Modeling mechanical fatigue of UD composite: multiple fiber breaks and debond growth

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Abstract. The objective of this paper is to analyze fiber/matrix debond crack growth in unidirectional (UD) composites during high stress cyclic tension-tension loading. High stress loading means that fiber breaks and consecutive fiber/matrix interface debond growth are expected. Fracture mechanics concepts are applied to analyze damage evolution

1. Introduction

When UD fiber reinforced polymer composites (CF/EP, GF/EP) are loaded in fiber direction in quasi-static or in a high stress cyclic tension-tension regime, multiple fiber breaks occur in random positions already during the first cycle. In cyclic loading with constant amplitude fibers are usually assumed not to experience fatigue. Therefore the damage evolution with increasing number of cycles is in form of development of interface cracks (debonds) growing along the fiber/matrix interface and thus fracture mechanics concepts (energy release rate G_{II}) may be used for the damage evolution analysis. The sequence of events at strain levels below the fiber breaking limit may be different, for example, initiation of small matrix cracks between neighbouring fibers could be the first mode of damage, which would be followed by fiber/matrix debond crack propagation along fibers. However, for the studied CF/EP and GF/EP composites the energy release rate is much smaller for matrix crack initiated debond growth, besides energy release rate decays with the distance from the primary matrix crack. Therefore only debonds initiated from fiber breaks are of interest if we consider high stress cyclic loading.

To analyze the interface debond crack growth using fracture mechanics the energy release rate related to the increase of the debond length has to be calculated. It has been done previously for debond growth analysis along a single fiber fragment in single fiber fragmentation (SFF) test. The used methods cover a wide spectrum from approximate analytical to numerical based on finite elements or boundary elements [1-2].

First an analytical solution for the strain energy release rate G_{II} in the self-similar debond propagation region is given. For short debonds, where the stress perturbation from the fiber break is interacting with the stress perturbation from the debond crack tip, FEM using virtual crack closure technique is used to calculate the magnification of G_{II} [4]. The calculations are summarized in simple expressions and used in simulations of the debond growth in tension-tension fatigue according to Paris law expression [7].

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2. Model representing composite with broken and partially debonded fiber

UD composite with a broken and partially debonded fiber is represented by three concentric cylinder assembly model (CCA) as shown in figure 1. The broken fiber in the middle is surrounded by a matrix cylinder and the fiber/matrix interface is debonded over distance l_d , as shown in figure 1b. This fiber/matrix block is embedded in large “effective composite” cylinder denoted as “C” in figure 1b.

Figure 1. a) Unidirectional composite subjected to mechanical tensile loading in fiber direction (σ_z) and thermal loading (ΔT); b) Concentric cylinder assembly model: F – fiber, M – matrix, C – effective composite, l_d shows the fiber/matrix debond length.

3. Strain energy based debond growth modelling

Since the loading type relevant to this study is axial tension and the applied temperature is negative, the radial stress on the fiber surface is compressive and thus the debond crack propagation is in Mode II. It is due to larger Poisson's ratio for the matrix and also due to larger thermal expansion coefficient of the matrix. The energy release rate G_{II} is obtained dividing the energy change due to crack growth by the new created surface area:

$$G_{II} = -\frac{dU}{dA} \Big|_{u=const} \quad (1)$$

where the new free surface area of the interface crack corresponding to debond crack growth by dl_d along the circular cross-section fiber with radius r_f is:

$$dA = 2\pi r_f dl_d \quad (2)$$

Four regions of the stress state can be distinguished along the path of the debond crack: a) a very complex stress state at the fiber crack; b) plateau region between the fiber crack and the debond tip far away from both of them; c) the debond tip region with stress singularity; d) a region in front of the debond tip (far away from it) where the fiber is perfectly bonded to the resin. For short debonds the regions b) and d) do not exist and only numerical methods can give reliable results. For long debond when it grows the plateau region b) becomes longer and the region d) correspondingly shorter leaving the other two regions unchanged: region c) is just shifted along the fiber. Therefore, long debond cracks propagate in a self-similar manner.

The virtual crack closure (VCC) technique [3] is a convenient method for obtaining the values of G_{II} from FE calculations. The calculated fiber/matrix relative tangential displacement behind the debond

crack tip and the shear stress values ahead of the crack tip can be obtained directly from the FEM model as shown in figure 2.

Figure 2. Concentric cylinder geometry showing debond crack at the fiber/matrix interface.

VCC technique states that the energy released due to debond crack growth by dA is equal to the work which is required to close the newly created surface from size $A+dA$ back to size A , where $dA = 2\pi r_f dl_d$. When closing the debond crack by dl_d (from $l_d + dl_d$ to l_d), points at the debonded surface in the region $z \in [l_d; l_d + dl_d]$, which have relative tangential displacement:

$$\Delta u^{l_d + dl_d}(z) = u_{fz}^{l_d + dl_d}(z) - u_{mz}^{l_d + dl_d}(z) \quad (3)$$

are moved back to coinciding positions applying tangential tractions. The upper index in this section indicates the crack size used in calculations. At the end of this procedure the shear stress in point z is equal to $\sigma_{rz}^{l_d}(z)$, which is the shear stress in front of the crack with size l_d . Then the work required to close the crack by dl_d can be expressed as:

$$\Delta W = 2\pi r_f \frac{1}{2} \int_{l_d}^{l_d + dl_d} \Delta u^{l_d + dl_d}(z) \sigma_{rz}^{l_d}(z) dz \quad (4)$$

Within the VCC technique it is assumed that due to small value of dl_d the relative sliding displacement at the tip of the crack with size $l_d + dl_d$ is the same as at the tip of the debond crack with size l_d :

$$\Delta u^{l_d + dl_d}(z) = \Delta u^{l_d}(z - dl_d) \quad (5)$$

The benefit of this assumption is that only one stress state calculation for a given debond length is required.

The following result is obtained using expressions (4) and (5):

$$G_{II}(l_d) = \lim_{dl_d \rightarrow 0} \frac{\Delta W}{2\pi r_f dl_d} = \lim_{dl_d \rightarrow 0} \frac{1}{2dl_d} \int_{l_d}^{l_d + dl_d} \Delta u^{l_d}(z - dl_d) \sigma_{rz}^{l_d}(z) dz \quad (6)$$

The strain energy release rate is a quadratic function of the applied mechanical strain ϵ_z^{mech} and ΔT :

$$G_{II} = k_m (\epsilon_{z0}^{\text{mech}})^2 + k_{m-th} \epsilon_{z0}^{\text{mech}} \Delta T + k_{th} (\Delta T)^2 \quad (7)$$

Paris law expression which will be used for debond growth description in fatigue is following:

$$\frac{dA}{dN} = B \left(\frac{\Delta G_{II}}{G_{IIC}} \right)^m \quad (8)$$

Using (2) and introducing normalized debond length $l_{dn} = l_d / r_f$ the expression (8) can be written in form:

$$\frac{dl_{dn}}{dN} = B^* \left(\frac{\Delta G_{II}}{G_{IIC}} \right)^m \quad (9)$$

where $B^* = \frac{B}{2\pi r_f^2}$

Obviously B^* is dimensionless. ΔG_{II} is the difference between the value corresponding to maximum and minimum load $\varepsilon_{\max}, \varepsilon_{\min}$. In mechanical fatigue according to (7) it is:

$$\Delta G_{II}(\varepsilon, T) = k_m (\varepsilon_{\max}^2 - \varepsilon_{\min}^2) + k_{m-th} \Delta T (\varepsilon_{\max} - \varepsilon_{\min}) \quad (10)$$

Coefficients k_m and k_{m-th} in (10) are larger for short debonds and approach the values for self-similar cracks for longer debonds. Therefore they can be written in form with magnification factors $k_m^*(l_{dn})$ and $k_{m-th}^*(l_{dn})$ as follows:

$$k_m(l_{dn}) = k_m^* k_m^\infty \quad k_{m-th}(l_{dn}) = k_{m-th}^* k_{m-th}^\infty \quad (11)$$

For simulation purposes (9) will be used in an incremental form:

$$\Delta l_{dn}^{(k)} = B^* \left(\frac{\Delta G_{II}(l_{dn}^{(k)})}{G_{IIC}} \right)^m \Delta N \quad l_{dn}^{(k+1)} = l_{dn}^{(k)} + \Delta l_{dn}^{(k)} \quad (12)$$

In equation (12) $l_{dn}^{(1)}$ is the initial debond length.

4. Energy release rate in self-similar debond propagation region

The debond crack propagation can be considered as self-similar when the tip of the fiber/matrix debond crack is far away from the fiber break where it was initiated and also far from another debond which may be approaching from the other end of the fiber. When this debond grows by dl_d the region b) (see section 3) is increasing by dl_d and the region d) decreasing by dl_d . The energy change in the system can be calculated as the difference between strain energies of these regions:

$$dU = U_d^{\text{tot}} - U_b^{\text{tot}} \quad (13)$$

In (13) $U_b^{\text{tot}}, U_d^{\text{tot}}$ are strain energies of the bonded region and debonded region of the model of length dl_d . Models representing these two regions are shown in figure 3. Note that in the debonded region, (figure 3b) sliding of the fiber with respect to the matrix is possible and hence its deformation is different than the deformation of the rest of the model.

Expressions for $U_b^{\text{tot}}, U_d^{\text{tot}}$ for the 3 phase model in figure 3 (fiber-1, matrix-2, effective composite-3) with debonded phase 1 are derived in [4]:

$$U^{\text{tot}} = \frac{V}{2} \left(\sum_{k=1}^N V_k \bar{\sigma}_{zz}^k (\varepsilon_{z0}^k - \alpha_z^k \Delta T) - \Delta T \sum_{k=1}^N V_k \alpha_r^k (\bar{\sigma}_r^k + \bar{\sigma}_\theta^k) \right) \quad (14)$$

Bars over stresses denote the average values in phases. Due to differences in boundary and interface conditions the average stresses are different in the bonded and in the debonded case leading to different values of $U_b^{\text{tot}}, U_d^{\text{tot}}$. In the bonded region $\varepsilon_{z0}^k = \varepsilon_{z0} = \varepsilon_{z0\text{mech}} + \varepsilon_{z0}^{\text{th}}$. It is different in the fiber within the debonded region (because of zero axial stress).

Figure 3. Schematic showing of bonded region (a) which due to debond crack growth by dl_d turns to debonded region with length dl_d .

5. Energy release rate and debond growth in fatigue

In the calculation example fiber radius r_f was assumed equal to $4 \mu\text{m}$, outer radius of the matrix r_m is easily calculated through fiber volume fraction V_f . The thickness of the outer effective composite layer that represents an infinite composite was set equal to $d_c = 5r_f$. Since the friction between fibers and resin is neglected, the contact behavior in the finite element model at the debonded interface can be simply modeled by coupling the adjacent fiber and matrix elements in the radial direction r and by allowing them to move freely in the longitudinal direction z . The possibility to avoid the use of contact elements is a benefit that significantly reduces the calculation time.

Using FEM based solution the integration is over finite region (which most probably is larger than the region where the solution is singular) and strictly speaking the obtained value is not G_{II} . It depends on the size of the integration region dl_d and rather should be called “energy release rate over distance dl_d ”. The integration region was adjusted to fit the exact G_{II} values in the self-similar propagation region.

6. Results and discussion

The properties of the studied materials are given in table 1.

Table 1. Elastic properties of materials.

Material	E_L [GPa]	E_T [GPa]	G_{LT} [GPa]	ν_{12} [-]	ν_{23} [-]	α_L [1/°C]	α_T [1/°C]
CF1 (CF2)	500 (300)	30	20	0.2	0.45	-1.0e-6	7.8-6
GF	70	70	29.2	0.2	0.2	4.7e-6	4.7e-6
EP	3	3	1.07	0.4	0.4	60e-6	60e-6

In table 1, CF – carbon fibers, GF – glass fibers, EP – epoxy resin.

The elastic properties of the effective composite were calculated combining Hashin’s and Christensen’s models [5,6].

Table 2 shows the calculated values of coefficients k_m^∞ , k_{m-th}^∞ and k_{th}^∞ in the self similar propagation region. The values are presented for the 3 studied materials and for 3 volume fractions $V_f = 0.4$, $V_f = 0.5$, $V_f = 0.6$. Using equation (7) mechanical strains have to be in %, temperature in °C.

It can be seen that coefficient k_m^∞ which is related to mechanical term of the energy release rate is almost volume fraction independent (within the observed range). However it varies a lot depending on the fiber material. Comparing with table 1 one can conclude that k_m^∞ is almost proportional to the axial modulus of the fiber.

Table 2. Calculated coefficients for equation (7), (11).

Mat.	k_m^∞			$k_{m-th}^\infty \cdot 10^3$			$k_{th}^\infty \cdot 10^7$		
Vf=	0.4	0.5	0.6	0.4	0.5	0.6	0.4	0.5	0.6
CF1/EP	49.985	49.984	49.982	6.860	4.806	3.352	2.354	1.155	0.562
CF2/EP	29.975	29.975	29.972	6.819	4.786	3.342	3.879	1.911	0.932
GF/EP	6.962	6.960	6.953	6.050	4.323	3.056	13.145	6.714	3.354

The other two coefficients, however, depend on fiber volume fraction quite significantly. Coefficient k_{m-th}^∞ is just weakly dependent on the fiber properties (about 10% variation in the considered range). Coefficient k_{th}^∞ is related to pure thermal term in the energy release rate and is in order of magnitude 10^8 smaller than k_m^∞ . Since $(\epsilon_{z0}^{mech}(\%))^2$ is of order of magnitude 10^0 whereas $\Delta T^2 \approx 10^4$, the influence of the temperature related term for the considered materials is insignificant and the effect of the mixed term varies from 1-10% dependent on fiber properties and volume fraction.

Using data in table 2 to plot the dependence of k_{m-th}^∞ and k_{th}^∞ on fiber content one can see that the relationships for each material are rather linear (linear fit is better for k_{m-th}^∞ which as analyzed above has more practical significance). In other words linear interpolation of these constants in the analyzed volume fraction region is possible, thus allowing for k_m^∞ , k_{m-th}^∞ and k_{th}^∞ estimation for any given fiber content.

In order to use Paris law expression (8) for an arbitrary (finite) debond length, the strain energy release rate was calculated using FEM and the virtual crack closure technique as described above [7]. The obtained G_{II} values were compared with values for the self-similar region and magnification coefficients k^* , (see equation (11)), were found and fitted with exponential function of type:

$$k^* = 1 + a \cdot e^{-b \cdot l_d} \quad (15)$$

where a and b are constants, k^* denotes the magnification factor, so that for an infinitely long debond length l_d the value of k^* is equal to 1. The fitting parameters a and b can be easily obtained taking the logarithm of the expression (15). Since mechanical fatigue loading is studied, according to equation (10) only k_m^* and k_{m-th}^* have to be known since ΔT is not changing. In figure 4 values of coefficients k_m^* and k_{m-th}^* from the fitting expression (15) are compared with the used FEM data showing that the fitting is good.

Since the values of coefficients k_m^* and k_{m-th}^* are now known for an arbitrary interface debond length, mechanical fatigue simulations can be performed according to equation (12). Simulations have been performed for two materials – CF1/EP and GF/EP (see table 2).

Figure 4. Coefficients k_m^* for CF1/EP and GF/EP with the increasing debond length l_d .

Parametric analysis has been performed analyzing the influence of values of parameters and constants such as: the presence of thermal strains (caused by $\Delta T = -100^\circ\text{C}$) (Figure 5); fiber radius r_f (Figure 5); the constant m (Figure 6); material constant G_{HC} (Figure 6). It can be seen from Figure 5.a that, since the thermal strains act opposite to applied tensile strains, the debond growth with the increasing number of load cycles is considerably slower than for purely mechanical case. For CF1/EP composite the effect of initial ΔT was not as evident as for GF/EP. Figure 5.b shows that fiber radius r_f is an important parameter and debond is growing considerably slower for thin fibers.

Figure 5. Debond crack growth with the increasing number of applied cycles in mechanical loading. The effects of : a) thermal strain b) fiber radius. $N^* = N \cdot B^*$

Figure 6. Debond crack growth with the increasing number of applied cycles in mechanical loading. The effects of : a) parameter m b) constant G_{IIc} . $N^*=N \cdot B^*$

7. Conclusions

Mode II energy release rate G_{II} for debond crack growth along fiber in UD composite has been calculated for an arbitrary thermo-mechanical loading and an arbitrary debond length by combining analytical model results for a self similar crack growth and FEM based results for relatively short debonds. Results for G_{II} are expressed through analytical solutions in the self-similar crack growth region, corrected by simple FEM analysis based fitting expressions for magnification factors for short debonds.

These expressions are used to simulate debond crack growth in tension-tension fatigue. Effect of thermal stresses, fiber type and fiber content, fiber radius and material parameters in the used Paris law has been investigated.

- [1] Wu W, Verpoest I and Varna J, Prediction of energy release rate due to the growth of interface crack by variational analysis 2000 *Composites Science and Technology* **60** 351-360
- [2] Graciani E, Mantič V, París F and Varna J, Single fiber fragmentation test. A BEM analysis 2003 *Collection of Technical Papers - AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Norfolk (Virginia), United States* **2** 988-997
- [3] Irwin G R *Fracture*, 1958 *Handbuch der Physik* **5** 551
- [4] Pupurs A, Varna J Energy release rate based fiber/matrix debond growth in fatigue. Part I: Self-similar crack growth 2009 *Mechanics of Advanced Materials and Structures*, Submitted
- [5] Hashin Z Analysis of composite materials – a survey 1983 *Journal of Applied Mechanics* **50** 481-505
- [6] Christensen R M, Lo K H Solutions for effective shear properties in three phase sphere and cylinder models *Journal of Mech. Phys. Solids*, **27**(4) 315-330
- [7] Pupurs A, Varna J Energy release rate based fiber/matrix debond growth in fatigue. Part II: numerical simulation for finite length debonds using Paris law 2009 *Mechanics of Advanced Materials and Structures*, Submitted