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NONLINEAR FINITE ELEMENT SIMULATIONS OF ICE SHEET FORCES ON CONICAL STRUCTURES

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ABSTRACT

The present paper deals with interaction between an ice sheet and fixed, conical structures. The ice sheet as well as the structure is discretized by finite elements. The interaction between the ice sheet and the conical structure is simulated using a special contact algorithm which makes it possible to follow the gradually developing contact between the ice sheet and structure. As the configuration of the ice sheet changes during the interaction process, the buoyancy forces change accordingly. This process is traced by introducing a continuous nonlinear foundation model to include the effects of buoyancy forces and specific weight of the ice. The mechanical behavior of ice is approximated using two different constitutive models. In the first material model the ice is treated as an isotropic, brittle material, while in the second model the ice is considered being a transversal isotropic, brittle material. When the state of stress at a material point in the ice reaches the failure surface, cracking or crushing is said to occur. After cracking or crushing, the post peak behavior of the ice is approximated as a rigid plastic material. The results obtained during the finite element simulations are compared with analytical methods for calculation of ice sheet forces on conical structures.

INTRODUCTION

Design of offshore structures in Arctic waters is strongly dependent on local and global ice loads. These loadings are, in general, contact forces transmitted to the structures during interaction with ice floes, ice ridges or icebergs. The prediction of ice forces on structures relies heavily on a thorough understanding of ice mechanics as well as on in-depth knowledge of interaction between ice features and structures. The complexities of the constitutive behavior of ice and the variations in ice failure modes are major obstacles in the development of efficient and reliable methods for the estimation of ice loads on fixed or compliant structures.

A great number of geometric and material parameters will affect the magnitude of ice sheet loads on conical structures; moreover, the displacement boundary conditions as well as the application of the external loading also influence the results. A special intricate problem is the pile up of rubble. To consider all these effects is beyond the scope of this study, which restricts attention to the effects of slope angle of the conical structure and the friction between the ice sheet and the surface of the structure.

The use of computerized spatial discretization techniques, such as the finite element method, in simulation of ice-structure interaction phenomena may relieve some of problems inherent in the existing empirical procedures. In particular, the finite element method can effectively be utilized to study the effects of different geometry and boundary conditions, varying velocity, shape and mass of ice features, and so forth. On the other hand, the practical value of results from numerical simulations depends on several factors of which the accuracy of the constitutive model is the most important. Thus, a close collaboration between scientists involved in experimental ice research and specialists in numerical analysis is a prerequisite for the successful application of nonlinear finite element analysis to ice-structure interaction problems.

In this study, the finite element method is adopted to calculate ice forces on conical shaped structures. The effects of material nonlinearities and friction between ice sheet and structure are taken into account. The ice is treated as a transversely isotropic or isotropic, nonlinear material capable of cracking and crushing and the contact interaction between the ice sheet and the structure is simulated with a contact algorithm that permits surface-to-surface contact with Coulomb friction sliding. The weight and buoyancy of the ice is taken accounted for by employing a nonlinear foundation model. The numerical results presented in the present study were obtained by use of the general finite element code ANSYS Release 5.6.

CONSTITUTIVE MODELING OF ICE BEHAVIOR

To be able to represent the behavior of ice over a broad range of strain rates, elastic, time-dependent (creep) and instantaneous, inelastic components of deformation must be considered. The resulting constitutive model is commonly referred to as a visco-plastic formulation. Here, only the instantaneous, inelastic part is considered and it is interpreted on the basis of the theory of plasticity. The theory of plasticity represents a general mathematical framework for the description of ductile failure of materials and it has been applied to a wide variety of materials, such as metals, polymers, concrete and soils. The theory of plasticity includes three basic sets of relations: a failure (yield) criterion, a flow rule and a hardening rule. With the modeling of ductile ice failure in mind, adequate experimental data are limited and do not seem to justify more sophisticated assumptions than associated flow rule and isotropic hardening. The major questions to be answered are related to the failure criterion which therefore will be the focal point in the subsequent discussion.

In general, the failure criterion can be defined in terms of the components of the Cauchy stress tensor, σ_{ij} and interpreted as a hyper surface in the six-dimensional stress space spanned by σ_{ij} . The majority of the proposed failure criteria for ice have been limited to isotropic ice, for which the failure criterion can be expressed in terms of the invariants of the stress tensor. Reinicke and Remer (1978) introduced an elliptic yield surface for granular, isotropic ice. This failure surface, which describes an elliptic strength increase with increasing hydrostatic pressure, is expressed as

$$f(I_1, J_2, k_1, k_2, k_3) = k_1 J_2 + k_2 I_1 + k_3 (I_1)^2 - 1 = 0 \quad (1)$$

where I_1 is the first stress invariant of the stress tensor σ_{ij} , and J_2 is the second stress invariant of the deviatoric stress tensor s_{ij} , and k_1 , k_2 and k_3 are material parameters that must be determined experimentally. In principal stress space, Eq. 1 represents a closed ellipsoidal surface and is often referred to as the three-parameter failure criterion. Since a linear term of I_1 is included in this criterion, different failure stresses in tension and compression can be predicted. The quadratic term of I_1 assures that a strength variation of elliptic form can be predicted on a hydrostatic plane. This is important as the failure criterion limits the ice strength at high hydrostatic pressure. This is illustrated in Fig. 1, where the shapes of the yield surface on a hydrostatic plane and on a deviatoric plane intersecting the yield surface are depicted. Note that the yield surface on a deviatoric plane is a circle, implying that yielding is governed by the shear distortion energy. Reinicke and Remer's (1978) failure criterion, given by Eq. 1, has been employed by Horrigmoe and Zeng (1994) to simulate indentation tests of ice sheets. The failure criterion was fitted to data reported by Jones (1978) and the following data was used to determine the three parameters in Eq. 1: uniaxial compressive strength $f_c = -4.5$ MPa, uniaxial tensile strength $f_t = 0.93$ MPa and a set of confined compression test on the strength of a cylindrical sample is $(p, q) = (9.18 \text{ MPa}, 16.39 \text{ MPa})$, see

Fig. 2 for notation. From these tests, the three material parameters in Eq. 1 were determined as follows: $k_1 = 0.674 \text{ MPa}^{-2}$, $k_2 = 0.853 \text{ MPa}^{-1}$ and $k_3 = 0.014 \text{ MPa}^{-2}$.

Figure 1. Failure surface for isotropic materials, see Eq. 1. (a) On a hydrostatic plane, (b) On a deviatoric plane

Figure 2. Confined compression test on cylindrical specimen.

The microstructure of columnar-grained ice produces anisotropic mechanical behaviour. Laboratory experiments, carried out by Kuhn et al. (1990), on columnar, saline ice (S2-type) conducted at a strain rate of 10^{-3} s^{-1} showed that the modulus of elasticity is greater (about 30%) along the columns than across the columns. In the same experimental study, it was found that the tensile strength along the columns was about four times greater than in the transverse direction. Gratz and Schulson (1994, 1997) reported results from compressive tests revealing that the uniaxial compressive strength was approximately three times greater along the columns than across the columns at a strain rate of 10^{-3} s^{-1} , whereas this ratio was reduced to 1.6 when the strain rate was increased to $6 \cdot 10^{-3} \text{ s}^{-1}$. Despite the fact that the strength properties of columnar ice depend on a number of parameters and that there is a certain scatter in the experimental data, there is ample evidence of the anisotropic behaviour. Thus, application of the isotropic failure criteria examined above is limited in scope. In order to achieve realistic predictions of the failure of columnar ice it is necessary to account for anisotropy.

Horrigmoe and Zeng (1994) proposed an anisotropic failure criterion of elliptic type. This criterion is capable of predicting different strengths in tension and in compression and also accounts for the nonlinear effects of hydrostatic stress on the multi axial strength of ice. This failure criterion is defined as

$$\begin{aligned}
f(\sigma_{ij}, a_1, \dots, a_{12}) = & a_1 \sigma_x^2 + a_2 \sigma_y^2 + a_3 \sigma_z^2 \\
& + a_4 \sigma_x \sigma_y + a_5 \sigma_y \sigma_z + a_6 \sigma_z \sigma_x \\
& + a_7 \sigma_x + a_8 \sigma_y + a_9 \sigma_z \\
& + a_{10} \tau_{xy}^2 + a_{11} \tau_{yz}^2 + a_{12} \tau_{zx}^2 - 1 = 0
\end{aligned} \quad (2)$$

Where a_i ($i = 1, 2, \dots, 12$) are material constants which must be determined by twelve independent tests. These tests are: (i) three uniaxial tension tests in the x -, y - and z -directions, (ii) three uniaxial compression tests in the x -, y - and z -directions, (iii) three simple shear tests along the xy -, yz - and zx -planes, and (iv) three confined compression test with confining pressure in the x -, y - and z -directions, respectively. See Fig. 3 for notation. The function, given by Eq. 2, may be interpreted as a closed hyper-ellipsoidal surface in the space spanned by the stress components σ_x , σ_y , σ_z , τ_{xy} , τ_{yz} and τ_{zx} as illustrated in Fig. 4. The linear normal stress terms present in the failure criterion allows different strengths in tension and in compression to be predicted in three different directions whereas the quadratic terms take account of the nonlinear effects of hydrostatic stress and limits the ice strength at high hydrostatic pressure. The anisotropic failure criterion defined by Eq. 2 is a second order differentiable function. This implies that when this criterion is used in conjunction with computer-based discretization procedures, e.g. the finite element method, a single subroutine that incorporates several material models can be written in compact format.

Figure 3. Multiaxial ice strength testing on columnar ice and cubic specimens

The hyper-elliptic failure criterion, Eq. 2, is valid for general orthotropic materials. Any type of isotropy, such as transverse isotropy (rotational symmetry) or complete isotropy, will reduce the number of material constants and, thus, the number of material tests required. For the case of transverse isotropy with the z -axis taken as the axis of rotational symmetry, the number of material constants reduces to seven and we have the following relations:

$$a_1 = a_2, a_5 = a_6, a_7 = a_8, a_{11} = a_{12}, a_{10} = 2a_1 - a_4 \quad (3)$$

No major difference has been found in shear strength along and normal to the columns of transversely isotropic ice (Cammaert and Muggeridge, 1988, Michel, 1978, Sanderson, 1988, Frederking and Timco, 1984). Thus, the shear strengths S_{yz} and S_{zx} , in the yz - and zx -planes, respectively, may assumed

to be equal to that in xy -plane, i.e. $S_{xy} = S_{zx} = S_{yz}$. This gives the following relation:

$$a_{11} = a_{12} = a_{10} = 2a_1 - a_4 \quad (4)$$

Columnar-grained ice of S2- type can be considered as transversely isotropic. Hence, the number of material parameters in the failure criterion, Eq. 2, reduces from twelve to six, cf. Eqs. 3. and 4. In general, test data are available for different stress ratios, temperatures, strain rates, etc., and the number of test data is much greater than the number of undetermined material parameters. There is considerable scatter in the test results and the failure criterion has been shown to be somewhat sensitive to the selection of test data to define the material parameters. Thus, the selection of a small subset of these data points to determine the six material constants does not constitute a viable option. For the purpose practical use, it is important that the failure criterion is representative for columnar ice over a certain range of grain sizes, temperatures and strain rates. All of this points to the use of some sort of optimization technique to determine the material parameters from a relatively large bulk of test data.

Figure 4. Hyper-elliptic failure criterion proposed by Horrignoe and Zeng (94), see Eq. 2.

If the material is completely isotropic, the general criterion (Eq. 2) becomes identical to the so-called three-parameter failure criterion introduced by Reinicke and Remer, see Eq. 1. The relations between the material parameters in the orthotropic failure criterion (Eq. 2) and those in the isotropic function (Eq. 1) are simply:

$$\begin{aligned}
a_1 = a_2 = a_3 = 2k_1 + k_3; & \quad a_4 = a_5 = a_6 = 2(k_3 - k_2) \\
a_7 = a_8 = a_9 = k_2; & \quad a_{11} = a_{12} = 6k_1
\end{aligned} \quad (5)$$

The criteria suggested by Hill (1950) and Pariseau (1968) can also be obtained by appropriately setting the material parameters in the general function given by Eq. 2.

Nonlinear least-square fitting of the general failure criterion, Eq. 2, can be obtained by several alternative procedures such as the Gauss-Newton method and the Levenberg-Marquardt algorithm (Matlab, 1999, Levenberg, 1944, Marquardt, 1963). The latter was chosen in the present study due to its superior robustness. Weights may be associated

with the individual data points according to their relative importance for the particular case under consideration. For a more comprehensive treatment on the fitting of the general failure criterion, Eq. 2, to test data for ice, the reader is referred to Sand (in preparation). Resulting values for the material parameters obtained by nonlinear least square fitting of Eq. 2 to experimental data reported by Schulson and co-workers (Kuehn, Lee, Nixon and Schulson, 1990, Nickolayev and Schulson, 1993, Schulson and Nickolayev, 1995, Smith and Schulson, 1994, Gratz and Schulson, 1994, 1997) are: $a_1 = 0.768 \text{ Mpa}^{-2}$, $a_3 = 0.108 \text{ Mpa}^{-2}$, $a_4 = -1.123 \text{ Mpa}^{-2}$, $a_5 = -0.117 \text{ Mpa}^{-2}$, $a_7 = 2.533 \text{ Mpa}^{-1}$, and $a_9 = 0.672 \text{ Mpa}^{-1}$. After the numerical least-square fitting, the uniaxial tensile and compressive strengths were calculated from Eq. 2, by using the conditions given by Eqs. 3 and 4. The analytical results for these strengths are summarized Table 1 together with the corresponding experimental values reported by Schulson and co-workers. The uniaxial tensile strengths in x-, y- and z-directions, are denoted by T_x , T_y , and T_z while the compressive strengths are denoted by C_x , C_y , and C_z , for notation see Fig. 3.

Table 1. Uniaxial compressive, tensile and shear strengths calculated from Eq. 2, by using the conditions given by Eqs. 3 and 4.

	$C_x = C_y$ [MPa]	$T_x = T_y$ [MPa]	C_z [MPa]	T_z [MPa]	$S_{xy} = S_{yz} = S_{zx}$ [MPa]
Eq. (2)	-3.65	0.36	-7.48	1.24	0.61
Exp.data	-3.62	0.35	-7.49	1.25	-

It is seen that the failure criterion, Eq. 2, by using the conditions given by Eqs. 3 and 4, predicts values for the uniaxial tensile and compressive strengths that are in very good agreements with the experimental data. Unfortunately, Schulson and co-workers have not conducted shear strength tests on this type of ice. Therefore, it is difficult to make any comparisons of the shear strength S_{xy} , S_{yz} and S_{zx} , in the xy-, yz- and zx- planes.

CRACKING AND CRUSHING TREATED AS TRANSFORMATION OF STATE

Ice exhibits two kinds of inelastic behavior under compression and the transition is defined macroscopically, in terms of the shape of stress-strain curve and the behavior of the material, see Fig. 5. At lower strain rates ($\dot{\epsilon} < \dot{\epsilon}_{D/B}$) the material is ductile and exhibits strain hardening, followed by strain softening, while at higher strain rates ($\dot{\epsilon} > \dot{\epsilon}_{D/B}$) ice behaves in a brittle manner. Accordingly, ice considered to be ductile when its stress-strain curve exhibits either a rounded peak or plateau and the material does not suddenly lose its load-bearing ability. Ascending and descending branches characterize its stress-strain curve, and plastic strain in excess of 0.1 can be obtained without macroscopic collapse. The ductile compressive strength or “peak” stress increases with increasing strain rate and with decreasing temperature. The “peak” stress decreases with increasing salinity/porosity, but exhibit little dependence upon grain size. Within the ductile regime, dislocation processes are

at play: simple atomic bond deformation and grain boundary sliding gives rise to immediate and delayed elastic deformations. Secondary creep is associated with dislocation glide and climb within grains. As a result of grain boundary sliding and dislocation movements, dislocations tend to pile-up at grain boundaries, which may lead to formation of cracks and this is one cause of tertiary creep. These processes are described and discussed elsewhere, see Sanderson (1988).

On the other hand, ice is considered to be brittle when its stress-strain curve is pseudo-linear in shape and terminates suddenly owing to the onset of a mechanical instability. At higher deformation rates, i.e. above 10^{-4} s^{-1} to 10^{-3} s^{-1} , ice exhibits brittle behavior. Under these conditions, an ascending branch only characterizes its stress-strain curve, and failure occurs after inelastic strain less than 0.003. Unconfined ice fails via longitudinal splitting, while confining ice fails by faulting or spalling, here termed as crushing. The terminal stress or failure strength is function of temperature, strain rate and grain size. The failure strength decreases with increasing temperature (by a factor of approximately 2.5 from -40°C to -10°C), with increasing strain (by a factor of 0.7 from 10^{-3} s^{-1} to 10^{-1} s^{-1}) and with increasing grain size (by a factor of 3 from 1 mm to 10 mm). The brittle strength of ice appears to be independent of the salinity/porosity encountered in sea ice, at least at -10°C .

Figure 5. Schematic sketch showing the effect of strain rate on the compressive stress-strain behavior of ice.

The transition between ductile and brittle behavior of ice is very important because it sets the conditions under which the compressive strength reaches a maximum and thus the force of a given ice feature against a engineered structure is also a maximum. The strain rate $\dot{\epsilon}_{D/B}$, which marks the transition between ductile and brittle behavior is between 10^{-4} s^{-1} to 10^{-3} s^{-1} at temperatures of practical interests (from -40°C to -5°C), but increases with confinement, and for columnar ice, with change of loading from parallel to perpendicular to the columnar grains. Because of the transition between ductile and brittle behavior of ice, the mechanical behavior can therefore be approximated from the ductile or the brittle end.

In the proposed approach for constitutive modelling of ice, the formation of crushing or cracking is treated as a transformation of state. In this manner, the mechanical behaviour of ice is approximated from the brittle end and it is treated as a rate-independent, elastic-brittle material. Under

predominantly tensile stress, cracking occurs if the stress exceeds the tensile strength of the solid virgin ice. After cracking has taken place, the material cannot sustain tensile stresses due to cracking of the material. When ice is subjected to compressive states of stress, a large number of small cracks form. Under increasing load, these cracks coalesce and produce crushed ice, in many cases resembling a powder. In virtually all ice-structure interactions, crushing has been observed, often combined with other failure modes such as flexural failure.

The idea described in this section is to employ two separate constitutive models to simulate the transformation of state due to cracking or crushing of the ice. The first constitutive model describes the material behavior of unbroken or virgin ice and the second one describes the behavior of crushed or cracked ice. The total strain rate $\dot{\epsilon}_{ij}$ is assumed to be the sum of elastic strain rate $\dot{\epsilon}_{ij}^e$ and inelastic strain rate $\dot{\epsilon}_{ij}^{cr}$:

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^e + \dot{\epsilon}_{ij}^{cr} \quad (6)$$

where the inelastic strain $\dot{\epsilon}_{ij}^{cr}$ is due to cracking or crushing of the ice. The stress rate $\dot{\sigma}_{ij}$ is related to the incremental elastic strain rate $\dot{\epsilon}_{ij}^e$ through Hooke's law:

$$\dot{\sigma}_{ij} = C_{ijkl} \dot{\epsilon}_{kl}^e \quad (7)$$

where C_{ijkl} is the fourth order symmetric tensor of elastic stiffness moduli. For virgin, transversely isotropic ice with the z-direction taken as the axis of rotational symmetry, Eq. 7 may be expressed in matrix notation as:

$$\begin{bmatrix} \dot{\sigma}_x \\ \dot{\sigma}_y \\ \dot{\sigma}_z \\ \dot{\tau}_{xy} \\ \dot{\tau}_{yz} \\ \dot{\tau}_{zx} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \dot{\epsilon}_{xx}^e \\ \dot{\epsilon}_{yy}^e \\ \dot{\epsilon}_{zz}^e \\ \dot{\gamma}_{xy}^e \\ \dot{\gamma}_{yz}^e \\ \dot{\gamma}_{zx}^e \end{bmatrix} \quad (8)$$

The elastic constants in the above equation are expressed by the elastic modulus in x- and z-direction E_x and E_z , Poisson's ratios ν_{xy} and ν_{xz} , shear modulus G_{xy} and G_{xz} as follows:

$$C_{11} = \frac{E_x}{E_R} (1 - \nu_{zx}^2 \frac{E_x}{E_z}), \quad C_{12} = \frac{E_x}{E_R} (\nu_{xy} - \nu_{zx}^2 \frac{E_x}{E_z}) \quad (9)$$

$$C_{13} = \frac{E_x}{E_R} (\nu_{zx} - \nu_{zx} \nu_{xy} \frac{E_x}{E_z}), \quad C_{33} = \frac{E_z}{E_R} (\nu_{zx} - \nu_{xy}^2)$$

$$C_{44} = G_{xy}, \quad C_{66} = G_{xz}$$

where

$$E_R = 1 - \nu_{xy}^2 - 2\nu_{xy}^2 \frac{E_x}{E_z} - 2\nu_{xy} \nu_{zx} \frac{E_x}{E_z} \quad (10)$$

For isotropic ice, the relation given by Eq. 8 reduces to the well-known expressions:

$$C_{11} = C_{33} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}, \quad C_{12} = C_{13} = \frac{E\nu}{(1+\nu)(1-2\nu)} \quad (11)$$

$$C_{44} = C_{66} = \frac{E}{2(1+\nu)}$$

where E and ν are the elastic modulus and Poisson's ratio, respectively. When the stresses σ_{ij} in the virgin ice reaches the failure criterion, i.e. $f = f(\sigma_{ij}) = 0$, cracking or crushing is said to occur. Hence, the tangent relation between the stress rate $\dot{\sigma}_{ij}$ and total strain rate $\dot{\epsilon}_{ij}$ is

$$\dot{\sigma}_{ij} = C_{ijkl}^c \dot{\epsilon}_{kl} \quad (12)$$

where C_{ijkl}^c is the fourth order symmetric tensor of elastic stiffness moduli similar to Eq. 8. The constants in Eq. 12 are expressed by the post failure modulus E^c Poisson's ratio ν^c of cracked or crushed ice as follows:

$$C_{11}^c = C_{33}^c = \frac{E^c(1-\nu^c)}{(1+\nu^c)(1-2\nu^c)} \quad (13)$$

$$C_{12}^c = C_{13}^c = \frac{E^c\nu^c}{(1+\nu^c)(1-2\nu^c)}$$

$$C_{44}^c = C_{66}^c = \frac{E^c}{2(1+\nu^c)}$$

In the finite element method, transformation of state can be represented by modeling the ice with dual sets of elements, which occupies the same volume by using coincident nodal points as illustrated in Fig. 6. The first set is the virgin elements and represent unbroken ice. The stress-strain relation of these elements depends on the structure of the ice. In the case of columnar ice, the stress-strain relation is given by Eqs. 8, 9 and 10. For isotropic ice, Eqs. 8 and 11 are employed. The second set of elements represents the post-failure behavior of cracked or crushed ice. The stress-strain relation of these elements is given by Eqs. 12 and 13. At the start of the analysis, the virgin elements (unbroken) are active and the damaged elements are deactivated. This is achieved by multiplying the stiffness of the damaged elements by a severe reduction factor, which is set equal to 10^{-6} . When the state of stress at a point reaches the failure surface, i.e. $f = f(\sigma_{ij}) = 0$, failure is said to occur and the virgin elements are deactivated and the damaged elements are reactivated simultaneously.

The concept of treating cracking and crushing as transformation of state is demonstrated by a uniaxial stress-strain curve drawn in Fig. 7. At the start of the analysis, the damaged element is deactivated and only the virgin element is active. The stress σ of the virgin element follows Hooke's law and is defined by the elastic modulus E of the virgin element. As the virgin element is deformed, the stress σ increases in a linear manner until the stress reaches the uniaxial tensile strength T_g . At this stress level cracking or crushing is said to occur, and the virgin element is deactivated and damaged element is reactivated simultaneously. The stress σ in the damaged element is calculated as $\sigma = E^c \epsilon$, where E^c post

failure modulus of the damaged element. Because the elastic modulus E of the virgin element is much greater than the post failure modulus E^c , a sudden drop in the material stiffness occur and the material can only sustain small stresses in the in the post cracking state.

Figure 6. Simulation of transformation of state by using virgin and damaged elements.

Figure 7. Uniaxial stress-strain curve demonstrating the concept of simulating cracking or crushing by transformation of state.

The present approach makes use of a failure surface in stress space to determine when cracking or crushing takes place. This approach is based on deactivation of virgin elements; the failure surface must be evaluated from the mean values of the stress components in each element. In the finite element method, the stresses within each element are computed in the Gaussian integration points. The mean values of the stress components σ_{mx} , σ_{my} , σ_{mz} , τ_{mxy} , τ_{myz} and τ_{mzx} are calculated based on the stresses in the Gauss integration points in the virgin elements in the following manner:

$$\begin{aligned} \sigma_{mx} &= \frac{1}{n} \sum_{i=1}^n \sigma_{x,i}, & \sigma_{my} &= \frac{1}{n} \sum_{i=1}^n \sigma_{y,i}, & \sigma_{mz} &= \frac{1}{n} \sum_{i=1}^n \sigma_{z,i} \\ \tau_{mxy} &= \frac{1}{n} \sum_{i=1}^n \tau_{xy,i}, & \tau_{myz} &= \frac{1}{n} \sum_{i=1}^n \tau_{yz,i}, & \tau_{mzx} &= \frac{1}{n} \sum_{i=1}^n \tau_{zx,i} \end{aligned} \quad (14)$$

where $\sigma_{x,i}$, $\sigma_{y,i}$, $\sigma_{z,i}$, $\tau_{xy,i}$, $\tau_{yz,i}$ and $\tau_{zx,i}$ are the stress components in Gaussian integration point number i and n is the total number of integration points in the element. For a

hexahedral element with eight nodal points, standard Gaussian integration implies $2 \times 2 \times 2$ integration points ($n = 8$).

KINEMATIC MODEL

The most commonly employed method to account for buoyancy forces on an ice floe is the use of an ordinary Winkler foundation. This is a satisfactory model provided that the ice sheet is not submerged or lifted out of the water. When large vertical motions of the ice sheet occur due to contact, as would be the case when the ice sheet interacts with a conical structure, a more sophisticated model is required, e.g. a continuous nonlinear foundation model, or discrete nonlinear springs attached to nodal points. The latter approach was employed in the present study to include the effects of buoyancy forces and specific weight of the ice sheet. This approach has earlier been employed by Sand and Horrignoe (1998) to simulate ice sheet forces on sloping structures. In the discrete, nonlinear spring model, as shown in Figure 8, the spring stiffness K_i is a function of the element area A_i , the densities of ice ρ_i (910 kg/m³) and water ρ_w (1027 kg/m³), ice thickness h and the vertical displacement w of the ice sheet.

Figure 8. Discrete, nonlinear spring model. Spring stiffness versus displacement.

In the present paper, the structure is considered as fixed and rigid. The contact interaction between the ice ridge and the structure may then be described by a model, which is based on finite sliding between a deformable body and a rigid body. Contact kinematics is concerned with the precise tracking of contact nodes and surfaces in order to define clear and unambiguous contact conditions. The primary aim is to distinguish between open (i.e. not in contact) and closed (in contact) situations. This task is accomplished by two different algorithms, the pinball algorithm and pseudo-element algorithm, see Kohnke (1998). Contact forces can be split into normal and tangential components. Forces normal to the structure are calculated with a combined penalty and Lagrange multiplier method Kohnke (1998). In this method, the contact stiffness is variable in order to achieve contact compatibility to a user-defined precision. Tangential forces are due to friction that arises as the nodal points on the ice ridge meet the target surface of the structure and slides along it. The most commonly used friction law is the Coulomb friction model. The sticking force limit of the Coulomb friction model is a function of the

friction coefficient and the contact forces normal to the target surface. This kinematic model has earlier been employed by Sand and Horrigmoe (2001) to simulate ice ridge forces on conical structures.

NUMERICAL RESULTS

The geometry of the conical structure is defined by the waterline diameter D , slope angle α , height above waterline $h_t = D/4$, and submerged depth $h_b = D/4$ as illustrated in Fig. 9a. The diameter $D = 11.6$ m, whereas three different values $\alpha = 45^\circ, 60^\circ$ and 75° were selected for the slope of the conical structure. The cone was modeled as a fixed and rigid structure with a total number of 28 eight noded hexahedral elements with three translational degrees of freedom associated with each nodal point. The element mesh of upward bending cone and the ice sheet is shown in Fig. 9b. The ice sheet had length L and width $2L$. In the numerical studies, $L = 45$ m was used together with an ice sheet thickness $h = 0.8$ m. The ice sheet was moved into contact with the conical structure by applying a uniformly distributed force in the y -direction along the edge ABC of the ice. The boundary conditions of the edge ABC were such as to keep the ice sheet in a horizontal position. The remaining edges of the ice sheet and ridge were free.

In the present study, it is assumed that the ice is made of either columnar or granular ice and the strength of ice is constant through the thickness of the ice sheet. When ice behaves in a brittle manner, cracking or crushing occurs during most interaction with structures. The formation of crushed or cracked ice may be seen as a phase change, where the ice changes from a solid to a cracked or crushed material as described earlier. To achieve this phase change, the ice sheet was discretized with a dual set of elements using an element mesh of 1344 isoparametric hexahedral elements to model the behavior of solid ice and equal number of elements to approximate the post failure behavior of ice.

In the case of granular, isotropic ice, Reinicke and Remer's failure criterion, Eq. 1 was employed and the incremental stress-strain relation of these elements is given by Eqs. 8 and 11. The following values for the elastic material constants were chosen: elastic modulus $E = 6$ GPa, shear modulus $G = 2.256$ GPa and Poisson's ratio $\nu = 0.33$. In the case of columnar ice (transversely isotropic) the z -direction normal to the plane of the ice beam is taken as the axis of rotational symmetry of the material properties. Therefore, the ice is isotropic in the x - y plane but its properties are different in the z -direction.

To predict failure of transversely isotropic ice, Horrigmoe and Zeng's (1994) hyper-elliptic failure criterion, was employed, see Eqs. 2, 3 and 4. The incremental stress-strain relation for transversely isotropic ice is given by Eqs. 8, 9 and 10 and the following values elastic constants are used: elastic moduli; $E_x = E_y = 6$ GPa, $E_z = 8.2$ GPa, shear moduli $G_{xy} = G_{zx} = 2.26$ GPa, and Poisson's ratios $\nu_{xy} = \nu_{zx} = 0.33$. Values for the elastic moduli and shear moduli and Poisson's ratios are based on test data for columnar saline ice at a strain rate of 10^{-3} s^{-1} .

The second sets of elements (1344 elements) are the damaged ones that represent the post-failure behavior of failed ice. The stress-strain relation of these elements is given by Eqs. 12 and 13 with post failure modulus $E^c = 20$ MPa and Poisson's ratio $\nu^c = 0.33$.

Figure 9. a) Geometry of upward bending cone. b) Geometry and element mesh of the ice sheet and upward bending cone

At the start of the analysis, the virgin elements are the active ones and the damaged elements are inactive. During the numerical simulations, the failure criteria were evaluated based on the mean values of the stress components in the finite elements given by Eq. 14. When the state of stress reaches the failure surface, i.e. failure is said to occur and the virgin elements are deactivated, and simultaneously, the damaged elements are reactivated. To obtain meaningful results, the finite element analysis is based on an updated Lagrange formulation. The geometries of the newly reactivated damaged elements are then evaluated in the current geometric configuration of the deactivated virgin elements. In this way the current strains in the virgin elements are transferred to the damaged elements.

Fig. 10 show force vs. displacement curves obtained during finite element simulations based on Horrigmoe and Zeng's failure criterion for transversal isotropic ice. Similar force vs. displacement curves obtained with Reinicke and Remer's failure criterion for isotropic ice are shown in Fig. 11. In these diagrams the horizontal force P_H and vertical force P_V are plotted versus the displacement in y -direction at point B of the ice sheet as shown in Fig. 9b. The slope angle of the upward bending cone was 45° and the diagrams also demonstrate the effect of different magnitudes of the ice-structure friction coefficient. In general, the horizontal and vertical forces increased as the magnitude of friction increases. When Horrigmoe and Zeng's failure criterion was employed, the maximum horizontal force increased by 23% from 1175 kN to 1451 kN as the friction increases from 0.0 to 0.1. But the maximum vertical force only increased by 1%, from 1289 kN to 1304 kN. Similar trend was also observed when Reinicke and

Remer's failure criterion was applied. The maximum horizontal force increases from 1242 kN to 1544 kN, i.e. increased by 23%. But, the maximum horizontal remains constant and was predicted to 1376 kN.

Figure 10. Horizontal and vertical ice forces vs. horizontal displacement (Horrigmoe and Zeng's failure criterion.)

Figure 11. Horizontal and vertical ice forces vs. horizontal displacement (Reinicke and Remer's failure criterion)

Contours of the calculated vertical displacements and deformed element mesh of the ice sheet at different load levels are shown in Figure 12. In this case Horrigmoe and Zeng's failure criterion was employed to predict failure of columnar ice. The cone angle and friction coefficient was set to 45° and 0.0, respectively. The development of the ice failure process or calculated ice failure patterns are portrayed in Figure 13 (top view in a-c and bottom view in d-f). The failure process starts as bending cracks are formed at the top and bottom of the ice sheet as shown in Figures 13a and 13d. The cracks start near the cone and divide the ice sheet into wedges as they propagate in radial direction. The radial cracking is followed by the formation of a circumferential crack as clearly seen at the bottom if the ice sheet as portrayed in 13d. Circumferential cracking of the ice sheet is characterized by the first change in the slope of the force-displacement curves as shown in Figure 10. Thereafter, the ice sheet rides further up the face of the cone, and the weight of the ice become more important and starts to affect the ice forces on the structure. This is seen as the increased stiffness of the force-displacement curves. Maximum

horizontal and vertical forces occurred as the front of the ice sheet reached the top of the cone as shown in Figure 12c. At this stage the ice sheet fails completely, as shown in Figures 1da and 13d, and the ice sheet can be considered broken into ice blocks. A similar trend in the ice failure process was also calculated when Reinicke and Remer's failure criterion was employed to predict failure of granular ice.

Figure 12. Side view of the deformed element mesh showing contours of vertical displacements at different load levels

Figure 13. Calculated failure pattern at different load levels obtained by Horrigmoe and Zeng's failure criterion; slope angle $\alpha = 45^\circ$ and friction coefficient $\mu = 0.0$.

COMPARISON OF NUMERICAL RESULTS AND ANALYTICAL SOLUTION

To verify the numerical results obtained during finite element simulations, ice sheet forces were also calculated using the plastic limit analysis proposed by Ralston's (1980). Ralston's model is based on the assumption that the ice behaves in an elastic ideal-plastic manner and Johanson's moment yield criterion is applied. This moment yield criterion depends on the

flexural strength σ_f of the ice sheet, which was set equal to 0.71 MPa. In Ralston's approach, it is assumed that the conical structure is totally surrounded by an ice sheet of infinite size. As shown in Fig. 9b the part of the ice sheet behind the conical structure were neglected when the ice sheet was discretized by finite elements. Therefore, the ice forces associated with the formation of side cracks is neglected in Ralston's model. In this manner, Ralston's approach is closer to the finite element model employed to simulate the ice sheet interaction with a conical structure.

Fig. 14 shows comparisons between maximum horizontal forces obtained by Ralston's method and by finite element computations as function of the slope angle for the upward bending cone. This diagram also demonstrates the effect of varying the ice-structure friction coefficient. When Horrigmoe and Zeng's criterion is applied to model the behavior of columnar, transversal isotropic ice, the horizontal force is 1% greater than obtained by Ralston's method. As the cone angle becomes steeper ($\alpha = 70^\circ$) Ralston's method predicts from 24% ($\mu=0.0$) to 31% ($\mu=0.1$) greater horizontal forces than Horrigmoe and Zeng's criterion. Similar trend was also observed for granular, isotropic ice. Reinicke and Remer's failure criterion predicts from 5% ($\mu=0.1$) to 7% ($\mu=0.0$) greater horizontal forces than Ralston's method. As the cone angle becomes steeper, Ralston's method resulted in horizontal ice forces that are 15% ($\mu=0.1$) and 22% ($\mu=0.0$) greater.

Figure 14. Comparison of horizontal forces on upward bending cone obtained by Ralston's method and by finite element simulations.

Values for the vertical forces on upward bending cone are presented as function of cone angle and friction coefficient in Fig. 15. In general, Ralston's method predicts higher vertical forces than obtained by finite element simulations. In the case of columnar, transversal isotropic ice, Horrigmoe and Zeng's failure criterion predicts from 15% ($\alpha = 70^\circ$) to 34% ($\alpha = 70^\circ$, $\mu=0.0$) and 39% ($\alpha = 70^\circ$, $\mu=0.1$) lower vertical ice forces than obtained by employing Ralston's method. Similar trend was also observed for granular, isotropic ice. Reinicke and Remer's failure criterion predicts from 10% ($\alpha = 70^\circ$) to 25% ($\alpha = 70^\circ$, $\mu=0.0$) and 32% ($\alpha = 70^\circ$, $\mu=0.1$) lower vertical ice forces than obtained by employing Ralston's method.

Figure 15. Comparison of vertical forces on upward bending cones obtained by Ralston's method and by finite element simulations.

Even if there is a three-dimensional stress state in the ice sheet in vicinity of the conical structures, bending and buckling are dominated by tensile failure. This is motivated by the fact that the uniaxial compressive strength of ice is 4-5 times the uniaxial tensile strength. The in-plane tensile strength of granular ice was 0.93 MPa, while the in-plane tensile strength of columnar ice was 0.35 MPa. The tensile strength of granular ice is therefore 2.7 times the in-plane tensile strength of columnar ice. It is therefore obvious that Reinicke and Remer's criterion lead to larger ice forces than obtained for Horrigmoe and Zeng's failure criterion. But it is difficult to explain the reason for the fact that Reinicke and Remer's predicts maximum forces on upward bending cone that are only 6% to 14% larger compared to Horrigmoe and Zeng's criterion. The deviation between the ice forces predicted by Ralston's method and those obtained during finite element simulations might be due to several factors. In the present finite element approach, the complex constitutive behavior of ice is formulated as elastic-brittle materials. In Ralston's approach, the ice is treated as an elastic, ideal-plastic material and the behavior is therefore considerable more ductile compared to the elastic brittle approach adopted in the finite element simulations. In addition, Ralston assumed full contact between the ice sheet and structure. Observations of the ice sheet failure processes obtained during the finite element computations show that the effective contact area of the ice sheet decreases as the advancing ice sheet slides along the surface of the conical structure. In Ralston's method, in-plane forces are neglected and only consider bending failure of the ice sheet and therefore do not consider buckling of the ice sheet. In the present study, all the finite element computations predicted a combined buckling-bending mode of failure. In the case of upward bending cone, the upward movement of the ice sheet induces high buckling stresses, which in turn lead to initiation of radial surface cracks at the top of the ice sheet. Bending failure, however, induces by high stresses at the bottom of the ice sheet and leads to the

formation of circumferential surface cracks at the bottom of the ice sheet.

CONCLUDING REMARKS

The present paper has been devoted to the development of nonlinear finite element simulations as a means of obtaining ice sheet forces on conical structures. The effects of material nonlinearities and friction between ice and structure are taken into account. The key ingredients of the present approach have been described, i.e., realistic representation of the complex constitutive behavior of ice, accurate tracking of contact between the ice sheet and the structure including friction, and computation of buoyancy forces on a partially or completely submerged ice sheet.

The analysis reported herein deal ice forces on upward bending cones. Particular attention was given to the effect of varying cone angle and friction coefficient between the ice and the structure. The numerical results have been compared with an analytical procedure based on plastic limit analysis. The finite element results are in good agreement with the analytical solutions. It is believed that nonlinear finite element simulations can be developed into a reliable tool for predicting ice forces on offshore structures. Further research is needed to improve the constitutive model of ice to include the effect of strain rate on the ice strength.

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