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SEASONAL EFFECTS ON THE FLEXURAL STRENGTH OF RIVER ICE

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ABSTRACT
River ice at the same location has been tested (through 4-point bending of simply supported beams) on a regular basis during three winter seasons. This paper describes the method briefly and presents the seasonal variations of the flexural strength and elastic modulus. In general, the flexural strength of beams tested with the bottom surface in tension stayed constant until solar radiation penetrated the ice cover. A possible effect due to the beam size was indicated. The results for the flexural strength of beams tested with the top surface in tension were on the other hand scattered, with high values in December and much lower values in March. In April, after melting periods, the top layer strength recovered. The elastic modulus was usually independent of the bending direction, but for certain snow-ice beams there was a strong relationship between the elastic modulus and the flexural strength.

INTRODUCTION
In ice engineering the flexural strength of the ice sheet is used in formulas for bearing capacity, ice loads on sloping structures, ice breaking resistance, etc. Over the years a large number of flexural strength data on natural ice, commonly obtained from field tests of in-place cantilever beams, have been reported (Butiagin, 1955; Butkovich, 1956; Frankenstein, 1961; Drouin and Michel, 1972; Tryde, 1979; Saeki et al., 1981; Shen et al., 1988, and many others). This extensive store of data has made it possible to compare the typical ice conditions and how they vary during the winter at many places in the world. However, one must mention two important problems with the testing technique that make such comparisons, especially concerning freshwater ice, more complicated. Firstly the often indicated size-effect, statistically quantified by Parsons et al. (1992), and secondly the influence of stress concentrations. The effect of different stress concentrations was illustrated when Gow et al. (1978) showed that in-place cantilever beams, for lake ice, yielded significantly lower strengths than simply supported beams when analysed with elastic beam theory. This paper is based on data from 100 standardized four-point bending tests carried out during three winters in northern Sweden. Simply supported beams, cut and tested in the water, were used in an attempt to emulate the behaviour of a vertically loaded ice sheet. The aim was to monitor seasonal effects on the flexural strength, which were assumed to be more pronounced when fracturing was allowed at natural weaknesses in the ice beam.

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ICE FORMATION AT THE TEST SITE

The tests took place at Ängesholmen in the Lule River in northern Sweden. In this regulated river the water level was fairly constant over the winters and the water velocity was less than 0.6 m/s within the test area. Ice formation close to the shore started in the middle of November and the ice cover expanded gradually towards the main channel. During the test period, 1995-98, the coldest recorded air temperature was –34°C (30 Dec 1995). Mild periods with temperatures above zero could occur at any time during the winter, but the spring melting did not start before April. The cumulative daily air temperatures for the three test winters are shown in Figure 1. The ice growth was calculated as proportional to the square root of the negative degree-days after the ice formation date. Due to the influence of snow and snow-ice, the empirical constant was chosen differently from one year to another to obtain the best fit. The constant, used in Figure 2, was 25 mm $C^{-½}$ $day^{-½}$ for 95/96, 20 for 96/97 and 23 for 97/98. The ice thickness was measured about 50 m from the shore with a weight on a thin wire through the ice sheet. The wire was heated with a car battery until the weight could be pulled up to check the position of the ice bottom.

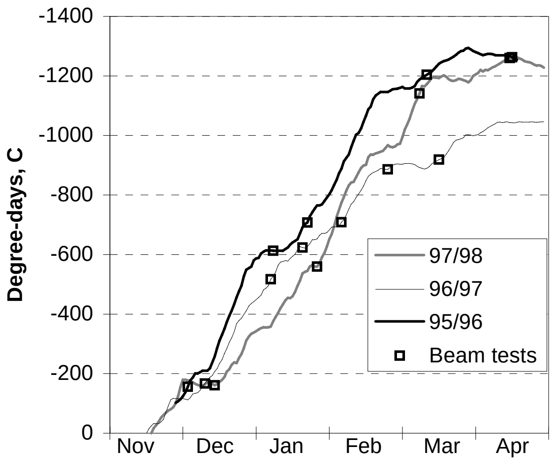

Figure 1: Degree-days at the Lule River.

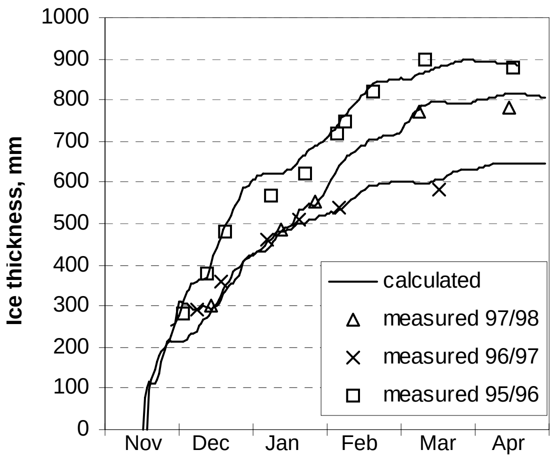

Figure 2: Ice thickness on the Lule River.

Table 1: Classification of tested ice covers

Test winter	Total ice thickness mm	Depth mm	Ice type*	Grain size mm	Density kg/m³	Tensile Strength MPa	Elastic Modulus GPa
95/96	890	0-150	G/M	1-20			
		150-890	K	6-35	~910	1.0	6.5
96/97	590	0-160	G	2-5			
		160-220	G/M	2-10			
		220-350	K	5-15			
		350-490	G	1-5			
		490-590	K	5-20			
97/98	295	0-93	G/M	3-7	863	0.9	3.7
		93-295	K	5-9	922	1.1	5.4
97/98	770	0-100	G		882		
		100-200	G		906		
		200-300	G		871		
		300-400	G		830		
		400-500	G/M		853		
		500-770	K		915		

* The ice types were (G) granular, (K) columnar and (M) mixed or crushed ice.

Ice cores and ice blocks from the test area were taken to Luleå University of Technology for classification and strength testing. The crystal shape and size were obtained from thin sections and the density of the snow-ice was measured from the volume and weight on 70 mm horizontal cylinders at –10 °C. The tensile strength and elastic modulus in the horizontal direction were obtained from 31 loading tests performed on the ice cylinders at –10 °C with a loading rate of 0.5 kN/s. The results are given in Table 1. Granular ice on top of the first grown columnar ice was assumed to be snow-ice, but the depth of the snow-ice layer was only estimated in the field as the whitish, non-transparent portion. The transition between snow-ice and columnar ice was fuzzy, because mixed ice and water-rich snow-ice were both quite transparent.

BEAM TESTS

Beams with a length of about 10 times the ice thickness were cut with a chain saw attached to a sledge. Usually six beams in a test series were cut next to each other, parallel with the shoreline, and floated into the place where the loading equipment was installed. The load was applied manually with a lever when the ice was thin, and later on with the use of an electro-hydraulic cylinder. In a typical test the mid-section of the ice beam was rapidly pulled upward, resulting in a bending failure (after 2-4 seconds) with one single crack propagating vertically from the surface. The beams were rapidly broken because the brittle failure mode was of major interest even though some hydrodynamic effects were seen. About half of the tests were performed with the beams upside down in order to put the ice bottom in tension with the same test set-up. The curvature of the ice beam was measured with a displacement transducer and the pulling force was measured with an electrical load cell. The load, displacement and time were stored on a PC with a sample rate of 500 or 1000 Hz. This type of four-point bending test on floating ice beams was carried out with the geometry shown in Figure 3.

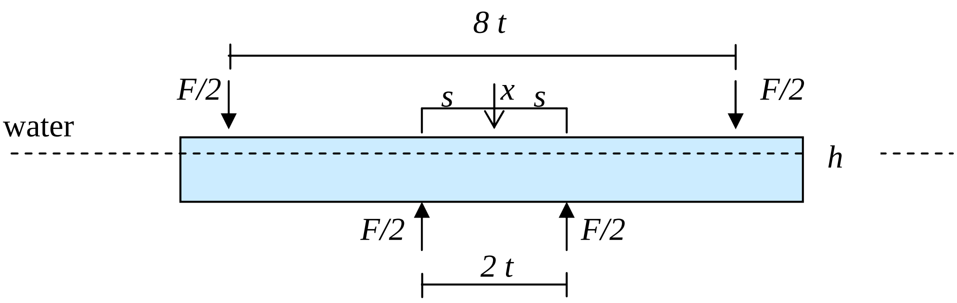

Figure 3: Geometry of the four-point bending test used.

The flexural strength σ_f was calculated with the formula

$$\sigma_f = F\frac{9t}{bh^2} \tag{1}$$

where F is the failure load, t is a constant about equal to the average ice thickness in a test series, b is the beam breadth and h is the beam height. The surface strain ε at the mid-section of the beam was calculated from the curvature deformation using the formula

$$\varepsilon = x\cdot\frac{h}{s^2} \tag{2}$$

where x is the relative displacement (see Fig. 3) and s is half the span length of the curvature rig (usually $s = t$). The elastic modulus $E = \Delta\sigma/\Delta\varepsilon$ was obtained from the slope of the load-deformation curve $\Delta F/\Delta x$. In an arbitrary case the Elastic modulus or Young's modulus is calculated as

$$E = \frac{\Delta F}{\Delta x} \cdot \frac{9ts^2}{bh^3} \qquad (3)$$

More details about the testing program are found in Andersson et al (1996).

Results

An example of a load-displacement curve (96/97 No. 5.1) is shown in Figure 4. The flexural strength of the beams tested with the top surface in tension (TT) and that of the beams tested with the bottom surface in tension (BT) are given for each test day in Table 2, together with the elastic modulus and other averaged results. In most test series the standard deviation of the strength was less than 10 % (± 0.05 MPa) when the bottom was in tension and 10-30 % when the top was in tension. The standard deviation of the calculated modulus was usually less than 15 %, but the properties of the snow-ice could change with time during the test day, which will be discussed later.

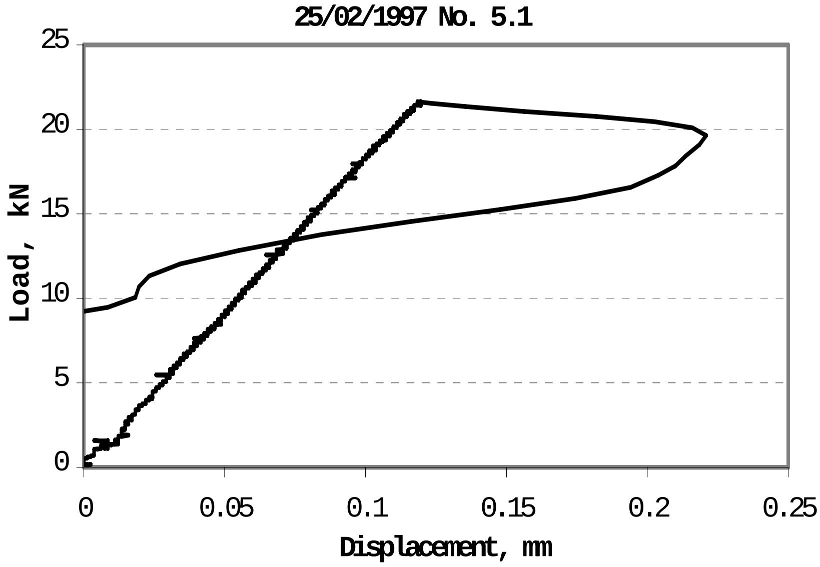

Figure 4: Load-displacement curve of an ice beam tested with the top surface in tension.

DISCUSSION OF RESULTS

First test winter 95/96

This winter was cold with less snow than usual and consequently blue transparent columnar-grained ice formed. The flexural strength of the beams tested with the top surface in tension (TT) was 1 MPa in December and decreased for every test date to a minimum of 0.2 MPa in March. It was observed that the ice surface, which was exposed to temperature variations, sunshine and sublimation, became more uneven with a net of fine cracks. In the first test series the fracture surface was curved with a thin horizontal crack 90 mm from the bottom, intersecting with a plane of small air bubbles. In later tests most fractures were more or less vertical with shell-like brittle surfaces. The fracture started at different points close to and above one load, and moved towards the load application line. A few beams were broken in the middle into two equal halves. The flexural strength of the beams tested with the bottom surface in tension (BT) was about constant at 0.5 MPa during the ice-growing period, but decreased drastically at the end of the season when the beams broke along the crystal boundaries. The elastic modulus obtained was 6 GPa, independently of the bending direction. Lower values of the modulus were obtained later in the winter, which probably can be explained by an increased influence of fine cracks at the ice surface.

Table 2: Seasonal variations of the ice properties in the Lule River 1995-98.

Series No.	Test Date dd/mm/yyyy	Tested Beams	Air Temp	t mm	b mm	h mm	Snowice mm	σ_f (TT) MPa	σ_f (BT) MPa	E Gpa
1	05/12/1995	7	-13	300	301	280	0	1.019	0.527	6.21
2	09/01/1996	6	0	500	442	524	0	0.614	0.589	6.18
3	23/01/1996	4	-15	600	641	583	0	0.537	0.527	5.92
4	13/03/1996	6	-5	800	822	832	20	0.214	0.369	3.52
5	17/04/1996	5	3	800	842	794	10	0.326	0.112	2.67
1	10/12/1996	8	-6	300	284	290	141	0.740	0.653	5.21
2	08/01/1997	6	-21	400	423	431	137	0.333	0.491	4.69
3	21/01/1997	8	4	500	493	529	180	0.337	0.397	4.96
4	06/02/1997	7	-12	600	590	646	149	0.489	0.483	2.85
5	25/02/1997	6	-1	600	597	538	152	0.475	0.400	3.71
6	18/03/1997	6	-5	600	598	583	133	0.974	0.631	4.79
1	16/12/1997	6	2	300	297	302	90	0.455	0.511	3.61
2	13/01/1998	6	-10	400	457	485	155	0.405	0.582	4.62
3	27/01/1998	7	0	400	478	551	179	0.685	0.421	3.51
4	10/03/1998	6	-12	700	703	770	210	0.168	0.379	3.08
5	16/04/1998	6	1	800	775	783	532	0.420	0.339	4.67

Second winter 96/97

Snow, rain, frazil slush and mild temperatures caused a much more complex ice cover than the previous winter. In December 140 mm of snow-ice had been formed on top of 150 mm of transparent ice of mixed ice types. The top surface was smooth without visible cracks, but the first flexural strength (TT) was less than for early columnar ice. In January, after a cold period, the strength had dropped to 0.3 MPa and this type of snow-ice stayed unpredictable with strengths between 0.3 and 0.5 MPa. This scatter was eventually enlarged by cracks created with a wheeled truck when the test area was cleaned from snow. The highest strength this winter, 1.0 MPa, was obtained in the last test series when the snow-ice had been refrozen after weeks of air temperatures above zero. The fractures in the first tests were strongly curved in the middle between the two ice layers. Taking the whole season in consideration, most fractures were vertical and quite planar. The flexural strength of the beams tested with the bottom in tension showed larger scatter than expected, probably due to the inclusion of frazil slush, but on average the flexural strength was about 0.5 MPa, as obtained during the first year for columnar ice. The elastic modulus was 4-5 GPa with the exception of test series 4, when the modulus systematically decreased from 4 to 2 GPa during the test day. It is possible that the water that flooded the beams early in the morning during sawing was absorbed in the snow-ice and slowly reduced the stiffness.

Third winter 97/98

The ice sheet comprised a great amount of snow-ice, and the typical flexural strengths were 0.4-0.6 MPa independent of the bending direction. Any seasonal effects on the strength (TT) that might have taken place on the top surface, were masked by the areal variation. In test series 4 the average strength was only 0.17 MPa, which was lower than the minimum strength of all the other beams tested that winter. The elastic modulus was rather constant in the interval 3-5 GPa. It was observed in the last test series in April that both the strength and modulus of the snow-ice beams decreased rapidly during the day, probably due to radiation heat and melting, see Figure 5. Localized strain and reduced effective height in the mid-section were probably measured with the curvature instrument and evaluated as a decreased elastic modulus. An empirical relationship between the elastic modulus and the strength of the type

$$E = k\sigma_f^{\ n} \tag{4}$$

is proposed where k is the elastic modulus for a flexural strength of 1MPa and n is a constant between 0 and 1. The evaluated parameters for all the test series during this winter are found in Table 2.

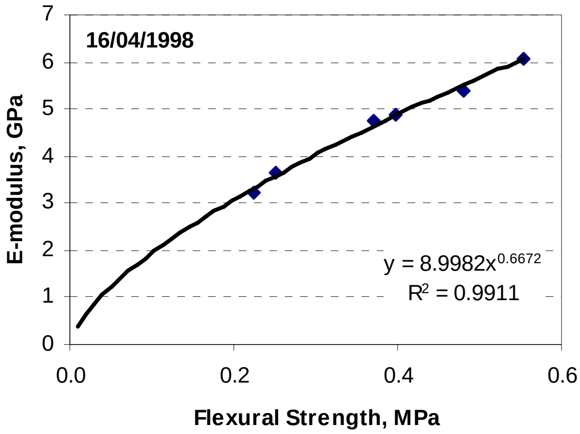

Figure 5: Flexural strength vs. elastic modulus for deteriorating snow-ice.

Table 2: Best fit parameters with equation (4)

Test date	k (GPa)	n	R^2
16/12/1997	7.2	0.8994	0.88
13/01/1998	4.9	0.0850	0.57
27/01/1998	4.3	0.4389	0.17
10/03/1998	2.8	-0.053	0.14
16/04/1998	9.0	0.6672	0.99

General trends

The general seasonal trend of the flexural strength is depicted in Figure 6. One test series (18/03/1997) of a total of 16 has been excluded because the high values obtained on that day could have been an areal variation rather than a seasonal variation. The flexural strength obtained for the beams with the bottom surface in tension was 0.4–0.6 MPa as long as the ice was growing, usually until the end of March. The strength decreased gradually in April, when solar radiation started to penetrate the ice cover. Different crystal sizes of the columnar ice and sometimes inclusions of frazil slush had a relatively small influence. In Figure 7 the flexural strength, for columnar bottom ice before it was candling, is plotted vs. the cross-sectional beam area. It is, however, unclear whether the obtained decrease was influenced by the beam size or the crystal size, or both. Gow et al. (1978) obtained about the same strength for the bottom layer of lake ice with much larger grains. A few beams in their tests were stronger (max 1 MPa), which could be connected with the slower loading rate.

The results for the flexural strength obtained for the beams with the top surface in tension were scattered (0.2–1.0 MPa) and dependent on many different factors. The young ice was stronger than the old ice, but the surface strength could increase substantially after a melting period. The surface ice without a snow cover became weaker than the bottom ice, with a minimum strength in March. The snow-ice was as strong as the columnar ice, but the elastic modulus and strength were sensitive to heating. The strength of the melting and refrozen top layer was not further reduced during the spring melting, at least not before the middle of April. For intact snow-ice on top of columnar ice, Gow et al. (1978) consistently obtained values of about 1–1.5 MPa with simply supported beams. Cantilever beams of the same ice, pushed downward, yielded 0.5–0.9 MPa. The strength decreased drastically at the end of the season when melting and granulation of the snow-ice layer were observed.

The elastic modulus was 6 GPa for the pure columnar ice and 3–5 GPa for the combination of snow-ice and columnar ice. The modulus was a function of the strength when the beam contained a great amount of snow-ice. All the measured deformation up to the maximum load level was not linear-elastic, and a low bulk modulus could be the result of cracks and increased non-elastic deformations. No dependency on the bending direction could be traced.

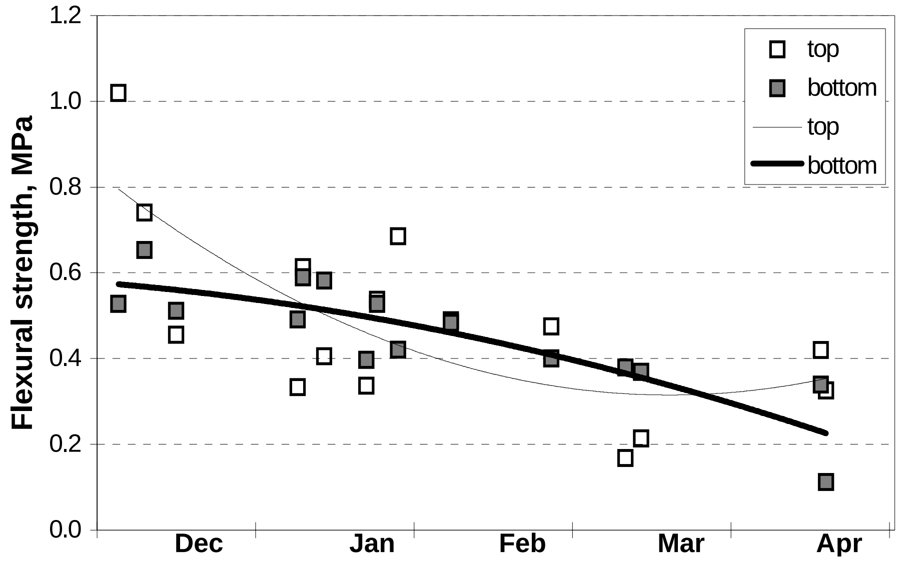

Figure 6: Seasonal effects on the flexural strength of ice beams. Lule River, 1995–98.

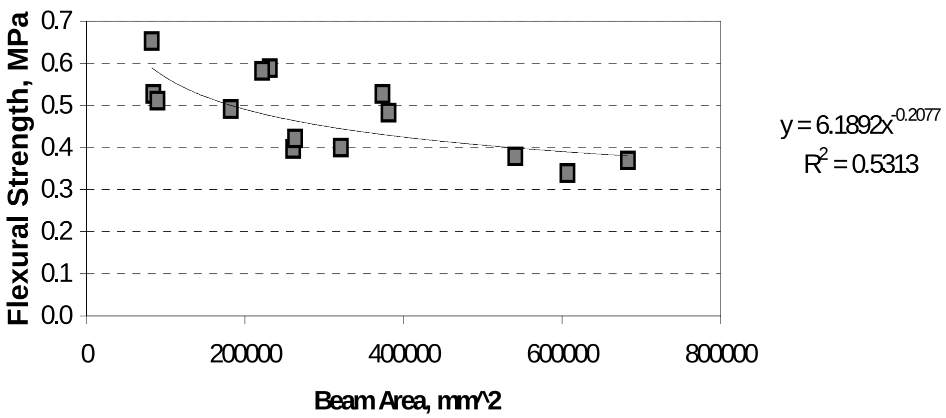

Figure 7: Flexural strength vs. beam area for river ice tested with the bottom surface in tension.

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