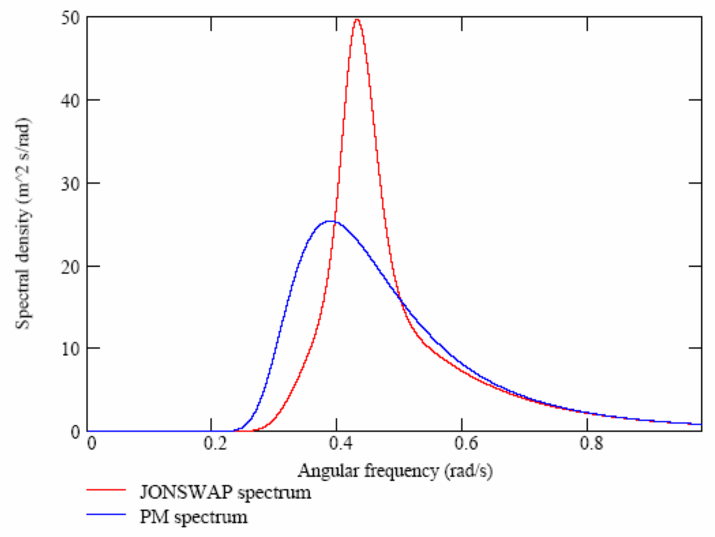

Figure 5. A JONSWAP spectrum and PM spectrum with the same significant wave height H_s.

Gran (1992) gives the following recommendations for the parameters in the JONSWAP spectrum based on field data (Houmb and Overvik, 1977).

$$\gamma = 7(1 - \frac{0.0027}{\alpha}) = 7(1 - 2.18 \cdot 10^{-5} \frac{g^2 T_z}{H_s^2}) \quad \text{(Sic. Dimensionally incorrect)}$$

$$\sigma_a = 0.07$$

$$\sigma_b = 0.09$$

$$T_p = \frac{T_z}{\sqrt{\frac{5+\gamma}{10.89+\gamma}}}$$

$$\alpha = \frac{30 H_s^2 \pi^4 f_p^4}{g^2 (5+\gamma)}$$

This spectrum parameterisation needs two input parameters: mean period, T_z, and significant wave height, H_s, and is the JONSWAP model originally implemented in Vidyn (Carlén, 2001)

While the JONSWAP spectrum originally was developed for developing sea in deep water, the waves in shallow areas are often waves coming in from deeper areas into an area where the waves are much affected by the limited water depth. Hughes (1984) gives for such cases the following modified JONSWAP spectrum in shallow water called the TMA spectrum. It is based on the fact that low-frequency or, equivalently, long-period waves must have a limited height in shallow water. Therefore the spectrum is multiplied by a function for limited depth Eq. (6) and Figure 6.

$$\phi(\omega,h)=\begin{cases}0.5\left(\omega\sqrt{h/g}\right)^2 & \text{if } \omega\sqrt{h/g}<1\\ 1-0.5\left(2-\omega\sqrt{h/g}\right)^2 & \text{if } 1\le\omega\sqrt{h/g}<2\\ 1 & \text{if } \omega\sqrt{h/g}\ge 2\end{cases} \quad \ldots(6)$$

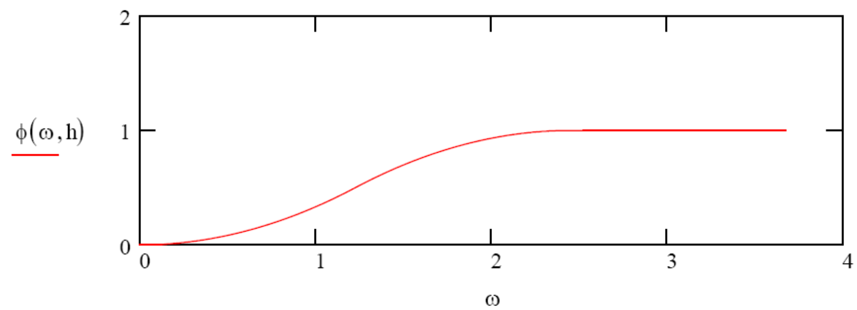

Figure 6. The limited depth function in the TMA spectrum as function of angular frequency, ω, and depth, h.

The expression for the TMA spectrum is then:

$$S_{TMA}(\omega)=\alpha g^2\omega^{-5}\exp\left[-\frac{5}{4}\left(\frac{\omega_p}{\omega}\right)^4+(\ln(\gamma))\exp\left(-\frac{1}{2}\left(\frac{\omega-\omega_p}{\sigma(\omega)\omega_p}\right)^2\right)\right] \quad \ldots(7)$$

Hughes gives e.g. the following possible expression for the parameters α and γ in the TMA spectrum based on shallow-water data.

$\varepsilon\varepsilon=\dfrac{H_s}{4L_p}$,

$\alpha=16\pi^2(\varepsilon\varepsilon)^2$ and

$\gamma=6614(\varepsilon\varepsilon)^{1.59}$,

where L_p is the wavelength at the peak frequency for the water depth at hand and H_s the significant wave height.

EXAMPLE

In Table 1 and Figure 7 below comparisons for one measured case at 6 m water depth are given. Results from the PM spectra with measured inputs of wind velocity, S_{PMU}, mean period, S_{PMTz}, or with significant wave height, S_{PMHs}, are given together with results from the deep-water JONSWAP spectrum with H_s and T_z and the shallow-water TMA spectrum. The spectra with significant wave height as input give approximately the correct significant wave height as output, as expected, but the wave energy is very differently distributed over the frequency.

Table 1. Significant-wave output yielded from the spectra with H_s= 0.664 m, T_z =5 s and $U_{19.5}$ = 6.81 m/s (8 m/s at anemometer height) as input.

Spectrum	$S_{JONSWAP}$	S_{PMTz}	S_{PMHs}	S_{PMU}	S_{TMA}
H_s (m)	0.66	1.49	0.66	0.99	0.58

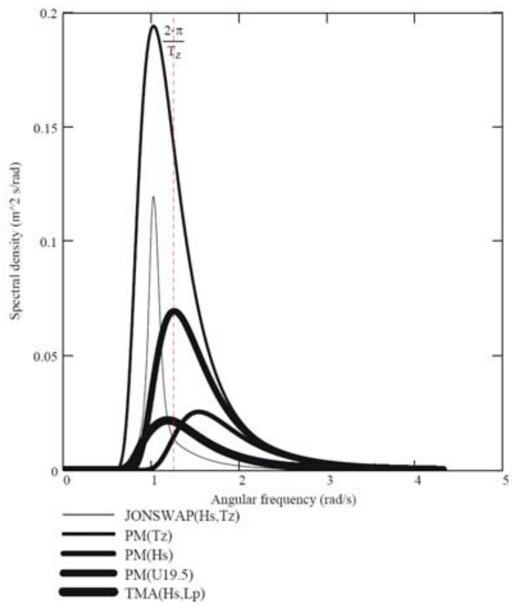

Figure 7. Variance density spectra for the JONSWAP spectrum with H_s and T_z as input, PM spectra with T_z, H_s or $U_{19.5m}$ as input, and TMA spectrum with H_s and f_p as input.

COMPARISON WITH MEASURED SPECTRA

In Figure 8 below the measured spectra, between midnight and 01:00 hrs 2004-07-11, from the Wave radar and the Valeport measurements are compared with the TMA spectrum using the recommended parameterisation with input significant wave height, H_s = 1.55 m and peak frequency f_p = 0.135 s estimated by inspection from the wave radar measurement. Looking at Figure 7 and Figure 8 it is obvious that only the TMA spectrum, of the spectra investigated here, is capable of reasonable approximation of this sea state at the Bockstigen site. The conformity between the measured spectrum and the TMA spectrum may be improved if the parameters are adjusted for each measured spectrum.

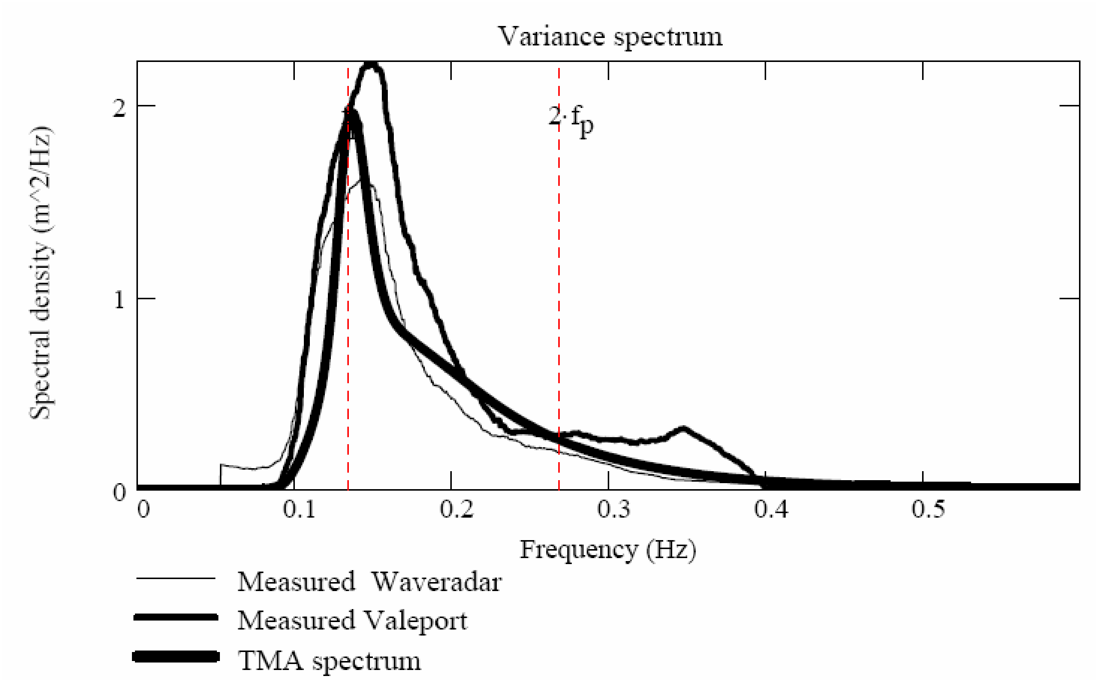

Figure 8. Smoothed density spectra of Waveradar and Valeport measurements compared with the TMA spectrum with standard parameterisation and input significant wave height, H_s = 1.55 m and peak frequency f_p = 0.135 s estimated from the wave radar measurement.

Spectra from 58 one-hour wave radar measurements have been compared with TMA spectra with standard parameterisation and input significant wave heights, H_s, and peak frequencies, f_p, estimated from the measurements. By looking at the graphs one can conclude that for high waves (> 1.5 m approximately) originating from steady winds in one direction the TMA spectrum gives a good fit. For lower waves the fit is poorer. For waves originating from turning or strongly changing winds with two spectral peaks, theoretical spectra with two peaks should be adapted for shallow water. These spectra are documented in Appendix 2 in Bergdahl *et al.* (2006).

CONCLUSION

It is obvious that only the TMA spectrum, of the spectra investigated here, is capable of reasonable approximation of the sea state at the Bockstigen site. The conformity can be improved if the parameters are adjusted for each measured spectrum.

ACKOWLEDGEMENTS

The wave radar and the wave measurements were financed within the STEM project 13080-4, VÅGVIT (Bergdahl *et al.*, 2006). Further comparisons of the spectra and evaluation of the TMA spectrum was financed within the Marie-Curie project, MRTN-CT-2004-505166, WAVETRAIN, (Bergdahl, 2006).

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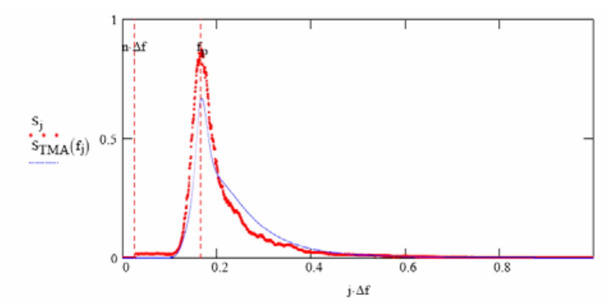
APPENDIX

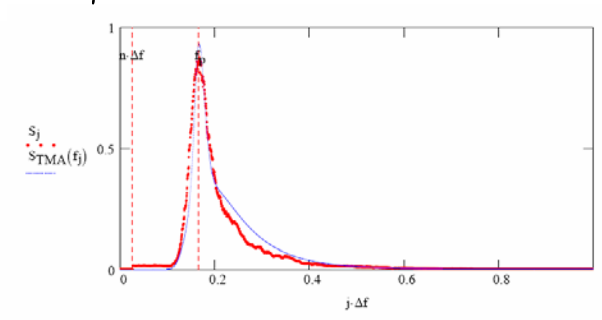
Three measured occasions

"DATA\wave_040625_23.txt"Hs = 1.06 m, fp = 0.165 Hz, $\alpha = 6.447 * 10^{-3}$,

$\gamma = 2.14$

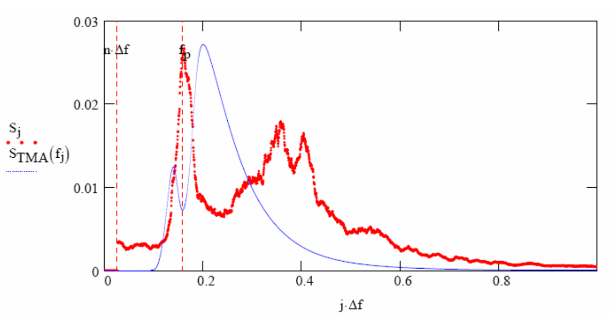


$\gamma = 3$

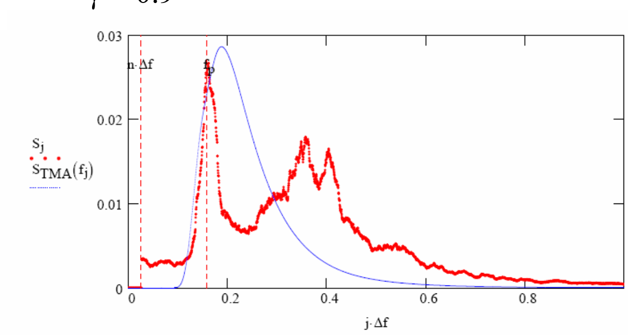


"DATA\wave_040710_0.txt" fp := 0.16 Hz Hs = 0.31 m α=5.006 10^{-4}

$\gamma = 0.281$

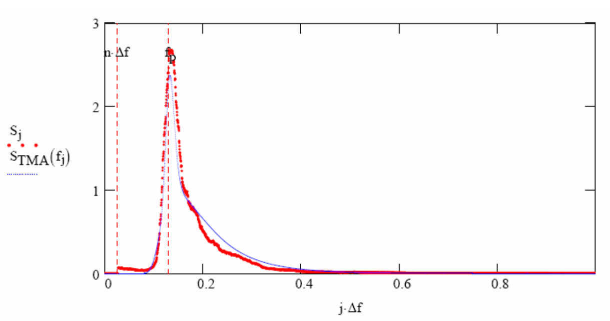


$\gamma = 0.9$



"DATA\wave_040711_0.txt"fp := 0.131 Hz, Hs = 1.638 m, $\alpha = 8.913\ 10^{-3}$

$\gamma = 2.773$



$\gamma = 3.1$