PEEC Models for Air-core Reactors Modeling Skin and Proximity Effects

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Abstract - This paper presents a partial element equivalent circuit (PEEC) model for air-core reactors modeling skin and proximity effects at higher frequencies using the volume filament approach. Modeling results are compared to measurements in both time domain and frequency domain, and show good agreement.

Keywords - Air-core reactor, Equivalent circuit modeling, Skin effect, Proximity effect.

I. INTRODUCTION

Air-core reactors are commonly used in current limiting applications and as harmonic filters. In order to model the propagation of fast transients ($\sim ns$) through air-core reactors there is a need for high frequency models. For such high frequency models, it is necessary to consider skin and proximity effects. High frequency models for air-core reactors using the Partial Element Equivalent Circuit (PEEC) [1], [2], [3] approach have been reported in [4]. In the PEEC model, each turn in the reactor winding is represented by a finite number of bars, and only currents flowing along the air-reactor windings were considered. Such models are good enough for reactors with very thin conductor cross sections and at lower frequencies (a few megahertz), and does not account for skin effects. For realistic reactors it is necessary to model the current along the windings as well as the transverse currents.

Several methods of modeling skin and proximity effects for a multi-conductor geometry have been reported. For example, the volume filament approach [1], [5], which requires a 3D discretisation of the conductors. There is the global surface impedance approach [6], which is more of a surface technique; and macro-models which involve a reduced decoupled RL circuits to model skin and proximity effects [7]. Other skin effects models for conductors are found in [8]. In this paper the volume filament technique is used to make a 3D model of air-core reactors accounting for skin and proximity, using the PEEC modeling approach. PEEC model results are compared with measurements in both time domain and frequency domain.

An alternative method for modeling air-core reactors, namely the lumped modeling approach, has been considered in [4]. In this approach, the reactor is represented by a number of mutually coupled lumped sections, with each section consisting of a given number of turns. A comparison with measurements showed that the lumped models are good only

at frequencies less than 1 MHz, but fail to capture the high frequency variations. A more detailed distributed system which accounts for couplings between all parts of the individual winding is necessary at higher frequencies.

II. BASIC PEEC THEORY

The PEEC approach is more detailed described in [1], [2], [3]. Only the time domain derivation is shown here. The frequency domain derivation is similar.

Consider the electric field on a conductor given by (1)

$$\vec{E}^i(\vec{r}, t) = \frac{\vec{J}(\vec{r}, t)}{\sigma} + \frac{\partial \vec{A}(\vec{r}, t)}{\partial t} + \nabla \phi(\vec{r}, t), \quad (1)$$

where $\vec{E}^i$ is an incident (externally) applied electric field, $\vec{J}$ is the current density in the conductor, $\vec{A}$ is the magnetic vector potential, ϕ is the scalar electric potential, and σ the electrical conductivity. By using the basic definitions of the electromagnetic potentials as in (2) and (3),

$$\vec{A}(\vec{r}, t) = \mu \int_{v'} G(\vec{r}, \vec{r}') \vec{J}(\vec{r}', t_d) dv' \quad (2)$$

$$\phi(\vec{r}, t) = \frac{\nabla}{\epsilon_0} \int_{v'} G(\vec{r}, \vec{r}') q(\vec{r}', t_d) dv'. \quad (3)$$

where the Green's function $G(\vec{r}, \vec{r}') = \frac{1}{|\vec{r} - \vec{r}'|}$, and substituting in (1) the electric field integral equation (4), at the point $\vec{r}$ in the conductor is obtained according to

$$\begin{aligned} \vec{E}^i(\vec{r}, t) &= \frac{\vec{J}(\vec{r}, t)}{\sigma} \\ &+ \mu \int_{v'} G(\vec{r}, \vec{r}') \frac{\partial \vec{J}(\vec{r}', t_d)}{\partial t} dv' \\ &+ \frac{\nabla}{\epsilon_0} \int_{v'} G(\vec{r}, \vec{r}') q(\vec{r}', t_d) dv'. \end{aligned} \quad (4)$$

Integrating (4) using a suitable inner product, followed by some algebraic manipulations, the Kirchhoff's voltage law for a PEEC cell is obtained as

$$V = RI + sL_p I + PQ, \quad (5)$$

where L_p contains partial inductances while P contains the coefficients of potential. The partial elements are evaluated using analytical routines for orthogonal geometries and numerical integration for non-orthogonal geometries, [4] - [3]. Fig. 1 (b)

shows the PEEC model representation of a the conducting bar in Fig. 1 (a), and is referred to as a basic PEEC cell. This type of PEEC cell is the building block of all PEEC models for conductors. The magnetic field couplings are considered through the mutual partial inductances represented in a voltage source V_{mm}^L while the electric field couplings are considered by the mutual coefficients of potentials represented in a current source I_C^i .

Fig. 1. Conducting bar in (a) and corresponding PEEC model representation (b).

A. Skin and Proximity effects

Skin and proximity effects bring about non-uniformity in the current distribution along a cross section of a conductor. The increase in current density towards the conductor surface and around edges, due to changing fields within the conductor itself only, is termed skin effect [8]. This phenomenon depends on the conductor geometry and frequency.

In this study, the volume filament (VFI) technique is applied. Skin effect is modeled by making a 3D discretisation of the conductor windings into volume cells of width $\delta/2 = \sqrt{\frac{1}{4\pi\mu\sigma f_m}}$, where δ is the skin dept of copper at the maximum frequency of interest f_m . An optimal way is to use a non-uniform meshing scheme, where a coarser mesh is used in the center and a finer mesh close to the edges, respecting the $\delta/2$ rule.

The current distribution in one volume filament could be influenced by changing fields in adjacent filaments. This is termed proximity effects [8]. Fig. 2 is an illustration of the current distribution in volume filaments due to skin and proximity effects. In the VFI PEEC model, the current directions in the volume cells is not assumed a priori. The electromagnetic coupling between all volume cells is considered, thus properly modeling proximity effects. The current distribution in a multi-conductor system is a combination of skin and proximity effects. Efforts are on the way to create a less expensive surface skin-effect model for reactors which requires just a surface discretization of the windings, a method similar to that described in [6].

Fig. 2. Current distribution due to skin and proximity effects. (a) Volume cell currents in same direction. (b) currents in opposite directions (c) Only skin-effect, no proximity effect.

III. AIR-CORE REACTOR MODELING

This section deals with the creation of the electromagnetic models for air-core reactors using PEEC cells.

A. Reactor structure

In order to validate the modeling approach, a simple air-core reactor model was constructed by winding 65 turns of thin copper tape of width 6.35 mm and thickness 0.076 mm around a sparse rectangular wooden support (low ϵ_r), with a constant separation of 1 cm. Unlike in [4], copper tape of large surface area was chosen in order to observe variations due to skin and proximity effects. Fig. 3 is a picture of the air core reactor model.

Fig. 3. Constructed tape reactor.

B. PEEC air-core reactor model

In the corresponding PEEC model of the reactor, each turn is represented by four rectangular bars. The end of the first turn is connected to the start of the second turn by a short circuit. In a similar fashion, the second turn is connected to the third, the third to the fourth until the last turn, modeling a spiral winding.

Each bar represents a PEEC volume cell, and is used in the calculation of the partial inductance and partial coefficients of potential. The partial inductance Lp_{ii} is calculated from the i :th volume cell, while the partial coefficient of potential P_{ii} is obtained from the corresponding surface cells.

C. Measurement setup

Impedance response over a specific frequency range is obtained directly using a vector network analyzer. For the time domain response, a low voltage impulse test is performed. A schematic of the time domain setup is described in Fig. 4. The input terminal of the reactor is excited with a fast trapezoidal pulse, of amplitude in the order of 10 V, from an impulse generator. The internal resistance of the pulse generator is 50 Ω . The output terminal of the reactor is in series with a 50 Ω resistor. The input and output pulses are observed using an oscilloscope.

Fig. 4. Setup for low voltage impulse test.

IV. RESULTS

The PEEC simulations are run on a Linux machine with a dual Intel Xeon CPU 2.8 GHz, and 3 GB RAM. Using the PEEC approach, the same model can be used both in the time domain and the frequency domain.

A. Time domain results

The reactor was excited with a fast trapezoidal pulse of rise time 32 ns, and peak voltage level of 9.2 V from the pulse generator. The meshing used is 3 volume cells per side (cell size = 16.67 cm) and the cell count is presented in Table I. The output current flows through a 50 Ω resistor while the generator has internal resistance is 50 Ω . The input voltage and output voltage were observed and recorded using an oscilloscope. Fig. 5 presents the time domain results.

Fig. 5. Terminal voltage response for the 65 turn tape reactor, excited at the input terminal with a fast trapezoidal pulse of rise time 32 ns, while the output is terminated through a 50 Ω resistor.

Fig. 7. Current distribution in reactor winding at 150 MHz using current vectors.

B. Frequency domain results

The frequency domain model considers all the electric and magnetic field couplings between all volume cells. A quasi-static simulation is performed, meaning the phase shift is updated every frequency step. This is good enough due to the large electrical length of the reactor. The PEEC model was excited with a unitary current source and the meshing is the same as for the time domain test in the previous section. However, for the skin effect model each volume cell is subdivided along the width into 7 cells. This gives the cell count as presented in Table I which is a considerable increase. The frequency response was obtained for 10 kHz to 5 MHz. Fig. 6 presents the simulated and measured results in the frequency domain. The influence of skin effect on the current distribution in the reactor windings is more pronounced at higher frequencies, as shown in Fig. 7

Fig. 6. Impedance response for frequencies 10 kHz to 5 MHz comparing measurements, PEEC model, and PEEC-VFI model for skin effect.

TABLE I
CELL COUNTS FOR TEST CASES.

	Part inductances		Coefficients of potential	
	self	mutual	self	mutual
Test IV-A & B: 65 turn, rectangular, tape reactor	780	303 810	1 040	540 280
Test IV-B: 65 turn, rectangular, tape reactor Skin effect model by VFI-PEEC	5720	16 356 340	4 160	8 650 720

C. Time complexity

The time required for each step in the PEEC solver for Test IV-A and IV-B are detailed in Table II. While the time for the volume filament (VFI) model is given in Table III.

TABLE II
TIME COMPLEXITY FOR 65 TURN, RECTANGULAR, TAPE REACTOR (TIME IN SECONDS).

Step	Time [s]	Time [s]
Solver type	Test IV-A	Test IV-B
Parsing & Meshing	2	2
Calc. partial inductances	3	3
Calc. coefficient of potentials	5	5
Solver	410 †	40 ‡
Total	420	50

TABLE III
TIME COMPLEXITY FOR 65 TURN, RECTANGULAR, TAPE REACTOR WITH VOLUME FILAMENTS FOR SKIN EFFECT MODELING (TIME IN MINUTES).

Step	Time [min]	Time [min]
Solver type	Test IV-A	Test IV-B
Parsing & Meshing	0.15	0.15
Calc. partial inductances	1	1
Calc. coefficient of potentials	2	2
Solver	235 †	19 ‡
Total	240	22

† for 100 frequencies

‡ for 1 000 time steps

V. DISCUSSION AND CONCLUSIONS

There is a fairly good agreement between the measurement results and the PEEC model results. A slight shift in the resonance peaks which increases as one goes higher up in frequency, is observed. This can be explained from the discrepancies between the model and the constructed reactor. As previously seen, the amplitudes are hard to capture in the modeling results leaving the measurement response more damped. Up to 5 MHz, the influence of skin effect on the measured impedance response is small. Possibly proximity effects are more important at these frequencies. The reactor model created here gives the voltage and current distribution in the reactor which could be used for example in estimating the near and far field interactions in and from the reactor, studying the mechanical stress on the reactor windings and other dynamic analysis.

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