What does the fragment size distribution of blasted rock look like?

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ABSTRACT: Rock fragmented by blasting follows a different size distribution than the commonly used Rosin–Rammler distribution, especially in the fines range. Sieving data from many different blasting operations have led to a new distribution function, which can describe the fragmentation over the range 1–500 mm quite accurately. It is called the Swebrec© function and works also for crushed rock. It contains three parameters, the median fragment size \(x_{50}\), the maximum fragment size \(x_{\text{max}}\) and an undulation parameter \(b\), which is shown practically to be a function of the other two. The fit to the sieved data is often better than \(r^2 = 0.995\) over 2–3 orders of magnitude of fragment size. The Swebrec function could be used to estimate the complete size distribution from a sample in the fines range or the fines tail from coarse range data. It is used to construct a blast fragmentation model, the KCO model, which improves greatly on the Kuz–Ram model. In a first application it is shown how, in a given production situation, it is possible to predict the change in average fragmentation that is required to increase (or decrease), for example, the road base fraction 0–32 mm in the muckpile by a desired amount.

1 INTRODUCTION

The whole fragment size distribution of a full-size blasted round has only been measured on a few occasions. Sieving is so costly that it is hard to motivate. If only a part of the muckpile is sieved, the matter of a representative sample arises as, for example, coarse material often stays on top. Nevertheless, sieving is the norm, which decides if, for instance, belt material meets the requirements of a customer.

Hence, much effort has been spent on developing fast indirect methods to measure fragmentation, going from a comparison of the appearance of the muckpile with calibrated photos (van Aswegen & Cunningham 1986) to an analysis of images of the muckpile or parts of it.

A survey of image analysis methods has been carried out by Ouchterlony (2004). A number of the conclusions are listed in Appendix 1. The most important ones are firstly that it is more difficult to measure material in muckpiles than material on a belt and that the size range seldom exceeds a ratio of 20–30 factors between the largest and the smallest fragment. Given a boulder size of about 1 m, the resolution would be 30–50 mm, and smaller fragments could not be measured without artifices like ‘zoom merge’ or fines correction techniques.

Perhaps the best-known image analysis systems like Split® (Kemeny et al. 1999) and WipFrag® (Maerz & Zhou 2000) have such possibilities. The extrapolations that are made are by necessity based on assumptions about the mathematical form of the size distribution. These are, however, much more often erroneous than correct, as this paper will show.

SveBeFo (Swedish Rock Engineering Research) has studied blast fragmentation for some time (see, for example, Gynnemo 1999, Svahn 2003, Olsson et al. 2003). The focus has been on how the fines part is generated and how it could be controlled. The same was true for the EU-funded project ‘Less Fines Production in Aggregate and Industrial Minerals Industry’ (Project GRD-2000-25224). A number of papers from this project were presented at the Third EFEE Conference in Prague (Holmberg 2003).

The work in the ‘Less Fines’ project entailed both blasting of cylindrical lab models and full-scale production blasts in three quarries.
The information gathered from SveBeFo’s research and the ‘Less Fines’ project has resulted in a new size distribution function for fragmented rock and a considerably revised fragmentation model (Moser et al. 2003b, Ouchterlony 2003 & 2005).

2 NEW FRAGMENT SIZE DISTRIBUTION, THE SWEBREC FUNCTION

The fragment size distribution commonly used to describe the cumulative mass or volume of blasted rock passing through a square mesh of given size \( x \) is the Rosin–Rammler (RR) or Weibull function, which may be written as

\[
P_{RR}(x) = 1 - e^{-\left(\frac{x}{x_{50}}\right)^n} = 1 - 2^{-\left(\frac{x}{x_{50}}\right)^n} \quad (1)
\]

It contains two parameters, the average (median) fragmentation \( x_{50} \) and the uniformity exponent \( n \), which is a measure of the slope of the distribution. It is a key part of the Kuz–Ram model (Cunningham 1987) – see Appendix 2. The Kuz–Ram model is probably the most referenced fragmentation model for rock blasting, especially if its spin-offs, the CZM and TCM models (Kanchibotla et al. 1999, Djordjevic et al. 1999), are included.

The models have equations for how \( x_{50} \) and \( n \) depend on specific charge, rock properties, rock mass jointing, blast pattern, etc. There are very few cases showing that the muckpile actually looks like this when sieved. One is given in Figure 1 (Kojovic et al. 1995).

![Figure 1](image1.png)

Figure 1. Size distribution after bench blasting of hornfels, together with the fitted Rosin–Rammler curve (Kojovic et al. 1995). Data range 0.35–2000 mm. Curve parameters: \( x_{50} = 116 \) mm and \( n = 0.572 \) with \( r^2 = 0.9958 \).

The sieving data cover nearly four orders of magnitude, which is unusual. The fitting curve reproduces the data with a very high accuracy. The Rosin–Rammler distribution is characterized by a straight-line asymptote with slope \( n \) in a log–log diagram for small fragment sizes. The corresponding asymptote \( (x/x_{max})^n \) is often called the Gaudin–Schuhmann distribution.

The sieved volume, however, consisted only of two truck loads of 32 t, one each from a round of about 70,000 t, so the room for errors is relatively large.

My experience is that most distributions deviate from the RR behaviour in two major ways: firstly there is naturally a largest boulder size \( x_{max} \), and secondly the fines tail is not linear in a log–log diagram (see Fig. 2).

The figure shows data from the Bårarp dimension stone quarry where monitored bench blasting tests were carried out (Olsson et al. 2003, Moser et al. 2003b). The seven single-row rounds contained 250–400 t each and were blasted with the same specific charge, about 0.55 kg/m\(^3\), but using different hole diameters, \( \varnothing = 38–76 \) mm. The bench was about 5 m high and 12–15 m wide. All material down to 25 mm was sieved; the rest was sampled. The lower set of curves in Figure 2 show the results.

![Figure 2](image2.png)

Figure 2. Size distributions from bench blasts and model blasts in granitic gneiss, Bårarp (Olsson et al. 2003, Moser et al. 2003b – Fig. 13). These curves are typical of blasted rock.

The upper set of curves was obtained from lab blasting of cylinders with different diameters, \( \varnothing = 100–290 \) mm, but with the same linear charge concentration in each, 20 g/m of PETN approximately.

None of the size distributions displays linear RR behaviour, except possibly in the ultrafine –
0.5 mm region. They look quite similar, especially in the range –20 mm. The model blasting curves appear to be parallel displaced in the vertical direction, just like the bench blasting curves. This behaviour is referred to as the natural breakage characteristics (NBC) (Steiner 1998, Moser et al. 2003b).

Hundreds of sieved size distributions from bench blasting in quarries, reef blasting, model blasting and crushing of many different rock types and concrete and mortar were studied by Ouchterlony (2003). They look like the curves in Figure 2, almost without exception.

A function that reproduces the main behaviour well is the Swebrec function

\[
P(x) = \frac{1}{1 + [\ln(x_{max}/x)/\ln(x_{max}/x_{50})]^b} \]

Function \(P(x)\), valid when \(0 < x < x_{max}\), is the cumulative distribution function, which may take any value in the range 0–1. The logarithm term is chosen so that its value is 0 when \(x = x_{max}\) and 1 when \(x = x_{50}\). The necessary conditions \(P(x_{max}) = 1\) and \(P(x_{50}) = 0.5\) are thereby met.

The Swebrec function has three parameters, \(x_{max}\), \(x_{50}\) and the curve undulation exponent \(b\). Parameter \(x_{max}\) limits the fragment size. The logarithm term dominates for small \(x\) values.

Take round 4 from Bårarp (Olsson et al. 2003). Six Ø51 mm holes were drilled in a 5.2 m high bench. The burden and spacing were 1.8 and 2.1 m respectively, the subdrilling 0.3 m and the charge length 4.2 m. The specific charge was 0.55 kg/m³ of Kemix emulsion explosive. A total of 309 t of rock was blasted and sieved (see Fig. 3).

The figure shows that in the data range 0.5–500 mm, three orders of magnitude in fragment size, deviations are less than 1.8% and the coefficient of determination is better than \(r^2 = 0.997\). Fitting the Swebrec function to the data for the other Bårarp rounds gives equally good results.

It is also possible to reproduce the fast decaying size distribution in the ultrafine range –0.5 mm by extending the Swebrec function with an extra term containing two parameters, the factor \(a\) and the exponent \(c\):

\[
P_2(x) = \frac{1}{1 + a}\left[\frac{\ln(x_{max}/x)/\ln(x_{max}/x_{50})}{\ln(x_{max}/x_{50})}\right]^b + (1 - a)\left[\frac{(x_{max}/x - 1)/(x_{max}/x_{50} - 1)}{\ln(x_{max}/x_{50})}\right]^c \]

The new term has the effect that \(P_2(x)\) behaves like a Gaudin–Schuhmann function \((x/x_{max})^c\) for small values of \(x\). It retains the properties \(P_2(x_{max}) = 1\) and \(P_2(x_{50}) = 0.5\) though.

The extended Swebrec function could be used for estimating the cumulative area of the fragments in a muckpile as the ‘surface integral’ between 0 and \(x_{max}\) based on \(P(x)\) does not converge. Furthermore, surface measurements for model blasting tests like those displayed in Figure 2 show that 75–80% of the fresh surface resides on grains smaller than 0.1 mm, and 85–90% on grains smaller than 1 mm (Moser et al. 2003a).

Three other examples of size distributions for blasted rock are given in Figures 4a–c.

An expression that will be used below is the slope \(s_{50}\) of the size distribution at \(x_{50}\):

\[
s_{50} = \frac{P(x_{50})}{P(x_{max})} = \frac{b}{4\ln(x_{max}/x_{50})} \]

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\[
s_{50} = \frac{P(x_{50})}{P(x_{max})} = \frac{b}{4\ln(x_{max}/x_{50})} \]
Presumably this slope value is easier to determine accurately than, for example, \( x_{\text{max}} \). Firstly, the number of fragments in a muckpile decreases with increasing size, and eventually the distribution ceases to be continuous. Secondly, the function \( P(x) \) normally joins the line \( P = 1 \) at a tangent, which makes the fitting procedure sensitive. The corresponding expression for the RR equation is

\[
P_{\text{RR}}(x_{50}) = \frac{n}{2 \ln 2x_{50}}.
\]

Common expressions for the range of variance of the Swebrec function such as \( x_{80}/x_{30} \) or \( x_{80}/x_{50} \) may be easily derived. The mesh size corresponding to a percentage passing \( P_p \) is given by

\[
x_p/x_{\text{max}} = (x_{50}/x_{\text{max}})^{(1/P_p - 1)/b}
\]

By choosing \( p = 30 \) or 80%, i.e. \( P_p = 0.3 \) or 0.8, the correct expression is obtained. As can be seen, the dependence on \( b \) and \( x_{\text{max}} \) is relatively complicated. It becomes easier to work with the expression for the slope, \( P'(x_{50}) \).

In spite of a literature search, I have not found any use of the Swebrec function to describe the fragment size distribution of blasted or crushed rock. I chose the name of the newly founded competence centre for blasting at Luleå University of Technology for it. Some examples of how it may be used are given below.

3 PRACTICAL APPLICATION OF THE SWEBREC FUNCTION

The first case concerns incomplete data. The Swebrec function could be used to predict the whole distribution. Again, consider round 4 from Bårarp. The sieving was done in three steps. Firstly, the boulders were put aside. The whole muckpile was then run through a rotary Hercules sizer, which gave \(-200, 200–350, 350–400, 400–500 \) and \(+500 \) mm fractions. The \(-200 \) mm material was run through an Extec sizer, which gave \(0–25, 25–90, 90–120 \) and \(+120 \) mm fractions. The \(0–25 \) mm fraction was quartered in two steps, and samples were lab sieved so that a total of 19 fractions between \(-0.075 \) mm and \(+500 \) mm were obtained.

If only coarse range data were known, \(90–500 \) mm, a Swebrec function fit to the data could be used to estimate the amount of fines. The result is shown in Figure 5, together with the corresponding RR fit.

![Figure 4b](image1.png)

**Figure 4b.** Size distribution data for reef blasting in quartzite with gelatine. Data range 0.6–215 mm. Swebrec function parameters: \( x_{50} = 37.8 \) mm, \( x_{\text{max}} = 215 \) mm and \( b = 2.067, r^2 = 0.9962 \) (Cunningham 2003).

![Figure 4c](image2.png)

**Figure 4c.** Size distribution data for bench blasting in dolomite with dynamite. Data range 9.5–457 mm. Swebrec function parameters: \( x_{50} = 147 \) mm, \( x_{\text{max}} = 571 \) mm and \( b = 2.150, r^2 = 0.9990 \) (Otterness et al. 1991).

![Figure 5](image3.png)

**Figure 5.** Estimating the amount of fines in a muckpile, based on coarse range data (filled symbols). Compare the use of the Swebrec function with the use of the RR function.
pict those that were not used. The Swebrec function makes a good fines prediction down to about 0.1 mm, while the RR function already deviates considerably from the measured data at 25 mm.

An estimate of the amount of –4 mm fines in the muckpile, using the Swebrec function, yields about 2%, while the RR estimate would be only 0.4%, the correct value being about 2.5%.

Conversely, if only a sieved fine material distribution and its percentage of the whole muckpile were known, then the coarse range could be predicted. Figure 6 shows the result of fitting the Swebrec function to the 0–22.4 mm material data from round 4 at Bårarp.

Figure 6. Estimating the coarse range data from knowledge of the fines range. In the figure, 1–22.4 mm data were used and $x_{\text{max}}$ was set equal to the burden.

For the coarsely blocky granitic gneiss at Bårarp, a burden value of 1.8 m was used as an estimate of $x_{\text{max}}$. Note that only the concave part of the fines data was used – the 0.075–0.5 mm data were excluded. The reproduction of the coarse range data is excellent.

In theory it may be sufficient to have a sample of the fines material of a muckpile plus an estimate of its percentage of the whole. Of course a 0–90 mm sample is better than one covering 0–22.4 mm. Furthermore, a correct notion of the size of $x_{\text{max}}$ will steer the curve fitting. A first estimate of $x_{\text{max}}$ is probably obtained by taking the largest in situ block size if this value is smaller than the burden.

The procedure for estimating the coarse range data probably works better on crushed rock on a belt. Firstly, $x_{\text{max}}$ may be estimated from the crusher settings, and secondly there are often belt scales that give certain fraction weights.

Figure 7. Estimating the coarse range data of crushed granite, based on the 0.5–22.4 mm fines. $x_{\text{max}} \approx 300$ mm and a fines percentage of 18% were used. The resulting fitting parameters became $x_{50} = 77$ mm and $b = 2.33$.

Figure 7 shows an example of how sieving samples for a classified roadbed fraction of crushed 0–32 mm material of granite in the Vändle quarry have been used to estimate the coarse range data (Optimal fragmentation 2004).

Our work at the Nordkalk quarry (Ouchterlony et al. 2003) shows that caution is needed, though, when the product on the belt consists of blasted and crushed rock, i.e. the flow is bimodal. In this case the –250 mm material is scalped with a roller grizzly at the crusher intake; it bypasses the crusher and is fed to the belt before the crushed product is put on. Even if the flow is mixed at two belt stations, the bimodal character is still present.

In this case the 1–100 mm fraction is described by the Swebrec function to within 0.75%. An estimate of the amount of material passing the 200 mm mesh would be much too high, however (see Fig. 8). The curve predicts more than 90% passing when the sieved value is less than 80%.

This experience injects a note of caution in the analysis of muckpiles from rounds with two vastly different fragmentation conditions, such as a long uncharged stemming part and a charged part with very different rock mass properties.

It is worth noting that the Swebrec could reproduce the main part of the size distribution in spite of its bimodal character. We conclude, firstly, that the Swebrec function is robust, and secondly that the blasted and crushed distributions are reasonably similar in character.

A comparison of Figures 4-8 shows that the size distributions for blasted and crushed rock look quite similar. This was also my experience in earlier work (Ouchterlony 2003), where data were
obtained from many types of rock, both blasting and crushing.

Another example of the robustness of the Swebrec function is given by Svahn’s data (2003). She blasted three Ø300 mm cylinders of mortar with 40 g/m PETN cord. The cylinders consisted of three layers of differently pigmented mortar: black, yellow and green. After blasting in a closed container, the fragments were collected, sieved and colour separated. The resulting curves are shown in Figure 9.

The fragment size distributions for each layer, \( P_b, P_y \) and \( P_g \) are reproduced nicely by the Swebrec function, \( r^2 < 0.9978 \) over the range 0.25–32 mm (Ouchterlony 2003). The composite fragment size distribution \( P_{\text{tot}} \) was equally well reproduced by the Swebrec function, even if the weighted average of the three individual functions does not add up to the composite function mathematically, \( a_1P_b + a_2P_y + a_3P_g \neq P_{\text{tot}} \) with \( a_1 + a_2 + a_3 = 1 \).

It may also be possible to use the Swebrec function together with an image analysis instrument, which provides data for the coarse material. At Nordkalk (Ouchterlony et al. 2004) the size range is 63–1600 mm for an installation that captures truck loads prior to crushing, and 12.5–315 mm for the one that is mounted over the belt.

A comparison with Figure 8 shows that, if image analysis could give a proper representation of the 20–315 mm material, then an extrapolation based on the Swebrec function would increase the measured size range down to about 1 mm, i.e. in practice double the relative range.

The analysis of more than 25 belt cuts shows that the size distribution over the interval 0.5–100 mm could be reproduced by a Swebrec function where the \( x_{50} \) value lies in the range 48–108 mm while \( x_{\text{max}} = 315 \) mm and \( b \approx 2.25 \pm 0.10 \), i.e. is practically constant. It is not known if the same regularity applies to other streams of crusher product.

4 PREDICTION EQUATION FOR THE SIZE DISTRIBUTION OF BLASTED ROCK

The Kuz–Ram model of Appendix 2 contains three basic equations apart from the RR distribution itself, the most important being that for \( x_{50} \). Therefore we may write

\[
x_{50} = AQ^{1/6}(115/s_{\text{ANFO}})^{10/30}q^{0.8} \quad (6a)
\]

\[
A = \text{function of rock mass properties} \quad (6b)
\]

\[
n = \text{function of the geometry of blastholes,) \quad (6c)
\]

hole pattern and charges.

where \( Q \) = charge weight per hole, kg; \( q \) = specific charge, kg/m\(^3\); and \( s_{\text{ANFO}} \) = weight strength relative to ANFO. The rock factor \( A \) depends primarily on the jointing of the rock mass and the orientation of these joints. The uniformity exponent \( n \) depends on the hole diameter, the spacing to burden ratio \( S/B \), the charge distribution and the drill hole deviations. Parameters \( n \) and \( x_{50} \), according to Equations 6a–c, would thus be independent of each other, which is contradicted by our data.

Most people would agree that the \( x_{50} \) equation describes reality reasonably well. Practical evidence that the \( A \) and \( n \) equations are correct is
lacking, however, particularly in the case of $n$. Several versions of the $A$ and $n$ equations are given in the literature (see, for example, Kanchibotla et al. 1999, Djordjevic et al. 1999).

Cunningham (2003) sees the Kuz–Ram model more as a tool to structure experiences and to think of how different parameters could influence the blast fragmentation than as a quantitative prediction model.

As experience clearly shows that the RR function normally does not do a good job of reproducing the blast fragmentation, primarily in the fines range, it could be replaced with the Swebrec function. One way of doing this is to prescribe the same slope value at $x_{50}$ for the two size distributions.

With Equation 4 and the corresponding expression for the RR function, this gives the condition

$$b = n \cdot 2 \ln 2 \ln \left( \frac{x_{\text{max}}}{x_{50}} \right)$$  \hspace{1cm} (7)

If this expression and the other equations of the Kuz–Ram model are applied to round 4 from Báráp, the result in Figure 10 is obtained. Note that it shows the curves in lin–log scale so that the slope values at $x_{50}$ become more visible. The predicted value is very close to the measured value, so the model reproduces the data quite well.

$$s_{50} = 0.25 / x_{50}^{0.75} \text{ for cylindrical models}$$  \hspace{1cm} (8b)

Figure 11 shows the results from blasting of amphibolite, dolomite, granitic gneiss, limestone, dolomite and quartzite.

As $s_{50}$ depends on $n$, Equations 8a, b show that $x_{50}$ and $n$ are directly coupled, not independent of each other as the Kuz–Ram model would have us think. The use of Equations 4 and 8a yields

$$b \approx 0.5 x_{50}^{0.25} \ln \left( \frac{x_{\text{max}}}{x_{50}} \right)$$  \hspace{1cm} (9)

This shows that the development of a better fragmentation prediction model should focus primarily on the $x_{50}$ and $A$ equations, and on the equation for $x_{\text{max}}$. This last quantity could probably be related to the maximum block size of a blocky rock mass or the burden in a massive one.

An analysis of the fragment size distribution data from model and full-scale blasting that were used to confirm the Swebrec function (Ouchterlony 2004 & 2005) also show, within engineering accuracy, that the slope value at $x_{50}$, $s_{50}$ (1/mm with $x_{50}$ in mm) is given by

$$s_{50} = 0.12 / x_{50}^{0.75} \text{ for full-scale blasting}$$  \hspace{1cm} (8a)

Figure 11. Relationship between the slope $s_{50}$ at $x_{50}$ and $x_{50}$ itself.

The regularity expressed by Equations 8a, b and 9 is unexpected, and I presently lack a good physical explanation. Had the exponent in Equations 8a, b been equal to 1.0 instead of 0.75, it would have corresponded to a similarity transformation. In earlier work (Ouchterlony 2004) there are physical explanations that apply primarily to the fines region. Here, a practical consequence of these equations is described. Consider how fragmentation changes at a given site (see Fig. 12).

Assume that the average fragment size distribution in a quarry is known for a given blasting procedure, at least concerning $x_{50}$, $x_{\text{max}}$ and a fines measure like the amount of –32 mm or –64 mm material. With these data, an estimate of $b$ can be made and in principle the whole size distribution is known.

Equation (9) now makes it possible to describe the consequences of a changed blasting procedure in the following way.
Figure 12. Variations in the fragment size distribution in a given quarry – the Bårarp example. When \( x_{50} \) changes, the slope at \( x_{50} \) changes according to Equation 8a.

Figure 12 implies that, if \( x_{50} \) is decreased from 300 to 200 mm, then the percentage of –32 mm material will increase from about 9% to about 13%. Conversely, it can be inferred that, if we wish to decrease the amount of –64 mm material from 22% to 15%, then \( x_{50} \) should be increased from 200 to 300 mm.

Other numbers could be used but the reasoning is the same. Given a wanted change in either \( x_{50} \) or the percentage of some finer fraction, it is possible to read off how much the other quantity will have to change. Equation 9 does not tell how to achieve these changes, however, as that requires either production experience or a fragmentation model that is calibrated with respect to specific charge, etc.

If we were to merge the information from the Kuz–Ram model and what has been presented here, the RR function and the \( n \) equation would have to be replaced and the following would remain:

\[
P(x) = 1\left\{1 + \left[\ln(x_{\text{max}}/x)/\ln(x_{\text{max}}/x_{50})\right]^b\right\}, \\
0 < x < x_{\text{max}}
\]  
\( x_{50} = AQ^{1/6} (115/S_{ANFO})^{19/30} / q^{0.8} \) \text{ (in mm)}
\( A = 0.6(\text{RMD} + \text{RDI} + \text{HF}) \)
\( b \approx 0.5x_{50}^{0.25} \ln(x_{\text{max}}/x_{50}) \)
\( x_{\text{max}} = \text{min (in situ block size S}, S \text{ or B)} \)

By analogy with earlier work (Ouchterlony 2005) we might call this set of equations the new Kuznetsov–Cunningham–Ouchterlony or KCO model.

Some data indicate that Equation 8a may also be valid for certain types of crushing, maybe with a slightly different numerical prefactor. If so, the equation could be used to predict, for example, the effect on the fines percentage in the product of changes in the crusher settings, or vice versa.

The same reasoning is, in principle, also valid for the boulder volume in the muckpile, e.g. how much must \( x_{50} \) decrease for this volume to be halved? In practice, however, the uncertainties in this part of the distribution curve are probably much greater. One reason is that the accuracy in the determination of \( x_{\text{max}} \) is poorer than in the determination of \( x_{50} \), and another is that we do not really know yet how \( x_{\text{max}} \) depends on, for example, the specific charge.

5 CONCLUSION

Studies on the sieved size distribution of blasted rock have shown that the new Swebrec function will reproduce it quite well in the size range 1–500 mm. The main improvements on previous work lie in the fines range and the limited fragment size. The same function describes well a large number of sieved size distributions of crushed rock.

The Swebrec function has three parameters: the average (median) fragment size \( x_{50} \), the maximum fragment size \( x_{\text{max}} \) and the curve undulation parameter \( b \). Sieving data show that the parameters are not independent, and an interrelation is proposed.

In addition to the improved reproduction of sieved fragment size distributions, the Swebrec function may be used to predict the whole size distribution from sieve samples. Fines samples may be used to predict coarse range data, and vice versa. The function should also make it possible to improve the relatively modest size range of image analysis methods.

The relationship between the parameters \( x_{50} \), \( x_{\text{max}} \) and \( b \) may be used to predict the effect of changed blasting practices in a quarry. It would be possible to predict how much the average fragmentation should change in order to give a desired change in a given finer fraction, or vice versa. A similar prediction for the amount of oversize is possible in principle, but probably not accurate enough for practical purposes.
The new information makes it possible to improve on the Kuz–Ram model on two counts: an improved distribution function and replacing the $n$ equation by one that relates $b$ to $x_{50}$ and $x_{\text{max}}$. The new model is called the KCO model. To make this a validated model we still need a better verification of the $x_{50}$ and $A$ equations.

The large number of sieved distributions obtained in earlier work by the present author (Ouchterlony 2003 & 2005) could probably be used to make a statement about the dependence on a specific charge and the factors that make the numerical constants in the slope equations [Equations 8a, b] different. Improvement in the influence of rock mass jointing on the fragmentation probably requires both a much better theory and extensive tests.

Even if these data are lacking, we have come much closer to the goal of being able to control the amount of fines created by blasting. The understanding of where they originate in the blast has increased as well (Ouchterlony 2004). It is, for example, not likely that the so-called crushed zone around the blasthole is the major source of blasted fines.

ACKNOWLEDGEMENTS

The author is grateful for the generous help in providing data and for discussions going into this document by: Ingvar Bergqvist of Dyno Nobel, Claude Cunningham of AEL, Andreas Grasedieck at Montanuniversität (MU) Leoben, Jan Kristiansen of Dyno Nobel, Cameron McKenzie, Brisbane, Australia, Mats Olsson at Swebrec and Agne Rustan, formerly of Luleå Univ. Techn.

Part of this work was financed by the EU Project GRD-2000-25224 entitled ‘Less Fines Production in Aggregate and Industrial Minerals Industry’. The ‘Less Fines’ partnership includes MU Leoben with Hengl Bitustein, Austria, the CGES & CGI laboratories of Armines/ENSMP, France, and Universidad Politécnica de Madrid (UPM) with UEE and Cementos Portland, Spain. The Nordic partners were Nordkalk Storugns, Dyno Nobel and SveBeFo.

Special thanks go to my Less Fines colleagues Professor José Sanchidrián and his student Pablo Segarra at Universidad Politecnico in Madrid. Their work on size distributions of muckpiles inspired the discovery of the relationship between the curve parameters. Finally, Professor Peter Moser, MU Leoben, is thanked for his part in our continuing collaboration about all aspects of rock fragmentation.

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There are at least 100 on-line installations in production conditions.

They have the advantage of producing fast estimates of the fragmentation size distribution without interfering with the production. The analysis is so fast that decisions like whether the contents of truck loads meet certain size requirements can be made more or less directly.

The methods and systems are still in their infancy and there are several drawbacks with them, but to use sieving is only an alternative in exceptional situations. Sieving is still the norm, though, for fragment size distributions in specifications, etc.

This review of digital image based methods for determining the fragment size distribution may be summarised as follows.

1. It seems that, as rock moves from muckpile to crusher to conveyor belt, the digital image based methods improve in intrinsic effectiveness.

2. Many systems yield acceptable estimates of the central measures of the fragment size distribution, like $x_{50}$, provided that this point is not near the end of the resolution range.

3. Most methods tend to overpredict $x_{50}$ for photos with smaller $x_{50}$ values and to underpredict $x_{50}$ for photos with larger $x_{50}$ values.

4. Old rules of thumb may still be valid:
   - the largest fragment should not be larger than 20 times the smallest fragment;
   - the smallest fragment should be larger than 3 times the resolution;
   - the average fragment size should be small enough compared with the size of the image.

5. The results depend on even lighting conditions – bright sun or oblique lighting that creates shadows can cause errors, as can surface contamination by moisture, oil and snow. Dirty glass covers for video lenses create similar problems.

6. The methods are sensitive to segregation and masking, i.e. to the subsurface ‘hiding’ of fine material when the distribution has a low uniformity index.

7. The resolution is quite limited, 1–1.5 orders of magnitude in fragment size and, even in this narrow range, uncorrected data tend to give large errors near the lower size limit.

8. Zoom merging techniques, i.e. merging photos taken at different scales, can extend the range to maybe two orders of magnitude. This may be sufficient for many applications but probably not for fines problems.

9. Some systems, but not all, have incorporated fines correction algorithms. These generally need to be calibrated.

10. Two types of fines correction are used, either algorithms that enlarge tendencies observed in the image-based data or those that use a priori knowledge of the ‘true’, authentic fragment size distribution in the fines range.

11. The authentic distribution is usually of the Rosin–Rammler type, but there is little experimental evi-

APPENDIX 1: SUMMARY OF THE POSSIBILITIES AND LIMITATIONS OF IMAGE ANALYSIS TECHNIQUES

The following text is the summary section of Chapter 7 in work by Ouchterlony (2003) entitled ‘Methods for determining the fragment size distribution’.

It is clear that digital image based methods for determining the fragment size distributions of rock are becoming widely used. After the first trials in the early 1980s and method developments into the 1990s, systems are now being installed in mines and quarries worldwide.
dence that this distribution accurately describes the fines parts of blasted or crushed rock. One alternative is to use NBC-type curves.

To make accurate predictions outside the range of resolution, the systems need to have calibrated fines corrections.

Production situations may not need absolute (accurate) fragment size measurements to be carried out. Relative changes in fragmentation may be sufficient, and systems that give precise values with lower accuracy may be a better choice.

It is probably well to remember that sieving sizes a flow of oriented fragments, and the image of a pile of fragments captures something else.

**APPENDIX 2: THE EQUATIONS OF THE KUZ–RAM MODEL**

The equations of the Kuz–Ram model apply to bench blasting (Cunningham 1987). They may be written as follows. The cumulative fragments size distribution is

\[ P_{RR}(x) = 1 - e^{-\ln(2) (x/x_{50})^n} \]  

(2.1)

The average fragmentation is given by

\[ x_{50} = A Q^{1/6}(115/S_{ANFO})^{19/30}/q^{0.8} \]  

(2.2)

Here

- \( x_{50} \) = median value or mesh size through which 50% of the material passes, in cm
- \( Q \) = charge weight per hole (kg)
- \( q \) = specific charge (kg/m³)
- \( S_{ANFO} \) = weight strength of explosives relative to ANFO (%)

The rock mass factor \( A \) lies in the interval 0.8–21 and is given by

\[ A = 0.06(RMD+RDI+HF) \]  

(2.3)

Here

- \( RMD \) = rock mass description = 10 (if powdery or friable), JF (if vertically jointed) or 50 (if massive)
- \( JF \) = joint factor = joint plane spacing term JPS + joint plane angle term JPA
- \( JPS \) = 10 (if mean spacing \( S_j < 0.1 \) m), 20 (if \( S_j \) is within range from 0.1 m to oversize) or 50 (if \( S_j > \) oversize)
- \( JPA \) = 20 (if dipping out of face), 30 (if striking \( \perp \) to bench face) or 40 (if dipping into face)
- \( RDI \) = rock density influence = 0.025\( \rho \) (kg/m³) – 50
- \( HF \) = hardness factor = \( E/3 \) if \( E < 50 \) or \( \sigma_c/5 \) if \( E > 50 \) and depends on compressive strength \( \sigma_c \) (MPa) or Young’s modulus \( E \) (GPa)

The uniformity exponent is given by

\[ n = (2.2 - 0.014B/\Omega_h)(1 - SD/B)\sqrt{(1+S/B)/2} \]

\[ x \left( L_b - L_c/L_{tot} + 0.1 \right)^{0.1} \left( L_{tot}/H \right) \]  

(2.4)

Here

- \( B \) = burden (m)
- \( S \) = spacing (m)
- \( \Omega_h \) = bore hole diameter (m)
- \( L_b \) = length of bottom charge (m)
- \( L_c \) = length of column charge (m)
- \( L_{tot} \) = total charge length above grade (m)
- \( H \) = bench height or hole depth (m)
- \( SD \) = standard deviation for drilling error (m).

The first factor in Equation 2.4 (2.2 – 0.014\( \Omega_h \)) typically takes the value 1.5. It may be seen as a base value, and the other factors may then be seen as correction factors, in turn, for drill hole deviations, hole pattern and different bottom and column charges plus the charged part of the bench. If an alternating drill hole pattern were used instead of a rectangular one, \( n \) would increase by 10%.

The equation for \( n \) contains only geometric data and consequently is independent of the equations for \( x_{50} \) and \( A \). The rock mass factor \( A \) has the largest variations.

The Kuz–Ram model has many parameters and should be used with judgement. It is still incomplete in that it contains no information about the effect of timing. To obtain that information, often contradictory practical experience has to be relied upon (Ouchterlony 2003).