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SOCIAL INTERACTION AND LEARNING IN MATHEMATICS CLASSROOMS IN AUSTRALIA, GERMANY, HONG KONG, JAPAN, SWEDEN, AND THE UNITED STATES

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Introduction

The analysis reported in this paper is part of larger study of data from the Learners' Perspective Study (LPS), an international collaboration of researchers investigating practices in competently-taught eighth grade mathematics classrooms in nine countries. (For further information and references visit the website http://www.edfac.unimelb.edu.au/DSME/lps/). In each participating country, sequences of at least ten consecutive lessons in three urban public schools were documented. The data related to each lesson comprise classroom videotapes, teacher questionnaires, post lesson video-stimulated recall interviews with students, teacher interviews, field notes from classroom observation, students' productions, and resources used by the teacher. Some tentative results of the ongoing analysis are reported here.

The research reported in this paper aims to identify ways in which role-related asymmetries and culturally sanctioned ways of interaction serve as an orientation for the participants in mathematical reasoning discourses in classrooms and what consequences result in terms of the emerging different styles of mathematical reasoning.

In mathematics classrooms, an observer would expect to find a high proportion of reasoning discourse, the stylistic characteristics varying according to the educational rationales of learning and teaching mathematics: When a teacher explicates and elucidates the meaning of a new concept or demonstrates a mathematical proof, when students try to solve a problem in group work, or when in a "whole class" discussion mathematical meaning is negotiated. Explaining by inferring - without reference to any authority but the human mind and the stock of scrutinised results - is seen as a central feature of an ideal academic mathematical discourse. The re-contextualisation of this type of discourse in a classroom is problematic due to role-related asymmetries and other institutional constraints.

In this paper patterns of classroom interaction are compared to similar patterns from everyday contexts.

Methodology

As the main database, classroom videos and transcripts from two LPS classrooms in Germany, Hong Kong and the U.S.A. were used.

In a first step, „reasoning events" were identified on the level of utterances. The following definition of such an event was used:

A "Reasoning Event" is a part of a discourse in which a person offers a reason for something. The person who gives a reason seems to interpret something as being not evident, doubtful or disputable, or is asked by another person to give a reason. The attempt to increase evidence or acceptance has to be visible for the other participants in the discourse.

This definition does NOT take into account:
- Whether the attempt to increase evidence or acceptance (by providing reasons) is successful or unsuccessful (which could be seen from the reaction of the audience).
- Whether the reason provided is simple, sophisticated, correct, acceptable, unreasonable, not true, valid, invalid, wrong etc.

These events cannot be identified by surface linguistic characteristics. In most of the cases, linguistic indicators, such as *since, because, thus, consequently, accordingly*, or meta-communicative statements like *'Let me give you a reason'* or: *'The reason for this is…'* are missing. In the course of an argumentation, a reason might be introduced by *but* or *'No, I think…'*

Whether something is seen as an attempt at offering a reason is left to the interpretation of the participants and – in this study – of the coder[*]. Doubtful cases were discussed with the researcher.

As a strategy to identify reasoning events, reformulation was used:

Try to reformulate the utterance by filling in the missing parts that could have been said. If this can easily be done in a way so that it becomes clearly a reasoning event then it has to be coded as such.

Example 1 (Hong Kong)
Some minutes before this, the teacher stated a problem to work on: A farmer has some rabbits and some chickens. He does not know the exact number of rabbits and chickens...but in total there are ten heads...remember...ten heads...and there are twenty-six legs. The teacher shows a strategy to find the answer by trial and error: for example start with the assumption that half of them are chicken, then there are ten chicken legs …
[S wants to say something]
11:13 **Teacher** Okay...please say.
11:16 **S** Rabbits have only two legs as the other two are hands.
[whole class laughs]
11:18 **Keith** [laughs] He is impolite.
11:20 **Teacher** We usually say they are legs instead of hands. Don't argue about this point...okay?
The utterance of the student can be read as an attempt to argue that what the teacher said is not correct or is not obvious. He gives a reason why he thinks that it is not obvious. A possible reformulation of the utterance that suggests this interpretation could be:
I don't understand. I think the numbers are wrong because rabbits have only two legs as the other two are hands

The interpretation of this utterance as an event of *reasoning* does, of course, not say anything about the motives and goals of the student for uttering this assertion. Such an interpretation has to take into account the institutional setting of classroom discourse (see below).

These events were then grouped into two main categories: Self-initiated reasoning and reasoning as a reply. Self-initiated reasoning occurs when the speaker feels obliged to give reasons for an action or an assertion, even if nobody has explicitly prompted this by a question or an expression of disagreement or doubt. It may be an implicit rule of a cultural practice that, for example, apologising for an action (such as being late or not completing one's homework) is done by giving reasons. In mathematics classrooms we expect self-

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initiated reasoning episodes whenever the teacher expands on a new mathematical topic or demonstrates a solution to a problem. Another typical example is a student who gives reasons for each step in a problem solving process when presenting the solution because this has been established as a genre of discourse in a particular classroom. Reasoning as a reply was initially defined as non self-initiated reasoning as these codes were developed to exhaustively cover events of reasoning, It may be simply to answer a question which, interpreted as a lack of understanding, prompts an explanation, or an explicit request for a reason, or a reaction to an expression of doubt. If initiated by a comment only, it could as well be interpreted as self-initiated, the decision to be made by taking into account the interactional dynamics of the whole episode.

If the reasoning did not refer to *mathematics*, it is labelled as *organisational*. This happens, for example, when the teacher justifies decisions of classroom management (such as the way a mathematical task is set up or special arrangements to carry out a test), or vindicates disciplinary actions. On the part of the students it happens frequently when they explain a failing or give an account of their absence.

After retrieving single reasoning events, *reasoning episodes* where identified as the units of analysis. This points to a methodological problem. In this study, language is conceived as an action, that is, as intentional and purposeful. In order to interpret and analyse discourse, the purposes of these actions have to be reconstructed. However, actions are in general not units, but consist of "units" themselves that are repeated and form patterns so that they become predictable. The recognition of repetition presupposes the recognition of discrete units of actions but it is only the combination of the units that constitutes the realisation of the purpose of the action. Moreover, actions can be structured along different dimensions. Consequently, the identification of the reasoning episodes remains vague. Some patterns in classroom discourse are already visible in small units (e.g. the initiation-response-evaluation sequence), others can only be interpreted as parts of larger patterns (see below).

Everyday reasoning and reasoning in classrooms

Interaction in pedagogical institutions shows some distinct features as compared to everyday situations or interaction in other institutions. This does not mean that classroom talk should be characterised as completely different from everyday conversation. The functioning of everyday conversation can be seen as the pre-condition for all other institutionally constrained forms (Streek, 1983). This does not mean that forms of everyday conversation do not show role-related asymmetries (Drew & Heritage, 1992). A central assumption underlying this study is that at school, the everyday patterns of conversation are systematically transformed. Patterns of classroom interaction are variations of existing patterns known to the participants. These variations are not just concessions to the practical constraints of classroom teaching but serve the distinct goals of the institution school, that is, they are functional in terms of enculturation into distinct practices.

In this study, reasoning is not conceived as only a mental process, but as a mental process that is the consequence of an action performed by the means of language. This action is not just a single speech act, but occurs within relatively complex patterns of interaction. In a single utterance in which a speaker provides a reason, the history of the interaction is not visible any more. Thus, it is important to take the history of the interaction into account when analysing reasoning discourse.

In many languages there exist performative verbs referring to this action. The German word *begründen* refers to giving and accepting a reason; there is no English equivalent. A

dictionary gives as translations: *to give reasons for, to justify, to account for, to substantiate*. In Italian *begründen* would be a mix of *fondare* and *motivare*. In other languages the word is linked to quite different etymological contexts. The Arabian word *taclil*, for example, does not only mean *explanation* or *reason*, but also *conversation* and *diversion*. In Turkish, there is only the speaker-oriented version *sebeb göstermek*, which means to provide a cause or reason.

Reasoning as a pattern of interaction in an everyday context

Students and the teacher bring to the classroom their knowledge of the interactional pattern of reasoning in everyday contexts. This knowledge is not explicit. It usually becomes explicit only in situations in which the co-operation of the interlocutors is at stake or brakes down. To study the transformations and assimilation of everyday patterns in classrooms, a short outline of reasoning in everyday interaction is necessary. It draws on the analysis of Ehlich and Rehbein (1986).

If a person utters a reason, this aims at transforming the knowledge or values of the addressee in a way that he or she accepts or understands an action or an assertion of the speaker. This is only necessary if the speaker has a reason to think that otherwise the listener would not understand her action or assertion. If the speaker wants to prevent a breakdown of co-operation, which would lead to a breakdown of the system of actions in which the participants are engaged, she decides to utter a reason. Such an attempt can be successful or unsuccessful. If unsuccessful, several cycles may follow. Success is only possible if the participants share a system of knowledge and values. If this were not the case, the reasoning would theoretically lead to an infinite recursion. In everyday contexts the breakdown usually happens after a few unsuccessful cycles.

There are different types of reasoning in terms of what is to be substantiated. Ehlich and Rehbein (1986) describe the pattern of interaction as follows (though much more detailed):

1. The speaker performs an action or an assertion
2. The listener shows or utters a sign of a lack of understanding.
   If the listener precisely explains what he or she does not understand, this can be taken as a sign of co-operation.
3. The speaker decides to utter a reason and hopes this would transform the knowledge and values of the listener as far as the action/ assertion at stake is concerned.
4. The listener utters a sign of appreciation and understanding.

Or

4a. The listener shows/ utters a sign of a lack of understanding.
The steps 2 and 3 are repeated while the speaker utters the same or a modified reason until 4 happens.

Or

4b. The listener shows no reaction.
5. The speaker interprets this as success.

Or:

5a. The speaker infers that this means that there is still a lack of appreciation.
The steps 2 and 3 are repeated while the speaker utters the same or a modified reason until 4 happens.

A modified pattern occurs when the speaker refers to an <u>intended action</u> and she is already convinced that the listener is likely to have a lack of understanding. Another case (with a slightly modified pattern) occurs when the reasoning refers to an <u>action that is expected</u> to be performed by the listener. Ehlich and Rehbein (1986) do not list a similar case for <u>assertions</u>

intended to be uttered by the speaker or expected to be understood by the listener. However, some examples of classroom discourse can be described in this way, especially in the context of explanations given by the teacher:

Example 2 (Germany)
00:40:31 **S** Does the same rule count...like changing the signs round in the brackets...or what?
00:40:36 **T** Well if the...if I have a negative sign in front of the brackets...then I can look at it like this...as if to forget the minus sign and then simply change all the signs in the brackets.

If the speaker decides not to utter a reason, the lack of understanding remains unproductive. It is likely to lead to frustration or disapproval on the side of the listener. This might be the case in **Example 1**, assuming that the student did not act tactically (see below).
After an unsuccessful attempt, the speaker might alternatively choose to enter a meta-communicative discourse.

<u>Reasoning in the mathematics classrooms under study</u>

Tactical behaviour
In the classrooms under study, the version **4b** of the pattern of reasoning is very common, when the teacher is addressing the whole class. It is usually not the case that all students get a turn to express their understanding. In the Hong Kong classroom from the third school the teacher frequently addresses the whole class by asking something like "Did you understand, class?" and is then confronted with no reaction. It is likely that the students even use a non-reaction to pretend understanding in case of a lack of understanding. This helps them to circumvent or to undermine the formal and compulsive character of classroom interaction. This means the students are using their knowledge of the functioning of the reasoning pattern in order to pretend approval and co-operation. The teacher might use the same reaction, that is neither showing a sign of understanding nor of not doing so after a student has uttered a reason, to provoke further reasoning on the side of the student. Such a use of the knowledge about patterns of interaction can be called <u>tactical behaviour</u>. It even takes the form of showing a reaction that expresses the opposite of the mental state (e.g. approval in the case of disapproval) instead of only performing the ambivalent action of not reacting.
Consequently, when interpreting classroom discourse - be it as a participant or as a researcher - assuming sincerity can be problematic. Especially in a situation of unequal distribution of power and control over interaction, tactical behaviour is likely to occur. In mathematics classrooms this is often the case when the disapproval of a reason provided by a student can be taken as a lack of understanding, which, in turn, is evaluated in terms of achievement. This may establish a dynamic: the more tactical approval a student shows, the fewer reasons he/she is offered as explanations, the less chance he/she gets to make sense. The following short conversation is typical of one in the first German classroom from the LPS. The teacher offers an explanation because the student has got an incorrect result and the student acts like being convinced, though by analysing his written productions and from the interview it can be said that this is tactical. In the interviews some other ways of acting as a "professional student" were discussed.

Example 3 (Germany)
09:20 **Teacher** No that's not right...this negative sign here in the brackets...you have to...that means we have to envisage that it's multiplied by minus one...which means you write three minus z plus one half minus two.
09:38 **Günther** Oh right...so I have to.

More interestingly, tactical behaviour also appeared when students discussed a problem in group work. It was only the interview that gave information about Otto's tactical approval and Norbert's strategic behaviour – both of them could not make a lot of sense of the task:

Example 4 (Germany)
31:53 **Norbert** Yeah we've got that. I don't, but I don't know how we can – that's the measurement of the area. Oh we probably have to uh explain how we've arrived at *a* minus *b* in brackets squared. Here I mean.
32:15 **Tom** It says here we only have to make a picture.
32:17 **Albert** Finished?
32:17 **Norbert** No wait, give me another sheet of paper (to Albert). I haven't got one. I'm poor. No I need that, I don't want that spoiled.
32:20 **Otto** Haven't got a sheet.
32:24 **Norbert** Sheet, sheet...Right look now we've got the square.
32:33 **Tom** Yeah.
32:37 **Norbert** That's the size here.
32:39 **Otto** Yeah do it, get it done.
32:41 **Norbert** Right and this is *a* squared, this is *b* squared, this is minus *ab* and minus *ab* right?
32:47 **Otto** Yeah
32:48 **Norbert** So, because that's, and because that's minus, because that's now minus, um this has, this has to go.
32:58 **Otto** Yeah, yeah that's clear.
32:59 **Norbert** This here then stays.
33:02 **Otto** *a* squared?
33:03 **Norbert** *a* squared then, this thing here and *b* squared.
33:10 **Norbert** Yeah, yeah like that and then well like that, if you were to do that as a line, this here would be *a* and this bit *b*, then you take it away, then this piece here is *a* minus *b*.
33:25 **Otto** Yeah.
33:26 **Norbert** Yeah? now when you have this bit *a* minus *b* and this bit *a* minus *b* here, and then the square of *a* minus *b* in brackets, then that must be the area.
33:45 **Otto** Ah.
33:46 **Norbert** Now look, *a* times *a* must be the area or *b* times *b*.
33:52 **Norbert** (dismissive gesture in the direction of Otto and Seppi) Don't understand any of it, do you.
33:59 **Norbert** But it's logical, look.
34:06 **Tom** The microphone's not right.
34:09 **Norbert** a minus b is this distance here, this here (emphasized)
34:14 **Otto** And this as well (points to it on the sheet).

Example 1 can also be interpreted as tactical behaviour on the side of the student. That is, his expression of a lack of understanding is not taken to be sincere, but as a means to distract the teacher from the planned course of the events. This interpretation might have caused the teacher to decide not to give a reason.

Example 5 (Hong Kong)
04:43 **T** Class, please read the question. What does it remind you of?
04:59 **T** What does it remind you of, Patrick?
05:06 **Patrick** Addition and subtraction.
05:08 **T** It reminds you of adding and subtracting numbers. Thank you. Why do you think about addition and subtraction?
05:17 **Patrick** Simplify complicated things.
05:26 **T** Who's more imaginative? What comes to your mind when you're reading this question, factorizing polynomials? What do you know and what do you want to know?
05:41 **Mark** Learning factors.
05:42 **T** Mark...huh?
05:43 **Mark** Learning factors.
05:44 **T** Learning factors. Factors...Thank you. We've learnt factors in primary school, like common factors, largest common factors etc. In this lesson, we'll talk about factorizing polynomials. This is a

new topic. You haven't learnt it in form one or form two. First, we look at a simple example. [T writing m times brackets a plus b is equal to m times a plus m times b]

Unlike normal questions, which aim at retrieving knowledge from the addressee that the speaker does not have, the questions of the teacher aim at conducting the students' thinking. The questions exactly specify the domain in which to answer. In normal questions this is the knowledge that the questioner thinks is shared by the addressee. Each question contains a part indicating the known and another part indicating what is unknown (in normal questions: to the questioner).

The development of the topic in the episode can be reconstructed by breaking it down into known and unknown and distributing the respective parts to teacher and students:

Teacher Known	Students Unknown
The questions reminds us	of addition and subtraction.
It reminds us of adding and subtracting numbers	because it is about simplifying complex things.
When reading this question about factorizing polynomials	we want to learn factors.
Learning factors. We've learnt factors in primary school, like common factors, largest common factors etc. In this lesson, we'll talk about factorizing polynomials. This is a new topic.	

This episode can be interpreted as a strictly planned teacher presentation with allocated parts (cf. Ehlich and Rehbein, 1986). It can be read as a consistent chain of assertions uttered by one speaker. However, the teacher's questions aim at still maintaining a minimum of students' participation. In order to guarantee that it goes off smoothly, this pattern of interaction needs some skills and experience in formulating the directing questions. The decision about what exactly is stated as the known and as the unknown in the question is based on the expected mental operations performed by the students and aims at influencing these according to the direction in which the teacher wants the discourse to continue.

The reasoning involved in this pattern is not intended to be self-initiated by students. The teacher wants Patrick to *explain* his assertion; the answer does not fit the teacher's propositional plan. Consequently, a new directing question follows which is less ambivalent.

This pattern is common in all the classrooms under study. However, there are differences in the ways the teachers deal with unexpected comments and answers, that is with ruptures. Very tentatively it can be said that one of the German teacher and even more one of the Hong Kong teachers have a tendency to repair the pattern by switching more to the presentation mode, while one of the U.S. teachers tends to open it by introducing normal questions instead of directing questions. In an arbitrarily chosen Hong Kong lesson, an episode of teacher talk in the course of a demonstration of a problem solution takes the form of a "dialogue" in which the teacher introduces the reasons he provides by rhetorical questions or rhetorical contradictions. In the classical European tradition of rhetoric this would be interpreted as a stylistic means used in speeches that aim at convincing the audience.

The following episode reveals a different pattern:

Example 6 (U.S.A.)
31:05 **Amiri** Actually. What do they mean by "another name"? Why do they say "another name"?
31:08 **T** Why do they- well ... what do you guys think? Amiri's asking why do you think they're saying "another name". What- why is that important?
31:16 **Amiri** When they teach (your kids alphabets) they don't go, "seven plus two ... over seven plus two".

31:22 **T** Be- Belinda?
31:23 **Belinda** Huh?
31:23 **T** What were you going to say?
31:24 **Belinda** (I was just thinking).
31:25 **T** No, no, that's alright. What- what ( ).
31:27 **Amiri** [to other students] Stop it, you two.
31:28 **T** Anybody else? Why is it important to look for another name for one here? What's- what's- why is that important? Olivia? You have an idea?
31:36 **T** Why is it //important
31:37 **Olivia** //Why is it important to look for another what?
31:39 **Roberto** Cuz you might forget one of them.
31:41 **T** [laughs] You might forget one of them.
31:43 **Amiri** Good one. Who said that?
31:45 **Roberto** I did.
31:46 **T** Roberto. Olivia; what do you- do you have an idea? Why- why are they asking you to look for another name for one?
31:55 **T** Not sure? That's okay. Amelia?
31:57 **Amelia** ( ). I can't explain it but like-
32:02 **T** Try your best. That's okay.
32:04 **Amelia** Um ... example two, (minus two) equals one.
32:09 **T** Okay.
32:09 **Amiri** Gee-wiz.
32:10 **T** Why, though? Why is it important- okay, I'll agree with you right now. I'll say yes, that's true. Why?
32:16 **Amiri** Cuz they said that.
32:16 **T** Wh- why is it important to see that?
32:18 **Amiri** Cuz they said so.
32:19 T Why do you think- cuz A- A- Amiri's asking a good question. Why- you know, what are they trying to get us to think about here? And ... I think it's important to at least have an idea why they're asking it. What?
32:27 **Amelia** Trying to confuse us?
32:29 **T** Trying to confuse you? No. No. Certainly not. A- anybody else?
32:35 **Amiri** But that's what it's doing though.
32:38 **T** [laughs] That could be. Unfortunately, these are all things we have to consider. Unfortunately, we really have to consider these.

The development of the topic can be reconstructed as follows:

Teacher Known	Students Unknown
They mean by "another name", they say "another name", this is important because	[but] when they teach (your kids alphabets) they don't go, "seven plus two ... over seven plus two".
This is important	because you might forget one of them.
You might forget one of them said	Roberto.
Olivia is not sure	
They are asking for another name for one	[like] two (minus two) equals one.
It is important	because they said that.
It is important to see that	because they said so.
They are trying to get us think about, it is important to at least have an idea why they are asking it.	Because they want to confuse us.
They certainly do not want to confuse us. But that's what it is doing though. Unfortunately, these are all things we have to consider. Unfortunately, we really have to consider these.	

This cannot be read as a consistent chain of assertions uttered by one speaker. The pattern has more the form of a guessing game. The knowledge deficit is clearly on the side of the students and the question is mystified. It evokes different mental processes than a question in which

the domain of the known and unknown is clear. The teacher is indeed in a position that can be compared with the one who poses a riddle because he refers to a tradition (that is the one of assessment tasks) like quoting a traditional riddle as a problem to be solved or guessed. However, it is missing the step in which the one who poses the riddle reveals the solution if no one was able to guess it. Comparable episodes can be found in one of the German classrooms when traditional mathematical problems are posed.

A salient feature of many reasoning episodes in all the classrooms is that, in the context of solving tasks, the reasoning resembles the everyday pattern of a *post-hoc* justification of an action rather than the substantiation of an assertion.

<u>Reasoning and argumentation</u>

Not every reasoning is necessarily part of an argumentation. Giving a reason for the purpose of illuminating, evidencing or excusing is in general not subsumed under "argumentation". Though the aspect of "convincing" might be part of these situations, the connotation of conflict is missing. In this study "reasoning" is used as a general term, no matter in which of these situations the discourse appears. Krummheuer and Brandt (2001) employ the term "argumentation" to describe collective processes of reasoning in mathematics classrooms that are constituted by a series of interactions as a whole, unlike the description of argumentation from classical rhetoric as a type of conversation between two parties who autonomously defend their positions. Though not all theoretical constructs of argumentation refer to winning or losing as a central feature, the everyday conceptualisation of argumentation does (though the English terms "argumentation and argument" connote "fighting" more than the German ones "Argumentation and Argument" do: in German it is not possible to use "to have an argument" in the sense of having a conflict). In drawing on their ordinary language concepts many students do not conceptualise typical reasoning episodes in the classroom as "argumentation." Thus it seams unreasonable to use the term "argumentation" for a process not perceived as such by the participants. From the viewpoint of an observer, episodes which resemble a collective argumentation are extremely rare in the classrooms under study.

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Transcript protocol:

S	Unidentified student
//	Overlapping talk
...	Pause of three or less seconds, respectively where it would be in the English version of the original
(…)	Indecipherable words