RELATIVE GAIN ARRAY ESTIMATION BASED ON NON-PARAMETRIC FREQUENCY DOMAIN SYSTEM IDENTIFICATION

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ABSTRACT

Since the introduction of the Relative Gain Array (RGA) by Bristol in 1966, it has received a high level of attention as a practical tool for solving the input-output pairing problem in decentralized control. In order to remove the dependency of these tools on a parametric description and accurate knowledge of a nominal model this work proposes a method to calculate the RGA directly from a non-parametric frequency response matrix (FRM), derived from a frequency domain system identification approach.

WHAT WE ARE AIMING TO DO

- Estimate the elements of RGA matrix over a frequency range of interest using random excitation.
- Calculate the variance of the RGA elements over a frequency range of interest using Monte-Carlo principles.

WHAT WE WILL GAIN

Parametric modeling of systems usually consists of two steps: (i) determining the system structure and (ii) estimating the model parameters. Both are susceptible to different sources of uncertainty which affect the pairing results. Accordingly, using a non-parametric model in estimating the RGA will reduce the influence of the model uncertainty.

FUTURE DIRECTIONS

- Applying this approach to non-linear systems using best linear approximation technique.
- Applying this approach to time varying systems.

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FRM AND RGA ESTIMATION USING RANDOM EXCITATION

Suppose a general $m \times n$ system,

$$\mathbf{G}(k) = \begin{bmatrix} G_{11}(k) & \cdots & G_{1n}(k) \\ \vdots & \ddots & \vdots \\ G_{m1}(k) & \cdots & G_{mn}(k) \end{bmatrix}$$

the estimated $\mathbf{G}$ at each frequency k calculated by:

$$\hat{S}_{YU}(k) = \hat{\mathbf{G}}(k) \hat{S}_{UU}(k)$$

where $\hat{S}_{YU}$ and $\hat{S}_{UU}$ are the cross and the auto power spectrum which are calculated by

$$\hat{S}_{YU}(k) = \frac{1}{M} \sum_{s=1}^M \begin{bmatrix} Y^{[1,s]}(k) \\ \vdots \\ Y^{[m,s]}(k) \end{bmatrix} \begin{bmatrix} \bar{U}^{[1,s]}(k) & \cdots & \bar{U}^{[n,s]}(k) \end{bmatrix}$$

and

$$\hat{S}_{UU}(k) = \frac{1}{M} \sum_{s=1}^M \begin{bmatrix} U^{[1,s]}(k) \\ \vdots \\ U^{[n,s]}(k) \end{bmatrix} \begin{bmatrix} \bar{U}^{[1,s]}(k) & \cdots & \bar{U}^{[n,s]}(k) \end{bmatrix}$$

where M , Y^s , U^s and $\bar{U}^s$ are number of data blocks, Discrete Fourier Transform (DFT) of the output, input and input conjugate for segment s , respectively.

Once $\hat{\mathbf{G}}(k)$ is found, then the RGA can be easily estimated using:

$$\hat{\Lambda}(k) = \begin{bmatrix} \hat{\lambda}_{11}(k) & \cdots & \hat{\lambda}_{1n}(k) \\ \vdots & \ddots & \vdots \\ \hat{\lambda}_{m1}(k) & \cdots & \hat{\lambda}_{mn}(k) \end{bmatrix} = \hat{\mathbf{G}}(k) \otimes \hat{\mathbf{G}}(k)^{-T}$$

where $\otimes$ denotes the element-by-element product and $-T$ is the inverse of the transpose.

The variance of the estimated RGA can be calculated by repeating the steps for many times, then calculation of the variance over all these trials is based on Monte-Carlo principle.

SIMULATION EXAMPLE

Consider the distillation column given by:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} \frac{4.05e^{-27s}}{50s+1} & \frac{1.77e^{-28s}}{60s+1} & \frac{5.88e^{-27s}}{50s+1} \\ \frac{5.39e^{-18s}}{50s+1} & \frac{5.72e^{-14s}}{60s+1} & \frac{6.9e^{-15s}}{40s+1} \\ \frac{4.38e^{-20s}}{33s+1} & \frac{4.42e^{-22s}}{44s+1} & \frac{7.2}{19s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + \begin{bmatrix} \frac{1.44e^{-27s}}{40s+1} \\ \frac{1.83e^{-15s}}{20s+1} \\ \frac{1.26}{32s+1} \end{bmatrix} d$$

Here, y_3 is not a quality variable and there is no need to keep it constant. Therefore, the control configuration selection problem turns into a 2×3 system problem. In this example, the Hanning window and $M = 150$ are used over 100 trials. The exact Λ (black), the mean of $\hat{\Lambda}$ (blue), and $\pm\sigma_{\Lambda}$ bounds (red) for $SNR = 10$ are depicted in below

As shown in the figure, the mean of $\hat{\Lambda}$ coincides well with the exact Λ and the pairing selection $y_1 - u_3$ and $y_2 - u_2$ are chosen.