Idealized Two-Axis Model of Induction Machines under Stator Fault

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Abstract

In this paper, a low differential order model describing the dynamics of IMs with faulty stator is developed. The modelling is performed under well established idealizing assumptions regarding the magnetic and electrical properties of the motor. Typical motor faults as inter-turn short circuit and increased winding resistance are taken into account. The resulting model can be handled analytically and serve as a ground for fault detection and fault tolerant control of the IM. For each of the considered faults, very good agreement between simulations and experimental data collected from mass-produced industrial IMs is observed.

Nomenclature

Variables	Parameters
ψ - magnetic flux	R - electrical resistance
i - electrical current	L - lumped inductance
u - feeding voltage	M - mutual inductance
λ - Lagrange coeff.	J - moment of inertia
γ - rotor angle	D - rotor damping
M_l - mechanical load	p - number of pole-pairs

For both the variables and parameters, a capital letter subscript refers to the corresponding stator axis in a conventional three-axis system. Similarly, a small letter subscript refers to the corresponding rotor axis. The units for the variables and parameters are SI-units which are omitted for the sake of clarity. Throughout the paper, bold capital letters stand for matrices, whereas small/capital letters denote scalars and vectors. Finally, differentiation with

respect to time is denoted by the dot, *i.e.* $\dot{x} \triangleq \frac{dx}{dt}$.

1 Introduction

It is well known that under idealizing assumptions, the dynamics of a healthy Induction Machine (IM) can be well described by a set of ordinary differential equations, instead of the more physically motivated partial differential equations, [1]. For the squirrel cage IMs, the most accurate way to proceed with motor modelling is possibly the *winding function* approach, where one electrical equation is introduced for each mesh in the rotor and typically one electrical equation for each stator phase, [2]. This approach yields a detailed model of high differential order which in most cases is too complex for control engineering ends.

To reduce the differential order of the system dynamics, one can utilize the *space phasor* approach, observing the idealizing assumptions given in [3]. It generally starts with defining three axes for the stator, where the consecutive axes are shifted $\frac{2\pi}{3}$ ahead of each other, corresponding to the three stator phases. Then either two (spaced $\frac{\pi}{2}$) or three axes (spaced $\frac{2\pi}{3}$) are as well defined for the rotor. The space phasor approach results in either 6(7) or 7(8) differential equations, including the *mechanical* part of the IM, referred to in the sequel as *three-axis* models. Although space phasor models are not completely faithful with the rotor physics, the dynamics of the stator are well described.

By utilizing physical relations between currents and/or voltages, one then usually projects the model into two sets of orthogonal axes ($\frac{\pi}{2}$ apart). These

models are referred to as *two-axis* models and can be further reduced to a set of three or four complex-valued differential equations.

The modelling approach outlined above is utilized *e. g.* in [4], where the feasibility of describing a squirrel-cage rotor by only a few equations is advocated. The paper also describes how the individual currents in the rotor bars can be derived from the reduced order model.

Unfortunately, many of the idealizing assumptions introduced in the derivation of the two-axis models fail to hold under stator faults. However, in some cases, this approach can still be utilized with care. For instance, for increased resistance in one stator phase, the number and structure of differential equations describing the IM should clearly remain the same. Under the basic underlying assumptions, an inter-turn short circuit in the stator leaves the equations for the rotor dynamics unaffected. However, extra mesh for one stator phase is usually introduced in order to model this fault. Hence, the number of differential equations describing the IM is incremented by one compared to the healthy IM. Such modelling approaches are reported in [5], [6], [7] and most recently [8]. In the latter, voltage equations in two axes are provided, indicating that additional differential equations are indeed necessary. In [9] no additional meshes are introduced, but the model is kept in three-axes. In fact, the problem of singular transformations *always* arises in a three axes description of the IM, leading to differential-algebraic (descriptor) models, [10]. To circumvent this, as few differential equations as possible should be kept in the model, by utilizing known algebraic relationships between the model variables.

In this paper, modelling of IMs with electrical stator faults is considered under standard idealizing assumptions, without introducing additional meshes and hence additional differential equations. Despite potential drawbacks, this simple *intuitive* two-axis approach is used in order to keep the differential order low. Furthermore, non-standard transformations to two axes are used in order to achieve as parsimonious model as possible. The models show good agreement with experimental data and appear to be suitable for model-based fault detection, to name one perspective application. This paper is part of the work reported in more detail in [11].

2 Three-axis model

To start with, define three coordinate axes for the stator and three for the rotor. The angle between two consecutive axes is $\frac{2\pi}{3}$. To simplify notation, introduce the electrical angle $\bar{\gamma} = p\gamma$.

The stator phases A, B, C are connected in a Y -configuration and thus $i_A + i_B + i_C = 0$. A similar

algebraic relationship is needed for the rotor phases [11], *i.e.* $i_a + i_b + i_c = 0$. Fig.1 illustrates the model parameters definitions with respect to stator phase A and rotor phase a . Note the interaction between stator phase A and rotor phase a through M_{Aa} . The situation is similar for the other phases. Assume now

Figure 1: Stator phase A , rotor phase a and the mutual inductance M_{Aa} between phases.

that a fault occurs in stator phase A . Examples of motor faults that are covered by the mathematical model are: stator winding shortcut, increased stator winding resistance and decreased air gap in front of stator phase A . When a stator fault occurs, one or

Healthy IM Parameters invariant under Stator Fault

Value	Model parameter
R_S	R_B, R_C
R_r	R_a, R_b, R_c
L_S	L_B, L_C
L_r	L_a, L_b, L_c
M_S	M_{BC}
M_r	M_{ab}, M_{ac}, M_{bc}
M	$M_{Ba}, M_{Bb}, M_{Bc}, M_{Ca}, M_{Cb}, M_{Cc}$

Parameters Influenced by Stator Fault

Value	Model parameter
R_A	R_A
L_A	L_A
M_{AS}	M_{AB}, M_{AC}
M_{Ar}	M_{Aa}, M_{Ab}, M_{Ac}

Table 1: Model parameters for IM with faulty stator several of the following inequalities apply, see Table 1; $R_A \neq R_S$; $L_A \neq L_S$; $M_{AS} \neq M_S$; $M_{Ar} \neq M$.

The second Kirschhoff's law equations written for the stator phases A, B, C and the rotor phases a, b, c (see Fig.2) give

$$\begin{aligned}\dot{\psi}_{3S} &= -\mathbf{R}_{3S}\mathbf{i}_{3S} + B\lambda_S + u_{3S} \\ \dot{\psi}_{3r} &= -\mathbf{R}_{3r}\mathbf{i}_{3r} + B\lambda_r\end{aligned}\quad (1)$$

where

$$\begin{aligned}\psi_{3S} &= [\psi_A \ \psi_B \ \psi_C]^T & \psi_{3r} &= [\psi_a \ \psi_b \ \psi_c]^T \\ \mathbf{i}_{3S} &= [i_A \ i_B \ i_C]^T & \mathbf{i}_{3r} &= [i_a \ i_b \ i_c]^T \\ u_{3S} &= [u_A \ u_B \ u_C]^T & B &= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T \\ \mathbf{R}_{3S} &= \text{diag}(R_A, R_S, R_S) & \mathbf{R}_{3r} &= R_r \mathbf{I}\end{aligned}$$

Figure 2: Connections between the stator phases and rotor phases. The left figure is for the stator and the right figure is for the rotor. The quantities u_A , u_B and u_C are the feeding voltages of the stator.

The Lagrange coefficients λ_S , λ_r can be explicitly expressed from (1) as

$$\begin{aligned}\lambda_S &= (B^T \mathbf{R}_{3S}^{-1} B)^{-1} B^T \mathbf{R}_{3S}^{-1} (\dot{\psi}_{3S} - u_{3S}) \\ \lambda_r &= (B^T \mathbf{R}_{3r}^{-1} B)^{-1} B^T \mathbf{R}_{3r}^{-1} \dot{\psi}_{3r}\end{aligned}$$

These voltages guarantee that the stator currents as well as the rotor currents sum up to zero at all time instances. From geometric considerations on Fig.3,

Figure 3: Three-axis description of the IM. The stator and rotor frames are separated by the electrical angle $\tilde{\gamma}$.

the fluxes are

$$\begin{bmatrix} \psi_{3S} \\ \psi_{3r} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{3S} & \mathbf{M}_3 \\ \mathbf{M}_3^T & \mathbf{L}_{3r} \end{bmatrix} \begin{bmatrix} i_{3S} \\ i_{3r} \end{bmatrix} \quad (2)$$

where

$$\mathbf{L}_{3S} = \begin{bmatrix} L_A & 0 & 0 \\ 0 & L_S & 0 \\ 0 & 0 & L_S \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 0 & M_{AS} & M_{AS} \\ M_{AS} & 0 & M_S \\ M_{AS} & M_S & 0 \end{bmatrix}$$

$$\mathbf{L}_{3r} = L_r \mathbf{I} - \frac{1}{2} M_r \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{M}_3 = \mathbf{\Gamma}_{3c} \cos(\tilde{\gamma}) + \mathbf{\Gamma}_{3s} \sin(\tilde{\gamma})$$

and

$$\mathbf{\Gamma}_{3c} = \frac{1}{2} \begin{bmatrix} 2M_{Ar} & -M_{Ar} & -M_{Ar} \\ -M & 2M & -M \\ -M & -M & 2M \end{bmatrix}$$

$$\mathbf{\Gamma}_{3s} = \frac{\sqrt{3}}{2} \begin{bmatrix} 0 & -M_{Ar} & M_{Ar} \\ M & 0 & -M \\ -M & M & 0 \end{bmatrix}$$

Equations (1) and (2) complete the description of the *electrical* part of the IM.

The *mechanical* part of the IM can be expressed as

$$J\ddot{\tilde{\gamma}} = -D\dot{\tilde{\gamma}} + p(M_{em} - M_l) \quad (3)$$

where M_{em} is the electromagnetical torque of the IM. Consider the electrokinetic energy of the IM

$$\begin{aligned}T_e &= \frac{1}{2} [\iota_{3S}^T \psi_{3S} + \iota_{3r}^T \psi_{3r}] \\ &= \frac{1}{2} [\iota_{3S}^T \mathbf{L}_{3S} \iota_{3S} + \iota_{3r}^T \mathbf{L}_{3r} \iota_{3r}] + \iota_{3S}^T \mathbf{M}_3 \iota_{3r}\end{aligned}$$

For the frozen currents, the electromagnetic torque is

$$M_{em} = \frac{\partial T_e}{\partial \tilde{\gamma}} = p \frac{\partial T_e}{\partial \tilde{\gamma}} = p \iota_{3S}^T \frac{\partial \mathbf{M}_3}{\partial \tilde{\gamma}} \iota_{3r} \quad (4)$$

where $\frac{\partial \mathbf{M}_3}{\partial \tilde{\gamma}} = \mathbf{\Gamma}_{3s} \cos(\tilde{\gamma}) - \mathbf{\Gamma}_{3c} \sin(\tilde{\gamma})$. A complete IM model in three axes is given by (1), (2), (3) and (4).

3 Two-axis model

Introduce two orthogonal axes $\alpha S, \beta S$ connected to the stator where axis αS coincides with the axis of phase A and the axis βS lies $\frac{\pi}{2}$ ahead in the positive (*i.e.* anti-clockwise) direction. For the rotor, two orthogonal axes $\alpha r, \beta r$ are defined in a similar manner. The stator currents i_A, i_B, i_C and the rotor currents i_a, i_b, i_c are related to their two-axis counterparts as

$$i_S = \mathbf{T}_{23} i_{3S}; \quad i_r = \mathbf{T}_{23} i_{3r} \quad (5)$$

and the inverse transformation is given by

$$i_{3S} = \mathbf{T}_{32} i_S; \quad i_{3r} = \mathbf{T}_{32} i_r \quad (6)$$

where

$$\begin{aligned}i_S &= \begin{bmatrix} i_{\alpha S} & i_{\beta S} \end{bmatrix}^T & i_r &= \begin{bmatrix} i_{\alpha r} & i_{\beta r} \end{bmatrix}^T \\ \mathbf{T}_{23} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{bmatrix} & \mathbf{T}_{32} &= \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}^T\end{aligned}$$

The inverse transformations are deduced using $i_A + i_B + i_C = 0$ and $i_a + i_b + i_c = 0$, see [11]. Much of the fault effects in phase A are translated via (5) to phase αS .

In the same manner, for the fluxes, define

$$\psi_S = \begin{bmatrix} \psi_{\alpha S} & \psi_{\beta S} \end{bmatrix}^T; \quad \psi_r = \begin{bmatrix} \psi_{\alpha r} & \psi_{\beta r} \end{bmatrix}^T$$

Comparing the electrokinetic energy

$$T_e = \frac{1}{2} [\iota_{3S}^T \psi_{3S} + \iota_{3r}^T \psi_{3r}] = \frac{1}{2} [\iota_S^T \mathbf{T}_{32}^T \psi_{3S} + \iota_r^T \mathbf{T}_{32}^T \psi_{3r}]$$

with the natural one, $T_e = \frac{1}{2} [\iota_S^T \psi_S + \iota_r^T \psi_r]$, gives the unique direct transformations for the fluxes

$$\psi_S = \mathbf{T}_{32}^T \psi_{3S} \quad \psi_r = \mathbf{T}_{32}^T \psi_{3r} \quad (7)$$

Transformations (6) and (7) applied to the *electrical* part of the three-axis model give

$$\begin{aligned} \dot{\psi}_S &= -\mathbf{R}_S \iota_S + u_S \\ \dot{\psi}_r &= -\mathbf{R}_r \iota_r \\ \begin{bmatrix} \psi_S \\ \psi_r \end{bmatrix} &= \begin{bmatrix} \mathbf{L}_S & \mathbf{M} \\ \mathbf{M}^T & \mathbf{L}_r \end{bmatrix} \begin{bmatrix} \iota_S \\ \iota_r \end{bmatrix} \end{aligned} \quad (8)$$

($u_S = [u_{\alpha S} \ u_{\beta S}]^T = \mathbf{T}_{32}^T u_{3S}$) where

$$\begin{aligned} \mathbf{R}_S &= \text{diag}(\frac{3}{2}R'_A, \frac{3}{2}R_S); \quad \mathbf{R}_r = \frac{3}{2}R_r \mathbf{I}_2; \quad \mathbf{L}_r = \frac{9}{4}L'_r \mathbf{I}_2 \\ \mathbf{L}_S &= \text{diag}(\frac{9}{4}L'_A, \frac{9}{4}L'_S); \quad \mathbf{M} = \mathbf{\Gamma}_c \cos(\bar{\gamma}) + \mathbf{\Gamma}_s \sin(\bar{\gamma}) \end{aligned}$$

and

$$\begin{aligned} \mathbf{\Gamma}_c &= \begin{bmatrix} \frac{9}{4}M'_{Ar} & 0 \\ 0 & \frac{9}{4}M \end{bmatrix}; \quad \mathbf{\Gamma}_s = \begin{bmatrix} 0 & -\frac{9}{4}M'_{Ar} \\ \frac{9}{4}M & 0 \end{bmatrix} \\ \frac{3}{2}R'_A &= R_A + \frac{1}{2}R_S; \quad \frac{9}{4}L'_A = L_A + \frac{1}{2}L_S + M_{AS} - \frac{1}{4}M_S \\ \frac{9}{4}L'_S &= \frac{3}{2}\left(L_S + \frac{1}{2}M_S\right); \quad \frac{9}{4}L'_r = \frac{3}{2}\left(L_r + \frac{1}{2}M_r\right) \\ \frac{9}{4}M'_{Ar} &= \frac{3}{2}\left(M_{Ar} + \frac{1}{2}M\right) \end{aligned}$$

For the *mechanical* part, the same transformations give

$$J\ddot{\gamma} = -D\dot{\gamma} + p(M_{em} - M_l) \quad (9)$$

where $M_{em} = p \iota_S^T [\mathbf{\Gamma}_s \cos(\bar{\gamma}) - \mathbf{\Gamma}_c \sin(\bar{\gamma})] \iota_r$. A complete description of the IM dynamics is given by (8) and (9).

By taking derivative of the algebraic equations for the fluxes in (8) and combining them with the two upper differential equations of the same system, one arrives at

$$\begin{aligned} \dot{\psi}_{\alpha S} &= -\frac{3}{2}R'_A \iota_{\alpha S} + u_{\alpha S} \\ \dot{\psi}_{\beta S} &= -\frac{3}{2}R_S \iota_{\beta S} + u_{\beta S} \\ \mu' \frac{9}{4}L'_A \dot{\iota}_{\alpha S} &= \dot{\psi}_{\alpha S} - \frac{2}{3} \frac{R_r}{L'_r} \left(\frac{9}{4}L'_A \iota_{\alpha S} - \psi_{\alpha S} \right) \\ &\quad - \dot{\gamma} \frac{M'_{Ar}}{M} \left(\mu' \frac{9}{4}L'_S \iota_{\beta S} - \psi_{\beta S} \right) \\ \mu' \frac{9}{4}L'_S \dot{\iota}_{\beta S} &= \dot{\psi}_{\beta S} - \frac{2}{3} \frac{R_r}{L'_r} \left(\frac{9}{4}L'_S \iota_{\beta S} - \psi_{\beta S} \right) \\ &\quad + \dot{\gamma} \frac{M}{M'_{Ar}} \left(\mu' \frac{9}{4}L'_A \iota_{\alpha S} - \psi_{\alpha S} \right) \\ \frac{1}{p} M_{em} &= \frac{M}{M'_{Ar}} \psi_{\alpha S} \iota_{\beta S} - \frac{M'_{Ar}}{M} \psi_{\beta S} \iota_{\alpha S} \\ &\quad - \frac{9}{4} \left(L'_A \frac{M}{M'_{Ar}} - L'_S \frac{M'_{Ar}}{M} \right) \iota_{\alpha S} \iota_{\beta S} \end{aligned}$$

where $\mu' = 1 - \frac{M'_{Ar}}{L'_A L'_r}$; $\mu = 1 - \frac{M^2}{L'_S L'_r}$.

Normalizing the variables as $\psi_S = \frac{3}{2}u_m \bar{\psi}_S$; $\iota_{\alpha S} = \frac{2}{3} \frac{u_m}{L'_A} \bar{\iota}_{\alpha S} = K_A \bar{\iota}_{\alpha S}$; $\iota_{\beta S} = \frac{2}{3} \frac{u_m}{L'_S} \bar{\iota}_{\beta S} = K_S \bar{\iota}_{\beta S}$; $u_S = \frac{3}{2}u_m \bar{u}_S$, gives

$$\begin{aligned} \dot{\bar{\psi}}_{\alpha S} &= -\varepsilon_A \bar{\iota}_{\alpha S} + \bar{u}_{\alpha S} \\ \dot{\bar{\psi}}_{\beta S} &= -\varepsilon_S \bar{\iota}_{\beta S} + \bar{u}_{\beta S} \\ \mu' \dot{\bar{\iota}}_{\alpha S} &= \dot{\bar{\psi}}_{\alpha S} - \varepsilon_r (\bar{\iota}_{\alpha S} - \bar{\psi}_{\alpha S}) - \bar{\omega} \frac{M'_{Ar}}{M} (\mu \bar{\iota}_{\beta S} - \bar{\psi}_{\beta S}) \\ \mu \dot{\bar{\iota}}_{\beta S} &= \dot{\bar{\psi}}_{\beta S} - \varepsilon_r (\bar{\iota}_{\beta S} - \bar{\psi}_{\beta S}) + \bar{\omega} \frac{M}{M'_{Ar}} (\mu' \bar{\iota}_{\alpha S} - \bar{\psi}_{\alpha S}) \end{aligned}$$

for the *electrical* part where $\varepsilon_A = \frac{2}{3} \frac{R'_A}{L'_A}$; $\varepsilon_S = \frac{2}{3} \frac{R_S}{L'_S}$; $\varepsilon_r = \frac{2}{3} \frac{R_r}{L'_r}$ and $\bar{\omega} = \dot{\gamma}$. After normalization, the *mechanical* part is

$$\dot{\bar{\omega}} = -\alpha_m \bar{\omega} + \delta (\bar{M}_{em} - \bar{M}_l)$$

where

$$\begin{aligned} \bar{M}_{em} &= \frac{M}{M'_{Ar}} \bar{\psi}_{\alpha S} \bar{\iota}_{\beta S} - \frac{L'_S}{L'_A} \frac{M'_{Ar}}{M} \bar{\psi}_{\beta S} \bar{\iota}_{\alpha S} \\ &\quad - \left(\frac{M}{M'_{Ar}} - \frac{L'_S}{L'_A} \frac{M'_{Ar}}{M} \right) \bar{\iota}_{\alpha S} \bar{\iota}_{\beta S} \end{aligned}$$

and $\alpha_m = \frac{D}{J}$; $\delta = \frac{p^2}{J} \frac{u_m^2}{L'_S}$; $\bar{M}_l = \frac{1}{p} \frac{L'_S}{u_m^2} M_l$.

Finally, introducing the pseudo-fluxes

$$\bar{\iota}_{\alpha S} = \frac{1}{\mu'} \bar{\psi}_{\alpha S} - \frac{\mu}{\mu'} \frac{1 - \mu'}{\mu'} \bar{x}_{\alpha S}; \quad \bar{\iota}_{\beta S} = \frac{1}{\mu} \bar{\psi}_{\beta S} - \frac{1 - \mu}{\mu} \bar{x}_{\beta S}$$

the IM dynamics can be concisely expressed in the form

$$\begin{aligned} \dot{\bar{\psi}}_{\alpha S} &= -\alpha_A \bar{\psi}_{\alpha S} + \alpha_A \frac{\mu}{\mu'} (1 - \mu') \bar{x}_{\alpha S} + \bar{u}_{\alpha S} \\ \dot{\bar{\psi}}_{\beta S} &= -\alpha_S \bar{\psi}_{\beta S} + \alpha_S (1 - \mu) \bar{x}_{\beta S} + \bar{u}_{\beta S} \\ \dot{\bar{x}}_{\alpha S} &= -\alpha_r \frac{\mu}{\mu'} \bar{x}_{\alpha S} - \bar{\omega} \frac{k_0}{k_1} \bar{x}_{\beta S} + \alpha_r \bar{\psi}_{\alpha S} \\ \dot{\bar{x}}_{\beta S} &= -\alpha_r \bar{x}_{\beta S} + \bar{\omega} \frac{k_1}{k_0} \bar{x}_{\alpha S} + \alpha_r \bar{\psi}_{\beta S} \\ \bar{M}_{em} &= k_1 \left[\bar{\psi}_{\beta S} \bar{x}_{\alpha S} - \bar{\psi}_{\alpha S} \bar{x}_{\beta S} - \left(1 - \frac{\mu}{\mu'} \right) \bar{x}_{\alpha S} \bar{x}_{\beta S} \right] \end{aligned} \quad (10)$$

where $\alpha_A = \frac{\varepsilon_A}{\mu'}$; $\alpha_S = \frac{\varepsilon_S}{\mu}$; $\alpha_r = \frac{\varepsilon_r}{\mu}$ and $k_0 = \frac{1 - \mu}{\mu}$; $k_1 = \frac{M}{M'_{Ar}} \frac{1 - \mu'}{\mu'}$.

4 Model Validation

All experiments were performed on two 1.1 kW, two pole-pair induction machines (ASEA:MT 90 S 24-4, 26 bars). In one of the motors, some of the windings in one phase were inter-turn short circuited, for approximately $\frac{1}{6}$ of the particular phase. Another

one was used for experiments on symmetrical IM and with increased phase resistance.

The rotor angle γ , the three stator currents (i_A , i_B , i_C) and the three stator voltages (u_A , u_B , u_C) were measured. In both cases, the *electrical* part of the model is validated.

The IMs were connected to three-phase mains, with frequency 50 Hz, and the voltages lowered to 75 V, in order to reduce magnetic saturation of the cores in stator and rotor.

The model parameters of (10) were identified during a start-up with a Gauss-Newton based algorithm explained in [11]. In Fig. 4, the positive envelope of the stator phase current under start-up of a healthy IM is depicted.

Figure 4: Start-up of a healthy IM. Envelope of the stator currents, $i_{eS} = (i_{\alpha S}^2 + i_{\beta S}^2)^{\frac{1}{2}}$. The data sequence begins at 0.5 s.

4.1 Healthy IM

The stator and rotor time constants in the two-axis system are

$$\tau_A = \frac{L'_A}{R'_A}, \quad \tau_S = \frac{L'_S}{R_S}; \quad \tau_r = \frac{L'_r}{R_r} \quad (11)$$

From the parameters in Table 2 and the model parameters, the resistances of the stator phases in the two-axis system amount to $R'_A = 9.899$, $R_S = 9.393$.

Fault invariant par.		Fault influenced par.	
Parameter	Value	Parameter	Value
τ_S	0.03999	τ_A	0.03258
τ_r	0.08007	μ'	0.1020
μ	0.08726	L'_A	0.3225
L'_S	0.3757	$\frac{M'_{Ar}}{M}$	1.001

Table 2: Estimated values of model parameters for healthy IM.

The measured and simulated by the model stator currents are compared in Fig. 5. Since the IM dynamics are time-varying, three characteristic time

intervals are considered to highlight the model quality. The first one is in the beginning of the data sequence. The second one is in the middle of the data set where the amplitudes of the currents drop most rapidly, see Fig.4. Finally, the third one is at the end of the data sequence when the motor has achieved stationary conditions. As can be seen, the identified model of the symmetrical IM explains the measurements quite well. Notably, even for the healthy IM, some imbalance is indicated by the estimated parameters. However, the quotient $\frac{M'_{Ar}}{M} = 1.001$ proves the IM magnetically symmetrical.

Figure 5: Comparison of measured data and model output for the healthy IM at three different time intervals. Solid lines - measured data. Dash-dotted lines - simulations.

4.2 Increased Phase Resistance

In this experiment, an additional resistance of 10Ω was connected in series with phase *A* of an otherwise healthy motor to simulate a poor connection of the winding. For cold IM, the nominal value of R_S is about 8.5Ω . The identified from logged data model parameters are given in Table 3.

Similar to the case of healthy IM, one obtains $R'_A = 16.45$, $R_S = 9.825$. In [11], it is shown that $\frac{3}{2}R'_A = R_A + \frac{1}{2}R_S$ which yields $R_A = 19.76 \Omega$, *i.e.* mainly R_A changes (*c.f.* Section 4.1). The estimated increase in R_A is about 9.5Ω and close to the actual value. Comparing Table 2 and Table 3, it can be seen

Fault invariant par.		Fault influenced par.	
Parameter	Value	Parameter	Value
τ_S	0.03346	τ_A	0.02279
τ_r	0.07954	μ'	0.08867
μ	0.09933	L'_A	0.3748
L'_S	0.3287	$\frac{M'_{Ar}}{M}$	1.008

Table 3: Estimated model parameters for increased phase resistance.

that almost all estimated parameters have changed due to the fault. Yet $\frac{M'_{Ar}}{M} = 1.008$ and magnetic balance is intact.

A significant difference between R_S and R_A (or R'_A) with preserved magnetic symmetry, *i.e.* $\frac{M'_{Ar}}{M} \approx 1$, seems to be a good indicator for this kind of fault.

Figure 6: Comparison of measured data and model output for the IM with increased resistance in one phase at three different time intervals. Solid lines - measured data. Dash-dotted lines - Simulations.

4.3 Inter-turn Short Circuit

For inter-turn short circuit, the parameter estimates change differently to what intuitively might be expected. For example, from the estimates of $R'_A = 9.467$ and $R_S = 8.165$ one obtains $R_A = 10.12 \Omega$, *i.e.* $R_A > R_S$, despite of the short circuit. However, many of the assumptions for idealized IMs ([11]) are violated and the reasoning based on the symmetrical

motor theory is apparently of limited use here.

Fault invariant par.		Fault influenced par.	
Parameter	Value	Parameter	Value
τ_S	0.05430	τ_A	0.01487
τ_r	0.07761	μ'	0.2175
μ	0.08057	L'_A	0.1407
L'_S	0.4434	$\frac{M'_{Ar}}{M}$	0.8471

Table 4: Estimated model parameters for inter-turn short circuit.

Figure 7: Comparison of measured data and model output for the IM with inter-turn short circuit in one phase at three different time intervals. Solid lines - measured data. Dash-dotted lines - simulations.

Nevertheless, the estimated parameters clearly indicate a severe imbalance in the IM. The time constants τ_S , τ_A , the goodness factors μ , μ' and the inductances L'_S , L'_A significantly differ between the healthy and the faulty phases. A good indication of an inter-turn short circuit seems to be that the quotient $\frac{M'_{Ar}}{M}$ significantly differs from one.

To summarize, the estimated model parameters confirm significant imbalance, but the parameters behave in a fairly unexpected manner. Further research is needed to understand whether the reason for this is that significant phenomena have been neglected in the model derivation, or it is due to problems with parameter identifiability.

5 Discussion

Some of the estimated imbalance in the IMs is due to inherent asymmetry in all real-life IMs as well as calibration errors of the sensors used to obtain the identification data. Hence, a decision threshold has to be enforced to distinguish between nominal model uncertainty and a fault. A reasonable approach is to consider differences between fault independent parameters and parameters expected to be influenced by fault. In this way, much of the effects of heating and other uncertainties have less impact since they influence similarly all the motor phases.

By examining Fig. 5, Fig. 6 and Fig. 7, it can be concluded that simulations and measured data agree very well. Still, some systematic discrepancies between the models and experimental data can be noticed. In particular, at the end of each validation sequence, the simulated data contain higher frequency oscillations of unknown origin.

One possible explanation is inaccuracy of the rotor angular speed. The angle measurements are exact but time resolution error is present.

However, in Fig. 5 through Fig. 7, it can be observed that the measured currents also contain higher frequency components. For the time being, it is not decided yet to what extent they should be modelled, because one effect is hard to distinguish from the other.

Magnetic saturation is not modelled but should not be significant in the performed experiments, because of the precautions taken.

6 Conclusions

In this paper a simple two-axis model of IMs with stator faults has been presented. From the identification results it can be concluded that the transient behavior and the dominating frequencies of the currents are very well explained by the derived model. Possible applications of the models are fault detection and fault-tolerant control.

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