Two-Dimensional Finite Element Analysis of Elastic-Plastic Deformation in Asperity-Asperity Collision

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Abstract

The collision of cylindrical asperities is simulated by means of a commercial multiphysics software in this paper. The model combines a computational structure mechanics approach with a contact algorithm. Numerical results are first compared to classical Hertzian theory. Effects of the bulk material under the mating surfaces have been included in the simulation. Materials of top and bottom surfaces have been assigned different properties. Development of plastic deformation and friction are presented along the process of collision. Strain hardening is included in the elastic-plastic deformation. The effect of interference change and friction coefficient variation on deformation and friction has been studied. Deformed shape and subsurface residual stress in asperity are output parameters from this study.

Keyword: Asperity, elastic-plastic, friction

Notation

$\delta$ \hspace{1cm} Interference [m] \hspace{1cm} $F_n$ \hspace{1cm} Load per unit length [N/m]
$\mu$ \hspace{1cm} Adhesive friction coefficient \hspace{1cm} $a$ \hspace{1cm} Semi contact length [m]
$R_1$ \hspace{1cm} Radius for top asperity [m] \hspace{1cm} $P$ \hspace{1cm} Contact pressure [Pa]
$R_2$ \hspace{1cm} Radius for bottom asperity [m] \hspace{1cm} $\sigma_y$ \hspace{1cm} Yield stress [Pa]
$R'$ \hspace{1cm} Effective radius [m] \hspace{1cm} $\varepsilon_{eff}$ \hspace{1cm} Effective strain
$\nu_1$ \hspace{1cm} Poisson ratio for top asperity \hspace{1cm} $\varepsilon_{ep}$ \hspace{1cm} Effective plastic strain
$\nu_2$ \hspace{1cm} Poisson ratio for bottom asperity \hspace{1cm} $E_{h1}$ \hspace{1cm} Hardening modulus for top asperity [Pa]
$E_1$ \hspace{1cm} Modulus of elasticity for top asperity [Pa] \hspace{1cm} $E_{h2}$ \hspace{1cm} Hardening modulus for bottom asperity [Pa]
$E_2$ \hspace{1cm} Modulus of elasticity for bottom asperity [Pa] \hspace{1cm} $\sigma_{hard}$ \hspace{1cm} Hardening function [Pa]
$E'$ \hspace{1cm} Effective module of elasticity [Pa] \hspace{1cm} $f$ \hspace{1cm} Global friction force [N/m]
$\sigma_{exp}$ \hspace{1cm} Function of effective stress [Pa]
Introduction

In reality, most of the contacts between machine components are boundary lubricated. In boundary lubricated contacts most of the load is carried by asperities. The onset of wear due to plastic deformation or adhesion is thus very much dependent on the mechanisms of single asperity contacts. Such contacts can be seen as “collisions” when sliding motion is involved. Asperities on one surface will collide with the asperities on the other surface due to the speed difference.

K.L. Johnson [1] did an extensive review on asperity collisions. He summarized that the use of slip-line field method for plain strain deformation of rigid-perfectly plastic material works fine, but this method can not be used in three dimensional models. He contributed the work related to two important terms namely shakedown and ratcheting. Shakedown means that the yield stress will be increased due to residual stress, strain hardening and increased contact area due to repeated contact. Ratcheting means that fracture will appear as a result of repeated contact if plastic deformation is accumulated. In the end, Johnson established a critical plastic strain value dependent on ratcheting rate in order to decide the number of related critical cycles. Johnson concluded that the case where the topography of both surfaces change during sliding needs further thought in the end of his review paper. The shakedown limit given by Johnson is based on the statistical rough surface contact mechanics framework developed by Greenwood and Williamson [3]. The (GW) model utilizes the solution of the frictionless contact of a rigid flat plane against an elastic hemisphere and the local asperity deformation is calculated with Hertzian theory. In their rough surface contact mechanics approach, all the asperities are assumed to have the same radius and to be distributed statistically on the surface according to Gaussian distribution in order to simplify the global solution. Moreover, the GW model assumes that the asperities do not interfere with neighboring asperities which is wrong in reality.

Tabor [4] in 1959 first proposed the idea of junction growth in order to explain that when the tangential load is increased, the contact normal pressure will be decreased while at the same time the deformation is still in plastic regime. Following the classical model of Tabor, not many improvements have been made so far about the phenomenon of junction growth. Zhang and Chang et al. [5] studied the effects of friction on the contact and deformation behavior in sliding asperity contacts. They used two input variables, normal approach, which is interference in this paper, and friction coefficient, to carry out the analysis. Two normal approaches were given in their paper, one was for the asperity to achieve the yielding point and the other was for the asperity to achieve fully plastic deformation. They conclude that friction can significantly reduce the normal approach that initiates plastic yielding in the asperity and especially the normal approach that causes the asperity deformation to become fully plastic. They also studied the effects of friction on the contact size, pressure and load carrying capacity of the asperity, but strain hardening and bulk material effects were not considered. Brizmer et al. [6] developed Tabor’s work, they proved the empirical relation between the junction growth and the normal preload with their numerical simulation results. The effects of the combined normal and tangential load on the evolution of contact area were also investigated. The model they used was a deformable sphere in contact with a rigid flat. Jeng and Peng [7] also studied the junction growth. The effect of an adsorbed layer on

\[
f_a \quad \text{Adhesive component [N/m]}
\]

\[
f_p \quad \text{Ploughing component [N/m]}
\]
plastic deformation was investigated and they concluded that the adsorbed layer has a significant effect on plastic deformation for small interferences, but has no effects for interferences above 35 nm.

Most of the previous studies have used the elastic-perfectly plastic material model, while in reality work hardening materials are predominant. The topography of both surfaces changes during sliding are not considered in most of the previous studies. In this paper, an asperity-asperity collision model with asperities that exhibit different material properties is studied. The interference and the friction coefficient are two important input variables in the asperity-asperity collision model, and the effect on plastic deformation and residual stress of varying these two variables over a range of values is investigated in this work. Strain hardening is included and the effect from bulk material is also considered. Critical plastic strain related to repeated contact will be explained in the end.

The model is implemented in COMSOL, which is a commercial multiphysics code based on the finite element method.

2. Model Problem

The main idea of this work is to simulate the contact between machine elements at the micro scale as physically correct as possible. Asperities have been modeled with a cylindrical geometry, as depicted in Fig.1. Boundaries 1, 2, and 3 have been applied a prescribed displacement in the \( x \)-direction and constrained in the \( y \)-direction; boundaries 4, 5, and 6 are assumed being totally constrained in all directions. The boundaries 7 and 8 are set to be free and can thus move subject to elastic-plastic deformation. When the simulation starts, the top surface will move from left to right while the bottom surface remains stationary. Because of the interference between the two asperities on both surfaces, collision will occur. Since it is a simulation of contact occurring at the micro level, the macro friction is defined as global friction in this paper to distinguish from the local adhesive friction. The friction is a combination of two components, i.e., adhesion and ploughing. Adhesion is modeled as the force between boundaries 7 and 8, while the global friction is modeled as the integral of the stresses acting in the \( x \)-direction on boundaries 1, 2 and 3. The ploughing component is defined in Eq. (1).

\[
    f_p = f - f_a \tag{1}
\]

In this work, the friction coefficient is used to model the adhesive friction component. It is applied at the contact region between boundaries 7 and 8 meaning that a change in the friction coefficient will affect global friction through a change in the adhesive component.

Some important parameters controlling the material properties and geometry have been listed in Table 1. \( E_{h1} \) and \( E_{h2} \) in the table are the hardening modules for the top and bottom contact pair asperities respectively; these are used in the bilinear isotropic hardening model.

A stress-strain curve is given in Fig. 2 to

![Figure 1. Finite element mesh of asperity collision model](image-url)
Table 1 Some important parameters of material property and contact geometry

<table>
<thead>
<tr>
<th>$E_1$ [GPa]</th>
<th>$E_2$ [GPa]</th>
<th>$\sigma_y_1$ [MPa]</th>
<th>$\sigma_y_2$ [MPa]</th>
<th>$\delta$ [$\mu$m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>210</td>
<td>160</td>
<td>1000</td>
<td>600</td>
<td>0.1, 0.2, 0.3</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$E_{h1}$ [MPa]</td>
<td>$E_{h2}$ [MPa]</td>
<td>$R_1$ [$\mu$m]</td>
<td>$R_2$ [$\mu$m]</td>
</tr>
<tr>
<td>0, 0.1, 0.2, 0.3</td>
<td>4000</td>
<td>4000</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

demonstrate the hardening mechanism adopted for the simulations. A mathematical formula is also given in Eq. (2) to describe this hardening mechanism. For example, for a material with hardening modulus $E_h = 4000$ MPa the yield stress will increase with 120 MPa if subjected to a strain increase of 0.03.

For a line contact, the corresponding Hertzian theory can be formulated as in Eq. (3) to Eq. (6).

\[
R' = \left[ \frac{1}{R_1} + \frac{1}{R_2} \right]^{-1}
\]  

\[
E' = 2 \left[ \frac{(1-v_1^2)}{E_1} + \frac{(1-v_2^2)}{E_2} \right]^{-1}
\]  

\[
a = \sqrt{\frac{8F}{{\pi E'}}}
\]  

\[
P = \frac{F'E'}{2{\pi R'}} \times (1 - \frac{x^2}{a})
\]

The commercial finite element code Comsol Multiphysics is used in this paper to handle the process of modeling and simulation. Triangular mesh is used because of the curvature of asperities. Meshes are finer around boundaries 7 and 8 in order to reduce discretization errors and achieve convergence. The mesh in Fig 1. consist of totally 3236 elements connected by 1767 nodes.

**Result and discussion**

To verify the present model, the simulation result of purely elastic asperity collision has been compared with the corresponding analytical result from Hertzian theory. For purely elastic deformation, the isotropic material module, in Comsol Multiphysics, is used in order to avoid including plastic deformation.

Numerical and analytical solutions are compared. The solid blue curve is the numerical result and the dashed red curve is
the analytical result. The small deviation at the bottom maybe because the geometry of the model used in the simulation is different as the geometry of the model which Hertz used in his solution. Overall, both the contact pressure and contact length are in good agreement, and this is taken as a verification of the correctitude of the present model.

Maximum and average friction values are shown in Fig. 5. Average friction values are meaningful because they can represent how much energy that will be consumed during the asperity collision process.

The process of asperity collision at different time steps has been captured in Fig. 6. The time stepping procedure has been normalized in order to achieve convergence. Maximum von Mises stress has been set to 1000 MPa which is also the yield stress for the top asperity material. In the subfigure of Fig. 6, stress levels above 1000 MPa are colored as at 1000 MPa. In this way it becomes easier to analyze the plastically deformed area. At time 0.01 the collision reaches a critical point because both asperities are plastically deformed after this. Since the bottom asperity is softer than the upper one it will also exhibit a larger area of plastic deformation. Figure 7 shows the plastically deformed area in the bottom asperity after the collision has finished; Fig. 7 (left) shows the development of plastically deformed area for a fixed interference $\delta = 0.1\mu m$ and for four adhesive friction coefficients, It can be deduced that the increase is quite moderate as the boundary friction coefficient is increased for a small interference value (left); However, from the right subfigure, where the adhesive friction coefficient is held fixed at $\mu = 0.3$, it can be deduced that the plastically deformed area exhibits a substantial increase with increasing interference. The plastically deformed area when $\delta = 0.3\mu m$ is around six times larger than the area when $\delta = 0.1\mu m$. Thus, the result shows that when the top asperity is harder than the bottom asperity. The interference has
a larger influence on the development of plastic deformation than the adhesive friction coefficient. Plastically deformed area will cease to increase and become constant after the time around 0.6, and this time of ceased plastic deformation increase will be delayed when the plastically deformed area is larger, for example this time for $\delta = 0.3 \mu m$ appears later than it is for $\delta = 0.1 \mu m$. About the displacement to make the asperities cease to contact, the author has to use a bigger displacement value for elastic-plastic deformation than for purely elastic deformation, because the plastically deformed asperities will prolong the geometry in the $x$-direction, as clearly depicted in the last
subfigure in Fig. 8. Residual stress and the remaining plastically deformed shape of the asperities are shown. The contact pressure will be decreased for the lateral contacts because of increased contact area. Using Johnson’s [1] words; the new shakedown limit will be harder to pass and this will protect the surface from damage. If the cyclic contacts continually make the deformation be plastic, then the critical plastic strain value, which is also the fracture point, will be reached to cause crack of material.

Conclusion

The plastically deformed area was obtained after the asperity collision process. The bulk material and how the stresses distribute in the substrate will be affected by the asperity collision have been investigated. The residual stress was shown in the deformed asperities, and this is very helpful to the future work of cyclic contact. The contribution to plastic deformation from interference and adhesive friction coefficient was studied. This showed that interference increase has a larger effect on plastically deformed area than the boundary friction coefficient in this study. The average friction force curves related to energy needed to conquer one collision process have been made. This is also very meaningful because energy consumption is a very important aspect.

Reference

2. K.L. Johnson and H.R. Shercliff, Shakedown of 2-D asperities in sliding