

Reliability and Spare Parts Estimation taking into Consideration the Operational Environment - A Case Study

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Abstract - Spare parts provision is a complex problem and requires an accurate model to analysis all factors that may affect the required number of spare parts. The number of spare parts required can be effectively estimated based on the reliability performance of the item. The reliability characteristics of an item are influenced not only by the operating time, but also by factors such as the operational environment. Therefore, for spare parts provisioning to be effective, the impact of these influence factors on the reliability performance of the item should be quantified. Hence, the statistical approach selected for reliability performance analysis must be able to handle the effect of these factors. One of the important models for reliability performance analysis that takes influence factors into account is the proportional hazard model (PHM), which has received less attention in the field of spare parts provisioning. In this paper the application of PHM to spare parts provision is discussed and demonstrated by a case study.

Keywords - Spare parts, Reliability performance, Operational environment(s), proportional hazards model

I. INTRODUCTION

As a result of some limitations in the design phase (e.g. the state of the art of the technology used, economic limitations, environmental conditions, etc.) there are many cases where systems are not able to meet the users' requirements fully in terms of system performance and effectiveness. This is often due to poor design in respect of reliability and maintainability performance, combined with a poor maintenance and product support strategy. Spare parts are an important part of the product support activity and have a direct effect on the reliability and maintainability performance of the system and consequently on its performance and effectiveness. The lack of critical spare parts can cause an untimely stoppage in a system. Hence, adequate spare parts provision has to be considered seriously in the design and operational phases.

In general, with the exception of preventive maintenance activities, spare parts for maintenance tasks are usually required at random intervals. Spare parts provisioning is a complex problem and requires the identification of all influence factors and the selection of an appropriate model for quantifying their effect on the required number of spare parts.

One of the important influence factors affecting the number of spare parts required is the operational

environment (e.g. the surrounding environment, the skill of the operational and maintenance crew, the history of repair activities carried out on the machine, etc.) [1]. For example, the same machines working in different climate conditions (temperature, humidity, etc.) may require different amounts of spare parts. In the literature, there are a large number of research projects in the general area of spare parts provisioning, especially in spare parts logistics [2-3]. Most of these works deal with repairable systems and spare parts inventory management [4]. They mostly use the queuing theory approach to determine the required number of spare parts [5]. The queuing theory based model primarily deals with constant failure rates and constant repair rates (exponential time to failure and time to repair), which are not valid assumptions in many cases. Furthermore, this model cannot be used to quantify the effect of operational environment on the required number of spare parts.

The quantitative techniques based on reliability theory can be used for spare parts provision when the failure rate is not constant [6-7]. In reliability based statistical approaches the required number of spare parts can be calculated on the basis of the reliability performance of the item [8]. Although reliability based spare parts provision provides a suitable statistical approach to quantify the effect of operational environment on the required number of spare parts (as covariates or explanatory variables), the existing studies, in general, have paid less attention to this subject. Most of the available studies considered the life time of the item (i.e. time to failure or time between failures) as the only variable.

Hence, the available methods need to be modified in order to consider the effect of the operational environment on spare parts. For this, an appropriate statistical model must be selected in order to be able to analyze the effect of operational environment on the reliability performance and failure rates of the item. One of the important statistical approaches for modeling the effect of the operational environment on reliability performance is PHM. Hence, the aim of this paper is to show the application of PHM for modeling operational environments as covariates for the reliability performance of an item and then using this model to predict the required number of spare parts.

The rest of this paper is organized as follows. In Section 2 reliability based spare parts estimation and PHM is reviewed briefly. Section 3 demonstrates the application of reliability based spare parts estimation

taking into consideration the effect of covariates, using a case study. Section 4 provides the conclusions.

II. RELIABILITY BASED SPARE PARTS ESTIMATION

The most commonly used model for analyzing the non-repairable system is renewal process. In renewal process the repair time is assumed to be negligible so that the processes can be viewed as point processes. The other assumption for renewal process is that the times between failures are independent and identically distributed [8-9].

The renewal process model provides a way to describe the rate of occurrence of events (in our case the number of failures) over time. Because non-repairable components are discarded, it is reasonable to assume that the number of spare parts required is equal to the number of failures. If $N(t)$ represents the number of renewals that occur by time t , and the time-to-failure random variables $X_i, i \geq 1$ have a common distribution $F(t)$, then the probability distribution of the number of failures is given by [8]:

$$P(N(t) = n) = F^n(t) - F^{n+1}(t) \quad (1)$$

where $F^n(t)$ is the n -fold convolution of $F(t)$ and denotes the probability that the n -th renewal occurs by time t and given by:

$$F^n(t) = \int_0^t F^{n-1}(t-x) dF(x) \quad (2)$$

The expected number of failures, $M(t)$, during a length of t is given by:

$$M(t) = \sum_{n=1}^{\infty} F^n(t) \quad (3)$$

The above equation is known as the Renewal Function, and it gives the number of renewals (failures) during $(0, t]$ and can be also written as :

$$M(t) = F(t) + \int_0^t M(t-x) f(x) dx \quad (4)$$

If the operation time t of the system on which the non-repairable component is installed is quite long and several replacements need to be made during this period, then the average number of failures, $M(t) = E[N(t)]$, will stabilize to the asymptotic value of the renewal function as [9]:

$$M(t) = N_t = \frac{t}{\bar{T}} + \frac{1}{2} \left(\left(\frac{\sigma(T)}{\bar{T}} \right)^2 - 1 \right) \quad (5)$$

where the \bar{T} is the mean time to failure and $\sigma(T)$ is the standard deviation of time to failure. If time t in the above equation representing a planning horizon is large, then $N(t)$ is approximately normally distributed (based on

a central limit theorem). Then the approximated number of spares N_t needed during this period with a probability of shortage equal to $1-p$ is given by [10]:

$$N_t = \frac{t}{\bar{T}} + \frac{1}{2} \left(\left(\frac{\sigma(T)}{\bar{T}} \right)^2 - 1 \right) + \frac{\sigma(T)}{\bar{T}} \sqrt{\frac{t}{\bar{T}}} \Phi^{-1}(p) \quad (6)$$

where $\Phi^{-1}(p)$ is the inverse normal function. One of the important stages in calculating the Renewal Function is to obtain the $F(t)$ or probability distribution of the failure data of components. Different common distributions (e.g. Exponential, Weibull, Lognormal, etc.) have been used to model the failure data. The only variable in these distributions is the operating time. Hence, these models cannot be used to model the effect of operational environments on the probability of failure of components and consequently on the number of spare parts required. Furthermore, the main assumption in these models is that the data come from a type of probability distribution and that inferences are made about the distribution parameters. However, if the historical data does not follow the selected distribution, these models may be misleading.

Limited satisfaction of the assumptions about distribution fits, and the need to consider covariate effect on reliability performance, led to the development of non-parametric reliability models. These models are mainly based on the method suggested by Kaplan and Meier [11] and Nelson [12]. In fact, a major contribution to the concept of non-parametric models for modeling the effects of covariates was made by the method known as the proportional hazard model (PHM), which is suggested by Cox [13]. In PHM, the hazard rate for an item is a product of the baseline hazard function, $h_0(t)$ of the item and a function $\varphi(\beta z)$ incorporating the effect of covariates. The generalized form of PHM that is most commonly used is written as [13]:

$$h(t, z) = h_0(t) \varphi(\beta z) \quad (7)$$

where z is a row vector consisting of the covariates and β is a column vector consisting of the regression parameters. The baseline hazard rate is the hazard rate that an item experiences when there is no influence from covariates. Different functional forms of $\varphi(\beta z)$ can be used. However the exponential form of $\varphi(\beta z)$ is the most widely used because of its generality and simplicity. Using the exponential form of $\varphi(\beta z)$ the hazard rate can be written as:

$$h(t, z) = h_0(t) \exp(\beta z) = h_0(t) \exp\left(\sum_{j=1}^n \beta_j z_j\right) \quad (8)$$

The related probability of failure function for components is given by:

$$F(t) = 1 - R(t) = 1 - \exp\left[-\int_0^t h_0(x) dx\right]^{\exp\left(\sum_{j=1}^n \beta_j z_j\right)} \quad (9)$$

Originally the PHM is a non-parametric regression method where there is no assumption about the baseline hazard rate. However, parametric models such as the Weibull distribution can be used to model the baseline hazard rate. Ghodrati and Kumar [9] (2005) used the 2-parameter Weibull distribution in order to model the baseline hazard rate to estimate the required number of spare parts for the hydraulic jacks (lifting cylinders) of load-haul-dump (LHD) machines in the Kiruna mine.

III. CASE STUDY

An electricity meter is a device that measures the amount of electrical energy consumed by a residence, business, or an electrically powered device. An electricity meter is part of a permanent circuit, is connected directly to the mains without any protection and operates unattended. Furthermore, an electricity meter is usually a maintenance-free device. These conditions pose a number of challenges to the reliability engineer in the design phase [14]. The operational environment has a direct influence on the time taken for an electricity meter to wear out, its failure mechanisms and consequently its reliability performance [15]. Edwards and Sawcer [16] discussed the impact of environmental issues on the design and application of electricity meters.

Some methods such as part count (part stress) are used to assess the effect of the operational environment on the reliability performance of an electricity meter at the early design stage. However taking into consideration that field reliability performance may differ from predicted reliability performance by a factor of as much as 3 to 4, calculating the reliability performance based on the actual operational environment is very important for designers and manufacturers as well as for providers and the users of energy [14]. Reliability performance assessments using the filed data can reflect the actual operational environment.

Jajarm - a city in the northern part of Iran - is very cold during the winter and hot in summer. Jajarm is a collection of small villages and towns. Western parts of this city have a higher rainfall than Eastern part. The eastern part is desert and is dusty. In this situation, depending on their location, electricity meters may experience different failure rates and therefore different requirements for spare parts. The failure data from electricity meters in the power distribution system in Jajarm is used in this study.

A. Covariates-based reliability analysis for electricity meter

Power distribution in Jajarm is divided in to 8 different sections. Five of them are in the rural area

(Sec.10, Sec.11, Sec.12, Sec.13 and Sec.14) and three of them are in the urban area (S_{21} , S_{22} , and S_{23}). In this study locations of failure are considered to be the covariates. The location of failure may represent the ambient conditions (e.g. rain, wind, dust), protection method, the customer type and demand etc. In this study we used the parametric PHM in order to estimate the effect of the covariates on the reliability performance of electricity meters.

Initial analysis of the electricity meter data: Similar to the other statistical approaches PHM is built on certain assumptions. The assumptions for PHM may be listed as shown below [17]:

- The times to failures are independent and identically distributed (*iid* assumption).
- All influencing covariates are included in the model.
- The ratios of any two hazard rates are constant with respect to time as:

$$\frac{h_1(t, z_1)}{h_2(t, z_2)} = \frac{h_0(t) \exp(\beta z_1)}{h_0(t) \exp(\beta z_2)} = \exp(z_1 - z_2) \quad (10)$$

where z_1 and z_2 are any two different sets of covariates assumed to be associated with the system. The first assumption looks for the presence of trend and serial correlations. The *iid* assumption can be investigated by using graphical or numerical methods. Graphical methods have been used in this study and the result of such analysis shows that there is no reasonable evidence for the presence of the trend and serial correlations in the data. Therefore, the times to failures (*TTF*) data of the electricity meter are *iid* distributed.

The second assumption investigates the omission of covariates. However in this case study we assume that the only influence covariate is the location of failure. For the third assumption, graphical methods or numerical methods can be used to examine the proportionality assumption. Often the graphical method is sufficient for checking the model. Graphical methods generally are based on the stratification approach. In these methods, the time to failure data (*TTF*) is stratified into several layers determined by the differing value levels of the desired covariate. If the proportionality assumption is justified, the shape of the baseline hazard functions for all strata should be similar. Furthermore, under the proportionality assumption, Log minus log plot of estimated survival functions (*LML*) or log cumulative hazard for different strata must provide the parallel curves as well, for the different strata [17]. In this study the *LML* is selected for checking the proportionality assumption.

Formulation of the covariates is a prerequisite and must be carried out for PHM analysis and checking the third assumption. In order to formulate the covariates the location of failure were considered as binary covariates from z_{01} - z_{08} respectively. For a particular time to failure,

only one of these covariates will have a value of one, to indicate the location where the failure has occurred, and the other covariates will be zero. The result of the analysis (LML plot) shows there is no significant deviation from parallelism of the two curves for different strata. Fig. 1 shows one of these plots when the data is stratified based on the z_{01} .

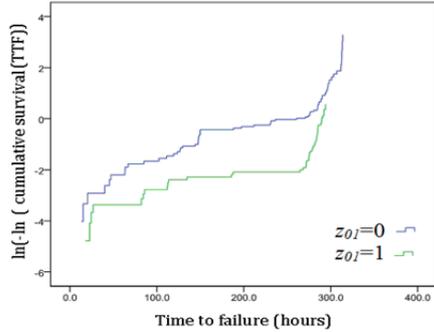


Fig. 1. Log minus log of cumulative survival function for z_{01}

As mentioned before in this study the parametric PHM is considered for reliability performance analysis of electricity meters. Hence, to find the best distribution for the baseline hazard rate, several distributions including, Weibull, Normal and Exponential distributions were nominated. The analysis of data was carried out using the software *ALTA7* [18]. The result for goodness of fit showed that the best fit distribution for the data sets is the two-parameter Weibull distribution. Under the Weibull distribution assumption for baseline hazard rate the hazard rate can be written as:

$$h(t, z) = \frac{\beta}{\eta} \left(\frac{t}{\eta} \right)^{\beta-1} \exp \left(\sum_{j=1}^n \beta_j z_j \right) \quad (11)$$

where β and η are the shape parameter and scale parameter of baseline hazard rate respectively. The result of analysis showed that only the three covariates z_{01} (Sec.21), z_{02} (Sec.23) and z_{03} (Sec.22) have a significant effect on the hazard rate for an electricity meter. Table 1 shows the distribution parameters and regression parameters of the covariates.

TABLE I
DISTRIBUTION AND REGRESSION PARAMETERS OF MODEL

β	η	z_{01} (Sec.21)	z_{02} (Sec.23)	z_{03} (Sec.22)
2.70	178.6	-0.88	-0.75	-1.11

Under this condition the hazard rate for an electricity meter can be written as:

$$h(t, z) = h(t, z) = \frac{2.7}{178.6} \left(\frac{t}{178.6} \right)^{2.7-1} \exp \left(\sum -0.88z_{01} - 0.75z_{02} - 1.11z_{03} \right) \quad (12)$$

Furthermore, the reliability functions, $R(t, z)$ and the probability distribution function, $F(t, z)$ for an electricity meter is given by:

$$F(t, z) = 1 - R(t, z) = 1 - \exp \left[- \int_0^t \frac{2.7}{178.6} \left(\frac{x}{178.6} \right)^{2.7-1} dx \right]^{\exp \left(\sum -0.88z_{01} - 0.75z_{02} - 1.11z_{03} \right)} \quad (13)$$

According to these results, the hazard rate for the electricity meter in the Sec. 21(z_{01}), Sec. 23(z_{02}) and Sec. 22 (z_{03}) will be reduced by 0.41($Exp(-0.88)$), 0.47 ($Exp(-0.75)$) and 0.33 ($Exp(-1.11)$) respectively and in the other section the hazard rate will be equal to baseline hazard rate.

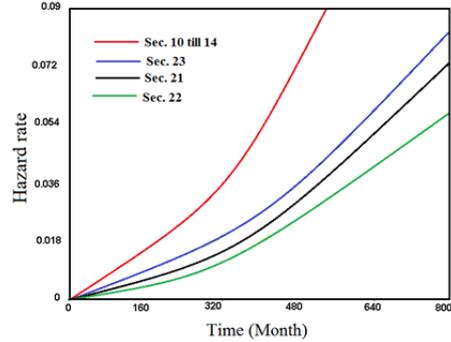


Fig. 2. Hazard rate of the electricity meters in different sections

Fig. 2 shows the hazard rate of the electricity meters in different sections. As shown in Fig. 2 the hazard rate in the rural area (Sec10 till Sec.14) is more than the hazard rate in the urban area (Sec.21 till Sec.23).

B. Reliability-based spare parts provision for electricity meters

An electricity meter is a non-repairable item. Furthermore, power distribution operations take place over quite a long time period and several replacements of electricity meter need to be made during this period. Hence an approximated number of spare electricity meters can be calculated based on (6). The mean time to failure and standard deviation of time to failure need to be calculated based on (12). From a statistical point of view, when the baseline hazard rate in the PHM is the two-parameter Weibull distribution, the influencing covariates only change the scale parameter baseline hazard rate and the shape parameter remains unchanged [9]. Based on this point, the reliability performance of the electricity meter in the presence of different covariates will be a two-parameter Weibull distribution with a shape parameter equal to the shape parameter of the baseline hazard rate (β_0) and a different scale parameter. The scale parameter of the Weibull distribution, η_c , taking into consideration the covariates effect, is given by:

$$\eta_c = \eta_0 \exp \left(\sum_{j=1}^n \alpha_j z_j \right)^{-1/\beta_0} \quad (14)$$

where, η_0 is the baseline scale parameter. As regards the 2-parameter Weibull distribution characteristics, the mean time to failure (\bar{T}) and standard deviation $\sigma(T)$ of electricity meter can be calculated by:

$$\bar{T} = \eta_c \Gamma \left(\frac{1}{\beta_0} + 1 \right) \quad (15)$$

$$\sigma(T) = \eta_c \sqrt{\Gamma\left(1 + \frac{2}{\beta_0}\right) - \left(\Gamma\left(\frac{1}{\beta_0} + 1\right)\right)^2} \quad (16)$$

The reliability characteristics of electricity meters in the different power distribution sections in Jajarm are shown in Table 2.

TABLE II
RELIABILITY CHARACTERISTICS OF ELECTRICITY METER

Section		Covariates	Distribution parameter	
Urban area	Sec.21	$z_{01}=1, z_{02}$ till $z_{06}=0$	$\beta: 2.69;$ $\eta:256.76$	$\bar{T}: 228.5;$ $\sigma(T): 91.8$
	Sec.23	$z_{02}=1, z_{01}$ and z_{03} till $z_{06}=0$	$\beta: 2.69;$ $\eta:244.65$	$\bar{T}: 217.7;$ $\sigma(T): 87.5$
	Sec.22	$z_{03}=1, z_{01}, z_{02}$ and z_{04} till $z_{06}=0$	$\beta: 2.69;$ $\eta:279.30$	$\bar{T}: 248.6;$ $\sigma(T): 100$
Rural area	Sec.10 till Sec.14	z_{01} til $z_{03}=0, z_{04}$ til $z_{05}=1$	$\beta: 2.69;$ $\eta:185.40$	$\bar{T}: 165;$ $\sigma(T): 66.3$

Using information from Table 2 and (6) the number of spare electricity meters required can be calculated. Table 3 shows the number of spare parts required in different areas with a probability of storage equal to 95% in the 10 years. According to Table 3 12,981 electricity meters are needed in Jajarm province over a 10-year period.

TABLE III
SPARE PART CONSIDERING COVARIATES EFFECTS

Section	Number of customers	$\sigma(T)$	\bar{T} (Month)	Number of spare parts
Sec. 21	3670	91.8	228.5	2146
Sec. 23	6154	87.5	217.7	3833
Sec. 22	2515	100	248.6	1316
Sec.10 till Sec. 14	6523	66.3	165	5686

If we ignore the effect of covariates on the hazard rate, then the electricity meter hazard rate will follow the two-parameter Weibull distribution with $\beta=1.84$ and $\eta=254.8$. Moreover, the mean time to failure and standard deviation of the electricity meter are equal to 226.3 and 126.8 respectively. Table 4 shows the number of spare parts required in different sections with a probability of storage equal to 95% in the 10 years. According to Table 4, when the effect of the covariate is not considered in the analysis, 16,278 electricity meters are needed for 10 years.

TABLE IV
SPARE PART FOR DIFFERENT SECTIONS WITHOUT CONSIDERING COVARIATES EFFECTS

Section	Number of customers	$\sigma(T)$	\bar{T}	Number of spare parts
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Sec.21	3670	126.8	226.3	3150
Sec.23	6154	126.8	226.3	5283
Sec.22	2515	126.8	226.3	2159
Sec.10 till Sec.14	6523	126.8	226.3	5686

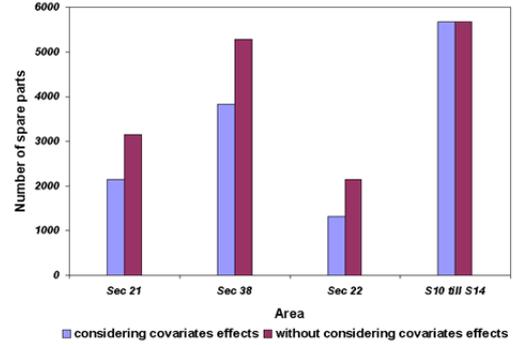


Fig. 3: Spare parts with and without the effect of covariates taken into consideration

Fig. 3 compares the number of spare parts with and without the effect of covariates being taken into consideration in the different sections. The result of this comparison shows that the number of actual spare parts in the urban area when the effect of covariates is taken into consideration is 25% less than when the covariates effect is not taken into account.

V. CONCLUSION

The operational environment may have a great effect on the reliability performance of an item and consequently on the required number of spare parts. Hence spare part estimation without considering operational environment can cause unplanned downtime in the system due to lack of availability of the required spare parts. By using reliability based spare part provision methods, the effect of the operational environment on spare parts can be assessed. Selecting the appropriate statistical approach for reliability analysis is an important stage in reliability based spare part provision methods. The selected method must be able to handle the effect of different operational environments. PHM is a suitable statistical approach that can be used to model the effect of operational environments on the reliability performance of the item.

In the Jajarm power distribution system, the number of spare parts required when the effect of the operational environment is not taken into consideration is 1.25 times more than when the effect of the operational environment is considered in the analysis. Furthermore the result of analysis shows that the operational environment in the rural area increases the hazard rate. It seems that the existence of good protection conditions is one of the main factors leading to the reduced hazard rate in the urban area.

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