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PERFORMANCE STUDY OF RAM-BASED DECISION FEEDBACK EQUALIZERS WITH APPLICATION TO NONLINEAR SATELLITE CHANNELS

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ABSTRACT

The performance of nonlinear decision feedback equalizer implementations based on the use a random access memory (RAM) look-up table are considered. RAM look-up tables admit the modeling, without approximation, of the nonlinear intersymbol interference of nonlinear channels with finite memory. Three equalizers are considered; the RAM-DFE; RAM-Canceler; and recently developed Pre-Cursor Enhanced RAM-DFE Canceler (PERC). In contrast to the classical "feedforward" Volterra equalizers, these equalizers have the advantage that the nonlinear processing element is out of the path of the noise, thus avoiding noise amplification. Results are presented for 16-QAM which compare and demonstrate the effectiveness of these equalizers, particularly the PERC, on the nonlinear satellite channel.

1. INTRODUCTION

The capacity of a satellite channel is limited by the bandwidths of various filters, the satellite's maximum output power and the nonlinearity of its amplifier. A number of approaches for mitigating the nonlinearity have been proposed over the years, including "backing-off" the amplifier away from its saturation point and predistorting the signal constellation prior to transmission [1]. However, backing-off to a lower output power clearly reduces the capacity of the satellite, and the effectiveness of predistortion techniques is hampered by the presence of the filters [2]. The alternative that has been developed is the class of so-called Volterra equalizers [3]. However, as shown in [4,5], while a Volterra equalizer can certainly "invert" a nonlinear satellite channel, it also amplifies and distorts the received noise is such a manner that improvements relative to baking-off are marginal.

We consider equalizers which circumvent this "noise gain" problem of the Volterra equalizeranalogous to ones first proposed for use in nonlinear voiceband and magnetic recording channels [6–9], which have not been examined on satellite channels to our knowledge. More recently, an generalization of this type of equalizer, the Pre-Cursor Enhanced RAM-DFE Canceler (PERC), has been introduced independently in [10].

2. THE CHANNEL MODEL

We assume the satellite is modeled by an amplifier preceded by a filter, $H_1(f)$, and followed by another filter, $H_2(f)$. The role of the pre-filter $H_1(f)$ is to limit out-of-band noise, whereas the post-filter $H_2(f)$ functions to limit emissions into neighboring channels. We shall assume throughout that $H_1(f)$ and $H_2(f)$ are identical 6^{th}-order Butterworth filters whose common cutoff frequency, f_c, is set to some fraction of the symbol rate, R_s, that is, $f_c = aR_s = a/T_s$, where T_s is the symbol period and $0 < a < 1$. We further assume an M-ary two-dimensional modulation scheme with a rectangular transmit pulse shape so that the baseband equivalent transmit signal is

$$s(t) = \sum_k s_k p(t - kT_s) \tag{1}$$

where s_k is one of M complex values and $p(t) = 1$ for $t \in [0, T_s)$, and zero otherwise.

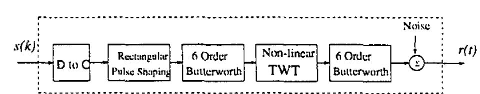

Figure 1: Nonlinear Satellite Channel Model

We assume throughout a traveling wave tube (TWT) amplifier and we adopt Saleh's TWT model [11]. According to the model, the TWT amplifier modifies the input signal amplitude and phase according to the equations

$$\begin{aligned} A(\rho) &= \frac{\alpha_a \rho}{1 + \beta_a \rho^2} \quad \text{(AM/AM conversion)} \\ \Phi(\rho, \theta) &= \theta + \frac{\alpha_\phi \rho^2}{1 + \beta_\phi \rho^2} \quad \text{(AM/PM conversion)} \end{aligned} \tag{2}$$

where ρ and θ are the amplitude (volts) and phase (degrees) of the complex baseband input, and $A(\cdot)$ and $\Phi(\cdot, \cdot)$ are the amplitude and phase of the complex baseband output. The values selected for the various parameters are $\alpha_a = 1.9638$, $\beta_a = 0.9945$, $\alpha_\phi = 2.5293$, and $\beta_\phi = 2.8168$ which correspond to normalized input/output characteristics (see [11]). Observe that the gain characteristic $A(\rho)$ is approximately linear for $\rho < 0.5$, implying that at least a 6 dB input back-off is required for linear operation. The resulting channel block diagram for additive white noise is shown in Fig. 1.

The impairments of this channel and its limiting of throughput is seen in Fig. 2. In this figure, the channel output is plotted in the *noiseless* case for constellations of increasing order. Typically in use today is QPSK which appears "open-eye". Higher order modulation using 8-PSK looks marginally possible without equalization, but 16-QAM is severely distorted, inhibiting high rate communications.

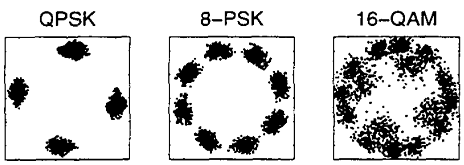

Figure 2: Noiseless Channel Output for Various Modulations

2.1. Volterra Representation

We now show how a Volterra series model derives from the above model assuming for simplicity real symbols s_k. Observe that the AM/AM conversion formula may be expanded as $A(\rho) = \alpha_a \rho - \alpha_a \beta_a \rho^3 + \alpha_a \beta_a^2 \rho^5 - \alpha_a \beta_a^3 \rho^7 + \cdots$ so that the amplifier whose input is $(s * h_1)(t) = \sum_k s_k (p * h_1)(t - kT_s)$ has output amplitude

$$\begin{aligned} A\big((s*h_1)(t)\big) &= \alpha_a \sum_k s_k \tilde{h}_{1k} - \\ &\alpha_a \beta_a \sum_{i,j,k} s_i s_j s_k \tilde{h}_{1i} \tilde{h}_{1j} \tilde{h}_{1k} + \\ &\alpha_a \beta_a^2 \sum_{i,j,k,l,m} s_i s_j s_k s_l s_m \tilde{h}_{1i} \tilde{h}_{1j} \tilde{h}_{1k} \tilde{h}_{1l} \tilde{h}_{1m} - \cdots \end{aligned} \tag{3}$$

where $\tilde{h}_{1k} \triangleq (p * h_1)(t - kT_s)$ and $h_1(t)$ is the impulse response of the pre-filter $H_1(f)$. The expression in (3) represents the Volterra series for $A\big((s*h_1)(t)\big)$ with expansion *kernels* $\alpha_a \tilde{h}_{1k}$, $-\alpha_a \beta_a \tilde{h}_{1i} \tilde{h}_{1j} \tilde{h}_{1k}$, and so on. The postfilter output amplitude will also have a Volterra series expansion

$$\begin{aligned} A\big((s*h_1)(t)\big) * h_2(t) = &\sum_k s_k h_k^{(1)} + \\ &\sum_{i,j,k} s_i s_j s_k h_{ijk}^{(3)} + \cdots \end{aligned} \tag{4}$$

where $h_k^{(1)} \triangleq \alpha_a \tilde{h}_{1k} * h_2$, $h_{ijk}^{(3)} \triangleq -\alpha_a \beta_a \tilde{h}_{1i} \tilde{h}_{1j} \tilde{h}_{1k} * h_2$, and so on. The satellite output phase will have a Volterra series expansion as well, but we need only focus on the amplitude to make our point.

Consider now the k^{th} sample, denoted $\tilde{s}_k$, of the output amplitude, obtained by setting $t = kT_s$ in (4):

$$\tilde{s}_k = \sum_q s_q h_{k-q}^{(1)} + \sum_{i,j,q} s_i s_j s_q h_{k-i,k-j,k-q}^{(3)} + \cdots. \tag{5}$$

Observe $\tilde{s}_k$ contains not only linear intersymbol interference (ISI) terms, but also third- and higher-order nonlinear ISI terms. (Note also this expression includes the special case for when the TWT is linear and $p * h_1 * h_2$ satisfies the Nyquist criterion, in which case $\tilde{s}_k = h_0^{(1)} s_k$, a scaled version of s_k.) Because the memory in the system can be expected to be short, the kernels in (5) are approximately zero except for small indices, and hence the summations involve only a few terms centered about k. Thus, we may write $\tilde{s}_k$ as some nonlinear function of symbols around s_k,

$$\tilde{s}_k \triangleq f(s_{k+d_1}, \ldots, s_k, \ldots, s_{k-d_2}), \tag{6}$$

for some d_1 and d_2. It is evident here that the noiseless satellite channel can be described by a state-machine and therefore be modeled by a shift register and a lookup table which implements (6) as in Fig. 3. The figure motivates RAM lookup-table-based equalizers which we now discuss.

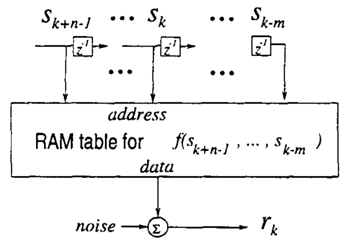

Figure 3: Nonlinear Channel Shift Register Model

3. RAM-BASED COMPENSATION TECHNIQUES

The deviation of the nonlinear distortion affecting $\tilde{s}_k$, away from the ideal s_k, may be written as

$$\begin{aligned} g(s_{k+n-1}, \ldots, s_k, \ldots, s_{k-m}) &\triangleq \\ f(s_{k+n-1}, \ldots, s_k, \ldots, s_{k-m}) &- s_k. \end{aligned} \tag{7}$$

Thus, the ideal receiver would implement the function $g(\cdot)$ so that $g(s_{k+n-1}, \ldots, s_{k-m})$ may be subtracted from the incoming sample $r_k = \tilde{s}_k + v_k$ to obtain a distortion-free sample $z_k = \tilde{s}_k + v_k - g(s_{k+n-1}, \ldots, s_k, \ldots, s_{k-m}) = s_k + v_k$ (here, v_k is a Gaussian noise sample).

The equalizer which embodies this idea is depicted in Fig. 4 where $g(\cdot)$ is implemented by a RAM table. Figure 4 as shown is a general RAM-based equalizer. Of course, the receiver shown in this figure is not implementable since it requires knowledge of the symbols $\{s_{k+n-1}, \ldots, s_k, \ldots, s_{k-m}\}$ which are not known.

Instead, the various RAM-based equalizers use various local decisions on these values. The different ways of obtaining and using these decisions is what separates the RAM-DFE, RAM-Canceler, and PERC. We remark that these types of equalizers typically work in cooperation with a feed-forward fractionally spaced equalizer (FSE) whose primary function is to act as a pseudo-matched filter (to optimize SNR) [6] and help in symbol synchronization issues. Note

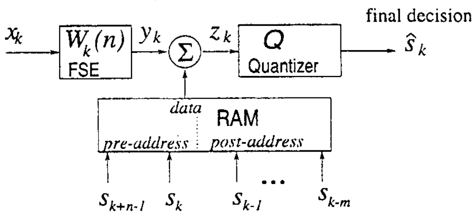

Figure 4: RAM-based Equalizer Structure

that $g(\cdot)$ and, hence, the RAM table, must be modified to account for its presence. We now discuss the RAM-based equalizers and their adaptation algorithms separately.

3.1. The RAM-DFE

The RAM-DFE only uses $\{s_{k-1}, \ldots, s_{k-m}\}$ and can then expected to be effective when most of the nonlinear ISI (NL-ISI) is postcursor. In this case, the estimated NL-ISI is a function only of the postcursor decisions $\{\hat{s}_{k-1}, \ldots, \hat{s}_{k-m}\}$. As for the conventional DFE, we can expect some error propagation, but the performance is still often better than that of a feedforward equalizer.

Writing $\hat{\mathbf{s}}_{k-1}$ for the RAM address $(\hat{s}_{k-1}, \ldots, \hat{s}_{k-m})$, the update algorithm for the RAM at location $\hat{\mathbf{s}}_{k-1}$ is

$$\mathrm{RAM}_{k+1}(\hat{\mathbf{s}}_{k-1}) = \mathrm{RAM}_k(\hat{\mathbf{s}}_{k-1}) + \mu_b \varepsilon_k \tag{8}$$

where $\varepsilon_k = z_k - \hat{s}_k$ is the estimated decision error and μ_b is the update stepsize . This algorithm was shown in [8] to be optimal in the minimum mean-squared error (MMSE) sense, yielding the *unique* global MMSE, except for some misadjustment. However, we may argue in favor of this algorithm heuristically as follows. If we suppose that $\varepsilon_k > 0$, then the RAM location will be increased by the algorithm, ensuring that the next time the channel is in that state (i.e., the next time that location is addressed), the error will be smaller.[1] This follows since $\varepsilon = y - \mathrm{RAM} - \hat{s}$, so that increasing RAM decreases ε. By symmetry the argument holds also $\varepsilon_k < 0$.

The convergence of the RAM to the MMSE solution is clearly slow since we can expect a large number of RAM locations (there are M^m of them), and only one location is updated per symbol. For example, assuming that 100 accesses are required for the convergence of a particular RAM location, $100 \cdot 16^4 \simeq 6,553,600$ uniformly distributed symbols are required for convergence of the entire RAM when $M = 16$ and $m = 4$. Fisher, *et al* [8], propose a "broadcast" training algorithm which speeds up convergence, but it is at the expense of increased misadjustment. The typical satellite channel can be expected to have stationary characteristics so that convergence speed is not an important issue.

The FSE which works in cooperation with the RAM-DFE is adapted in the usual manner [12]. That is, the $T_s/2$-spaced tap-weight vector W_k at time k is updated as

$$W_{k+1} = W_k + \mu_f \varepsilon_k R_k \tag{9}$$

[1] This argument ignores the noise. To include the noise, one would say that the error will be smaller *on average*.

where μ_f is the update stepsize, ε_k is defined above, and R_k is the tapped delay-line filter contents at time k. Various strategies exist for coordinating the FSE and RAM updates.

The role of the fractionally spaced feedforward filter in the conventional linear-ISI DFE is to ease symbol timing and reshape the channel response to be essentially causal, as the feedback filter is capable only of removing causal ISI. This is not possible for the NL-ISI channel since, in addition to being nonlinear, many of the distortion terms are neither causal nor anti-causal. For example, the nonlinear ISI term $s_{k+1}s_k s_{k-1}$ is possible (see (5)). Thus, the RAM-DFE is helpless against such terms since it can only remove terms of the form $s_{n-i}s_{n-j}s_{n-k}$, for $i,\ j,\ k > 0$. This motivates the RAM-Canceler and PERC equalizers which attempt to eliminate all of the significant nonlinear ISI terms.

3.2. The RAM-Canceler

The RAM-Canceler uses both precursor and postcursor symbol decisions to make nonlinear distortion estimates, it is faced with using tentative symbol decisions which can be expected to be much less reliable than the final decisions that are fed back in the RAM-DFE case (see Fig. 5). This is, of course, because the fed back postcursor decisions are made after the NL-ISI estimate has been subtracted out, whereas the tentative decisions are made in the presence of NL-ISI. Still, the RAM-Canceler, or functionally equivalent nonlinear cancelers, can still be effective in certain situations [6–9].

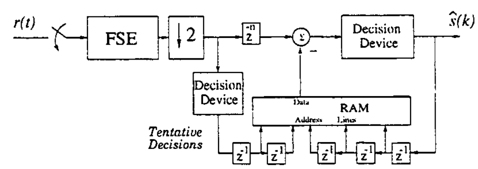

Figure 5: RAM-Canceler uses Tentative Decisions

A careful argument is given in [9], albeit for the situation where both tentative and final decisions are made by mismatched Viterbi detectors (we assume symbol-by-symbol detectors). The RAM-Canceler adaptation algorithm is identical to that of the RAM-DFE given in (8), with the exception that the RAM address is modified as $\hat{\mathbf{s}}_{n-1} = (\hat{s}_{n+d_1}, \ldots, \hat{s}_{n-d_2})$. Clearly, the convergence time is much longer now since there are now $M^{d_1+d_2+1}$ RAM locations.

It turns out that, even for relatively poor tentative decision error rates, it is possible to obtain a relative performance improvement with the canceler. However, better results may be obtained by considering a "block cursor" idea of the PERC.

3.3. Pre-Cursor Enhanced RAM-DFE Canceler

The basic structure of the PERC nonlinear equalizer is shown in Fig. 6 where RAM denotes a random access memory with address lines consisting of m past decision $\hat{s}_{k-1}$ to $\hat{s}_{k-m}$ (denoted post-address) and n present/future potential decisions $\tilde{s}_k$ to $\tilde{s}_{k+n-1}$. We denote such a configuration as

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PERC(n, m). The twist on the RAM-DFE is the incorporation of present and future decisions.

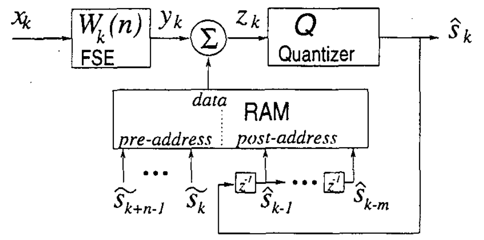

Figure 6: PERC Structure

The PERC must be run in a training mode first to learn the channel characteristics. In training mode, since the entire training sequence is known, it is clear that all "post", $\{\hat{s}_{k-1}, \ldots, \hat{s}_{k-m}\}$, and "pre", $\{\tilde{s}_{k+n-1}, \ldots, \tilde{s}_k\}$, components are available.

The feedforward equalizer is trained first to set the proper delay using the standard LMS update relation:

$$W_{k+1} = W_k + \mu_{\text{ff}} X_k e_k^* \qquad (10)$$

where $X_k = [x_k \ldots x_{k-\ell}]^T$, μ_{ff} is the stepsize and $e_k = s_k - \hat{s}_k$. After the delay has been set, the feedforward equalizer is then fixed and the PERC component is trained using

$$\text{RAM}_{k+1}(A) = \text{RAM}_k(A) + \mu_{\text{fb}} e_k \qquad (11)$$

where μ_{fb} is the stepsize, $\text{RAM}_k(A)$ denotes the contents at time k of address A. The address A is given by the bit representation of the vector $[\tilde{s}_{k+n} \ \ldots \ \tilde{s}_k \mid \hat{s}_{k-1} \ \ldots \ \hat{s}_{k-m}] = [A_{\text{pre}} \mid A_{\text{post}}]$.

After training, the PERC may be run in "fixed mode" where no adaptation takes place or in a "decision-directed" mode. We focus here on the performance of the fixed mode. The only difficulty here is to decide what is the proper "precursor" address component A_{pre} The idea is to test over all possible symbols of A_{pre} and choose the one that places z_k closest to a valid symbol value (i.e., the address that minimizes $|e_k|^2 = |z_k - Q(z_k)|^2$). Since the output is dependent on past decisions (which may be incorrect), error propagation is possible and MSE performance in non-training mode can be expected to be worse than in training mode.

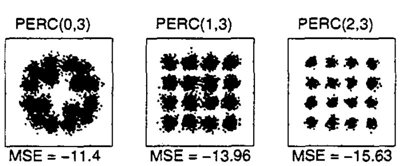

Figure 7: Decision Device Inputs for Various PERC's

4. SIMULATION RESULTS

A comparison of the decision device inputs for various PERCs are shown in Fig. 7. For the channel model under study, the linear and RAM-DFE (equivalent to PERC(0,m)) fails to provide acceptable improvement. The linear equalizer fails due to its inability to cancel any nonlinear components of the ISI. The RAM-DFE, while able to cancel *postcursor* components of the nonlinear ISI, is stymied by the cursor component (s_k) nonlinear terms. Allowing for the present cursor, the PERC(1,3) of the simulations shows a dramatic performance improvement in bit error probability, with PERC(2,3) showing little additional improvement.

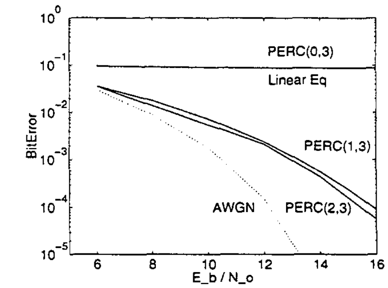

Figure 8: Bit Error Rate Comparison

5. REFERENCES

[1] Bhargava, *et al*, *Digital Communications by Satellite*, pp. 96-100, Wiley, 1981.

[2] G. Karam and K. Sari, "A data predistortion technique with memory," *IEEE Trans. Comm.*, pp. 336-344, Feb. 1991.

[3] S. Benedetto and E. Biglieri, "Nonlinear equalization of digital satellite channels," *IEEE J. Sel. Areas in Comm.*, pp. 57-62, Jan. 1983.

[4] A. Gutierrez, *Equalization and Detection for Digital Communication over Nonlinear Bandlimited Satellite Communication Channels*, Ph.D. Dissertation, New Mexico State University, Dec. 1995.

[5] A. Gutierrez and W. Ryan, "Performance of adaptive Volterra equalizers on nonlinear satellite channels, *Proc. 1995 Int. Conf. on Comm.*, pp. 488-492.

[6] E. Bigieri, *et al*, "Adaptive cancellation of nonlinear intersymbol interference for voiceband data transmission," *IEEE J. Sel. Areas in Comm.*, pp. 765-777, Sept. 1984.

[7] N. Holte and S. Stueflotten, "A new digital echo canceller for two-wire subscriber lines," *IEEE Trans. Comm.*, pp. 1573-1581, Nov. 1981.

[8] K. Fisher, *et al*, "An adaptive RAM-DFE for storage channels," *IEEE Trans. Comm.*, pp. 1559-1568, Nov. 1991.

[9] O. Agazzi and N. Seshadri, "On the use of tentative decisions to cancel intersymbol interference and nonlinear distortion (with applications to magnetic recording channels," *IEEE Trans. Inf. Theory*, pp. 394-408, March 1997.

[10] J.P. LeBlanc, R.A. Kennedy, W.E. Ryan, "Pre-Cursor Extended RAM-DFE Canceller (PERC) Nonlinear Equalizer for Nonlinear ISI Channels", *IEEE DSP Workshop, Aug. 1998.*

[11] A. Saleh, "Frequency-independent and frequency-dependent nonlinear models of TWT amplifiers," *IEEE Trans. Comm.*, pp. 1715-1720, Nov. 1981.

[12] G. Ungerboeck, "Fractional tap-spacing equalizer and consequences for clock recovery in data modems," *IEEE Trans. Comm.*, pp. 856-864, Nov. 1981.