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SMOOTHED PARTICLE HYDRODYNAMICS AND CONTINUOUS SURFACE CAP MODEL TO SIMULATE ICE RUBBLE IN PUNCH THROUGH TEST

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ABSTRACT

Recent trend in computational mechanics shows considerable development of numerical methods to simulate discrete materials such as ice rubble. Ice rubble has highly nonlinear behavior and to simulate shear properties requires a new numerical method. An attempt has been made to simulate a punch through test using the Lagrangian mesh-free partial based method formulation known as smoothed particle hydrodynamics. A newly implemented material model in LS-Dyna called the continuous surface cap model has been used in this simulation. A continuous surface cap model based on a combination of elastic-plastic and continuum damage mechanics formulation is used as constitutive model for ice rubble. The material model parameters are chosen to get best fit to test load displacement curve. A brief overview of the smoothed particle hydrodynamics is given. Finally, the results from simulations have compared with experimental results.

INTRODUCTION

This paper is extension of work done paper by Patil et al. (2015). The main purpose of this paper is to simulate punch through test event by using continuous surface cap model (CSCM) and smoothed particle hydrodynamics (SPH) for ice rubble. This material model is developed by Schwer and Murray (1994) and implemented by Murray (2007) in LS-Dyna as a general purpose nonlinear finite element code. A detailed theoretical description and comprehensive calibration procedure of CSCM is given in Murray (2007) and Murray, Abu-Odeh et al. (2007). For brief overview of mechanical properties of ice rubble material, model and simulation of punch through test, please refer Patil et al. (2015).

LAGRANGIAN MESH-FREE PARTIAL BASED METHOD FORMULATION

The advantage of the particle mesh free methods comparing to the conventional mesh-based methods are: (1) the analysed domain is discretised with particles that are not connected with a mesh, allowing for simple and accurate solution at large deformations; (2) the discretisation of complex geometries is less complicated; and (3) the physical values and paths of the particles are easy to follow and evaluate, consequently it is also simple to determine the free surface of movable interfaces or deformable boundaries Vesenjak and Ren (2007).

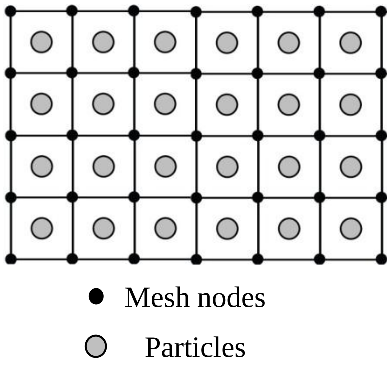

Figure 1: SPH particles with finite element mesh in the background

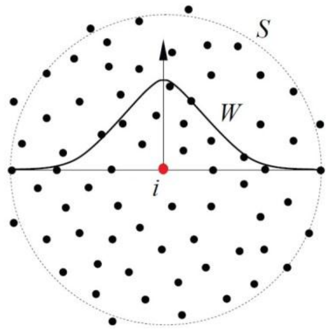

Figure 2: Particle approximation of centre particle' i' within the influence area (S) of the smoothing function W from Liu and Liu (2003)

Smoothed particle hydrodynamics (SPH) is a mesh free Lagrangian method developed by Lucy (1977) , Gingold and Monaghan (1977). It was originally proposed as Monte Carlo approach to calculate the time evolution of gaseous systems. This method was extended to Solid Mechanics by Libersky and Petschek (1991). The method was developed to avoid the limitations of mesh distortion issues in large deformation problems in finite element method. The main difference between finite element methods and SPH is absence of a grid. In the Smoothed Particle Hydrodynamics method, the state of the system is represented by a set of particles shown in which possess individual material properties and move according to the governing conservation equations.

It has some special advantages over the traditional mesh-based numerical methods. The most significant is the adaptive nature of the SPH method, which is achieved at the very early stage of the field variable (i.e. density, velocity, energy) approximation that is performed at each time step based on a current local set of arbitrarily distributed particles. Because of the adaptive nature of the SPH approximation, the formulation of the SPH is not affected by the arbitrariness of the particle distribution. Therefore, it can handle problems with extremely large deformations very well. Another advantage of the SPH method is the combination of the Lagrangian formulation and particle approximation. In Smoothed Particle Hydrodynamics (SPH), the particles have time-history variables such as density, displacement, velocity, acceleration, strain-rate, stress-rate, etc. and they act as interpolation points. The space and time dependent variable called smoothing length is used to determine the region of influence of the neighbouring particles. SPH formulation consists of the following general steps as given in Liu and Liu (2003):

(1) Generation of the mesh free numerical model, (Hopkins)

(2) Integral representation (kernel approximation),

(3) Hopkins particle approximation (Sandler et al.)

(4) Adaptation and Dynamic analysis

The SPH method consists of two key tasks. The first represents the integral representation and the second is the particle approximation. The concept of the integral representation of the function *f (x)*, used in SPH method, is based on the following presumption.

$$f(x) = \int f(y)\delta(x-y)dy' \quad (1)$$

Here $f(x)$ is the function of three-dimensional position vector x and $\delta(x\text{-}y)$ is the Dirac delta function. Above function can be rewritten in integral form with smoothing length function substitute for Dirac delta function.

$$f(x) \approx \int f(y)W(x-y,h)dy \tag{2}$$

W is the Kernel function and h is the smoothing length determining the influence domain of smoothing function. The Kernel function W is defined using the function θ by the relation given below.

$$W(x,h) = \frac{1}{h(x)^d}\theta(x) \tag{3}$$

d is the number of space dimensions and h is so called smoothing length which varies in time and space. $W(x,h)$ should be centrally peaked function. The most common smoothing kernel used by SPH community is cubic B-spline which is defined by choosing θ as:

$$\begin{aligned} &\theta(u) = C \times \left(1 - \frac{3}{2}u^2 + \frac{3}{4}u^3\right) \quad \text{for} |u| \le 1 \\ &\theta(u) = C \times \left(\frac{1}{4}(2-u)^3\right) \quad \text{for} 1 \le |u| \le 2 \\ &\theta(u) = C \times (0) \quad \text{for} 2 < |u| \end{aligned} \tag{4}$$

where C is constant of normalization that depends on the number of space dimensions.

This particle method is based on quadrature formulas on moving particles $(x_i(t),w(t))i \in P$, where P is set of particles, $x_i(t)$ is the location of particle i and $w(t)$ is the weight of the particle. The weight of particle varies proportionally to divergence of flow. The particle approximation of function can now be defined by

$$\prod^h f(x_i) = \sum_{j=1}^{P} w_j f(x_i) W(x_i - x_j, h) \tag{5}$$

In SPH method, the location of neighbouring particles is important. The sorting consists of find which particles interact with others at a given time. A bucket sort is used that consist of partitioning the domain into boxes where the sort is performed. With this partition the closet neighbours will reside in the same boxes where the sort is performed. With this partitioning the closet neighbours will reside in same box or in the closet boxes. This method reduces the number of distance calculations and therefore the CPU time.

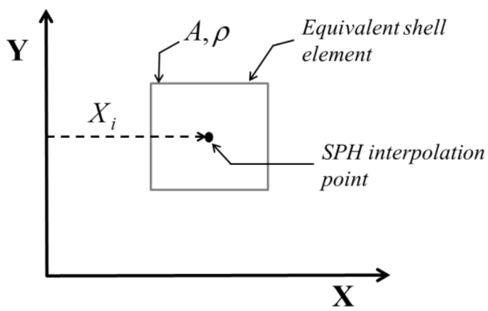

Figure 3: An axisymmetric SPH and equivalent shell element

Axisymmetric SPH is defined on global X-Y plane, with Y-axis as the axis of rotation. An axisymmetric SPH element has a mass of $A\rho$ where ρ is its density and A is the area of the shell element. The SPH element can be approximated by the area of its equivalent axisymmetric shell element, as shown in Figure 3.

The SPH elements are created with solid centre method with 100% fill, which mean each shell element will be replaced by a SPH element with 100 % mass.

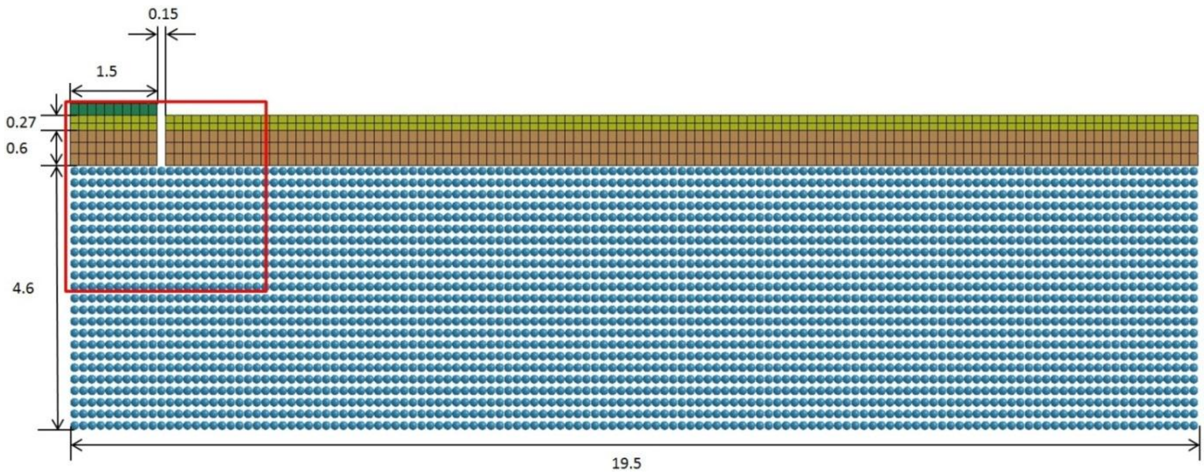

Figure 4: Axisymmetric punch through test model with SPH elements and major dimensions in m.

Now each shell element is having same mass as SPH corresponding element. Since no plastic deformation is assumed in consolidated layer, shell elements are used for consolidated layer. Node to node constrain is used to form coupling between shell and SPH elements. These constrains allow in plane movement only.

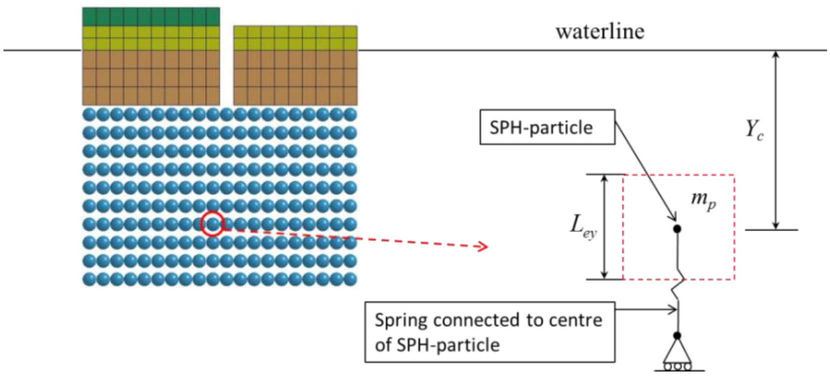

Figure 5: Illustrative sketch of beam elements employed to simulate buoyancy in SPH Model

Figure 4 shows axisymmetric punch through test model with SPH elements. Buoyancy force is applied same way as in Lagrange mesh model. Only difference is buoyancy force is applied to each SPH element and buoyancy force is calculated based on volume of corresponding shell element.

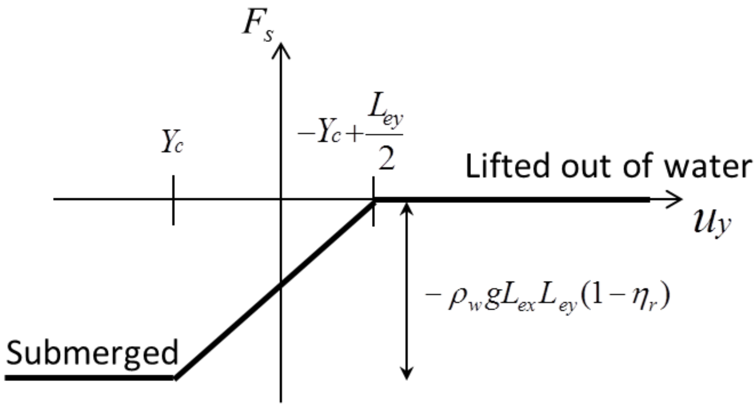

Figure 6: Force vs. displacement diagram for springs attatched at each particle in SPH model

CALIBRATION OF MATERIAL MODEL

The CSCM material model parameters were calibrated based on comparison of simulation results with chosen test data. For consolidated layer an elastic material model is used with material properties given in Table 1.

Table 1: Parameters used in simulations for consolidated layer

Parameter	Symbol	Value
Density (kg/m^3)	ρ_{cl}	871
Poisons ratio	v	0.3
Elastic modulus (MPa)	E	8000

The density of rubble is calculated based on its porosity given in Heinonen (2004). Typical force displacement diagram of punch through test can be divided into three parts. First part is elastic region. Until peak or yield strength, force is linear to displacement of platen. This can attribute to elastic properties of rubble. Elastic modulus was chosen based on parametric study against best fit to linear part of force displacement curve before peak. The shear modulus (*G*) and bulk modulus (K) were calculated based on relationship given in equation 1 as direct input to CSCM material model. In those relationship poisons ratio (ν) assumed to be 0.3. Given below are the parameters used in these simulations.

Table 2: Yield surface parameters of CSCM

Parameter	Symbol	Value	Parameter	Symbol	Value
Density (Kg/m^3)	ρ_r	541	Torsion surface terms	α_1	0.737
Elastic modulus (MPa)	E	45		θ_1	0
Shear modulus (MPa)	G	17.31		λ_1	0.16
Bulk modulus (MPa)	K	37.5		β_1	0
Triaxial compression surface terms	α	0.016	Triaxial extension surface terms	α_2	0.66
	θ	0.182		θ_2	0
	λ	0		λ_2	0.16
	β	0		β_2	0

The triaxial compression parameters such as α and θ were calculated based on relationship given by Schwer and Murray (1994) to Mohr-Coulomb parameters cohesion (c) and international friction angle (φ). Parametric study ensures that chosen α and θ gives approximately same peak force. Other two parameters λ and β, which represent nonlinear and exponent term of triaxial compression surface kept at 0.

Table 3: Cap hardening parameters of CSCM

Parameter	Symbol	Value
Cap ellipticity ratio	R	9.44
Initial intercept of the cap surface	X_D	0.595
The maximum plastic volumetric strain	W	0.05
The linear shape parameters	D_1	0.001
The quadratic shape parameters	D_2	0.65

To define cap-hardening laws five input parameters (X_D, W, D_1, D_2, and R) are selected from parametric study where simulated force displacement curve compared with modified test curve. Bottom displacement also compared.

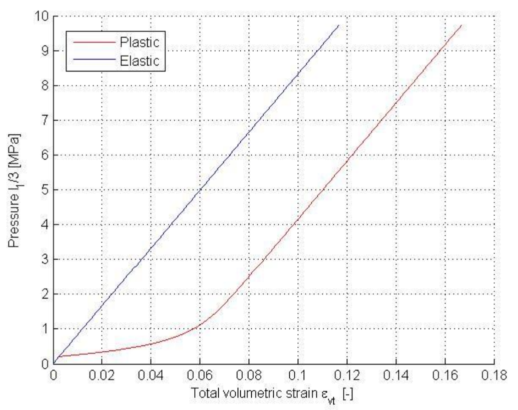

Figure 7: Plot of first invariant of stress tensor I_1 verses plastic volumetric strain ε_v^p for chosen value of X_0, W, D_1, D_2, and R

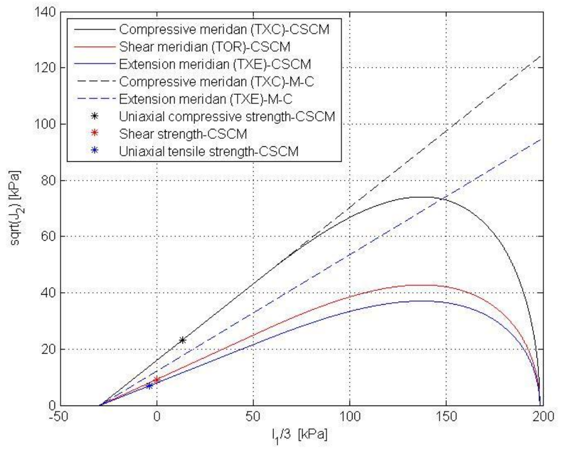

Figure 8: 2D yield surface plotting of CSCM criterion and Mohr-Coulomb criterion fitted to data for ice rubble

Softening part mainly controlled by Damage parameters. Given below are the values for selected parameters.

Table 4: Damage parameters of CSCM

Parameter	Symbol	Value
Ductile shape softening parameter	B	20
Fracture energy in uniaxial compression (J/m^2)	G_{fc}	0.4
Brittle shape softening parameter	D	1
Fracture energy in uniaxial tension (J/m^2)	G_{fs}	0.065
Fracture energy in pure shear (J/m^2)	G_{ft}	0.065

A 2D yield surface plotted with chosen parameters for CSCM material model. Figure 8 shows the plot. In this simulation damage parameters were selected based on fit to post peak part of experimental force displacement plot.

RESULTS ANALYSIS

Results are analysed based on failure modes described earlier. As platen moves down, the forces on platen increased with high rate and reached peak value for relatively small displacement. From simulation point of view this can be seen as failure of freeze bonding of ice blocks and peak value is direct indication of breaking those bonds.

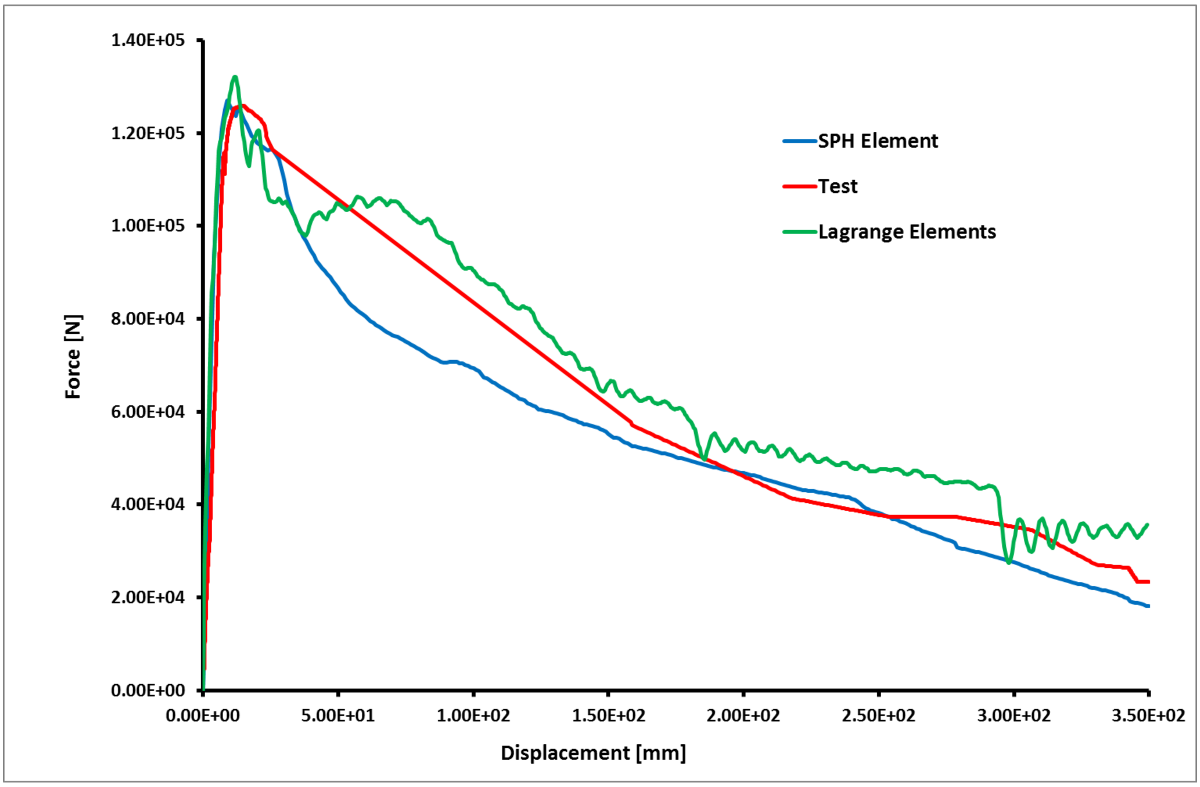

Figure 9: Force displacement diagram for test 0/2000 compared with simulated with Smooth particle hydrodynamics element and Lagrangian element.

Figure 9 shows comparison of test to simulation. As the peak force was seen clearly in actual force displacement plot, assumed peak from modified force displacement plot matches with simulated peak force.

Internal friction angle and cohesion are adjusted to match the peak force. Also Young's modulus was chosen to fit the slope of initial loading phase in force displacement diagram.

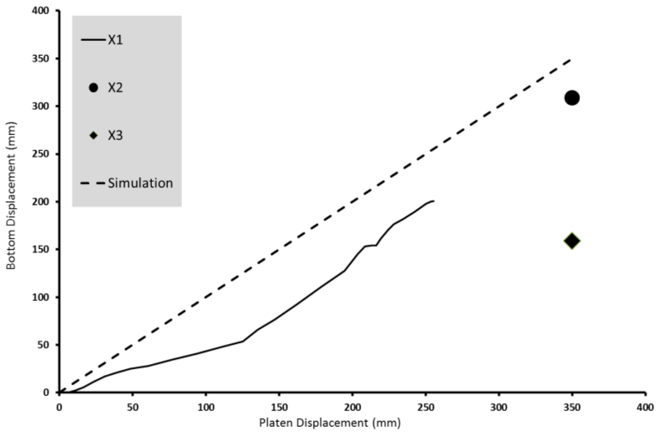

Figure 10: Bottom displacement of keel recorded by different sensors plotted against platen displacement.

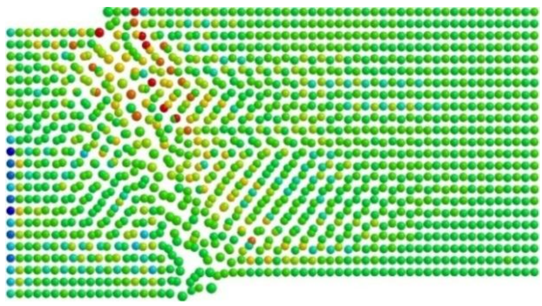

Figure 11: Stress distribution in XY plane at 350 mm displacement

In Figure 10, simulated deformation of sensor X1 in keel is much larger and more linear than test. The mesh sensitivity study was not performed in Lagrangian finite element mesh. In SPH formulation shell elements were replaced by integration points having same mass. The SPH formulation gives semi discrete nature to keel geometry.

DISCUSSION AND CONCLUSIONS

In total 22 parameters were needed to define continuous surface cap model. However, some approximations and simplification can reduce that number to 15. Material parameters were calibrated based on response to measured force displacement diagram resulted in good agreement in the load displacement relationship.

A 2D surface plotted for CSCM in compression, shear and extension meridian to ensure the validity of chosen values of material parameters. Those parameters also plotted for Mohr-Coulomb in compression and extension meridian.

An axisymmetric model with plane strain assumption gives reasonably good results. Although to get the clear view of rubble deformation 3D model is required. The displacement nodes at the bottom of keel were smaller than corresponding points in rubble obtained by sensors X1, X2 and X3 in Figure 10.

The major advantage of using SPH formulation over Lagrangian element is to avoid mesh tangling issues caused by large deformation. Despite using continuum definition of material model, SPH formulation can be used to simulate discrete nature of rubble. But other material properties like friction between particles cannot be introduced as the discretization domain is continuous. Therefore, all cohesive frictional material models like Mohr-coulomb cannot be used with this formulation.

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