Division of Signal Processing
Visiting address: Universitetsområdet, Porsön, Luleå
Postal address: SE-971 87, Luleå, Sweden
Telephone: +46 920 910 00. *Fax:* +46 920 720 43
URL: <http://www.sm.luth.se/csee/sp/>

Digital RFI Suppression in DMT-based VDSL Systems

Frank Sjöberg, Rickard Nilsson, Niklas Grip,
Per Ola Börjesson, Sarah Kate Wilson and Per Ödling

*In Proceedings of International Conference on
Telecommunications (ICT '98), Vol. II, pp.
189-193, Chalkidiki, Greece, June 1998.*

© 1998 IEEE. Personal use of this material is permitted.
However, permission to reprint/republish this material for
advertising or promotional purposes or for creating new collective
works for resale or redistribution to servers or lists, or to reuse
any copyrighted component of this work in other works must be
obtained from the IEEE.

Digital RFI Suppression in DMT-based VDSL Systems

Frank Sjöberg*, Rickard Nilsson*, Niklas Grip†

Per Ola Börjesson*, Sarah Kate Wilson* and Per Ödling*

*Division of Signal Processing †Department of Mathematics
Luleå University of Technology, SE-971 87 Luleå, Sweden

Abstract— In this paper we describe a method for suppressing Radio Frequency Interference (RFI) in Discrete Multitone (DMT) based Very High bit rate Digital Subscriber Line systems.

The method operates in the frequency domain of a DMT-system. It is based on a Taylor-expanded transfer function which maps the disturbance from the RF-signal onto the DMT-carriers. By measuring the RF-disturbance on a few unmodulated DMT-carriers we are able to subtract estimates of the disturbance from every modulated DMT-carrier. The RF-signal is assumed to be an unknown narrowband signal.

Simulation results show that this method, applied to an RFI-signal with the same average power as the VDSL-signal, reduces the SNR-loss from about 20 dB to less than 1 dB.

I. INTRODUCTION

Radio Frequency (RF) signals transmitted in the vicinity of an unshielded copper wire may cause *Radio Frequency Interference* (RFI) with signals in the wire [1]. In this paper we address the problem of RFI in *Discrete Multi-Tone* (DMT) [2] based *Very high bit rate Digital Subscriber Line* (VDSL) [3] systems and describe a method for suppressing the RFI-signal in the frequency domain.

VDSL is a transmission technology which can transmit up to 52 Mbit/s on unshielded twisted copper wires. To achieve these high bit rates, VDSL must use a large frequency band ranging from around 300 kHz up to 15 MHz. In this range VDSL will share some frequency bands, known as HAM-bands, with radio amateurs.

Since the twisted copper-wires are not shielded this will cause two types of related problems; *ingress* and *egress*, i.e., the reception and radiation of radio signals into and out from the wire, respectively. When a radio amateur transmits within a HAM-band, it will interfere with all DMT-tones including those outside the HAM bands. Hence, to avoid a large performance degradation it is desirable to suppress the RFI as much as possible.

Earlier suggestions for suppressing the RFI include the use of an analog pre-canceller using common mode to differential mode conversion [4], digital

notch filters and pulse shaping of the data-symbols [5]. These methods suppress the RFI to a certain extent but often not enough.

By expanding an idea, introduced in [6] by Bingham, we develop an RFI-cancelling method which is capable of suppressing general (unknown) narrowband RF-signals. This paper investigates the case when windowing is not used at the received data. When windowing is performed a similar approach is taken in [7] which gives similar results.

Our method operates in the signal's frequency domain after the *Analog to Digital Converter* (ADC). When we analyze the RFI at each DMT-tone we consider a transfer function which maps the RF-disturbance to the tones. Based on RFI measurements on a few *unmodulated* tones, we extrapolate the RFI that is present in all *modulated* DMT-tones.

Simulation results shows that our method often suppresses an RFI-signal, with the same average power as the VDSL-signal, from a 20 dB average *Signal-to-Noise Ratio* (SNR)-loss down to less than a 1 dB SNR-loss.

II. RADIO FREQUENCY INTERFERENCE

An unshielded twisted wire works as a *Common Mode* (CM) antenna that both generates and receives radio signals in the VDSL systems frequency range (300 kHz up to 15 MHz). VDSL signals are, however, transmitted and received as *Differential Mode* (DM)-signals. But unbalance in the wire will cause CM/DM conversion [1], resulting in both RF-ingress and RF-egress. In this paper we consider the problem of RF-ingress into VDSL systems.

The main source of RFI is believed to be amateur radio transmitters. This is because they can be located just a few meters from an unshielded wire [1] and the transmitting power is expected to be high – up to 400 W in the UK and 1.5 kW in the USA. Radio amateurs are allowed to operate at certain frequencies, called the *HAM bands*, which are listed in Table 1 for the UK [1]. The HAM bands for the USA are similar. VDSL systems are required to handle amateur radio signals with 4 kHz bandwidth [3].

Table 1. Four UK amateur radio bands overlapping the VDSL spectrum.

Band start (MHz)	1.81	3.50	7.00	10.10
Band stop (MHz)	2.00	3.80	7.10	10.15

III. SUPPRESSION OF RF-INGRESS

Since the signal power of the RFI may be high, it first has to be suppressed to the level of the desired signal by an analog RFI canceller to avoid clipping in the ADC. However, the interference after the ADC can be severe even if RFI power is reduced below the power of the desired signal. Thus, it is important to combat the RFI also in the digital domain.

DMT-tones in the HAM-bands must be unmodulated to avoid RF-egress [4]. Our method measures the RF-disturbance on a few unmodulated DMT-tones and then subtracts estimates of the disturbance from the other tones. This idea was introduced in [6] for a sinusoid RF-signal with known frequency. We expand on this idea and develop a method for any narrowband RF-disturbance with a roughly known center frequency.

In this section we develop a signal model over the received RF-signal. The model describes, with a transfer function, how the RF-signal interferes with each subcarrier in a DMT-system. After parametrizing the transfer function we calculate estimates of the RF-disturbance in a two stage process. First we make ML estimates of the unknown parameters using measurements from unmodulated tones. Then, we calculate and finally subtract estimates of the disturbance from the data.

A. Signal Model

In this subsection we derive the properties of the RFI in the *Discrete Fourier Transform* (DFT)-domain. This will be used in the subsequent derivation of the RFI-canceller.

Starting with the received signal expressed as

$$r(t) = x(t) * h(t) + s(t) + n(t),$$

where $x(t)$ is the transmitted baseband DMT-signal, $h(t)$ is the impulse response of the wire-channel, $s(t)$ is an unknown narrowband RF-signal and $n(t)$ represent other noise sources such as *far end crosstalk* (FEXT) and *additive white Gaussian noise* (AWGN).

Sampling and DFT-demodulation of $r(t)$ yields

$$\begin{aligned} R_k &= \sum_{n=0}^{2N-1} r(nT_s) e^{-j\frac{2\pi kn}{2N}} \\ &= X_k H_k + S_k + N_k, \quad (0 \leq k \leq N-1) \end{aligned} \quad (1)$$

where H_k is the wire channel's frequency response at frequency $k\frac{f_s}{2N}$, S_k are the narrowband RF-signal's DFT coefficients, N_k is the DFT of the noise.

Figure 1 – The DFT coefficients S_k of a disturber $s(t) = A \cos(2\pi f_c t + \theta)$.

To cancel the interference S_k we want to subtract an estimate, $\hat{S}_k$, from (1):

$$Y_k = R_k - \hat{S}_k = X_k H_k + (S_k - \hat{S}_k) + N_k.$$

To derive the estimates $\hat{S}_k$ we set up the following relationship between S_k and the Fourier transform of the disturbance $S(f) = \mathcal{F}\{s(t)\}$:

$$S_k = \int_{-\infty}^{\infty} S(f) G_k(f) df, \quad (2)$$

where we interpret $G_k(f)$ as a *transfer function* from the unknown RF-signals Fourier transform $S(f)$ to its DFT coefficients S_k . It can be written as

$$G_k(f) = \sum_{n=0}^{2N-1} e^{j\frac{2\pi n f}{f_s} - j\frac{\pi n}{N} k} = \frac{1 - e^{j\frac{4\pi f N}{f_s}}}{1 - e^{j(\frac{2\pi f}{f_s} - \frac{\pi}{N} k)}}. \quad (3)$$

We first consider the special case from [6], where the interferer is a sinusoid, $s(t) = A \cos(2\pi f_c t + \theta)$, with known frequency f_c . Then $s(t)$ has DFT coefficients

$$S_k = \frac{Ae^{j\theta} G_k(f_c) + Ae^{-j\theta} G_k(-f_c)}{2}.$$

Figure 1 shows the S_k 's for $N = 1024$ subcarriers. In this case the disturbance is completely characterized in the DFT plane by the two parameters A and θ . When these are known (or estimated) we can calculate S_k for all $k \in \{0, \dots, N-1\}$. For the general case, if we can find a parametrized model for an arbitrary narrowband RF-signal, then we can estimate the unknown parameters and calculate the RF-signals DFT-coefficients, S_k .

B. Parametrization of the disturbance

We assume that the disturbance $S(f)$ is narrowband with center frequency f_c and bandwidth $2B$, that is,

$S(f) = 0$ when $|f \pm f_c| \geq B$. Thus the DFT coefficients in (2) are

$$S_k = \int_{-B}^B S(f - f_c) G_k(f - f_c) df + \int_{-B}^B S(f + f_c) G_k(f + f_c) df. \quad (4)$$

We cannot compute S_k directly, since $S(f)$, f_c and B are unknown. We can, however, make a rough estimate, $\hat{f}_c$, of f_c from measurements R_k . Applying an idea from [8] (which also considers narrowband signals), we separate the known and unknown parts in (4) with a Taylor expansion around $\hat{f}_c$. (This is similar to the time domain approach in [7] in which they model the envelope of the disturbance as a first order polynomial.) The Taylor expansion is evaluated in Appendix A.. From this we find that

$$S_k \approx \tilde{S}_k \triangleq \sum_{l=0}^{L-1} A_l D_k^{(l)}(-\hat{f}_c) + \sum_{m=0}^{M-1} B_m D_k^{(m)}(\hat{f}_c), \quad (5)$$

where $\{A_l\}$ and $\{B_m\}$ are $L + M$ unknown parameters representing the disturbance and $D_k^{(m)}(\hat{f}_c)$ is the m^{th} derivative of the denominator of $G_k(\hat{f}_c)$

$$D_k^{(m)}(\hat{f}_c) = \frac{d^m}{df^m} \left[\frac{1}{1 - e^{j(\frac{2\pi f}{f_s} - \frac{\pi}{N} k)}} \right]_{f=\hat{f}_c}$$

Because the series-expansion (5) is truncated, we introduce a model error, $S_k - \tilde{S}_k$. Since the unknown parameters are independent of the tone index k , *i.e.* common for every subcarrier in one DMT-symbol, we are able to calculate the RF-disturbance on all subcarriers as soon as the parameters are estimated.

C. Model parameter estimation

In order to estimate the unknown RF-parameters, $\{A_l\}$ and $\{B_m\}$, we need to measure the RFI on some unmodulated subcarriers. Let the index set $\mathcal{U} = \{k_1, k_2, \dots, k_{|\mathcal{U}|}\}$ denote the set of subcarriers on which we measure the RF-disturbance. On these tones the received signal is

$$R_k = S_k + N_k, \quad \forall k \in \mathcal{U}$$

Our estimation problem from (5) is linear and can be written in matrix notation as

$$\mathbf{R}_{\mathcal{U}} = \tilde{\mathbf{S}}_{\mathcal{U}} + \mathbf{N}_{\mathcal{U}} = \mathbf{D}_{\mathcal{U}} \mathbf{C} + \mathbf{N}_{\mathcal{U}} \quad (6)$$

where

$$\mathbf{D}_{\mathcal{U}} = \begin{pmatrix} D_{k_1}(-\hat{f}_c) & \dots & D_{k_1}^{(L-1)}(-\hat{f}_c) & D_{k_1}(\hat{f}_c) & \dots & D_{k_1}^{(M-1)}(\hat{f}_c) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ D_{k_{|\mathcal{U}|}}(-\hat{f}_c) & \dots & D_{k_{|\mathcal{U}|}}^{(L-1)}(-\hat{f}_c) & D_{k_{|\mathcal{U}|}}(\hat{f}_c) & \dots & D_{k_{|\mathcal{U}|}}^{(M-1)}(\hat{f}_c) \end{pmatrix},$$

$$\mathbf{C} = (A_0 \ A_1 \ \dots \ A_{L-1} \ B_0 \ B_1 \ \dots \ B_{M-1})^T,$$

and

$$\mathbf{R}_{\mathcal{U}} = (R_{k_1} \ R_{k_2} \ \dots \ R_{k_{|\mathcal{U}|}})^T.$$

Assuming that the noise $\mathbf{N}_{\mathcal{U}}$ is independent identically Gaussian distributed, we obtain the ML estimate $\hat{\mathbf{C}}_{ML}$ of the parameters $\mathbf{C}$ as [9]

$$\hat{\mathbf{C}}_{ML} = \mathbf{D}_{\mathcal{U}}^\dagger \mathbf{R}_{\mathcal{U}}$$

where $\mathbf{D}_{\mathcal{U}}^\dagger$ is the pseudoinverse of $\mathbf{D}_{\mathcal{U}}$.

D. Disturbance estimation and cancellation

Now let the index set $\mathcal{M}$ indicate the modulated subcarriers. The expression in equation (6) is independent of the chosen index set $\mathcal{U}$, so by using $\hat{\mathbf{C}}_{ML}$, we get the estimates of the disturbance as

$$\begin{aligned} \hat{\mathbf{S}}_{\mathcal{M}} &= \mathbf{D}_{\mathcal{M}} \hat{\mathbf{C}}_{ML} = \mathbf{D}_{\mathcal{M}} \mathbf{D}_{\mathcal{U}}^\dagger \mathbf{R}_{\mathcal{U}} \\ &= \tilde{\mathbf{S}}_{\mathcal{M}} + \mathbf{D}_{\mathcal{M}} \mathbf{D}_{\mathcal{U}}^\dagger \mathbf{N}_{\mathcal{U}}. \end{aligned}$$

where we have assumed that $\mathbf{D}_{\mathcal{U}}$ has full column rank. Finally we use these estimates to reduce the effect of the disturber by subtracting it from the data

$$Y_k = R_k - \hat{S}_k = X_k H_k + (S_k - \hat{S}_k) + N_k.$$

IV. SIMULATIONS

We have simulated a DMT-system with 1024 subcarriers and using a sampling frequency of $f_s = 24$ MHz. The wire is assumed to be a 1000 m long TP1-wire [3]. The noise consists of 49 self-FEXT disturbers and AWGN with spectral density -140 dBm/Hz. The RFI-signal is assumed to have the same average power as the desired signal. In every simulation, the disturbance is measured on the same number of tones as the number of parameters in the sum (5), *i.e.* $|\mathcal{U}| = M + L$. We measure performance in terms of average SNR-loss due to the remaining RF-disturbance (the average SNR without RF-disturbance are in these cases 25 dB).

In the first simulation we let the bandwidth of the RF-signal vary between 0 and 20 kHz. The center frequency is 3.65 MHz and it is assumed to be known, *i.e.* $\hat{f}_c - f_c = 0$. Figure 2 shows the results for varying number of parameters in (5), $L = 1$ and $1 \leq M \leq 4$ (if $L > 1$, then the matrix $\mathbf{D}_{\mathcal{U}}$ is almost singular, resulting in numerical instabilities and noise enhancement). If the disturber is a pure cosine signal, then 2 tones ($M = L = 1$) is enough, as expected. But when the RF-signal becomes wider, there will be a performance improvement if we use more tones ($M > 1$). When the bandwidth of the RF-signal is less than 8 kHz the best choice is to use four tones, $M = 3$ and $L = 1$, resulting in an SNR-loss less than 1 dB.

Figure 2 – Average SNR-loss due to RFI at 3.65 MHz, with/without RFI-cancellation

Figure 3 – Average signal power before/after RFI-cancelling.

Figure 3 shows the impact from a 6 kHz wide RF-signal on different DMT-tones, before and after RFI-cancellation, by using 4 tones for RFI-measurement.

In a second simulation we use a disturber at 3.65 MHz with 6 kHz bandwidth and let the error in the center-frequency estimate, $\hat{f}_c - f_c$, vary between 0 and 20 kHz. Figure 4 shows performance, in terms of SNR-loss, when 4 parameters are used in the RFI-canceller. For frequency estimate errors up to 6 kHz (approximately 1/2 subcarrier spacing) the SNR-loss is less than 1 dB.

In the last simulation we use a single 6 kHz wide RF-signal whose center frequency error is 4 kHz. We measure the disturbance on 4 DMT-tones. Table 2 show the SNR-loss with and without RFI-cancellation when the RFI-signal is located in four different HAM-bands.

Figure 4 – SNR-loss after RFI-cancellation for $0 \leq |\hat{f}_c - f_c| \leq 20$ kHz. Disturber bandwidth is 6 kHz and $L = 3, M = 1$.

Table 2. Average SNR-loss with and without RFI-cancellation for a 6kHz wide RFI-disturber in different HAM-bands.

Center-frequency	With cancellation	Without cancellation
1.822 MHz	0.32 dB	19.9 dB
3.756 MHz	0.65 dB	19.0 dB
7.05 MHz	0.73 dB	14.8 dB
10.12 MHz	0.23 dB	10.4 dB

V. SUMMARY

We have derived a frequency domain RFI-suppression method for DMT-based VDSL-systems. The only assumptions made about the RF-signal are that it is narrowband and that its center frequency is approximately known or estimated.

By measuring the disturbance on a number of unmodulated DMT-tones we are able to subtract estimates of the RF's impact from all modulated DMT-tones. The estimation procedure parameterizes the transfer function between the RF-signal and the DMT-tones, by using Taylor expansion.

We have shown that this method is capable of reducing the SNR-loss, due to a narrowband RF-signal, from around 20 dB down to less than 1 dB. Four measurement-tones is the best choice for typical RF-disturbances but the method is also able to suppress wider RF-signals by using more tones.

A. LINEARIZATION OF NARROWBAND RFI

Note in equation (3) that only the denominator of the transfer function depends on the index k . Therefore, we can simplify differentiation below by splitting $G_k(f)$ into two parts, the numerator and the

denominator $D_k(f) \triangleq \frac{1}{1 - e^{j(\frac{2\pi f}{f_s} - \frac{\pi}{N}k)}}$:

$$G_k(f) = \frac{1 - e^{j\frac{4\pi f N}{f_s}}}{1 - e^{j(\frac{2\pi f}{f_s} - \frac{\pi}{N}k)}} = D_k(f) \left(1 - e^{j\frac{4\pi f N}{f_s}}\right)$$

A change of variables from f to $f \pm f_\Delta \triangleq f \pm (\hat{f}_c - f_c)$ in (4) yields

$$\begin{aligned} S_k &= \int_{-B+f_\Delta}^{B+f_\Delta} S(f - \hat{f}_c) G_k(f - \hat{f}_c) df \\ &\quad + \int_{-B-f_\Delta}^{B-f_\Delta} S(f + \hat{f}_c) G_k(f + \hat{f}_c) df \\ &= \int_{-B+f_\Delta}^{B+f_\Delta} S_-(f) D_k(f - \hat{f}_c) df \\ &\quad + \int_{-B-f_\Delta}^{B-f_\Delta} S_+(f) D_k(f + \hat{f}_c) df, \quad (7) \end{aligned}$$

where $S_\pm(f) \triangleq S(f \pm \hat{f}_c) \left(1 - e^{j\frac{4\pi(f \pm \hat{f}_c)N}{f_s}}\right)$. We can separate the dependence on k and f with a Taylor expansion around $f = 0$:

$$\begin{aligned} D_k(f - \hat{f}_c) &= \sum_{l=0}^{L-1} \frac{f^l D_k^{(l)}(-\hat{f}_c)}{l!} + O(f^L), \\ D_k(f + \hat{f}_c) &= \sum_{m=0}^{M-1} \frac{f^m D_k^{(m)}(+\hat{f}_c)}{m!} + O(f^M), \end{aligned}$$

where

$$D_k^{(m)}(\pm f_c) \triangleq \left[\frac{d^m}{df^m} D_k(f \pm \hat{f}_c) \right]_{f=0}.$$

Truncation of these series and insertion into (7) gives an approximation, $\tilde{S}_k$, of S_k as a linear combination of $L + M$ unknown parameters $\{A_l\}$ and $\{B_m\}$:

$$\begin{aligned} S_k \approx \tilde{S}_k &\triangleq \sum_{l=0}^{L-1} D_k^{(l)}(-\hat{f}_c) \int_{-B+f_\Delta}^{B+f_\Delta} S_-(f) \left(\frac{f^l}{l!}\right) df \\ &\quad + \sum_{m=0}^{M-1} D_k^{(m)}(+\hat{f}_c) \int_{-B-f_\Delta}^{B-f_\Delta} S_+(f) \left(\frac{f^m}{m!}\right) df \\ &= \sum_{l=0}^{L-1} A_l D_k^{(l)}(-\hat{f}_c) + \sum_{m=0}^{M-1} B_m D_k^{(m)}(+\hat{f}_c), \end{aligned}$$

where the approximation $S_k \approx \tilde{S}_k$ arises from truncation of the Taylor series and where

$$\begin{aligned} A_l &= \frac{1}{l!} \int_{-B+f_\Delta}^{B+f_\Delta} \left(1 - e^{j\frac{4\pi(f - \hat{f}_c)N}{f_s}}\right) f^l S(f - \hat{f}_c) df, \\ B_m &= \frac{1}{m!} \int_{-B-f_\Delta}^{B-f_\Delta} \left(1 - e^{j\frac{4\pi(f + \hat{f}_c)N}{f_s}}\right) f^m S(f + \hat{f}_c) df. \end{aligned}$$

The derivatives can be calculated from the following relations (with $D = D_k(f)$):

$$\begin{aligned} D' &= j\frac{2\pi}{f_s} D^2 (1 - 1/D) = j\frac{2\pi}{f_s} (D^2 - D) \\ D'' &= j\frac{2\pi}{f_s} (2DD' - D') = j\frac{2\pi}{f_s} D' (2D - 1) \\ D^{(3)} &= j\frac{2\pi}{f_s} (2DD'' + 2D'D - D'') \end{aligned}$$

REFERENCES

- [1] K. T. Foster, "The RFI environment for VDSL." Distributed at IEEE Globecom '96, VDSL Workshop, 18th November 1996, London.
- [2] J. A. Bingham, "Multicarrier modulation for data transmission: An idea whose time has come," *IEEE Commun. Mag.*, vol. 28, pp. 5–14, May 1990.
- [3] "Very-high-speed digital subscriber lines – system requirements, draft technical report,," Tech. Rep. T1E1.4/97-131R2, ANSI, Sacramento, CA, Sept. 1997.
- [4] J. A. C. Bingham, J. M. Cioffi, and M. Mallory, "SDMT methods of dealing with RFI." Contribution to ETSI Sub-Technical Committee TM6, June 1996.
- [5] P. Spruyt, P. Reusens, and S. Braet, "Performance of improved DMT transceiver for VDSL," Tech. Rep. T1E1.4/96-104, ANSI, Colorado Springs, Co, Apr. 1996.
- [6] J. A. C. Bingham, "RFI suppression in multicarrier transmission systems," in *Proc. Globecom*, vol. 2, (London, UK), pp. 1026–1030, Nov. 1996.
- [7] B. Wiese and J. Bingham, "Digital radio frequency cancellation for DMT VDSL," Tech. Rep. T1E1.4/97-460, ANSI, Sacramento, CA, Dec. 1997.
- [8] H. Eriksson, P. O. Börjesson, P. Ödling, and N.-G. Holmer, "A robust correlation receiver for distance estimation," *IEEE Trans. Ultras. Ferroelec. Freq. Contr.*, vol. 41, pp. 596–603, Sept. 1994.
- [9] L. L. Scharf, *Statistical signal processing: Detection, estimation, and time series analysis*. Addison-Wesley, 1991.