Response of a Rocket Nozzle to Power Spectral Density Loads

Mikael Helgesson
2015

Master of Science in Engineering Technology
Space Engineering

Luleå University of Technology
Department of Mechanical Engineering and Mathematics
MASTER THESIS
Response of a rocket nozzle to power spectral density loads

Mikael Helgesson

Luleå University of Technology
Dept. of Computer Science, Electrical and Space Engineering
Div. of Space Technology

Mars 2015
Supervisors

**Dr. Robert Tano**  
Engineering Method Specialist, Strength  
GKN Aerospace Engine Systems

**Jan Häggander**  
Senior Engineer Advanced engineering  
GKN Aerospace Engine Systems

**Prof. Lars-Göran Westerberg**  
Department of Engineering Science and Mathematics  
Division of Fluid Mechanics  
Luleå University of Technology
PREFACE

This thesis has been performed as the final part of the Master of Science program in Space Engineering at Luleå University of Technology. The master thesis was conducted at GKN Aerospace Engine Systems in Trollhättan at the department of nozzles and combustion chambers during 20 weeks starting from February 2012.

First of all I would like to express my sincere gratitude to my supervisor at GKN Aerospace Engine Systems, Dr. Robert Tano and Jan Häggander, for their insights and guidance during this thesis. Special thanks go to Marcus Almqvist and Jan Östlund for their topical expertise. I would also like to thank Joachim Steffenburg-Nordenström, Hanna Helmersson and Tomas Fernström for their help and assistance with ANSYS. Furthermore, a big thanks to everyone else at the department and GKN who helped me and made my stay a pleasant time. Additionally, I would like to send a special recognition to friends and family for the never ending support.

Finally, I would like to show my gratitude to my examiner at Luleå University of Technology, Prof. Lars-Göran Westerberg at the Division of Fluid Mechanics. I thank him for all his expertise and support throughout my education.

Trollhättan, Mars 2015

Mikael Helgesson
ABSTRACT

The aim of this thesis is to give a deeper understanding of Power Spectral Density analysis and how it can be used to predict the impact of fluctuating loads induced directly to a rocket nozzle. The initial study mainly deals with understanding how the response may vary with changed geometrical setup of the reinforcement structure. That is why a great deal of the time has been spent on parameterize the model to make it easy to automatically loop over many different geometrical and material properties of the nozzle extension. The work was implemented on a simplified FE-model of the next generation sandwich nozzle extension that was under development at GKN Aerospace Engine Systems.

The thesis will only address the impact of buffeting loads during atmospheric ascent as it is the most critical stage of the flight. The occurrence of buffeting loads during atmospheric ascent is due to varying pressure fields as it passes through the atmosphere. This load depends primarily on the shape of the vehicle, but the severity can change rapidly depending on the dynamic pressure, angle of attack and Mach number. To analyze the behavior of a random load it is important to understand which critical structure modes are present in the specified frequency range. Excitation of these modes can lead to structural failure of the nozzle due to excessive displacement.

Evaluation of the critical mode frequencies indicated a linear relationship, up to 60 mm stiffener height, between the eigenfrequency of the critical modes and the stiffener height, where both 3-wave and 4-wave increase modes more rapidly for higher stiffeners than for the ovalisation mode. The pendulum and torsion mode will have nearly constant frequencies. Both the torsion and pendulum mode are caused by motion at the nozzle throat and are unaffected by the increased stiffness of the lower part of the nozzle. Connecting these linear relations to the energy distribution of the PSD spectrum showed an increase of available energy for the ovalisation mode and a decrease for 3-wave and 4-wave modes for higher stiffeners.

A further study of how the reinforcement structure affects the eigenfrequencies and the magnitude of the critical modes states that the stiffener height has the largest contribution. The increased stiffener height gives an exponentially decreased radial displacement of the nozzle wall. The stiffener thickness has a minor effect on the mode frequencies and contributes only to an increased structural stiffness of the reinforcement structure.

This states that it is better to use higher stiffeners than an increased number of lower stiffeners. At the same time the allowed stiffener height is limited
by an increased stiffness of the wall at each stiffener due to increased risk for axial buckling between stiffeners.

An important notice is that the PSD spectrum for the Ariane 5 rocket was not available during the thesis. Instead a PSD spectrum from a slightly larger nozzle extension was used. How this PSD spectrum relates to the Ariane 5 PSD spectrum needs to be further investigated to verify the magnitude of these critical modes.

Finally, the study has shown that PSD analysis is a time efficient and competent tool that could be used as a first approach during the design phase. A problem that was stated early on was the lack of support to handle nonlinear behaviors such as plasticity in ANSYS. This meant that large temperature changes occurring during flight could not be accounted for in this PSD analysis. However LS-DYNA has recently implemented a non-linear solver for the aerospace industry, based on Boeing software N-FEARA, a NIKE3D-based finite element tool for structural analysis of vibro-acoustic loads. If the LS-DYNA non-linear solver can account for large thermal and structural loads, the PSD analysis might be an alternative to perform time history analysis.

It shall be emphasized that this is a reduced report; company sensitive results and developed methods are not included.
# TABLE OF CONTENT

TABLE OF CONTENT ................................................................. ix
LIST OF FIGURES ................................................................. xi
LIST OF TABLES ................................................................. xiii
ACRONYMS ................................................................................. xiv

1. INTRODUCTION ........................................................................ 1
   1.1. Assumptions and limitations .................................................. 4

2. THEORY ................................................................................. 5
   2.1. Random Vibration & Acoustic noise ...................................... 5
   2.2. PSD Analysis ........................................................................ 5
      2.2.1. Properties of Power Spectral Density (PSD) ..................... 7
      2.2.2. Wiener-Khinchine Theorem ........................................... 9
      2.2.3. Sound Pressure Level (SPL) .......................................... 11

3. MODELS .................................................................................. 14
   3.1. Geometry and elements ........................................................ 14
   3.2. Constraints ........................................................................... 15
   3.3. Nozzle wall Configuration ...................................................... 16
   3.4. Stiffener Configuration ........................................................ 17
   3.5. Material properties ............................................................... 17

4. METHOD .................................................................................. 18
   4.1. Static analysis ....................................................................... 18
      4.1.1. Flame pressure ............................................................. 19
      4.1.2. External pressure .......................................................... 19
   4.2. Modal Analysis ..................................................................... 20
      4.2.1. Applied Loads ............................................................... 21
      4.2.2. Mode Shapes ................................................................. 22
   4.3. PSD analysis ....................................................................... 24
      4.3.1. Applied Loads ............................................................... 24
      4.3.2. Structural damping ......................................................... 27
      4.3.3. Rayleigh damping .......................................................... 27
      4.3.4. Damping ratios from test measurements ....................... 28
      4.3.5. Define Rayleigh damping parameters ............................ 29

5. RESULTS ............................................................................... 30
5.1.  Modal analysis .................................................................................................................. 30
  5.1.1. Critical mode frequency vs. stiffener height ............................................................. 30
  5.1.2. Effective mass comparison ......................................................................................... 31
  5.1.3. Energy distribution .................................................................................................... 33
5.2.  PSD analysis .................................................................................................................... 35
  5.2.1. Radial displacement (RMS) of the nozzle contour .................................................... 36
  5.2.2. Radial displacement (RMS) vs. Stiffener height ....................................................... 37
  5.2.3. Radial displacement (RMS) vs. Orientation .............................................................. 39
  5.2.4. Radial displacement (RMS) vs. stiffener distance .................................................... 41
  5.2.5. Radial displacement (RMS) vs. stiffener position (z-direction) .............................. 42
  5.2.6. Comparison of fixture constraints ............................................................................ 43
5.3.  Response PSD .................................................................................................................. 44
6.    DISCUSSION AND CONCLUSION .................................................................................. 47
7.    REFERENCES ................................................................................................................... 48
8.    APPENDIX A .................................................................................................................... 49
LIST OF FIGURES

Figure 1-1 – FE model of the rocket Nozzle Extension.................................................1
Figure 1-2 The figure to the right show a snapshot of the ariane-5 Cp distribution and instantaneous stream traces. Cp is the coefficient of power used in wind turbine aerodynamics, ref.[2]. An overview of the launcher configuration is show to the left ....2
Figure 2-1 - Random process. .................................................................6
Figure 2-2 - Zero mean Gaussian probability distribution ....................................6
Figure 2-3 –The top signal is a sum of three sine waves. Using Fourier analysis it can be break down into its spectral components, see figures in the middle. Each sine wave has constant amplitude, see bottom figure.................................................................7
Figure 2-4 - Illustration of the Fourier transformation..............................................8
Figure 2-5 - Acoustic noise load in the one-third band..........................................11
Figure 3-1 - NE (Nozzle Extension). .......................................................................14
Figure 3-2 - NE (Nozzle Extension) & CC (Combustion Chamber). .....................15
Figure 3-3 - To the left, fixed constrain at the top of the TC. To the right, fixed constrain at the CC/NE I/F..........................................................15
Figure 3-4 - Nozzle wall thickness distribution. .......................................................16
Figure 3-5- Stiffener requirement............................................................................17
Figure 4-1 - PSD analysis cycle. .............................................................................18
Figure 4-2 Inner flame pressure as a function of axial location from the throat........19
Figure 4-3 – External surface pressure distribution. ..............................................19
Figure 4-4 Automated modal analysis cycle for the PSD analysis. .........................21
Figure 4-5 - Pendulum mode..................................................................................22
Figure 4-6 - Ovalisation mode. ..............................................................................22
Figure 4-7 - Torsion mode. ....................................................................................23
Figure 4-8 - Three-wave mode.............................................................................23
Figure 4-9 - Four-wave mode..............................................................................23
Figure 4-10 – PSD pressure spectrum. .................................................................24
Figure 4-11 Snapshot of the ariane-5 Cp distribution and instantaneous stream traces. Cp is the coefficient of power used in wind turbine aerodynamics. This allows one to make comparisons between different turbines, while eliminating the effects of size and wind conditions, ref.[2] .................................25
Figure 4-12 – PSD spectrum load factor distribution around the nozzle. ...............26
Figure 4-13 - Frequency effect of proportional damping on damping ratio. .............28
Figure 4-14 - Classical half-power bandwidth method..........................................28
Figure 5-1 Change of eigenfrequency for the nozzle extensions critical modes......30
Figure 5-2 - PSD response for 50 Modes...............................................................32
Figure 5-3 - PSD response for 200 Modes.............................................................32
Figure 5-4 - PSD response for 1000 Modes...........................................................32
Figure 5-5 – Eigen frequency of Ovalisation mode with increased stiffener height from left to right. ........................................................................33
Figure 5-6 - Eigen frequency of 4-wave mode with increased stiffener height from left to right. ........................................................................34
Figure 5-7 - Eigen frequency of 4-wave mode with increased stiffener height from left to right. .................................................................................................................................................. 34
Figure 5-8 Measurement points for radial displacement of the nozzle wall and stiffener tip displacement ........................................................................................................................................................................... 35
Figure 5-9 - Contour displacement for 20 mm stiffeners .................................................................................................................................................................................................................. 36
Figure 5-10 - Contour displacement for 40 mm stiffeners .................................................................................................................................................................................................................. 36
Figure 5-11 - Contour displacement for 60 mm stiffeners .................................................................................................................................................................................................................. 37
Figure 5-12 - Contour displacement for 80 mm stiffeners .................................................................................................................................................................................................................. 37
Figure 5-13 - Radial displacement at nozzle exit at zero degrees orientation ......................... 38
Figure 5-14 - Axial displacement at nozzle exit at zero degrees orientation ............................. 38
Figure 5-15 – Circumferential displacement for stiffener height 20 mm. ................................. 39
Figure 5-16 – Circumferential displacement for stiffener height 40 mm. ................................. 39
Figure 5-17 – Circumferential displacement for stiffener height of 60 mm. ............................ 40
Figure 5-18 – Circumferential displacement for stiffener height 80 mm. ............................... 40
Figure 5-19 Distance between stiffeners. .................................................................................. 41
Figure 5-20 – Displacement vs. stiffener distance, with 50 mm stiffeners ......................... 41
Figure 5-21 Axial location of the first stiffener. ................................................................. 42
Figure 5-22 - Displacement at nozzle exit, with 50 mm stiffeners ............................................ 42
Figure 5-23 Radial displacement for two different boundary conditions. ............................ 43
Figure 5-24 - PSD Response spectrum for 20 mm stiffeners ................................................. 44
Figure 5-25 - PSD Response spectrum for 40 mm stiffeners .................................................. 45
Figure 5-26- PSD Response spectrum for 60 mm stiffeners .................................................. 45
Figure 5-27- PSD Response spectrum for 80 mm stiffeners .................................................. 46
LIST OF TABLES

Table 3-1 Total nozzle wall thickness.................................................................16
Table 3-2 Material properties.................................................................17
Table 5-1 Displacement for different modes..............................................31
Table 5-2 Displacement for different modes..............................................31
## ACRONYMS

The following acronyms are commonly used throughout this report.

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC</td>
<td>Combustion Chamber</td>
</tr>
<tr>
<td>EAP</td>
<td>Etage d'Accélération à Poudre (Powder booster stage)</td>
</tr>
<tr>
<td>FE</td>
<td>Finite Element</td>
</tr>
<tr>
<td>GKN</td>
<td>Guest Keen Nettlefolds</td>
</tr>
<tr>
<td>LTU</td>
<td>Luleå University of Technology</td>
</tr>
<tr>
<td>MD</td>
<td>Metal Deposition</td>
</tr>
<tr>
<td>NE</td>
<td>Nozzle Extension</td>
</tr>
<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>RPSD</td>
<td>Response Power Spectral Density</td>
</tr>
<tr>
<td>SPL</td>
<td>Sound Pressure Level</td>
</tr>
<tr>
<td>TC</td>
<td>Thrust Chamber</td>
</tr>
</tbody>
</table>
1. INTRODUCTION

GKN Aerospace is one of four divisions under GKN plc; the remaining three are GKN Driveline, GKN Powder Metallurgy and GKN Land Systems. GKN Aerospace Engine Systems has its headquarters located in Trollhättan, Sweden and subsidiaries located around the world in India, USA, Norway and Sweden. GKN Aerospace Engine Systems develops and manufacture components for aircraft engines, space rockets and gas turbines, including maintenance facility for commercial engines, military engines and industrial gas turbines.

The nozzle extension is the part of a rocket engine that generates thrust, see Figure 1-1. The nozzle is attached to the combustion chamber which together is devoted the thrust chamber and allows the compressed exhaust gases from the combustion chamber to expand supersonically and thereby accelerates to create thrust. The nozzle considered in this thesis is based on GKNs next generation sandwich technology where the nozzle is milled in one outer and one inner shell and is later laser-welded together at each shared cooling channel wall. The upper part of the wall is strengthened by a thin layer MD. For the lower part, the MD layer is replaced by stiffeners that have a higher moment of inertia in tangential direction. MD is used at places where a high moment of inertia in axial direction is required.

![Figure 1-1 – FE model of the rocket Nozzle Extension.](image)

With the setup used on the Ariane 5 rocket today, the nozzle is located in the middle between two solid rocket boosters on each side. This creates a complex environment with multiple pressure fields to take into
consideration, see Figure 1-2. As the rocket passes through the atmosphere the nozzle extension will encounter fluctuating loads generated by the increased aerodynamic flow around the rocket aft body. The thesis will only address the impact of buffeting loads during atmospheric ascent which also involves random vibrations from EAP booster and core stage rocket engine nozzle flame pressure. The load induced by the EAP boosters has a major influence on the rocket nozzle during lift-off. But during the buffeting phase influence is decreased due the increased aerodynamic flow, ref.[1]. Concerning the inner flame pressure the incipient flow separation at the nozzle exit is hard to predict. It is when the flame pressure separates from the nozzle wall and allows ambient pressure to enter the nozzle, which can cause both axial and radial buckling, ref [1]. Instead an assumption is made to use a static circumferential inner flame will be assumed with a decreased pressure from the nozzle throat to the nozzle exit, due to the nozzle expansion.

Figure 1-2 The figure to the right show a snapshot of the ariane-5 Cp distribution and instantaneous stream traces. Cp is the coefficient of power used in wind turbine aerodynamics, ref.[2]. An overview of the launcher configuration is show to the left.

These random loads give rise to nozzle eigen modes such as ovalisation, three-wave and four-wave, see section 4.2.2. To analyze the behavior of these loads it is important to understand which critical modes of the structure are present in the specified frequency range of the aerodynamic load. Excitation of these modes can lead to structural failure of the nozzle due to excessive displacement. Structures that are most vulnerable for damage by acoustics loads are those with a high ratio of surface area to mass as they undergo large displacements while oscillating at low frequency, ref.[3]. This means that a fixed nozzle must be carefully designed and stiffened to avoid failure. Today these nozzle modes are often specified in terms of deflection at the exit of the nozzle based on whole engine analyses with condensed models of all subcomponents. How a change of the nozzle configuration affects the mode load is therefore difficult to predict and it is of high interest for the structural mechanics department to develop a method
to apply the acoustic noise directly to the nozzle. No previous work at GKN where available during this thesis.

The aim of this thesis is to create a parametric FE-model script in ANSYS Mechanical ADPL, with the same base characteristics as GKNs next generation sandwich nozzle extension. With help of the script the user can easily loop changes in geometrical and material properties of the FE-model.

This will later be used in the thesis to evaluate how changes of the nozzle configuration affect the eigenfrequencies of critical modes when exposed to random fluctuating loads during the buffeting phase. The response for different geometries will be used to setup guidelines for designing nozzle reinforcement structures. No previously defined load exists so a large part of this thesis is to evaluate the impact of different acoustic load spectrums and how to apply these loads onto the nozzle extension. The study will also involve a deeper understanding of Power Spectral Density Analysis, a function of frequency, and evaluating the possibility of using it to estimate the magnitude of critical modes, ref.[4].
1.1. Assumptions and limitations

One of the main goals of this thesis is to setup an accurate and fast method to easily see the contribution of different geometrical configurations. This means simplifications of the used FE model and applied loads.

- To reduce the computational time, multi-layer elements were used instead of modeling all sides of a channel with shell elements. Where each layer has the corresponding stiffness properties of the outer, mid and inner wall of the channels.
- The model does not account for inner pressure fluctuations as inception flow separation. This is a complicated phenomenon that is too extensive to account for in this thesis.
- A PSD analysis does not support nonlinear behaviors such as plasticity. This meant that large temperature changes occurring during flight could not be accounted for in a PSD analysis.
- Simplified material properties.
- The thesis is limited to one specific flight phase, buffeting. The only available load case.
- During the thesis only one PSD spectra were used for the entire nozzle. A more correct approach would be to use multiple PSD spectrums around the nozzle, instead of one PSD spectra with varying load factor. Using multiple PSD spectra would include the frequency varying energy distribution around the nozzle.
- When using a symmetric load distribution there is a risk that some modes will not be active e.g. the three-wave mode.

These assumptions have not been evaluated to see what contribution they have to the results. That is something that will need to be evaluated in the future.
2. THEORY

The theory chapter will describe the theories and procedures used throughout this thesis.

2.1. Random Vibration & Acoustic noise

Acoustic noise results from propagation of sound pressure waves through different media. During launch such noise is generated by the release of high velocity engine exhaust gases that sets engine components in motion. During ascent the noise is generated by separation of the aerodynamic flow field along the launch vehicle that creates turbulence around engine components. These fluctuating pressures can cause acoustic vibration of exposed structures over a broad frequency band.

Evaluation of a large frequency span as this is usually performed using random vibration analysis. Here one statistically looks at structure response to a given random vibration environment rather than at a specific frequency or amplitude at a certain moment in time. The random vibration analysis shows which frequencies that cause the largest random response, but most importantly the structures overall response for the whole frequency range.

2.2. PSD Analysis

PSD stands for Power Spectral Density. It is the most common way to describe the severity of damage for random vibration, ref.[4]. Power Spectral Density is the frequency response of a random or periodic signal. To predict the exact value of a periodic signal at any arbitrary point one can break it down to a wide range of trigonometric signals all with their own unique amplitude, frequency and phase.

When looking at a random vibration in the time domain the signal often has a non-periodic behavior. Over a wider time domain the mathematical complexity of working with these overlapping trigonometric signals makes it unpractical to find all the individual amplitudes as a function of time.

However, a more practical way of dealing with random vibrations is to average the amplitude over a large number of cycles using a statistical process to determine the probability of the occurrence of particular amplitudes. This together with the frequency content of the time domain will indicate where the averaged power is distributed in the frequency domain.
Figure 2-1 - Random process.

Most random processes will follow a zero-mean Gaussian probability distribution, see Figure 2-2. It shows the number of times the random signal reaches certain amplitude.

Figure 2-2 - Zero mean Gaussian probability distribution.

Since the random vibration load is statistically distributed, so are the structural responses. The response from the random signal can be described in terms of standard deviation, also known as sigma value of the distribution, ref.[4].

- (+/-) 1-sigma 68.3 percent of the time.
- (+/-) 2-sigma 95.4 percent of the time.
- (+/-) 3-sigma 99.73 percent of the time.

This indicates that all amplitudes are not equally probable.
2.2.1. Properties of Power Spectral Density (PSD)

As mentioned earlier a periodic random signal can be divided into a wide range of trigonometric signals all with their own unique amplitude, frequency and phase. To do this for a non-periodic random signal one need to assume that the signal has the same behaviors as for a periodic signal. That the signal is stationary; the expected value of the signal amplitude does not change over time. The signal needs also to be ergodic; the average value of one part of a signal is the same as the average value of the whole signal. Finally the signal can be represented by the sum of various sinusoids. When the signal has the same behaviors as for a periodic signal one can use Fourier analysis to break down a given signal into its spectral components. One can create some really complicated looking waves by just summing up simple sine and cosine waves. The signal in Figure 2-3 is a sum of three sine waves.

![Figure 2-3](image-url)

Figure 2-3 – The top signal is a sum of three sine waves. Using Fourier analysis it can be break down into its spectral components, see figures in the middle. Each sine wave has constant amplitude, see bottom figure.
**Fourier transform**

When a random signal is received in the time domain it is hard to get much useful information from it. The Fourier transform makes the signal much easier to understand by transforming it from the time domain into the frequency domain. This makes it possible to look at the amplitudes for each of the frequency components which make up the signal, ref.[5]. It is also important to recognize that this fundamental algorithm allows us to take a time domain signal into the frequency domain and back. A good demonstration of the transformation between time domain and frequency domain is illustrated in Figure 2-4. Here the time domain is given in the xy-plane of the coordinate system, where the amplitude goes up and down with time. Performing a Fourier transformation the signal will be divided into multiply signals with there own frequency. The frequency domain is shown the amplitudes of each signal component.

![Figure 2-4 - Illustration of the Fourier transformation.](image)
2.2.2. Wiener-Khinchine Theorem

The method will be discussed below and is based on the Wiener-Khinchine Theorem, [6] and [5], which states as earlier that the random time signal \( x(t) \) is a stationary process with autocorrelation \( R_{xx}(\tau) \) and that the power spectrum is the Fourier transform of auto-correlation \( R_{xx}(\tau) \). It also states that the signal is assumed to be ergodic.

The root mean square value (RMS) value of a random signal \( x(t) \) is defined by

\[
x_{\text{rms}} = \left( \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt \right)^{1/2}.
\]

(1)

The auto-correlation function of \( x(t) \) is defined by

\[
R_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x(t+\tau)dt.
\]

(2)

It can be observed at zero \( \tau \) the auto-correlation satisfies the random signal relation

\[
R_{xx}(0) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x(t)dt = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)dt = x_{\text{rms}}^2.
\]

(3)

The Perseval’s theorem states that the area under the energy spectral density curve is equal to the area under the square of the amplitude of the signal.

With the aid of the Perseval’s theorem the average power of the \( x(t) \) can be expressed in the frequency domain with use of complex conjugation

\[
\lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt = \lim_{T \to \infty} \frac{1}{2\pi T} \int_{-\pi}^{\pi} X(\omega)X^*(\omega)d\omega,
\]

(4)

\[
\lim_{T \to \infty} \frac{1}{2\pi T} \int_{-\pi}^{\pi} X(\omega)X^*(\omega)d\omega = \lim_{T \to \infty} \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\omega)|^2 d\omega.
\]

(5)

If the periodic time goes to infinity and the power spectrum density is positive then it can in general be represented with

\[
S_{xx}(\omega) = \lim_{T \to \infty} \frac{|X(\omega)|^2}{T}.
\]

(6)

The power can be written as follows

\[
x_{\text{rms}}^2 = R_{xx}(0) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt = \lim_{T \to \infty} \frac{1}{2\pi} \int_{-\pi}^{\pi} S_{xx}(\omega)d\omega.
\]

(7)
The power spectral density (PSD) function $S_{xx}(\omega)$ of $x(t)$ is defined as the Fourier transform of its auto-correlation function $R_{xx}(\tau)$

$$S_{xx}(\omega) = \mathcal{F}[R_{xx}(\tau)] = \int_{-\infty}^{\infty} R_{xx}(\tau)e^{-j\omega \tau} d\tau .$$

(8)

It can inversely transformed back to the auto-correlation function.

$$R_{xx}(\tau) = \mathcal{F}^{-1}[S_{xx}(\omega)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{xx}(\omega)e^{j\omega \tau} d\omega .$$

(9)

The PSD function is symmetric when $\omega = 0$ and the auto-correlation function can be rewritten as follows

$$R_{xx}(\tau) = \frac{1}{2\pi} \int_{0}^{\infty} 2S_{xx}(\omega)\cos(\omega \tau) d\omega = R_{xx}(\tau) = \frac{1}{2\pi} \int_{0}^{\infty} 2S_{xx}(\omega)\cos(\omega \tau) d\omega .$$

(10)

It is however more practical to define the power spectral density in frequency than angular velocity, such that

$$R_{xx}(\tau) = R_{xx}(\tau) = \int_{0}^{\infty} 2S_{xx}(\omega) \cos(2\pi f \tau) df = \int_{0}^{\infty} W_{xx}(f) \cos(2\pi f \tau) df .$$

(11)

Here $W_{xx}(f) = 2S_{xx}(\omega)$ is the PSD function in the frequency domain (unit²/Hz); unit depending if the time-hoysignal is given in acceleration or displacement.

The square root of the mean value $x(t)$ then reads

$$x_{rms} = \sqrt{R_{xx}(0)} = \sqrt{\int_{0}^{\infty} W_{xx}(f) df} .$$

(12)

The power in $x(t)$ can be calculated from a random signal over a given frequency band $f_1 - f_2$ as follows

$$P_{f_1-f_2} = \int_{f_1}^{f_2} W_{xx}(f) df .$$

(13)

The power is always positive, whereas amplitude can be negative.
2.2.3. Sound Pressure Level (SPL)

In the frequency domain acoustic noise is in general described as sound pressure level (SPL) expressed in decibels (dB). The difference between SPL and PSD is that a SPL spectrum is not generally provided at each and every frequency of the acoustic noise. The sound is measured in a certain center frequency with constant relative bandwidth based on one octave or one-third octave band. A typical acoustic noise load in the one-third octave band is illustrated in Figure 2-5.

![PSD Spectrum](image)

Figure 2-5 - Acoustic noise load in the one-third band.

The sound pressure level (SPL) is defined according to

\[
SPL(dB) = 10 \log \left( \frac{p}{p_{ref}} \right)^2,
\]

where the reference value of the sound pressure \( p_{ref} = 2.0 \times 10^{-5} \) Pa is the audible limit of the human ear and \( p \) is the effective value of the occurring sound pressure, both measured in Pascal (Pa).

To convert a SPL spectrum to PSD spectrum one need to know three things, the octave band, the center frequency and the relative bandwidth.
Octave band

Most SPL spectrums are defined in one octave or one-third octave band. The difference is that one-third octave band is an octave band split into three bands, giving more detailed description of the frequency content of the noise. For a constant relative bandwidth, the ratio between two following frequencies is defined as

\[
\frac{f_{\text{min}}}{f_{\text{max}}} = 2^k,
\]

where \( k = 1 \) for the octave band and \( k = 1/3 \) for the one-third-octave band.

Center frequency

The center frequency \( f_c \) is the geometric mean of the minimum frequency \( f_{\text{min}} \) and the maximum frequency \( f_{\text{max}} \) in the relative frequency band, and is dependent on the used octave band. The center frequency \( f_c \) (Hz) is defined by

\[
f_c = \sqrt{f_{\text{min}} f_{\text{max}}}. \tag{16}
\]

Relative bandwidth

The bandwidth \( \Delta f \) is the difference between the maximum frequency \( f_{\text{max}} \) and the minimum frequency \( f_{\text{min}} \) and is given by:

\[
\Delta f = f_{\text{min}} - f_{\text{max}}. \tag{17}
\]

Sound Pressure Level (SPL) to Power Spectral Density (PSD)

Combining the equations above the relative bandwidth \( \Delta f \) can be expressed in terms of the octave band \( k \) and the center frequency \( f_c \):

\[
\Delta f = \left(2^k - 2^{-\frac{k}{2}}\right) f_c. \tag{18}
\]

The effective pressure \( p \) can be calculated as follows

\[
p^2(f_c) = p^2_{\text{ref}} \cdot 10^{\frac{\text{SPL}(f_c)}{10}}. \tag{19}
\]

The reference pressure is \( p_{\text{ref}} = 2.0 \times 10^{-5} \text{ Pa} \), thus \( p_i \) can be written as follows

\[
p^2_i(f_c) = 10^{\frac{\text{SPL}(f_c)-94}{10}}. \tag{20}
\]
The power spectral density of the effective (RMS) sound pressure for a certain center frequency with relative bandwidth $\Delta f$, is calculated according to

$$W_p(f_c) = \frac{p^2(f_c)}{\Delta f}, \quad (21)$$

in which $W_p(f_c)$ is the power spectral density of the sound pressure (Pa$^2$/Hz) and $p^2(f_c)$ is the effective sound pressure.

Based on the Wiener-Kintchine theorem the noise strength over the entire frequency band is calculated from the relation

$$p^2_{rms} = \int_0^\infty W(f)df. \quad (22)$$

The PSD of the pressure field $W_i(f_c)$ in the frequency band with center frequency $f_c$, bandwidth $\Delta f$ and rms sound pressure $p(f_c)$ is defined as

$$p^2_{rms} = \int_0^\infty W(f)df = \sum_{i=1}^{k} W_i(f_c) \Delta f = \sum_{i=1}^{k} \frac{p^2(f_c)}{\Delta f} \Delta f = \sum_{i=1}^{k} p^2_i(f_c), \quad (23)$$

where $k$ is the number of one octave or one third octave bands.
3. MODELS

3.1. Geometry and elements

A parametric FE-model with the same base characteristics as GKNs next generation sandwich nozzle extension was scripted in ANSYS Mechanical ADPL, with the aim that the user does not need to rely on third-party software license. Both the setup of the geometry and the simulation are performed within the same software. With help of the input script the user can easily loop changes in geometrical and material properties of the FE-model. This will later be used in the thesis to evaluate how changes of the nozzle configuration affect the eigen frequencies of critical modes when exposed to random fluctuating loads. The response for different geometries will be used to setup guidelines for designing nozzle reinforcement structures. Some simplifications were made to lower the computation power. In the model each side of a channel is modeled by shell elements. These sides were replaced by one multi-layer element, where each layer has the corresponding stiffness properties of the outer, mid and inner wall of a channel. Also the NE inlet manifold is removed as it has minor impact on the radial stability of the nozzle. Additionally a refined mesh is used on the upper part of the geometry to avoid elements with to high aspect ratio. Two configurations are compared in the thesis; the first configuration is a parametric shell model of the nozzle extension, see Figure 3-1. The second configuration, see Figure 3-2, includes an arbitrary combustion chamber. These to configurations will be used to study the impacts of fixed boundary condition at the nozzle inlet in relation to a fixed boundary condition at interface of the combustion chamber. The origin is located at the throat.

![Figure 3-1 - NE (Nozzle Extension)](image)

Figure 3-1 - NE (Nozzle Extension).
3.2. Constraints

For the performed analyses the nozzle extension has a fixed interface. The nozzle extension is constrained in axial and tangential direction whereas the deformation in radial direction is left free. Depending on which FE-model that is used the fixed constrains are placed at the inlet of the nozzle extension or at the top of the thrust chamber when the combustion chamber is added to the nozzle extension, see Figure 3-1 and Figure 3-2.
3.3. Nozzle wall Configuration

The TC is divided into five sections in axial direction with different thickness for each section. The total wall thickness represents the combined thickness of the nozzle wall and the welded MD layer. Table 3-1 shows the thickness in percent relative section five at different axial locations from the throat.

Table 3-1 Total nozzle wall thickness.

<table>
<thead>
<tr>
<th>Section</th>
<th>Axial position (mm)</th>
<th>Total wall thickness (MD + wall thickness) (%)</th>
<th>TC/NE</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>-500-0</td>
<td>300</td>
<td>TC</td>
</tr>
<tr>
<td>S2</td>
<td>0-150</td>
<td>300</td>
<td>TC</td>
</tr>
<tr>
<td>S3</td>
<td>150-450</td>
<td>213</td>
<td>NE</td>
</tr>
<tr>
<td>S4</td>
<td>450-750</td>
<td>140</td>
<td>NE</td>
</tr>
<tr>
<td>S5</td>
<td>750-2500</td>
<td>100</td>
<td>NE</td>
</tr>
</tbody>
</table>

The figure below shows how the different wall thickness is divided long the nozzle, with more welded MD at the top of the nozzle where no stiffeners are added. The origin is located at the throat.
3.4. Stiffener Configuration

A requirement from manufacturing is to be able to weld each side of the stiffener to the nozzle wall. Then the stiffeners needs to be placed with at least a 45° angle between each other, see Figure 3-5. This is taken into consideration when evaluating the stiffener configuration. The stiffeners are in z-direction placed with its own length including a factor 1.5 between each other, to use the same configuration as today. This configuration keeps the mass close to constant for all stiffener heights.

![Figure 3-5 - Stiffener requirement.](image)

3.5. Material properties

General properties of steel at room temperature are used in the simulation. In general, properties such as density and thermal conductivity do not vary greatly with composition or heat treatment, whereas properties such as tensile strength, elongation, and hardness are in great extent dependent. General material properties for steel is used in this analysis, see Table 3-2.

Table 3-2 Material properties.

<table>
<thead>
<tr>
<th>General material properties of steel at room temperature</th>
<th>E [GPa]</th>
<th>Thermal exp. [1/K]</th>
<th>Poisson’s ratio</th>
<th>ρ [kg/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>1.00E-05</td>
<td>0.3</td>
<td>8900</td>
<td></td>
</tr>
</tbody>
</table>
4. METHOD

To perform a PSD analysis in ANSYS Mechanical ADPL one does first reuse the modal response from a modal analysis. Depending on the setup it is possible to perform a static analysis before the modal analysis, where in this case two loads are applied for inner and other pressure field. The modal response is combined in the PSD analysis to retrieve the response of each mode based on the input spectrum.

Figure 4-1 - PSD analysis cycle.

4.1. Static analysis

The static analysis is used in the modal analysis part of the PSD analysis. In the static analysis two pressure loads are generated. The first is the base surface loads caused by the aerodynamic flow during buffeting phase and the second one is the inner flame pressure load.
4.1.1. Flame pressure

The inner flame pressure load is unpredictable and changes constantly over time due to the incipient flow separation. Too extensive to be included in this thesis. To simplify the model an assumption is made to use a homogeneous pressure load in the tangential direction. The pressure decreases in axial direction from throat to nozzle exit, due to the nozzle expansion.

![Inner pressure as a function of axial location from the throat.](image)

4.1.2. External pressure

The used surface pressure load on the ambient side is the RMS value of the maximum applied surface load together with the ambient pressure. The pressure distribution in tangential direction has a cosine shaped distribution.

![External surface pressure distribution.](image)
4.2. Modal Analysis

Before performing a PSD analysis it is important to know how many modes to consider and to understand which frequencies that have the largest impact on the structure. The more modes you include, the more accurate your solution, but the longer it will take to solve. To know if one has chosen a sufficient number of modes, a useful check is to plot the mass participation factor or the effective mass. A mode with high mass participation is usually a significant contributor to the structural response. Some concluded guidelines concerning the mass participation factor and effective mass:

- For a modal analysis the effective mass should be close to 80% of the real mass in the predominant direction of vibration.
- One common rule of thumb is that a mode should be included if it contributes more than 2% of the total mass.
- If the requirement is to involve close to all modes, with an effective mass ratio close to one, the number of modes specified in the modal analysis should be in the area of the frequency range multiplied by a factor of 1.5.

To retrieve the desired number of modes that are included in the PSD analysis is really tricky. A first step is to specify the frequency range from where the modal analysis will retrieve its modes. In this case the response spectrum is defined from 1-10000 Hz, where most energy is distributed in the frequency interval from 1-1000 Hz. As well from earlier studies it is known that for current geometries the five most critical modes are located in the frequency interval 0-300 Hz, ref. [1]. If the frequency span is unknown it is necessary to look over the whole region to find the critical modes. This is a time consuming process and can be optimized by specifying a significance level that decide which modes to use.

This threshold value is compared with the ratio between the mode with the maximum effective mass contribution and the chosen mode. It goes through all modes in the frequency range and retrieves those modes that are above the significance level and those below are considered insignificant. The higher the significance level threshold, the fewer modes are included in the PSD analysis. The significance level varies for different geometries but is often in the range from 0.1-0.0001.

The number of modes retrieved is also limited by the specified maximum number of modes used in the analysis. The limited number of modes is hard to estimate and is retrieved from trial and error.

A recommendation is to start at a relatively high significance level, around 0.1. If not all modes are retrieved start to lower the significance level until the specified number of modes are fulfilled. If the effective mass ratio level is still not reached, reset the significance level and increase the number of
modes to find modes at higher frequencies. Repeat this step until the effective mass ratio is fulfilled. If the required ratio is already fulfilled after the first step, decrease the number of modes to speed up the solver time. Always keep the number of modes as low as possible. However as with all of these rules, some judgment has to be made and these can vary from case to case but are a good starting point. Figure 4-4 describe the automated iterative cycle that have been scripted in ANSYS APDL.

![Automated modal analysis cycle for the PSD analysis.](image)

4.2.1. Applied Loads

The static analysis described earlier is used in the pre-stress phase of the modal analysis to include the increased structural stiffness generated by the surface pressure.
4.2.2. Mode Shapes

Five critical mode shapes occur in the lower frequency domain below 300 Hz. The modes are:

- Pendulum,
- Ovalisation,
- Torsion,
- Three-wave and
- Four-wave

See Figure 4-5 to Figure 4-9.

Figure 4-5 - Pendulum mode.

Figure 4-6 - Ovalisation mode.
Figure 4-7 - Torsion mode.

Figure 4-8 - Three-wave mode.

Figure 4-9 - Four-wave mode
4.3. PSD analysis

A PSD analysis is performed in three phases. Firstly calculate the normal modes of the FE-model. Then apply the PSD load to the frequency response to get the Response PSD. Finally calculate the RMS value of the Response PSD, ref.[7].

4.3.1. Applied Loads

The PSD analysis applies a fluctuating outer pressure field that causes compression to act on the nozzle wall. Studies have shown that an increase or decrease in axial direction in comparison of a homogeneous outer pressure field has minor difference in load factor, see ref. [1]. So the pressure distribution will throughout this thesis be applied homogeneously in axial direction.

During ascend phase the nozzle will endure fluctuating pressure loads from the atmospheric flow generated around the rocket aft body. The acoustic load specified for the Ariane 5 rocket was not available during the thesis. So it was replaced with the external acoustic load for a slightly larger nozzle extension. How this affects the results can only be determined after comparison with the Ariane 5 load spectrum.

![PSD Spectrum](image.png)

Figure 4-10 – PSD pressure spectrum.
The flow field around the aft body is non-homogeneous distributed see ref. [2] and [8]-[11], where the solid rocket boosters’ shields the nozzle from most of the buffeting loads. See Figure 4-11.

Figure 4-11 Snapshot of the ariane-5 Cp distribution and instantaneous stream traces. Cp is the coefficient of power used in wind turbine aerodynamics. This allows one to make comparisons between different turbines, while eliminating the effects of size and wind conditions, ref.[2].

To illustrate the behavior, a sine pending load is applied around the nozzle. With maximum load on the non-protected sides, at position 90° and 270° see Figure 4-13. Defining the PSD spectrum for an entire nozzle was a problem stated early on. During the thesis only one PSD spectrum for one point were available for the entire nozzle. Instead of using multiple PSD spectrums around the nozzle; that includes the frequency varying energy distribution for each point on the nozzle. The PSD spectrum for the entire nozzle is defined using the known PSD spectrum and a known load pressure distribution and applies it as a load factor circumferentially around the nozzle, with the same load factor in axial location.
Figure 4-12 – PSD spectrum load factor distribution around the nozzle.
4.3.2. Structural damping

The recommendation from ANSYS support was to include the effect of damping in the PSD analysis to account for natural energy losses within the system. In ANSYS there are a few alternatives to include damping, see ref.[12]. In this section the methodology to apply Rayleigh damping (mass proportional and stiffness proportional damping) is explained. The damping ratios were extracted using the classical half-power bandwidth method. The damping ratios are used to correlate the Rayleigh damping parameters; $\alpha$ (mass proportional damping) and $\beta$ (stiffness proportional damping) [5].

4.3.3. Rayleigh damping

Rayleigh damping is applied in the direct integration approach. Unlike modal damping, which dampens the current mode, the Rayleigh damping dampens the system at the current frequency. Total damping is obtained through linear combination of the Rayleigh damping constants, Rayleigh mass proportional damping constant $\alpha$ and Rayleigh stiffness proportional damping constant $\beta$ according to

$$[C] = \alpha[M] + \beta[K] , \quad (24)$$

where $[M]$ and $[K]$ are mass and stiffness matrices, respectively. The relative effect of the coefficients $\alpha$ and $\beta$ on the effective damping ratio $\xi$ can be illustrated with a classical single-DOF system (mass $m$ and spring $k$ with damper written as, $c = \alpha m + \beta k$), where the natural frequency $f_n$, natural radian frequency $\omega_n$ and damping ratio $\xi$ are written

$$\omega_n^2 = \frac{k}{m} , \quad \omega_n = 2\pi f_n \quad (25)$$

$$\xi = \frac{c}{c_{cr}} = \frac{1}{2} \left( \frac{\alpha}{\omega_n} + \beta \omega_n \right) , \quad (26)$$

Based on Eq. (26), the proportional damping effect is illustrated in Figure 4-13, showing that the mass proportional damping term heavily damps the lowest modes and dominates in low-frequency applications. The opposite effect can be observed for the stiffness proportional damping term which damps the modes at high frequencies.
4.3.4. Damping ratios from test measurements

The damping ratios (relative to critical damping) can be retrieved from gauges with high sampled measurement data; vibration, strain gauges etc, using the classical half-power bandwidth method. The measurement data have been transformed into the frequency domain. The half-power bandwidth method is used to estimate the damping ratio for each natural frequency from the frequency domain. In this method, the amplitude of a natural frequency is obtained first. 3dB down from the peak there are two points corresponding to half power point, as shown in Figure 4-14.

The classical half-power bandwidth method is defined as the ratio of the frequency range between the two half power points to the natural frequency at this mode. The damping increases with the frequency range between these two points. The damping ratio $\xi_n$ is calculated from the relation

$$\xi_n = \frac{f_2 - f_1}{2f_n}.$$  (27)
4.3.5. Define Rayleigh damping parameters

Considering the frequency range from \( \omega_i \) to \( \omega_j \) with corresponding damping ratios \( \xi_i \) and \( \xi_j \). The two most dominant response frequencies are picked. This gives two equations from which we can solve for \( \alpha \) and \( \beta \)

\[
\xi_i = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} \tag{28}
\]
\[
\xi_j = \frac{\alpha}{2\omega_j} + \frac{\beta\omega_j}{2} \tag{29}
\]

the mass and stiffness proportional coefficients \( \alpha \) and \( \beta \) can be solved from

\[
\alpha = 2\omega_i\omega_j \frac{\xi_i\omega_j - \xi_j\omega_i}{\omega_j^2 - \omega_i^2} \tag{30}
\]
\[
\beta = 2 \frac{\xi_j\omega_j - \xi_i\omega_i}{\omega_j^2 - \omega_i^2} \tag{31}
\]
5. RESULTS

It shall be emphasized that this is a reduced report; company sensitive results and developed methods are consequently not included.

5.1. Modal analysis

5.1.1. Critical mode frequency vs. stiffener height

An important aspect of this study is to understand how the critical modes are connected to the geometrical properties. Figure 5-1 shows how the eigenfrequency for each critical mode changes with increased stiffener height. Note: Both the torsion and pendulum mode are caused by motion at the nozzle throat and are unaffected by the increased stiffener height. Therefore, are neither modes included in Figure 5-1. If the pressure load were asymmetrically distributed in tangential direction, an increased eigenfrequency of both modes would be encountered.

![Eigenmode frequency vs. Stiffener height](image)

Figure 5-1 Change of eigenfrequency for the nozzle extensions critical modes.
5.1.2. Effective mass comparison

The results from the RPSD analysis need to be verified to make sure that the included modes capture the whole response profile and at the same time limited to prevent excessive solver time. The RPSD plots, Figure 5-2 to Figure 5-4, show that the increased numbers of modes (50 → 1000) are located at higher frequencies (300 Hz and above), i.e. frequencies above the critical modes.

It is shown in Table 5-2 that increasing the number of modes show minor impact on the deformation at critical areas as the nozzle wall and at the stiffener tip. It is concluded that even if the effective mass ratio is not fulfilled for fewer modes, see Table 5-1; effective mass ratio equivalent to 80 %. It is enough to cover the critical modes (0-300 Hz) that are of interest.

Table 5-1 Displacement for different modes.

<table>
<thead>
<tr>
<th>Number of Modes</th>
<th>x-direction (%)</th>
<th>y-direction (%)</th>
<th>z-direction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>65.2</td>
<td>65.1</td>
<td>73.7</td>
</tr>
<tr>
<td>200</td>
<td>79.7</td>
<td>79.6</td>
<td>73.9</td>
</tr>
<tr>
<td>1000</td>
<td>84.8</td>
<td>84.8</td>
<td>80.4</td>
</tr>
</tbody>
</table>

Table 5-2 Displacement for different modes.

<table>
<thead>
<tr>
<th>Number of Modes</th>
<th>Runtime (min)</th>
<th>Stiffener tip (%)</th>
<th>Nozzle wall (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>14 (5.2%)</td>
<td>99.982</td>
<td>100.008</td>
</tr>
<tr>
<td>200</td>
<td>34 (12.6%)</td>
<td>99.977</td>
<td>100.004</td>
</tr>
<tr>
<td>1000</td>
<td>269 (100%)</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>
Figure 5-2 - PSD response for 50 Modes.

Figure 5-3 - PSD response for 200 Modes.

Figure 5-4 - PSD response for 1000 Modes.
5.1.3. Energy distribution

An important objective when designing a component that will encounter an acoustic environment is kept outside the most critical regions of the spectrum; frequencies with the highest energy contribution. Increasing the stiffener height increases the stiffness of the structure and therefore the eigenfrequency. The three red lines in Figure 5-5 to Figure 5-7 show the increased stiffener height from left to right. A study has been performed for three different stiffener heights: 25 mm, 50 mm and 75 mm.

Note: Both Torsion and Pendulum mode are caused by motion at the nozzle throat and are unaffected by the increased stiffener height. If the pressure load were asymmetrically distributed in tangential direction, an increased eigenfrequency of both modes would be encountered.

For this specific PSD spectrum, the energy contribution for the ovalisation mode increases with increasing stiffener height, see Figure 5-5.

![PSD Spectrum](image)

**Figure 5-5** – Eigen frequency of Ovalisation mode with increased stiffener height from left to right.

The 3-wave mode increases up to a stiffener height of 50 mm where the 3-wave mode reaches its maximum energy contribution. Thereafter it will decrease with increased stiffener height, see Figure 5-6.
Figure 5-6 - Eigen frequency of 4-wave mode with increased stiffener height from left to right.

On the contrary, the 4-wave mode has its peak energy contribution at 25 mm stiffeners and decreases for higher stiffeners, see Figure 5-7.

Figure 5-7 - Eigen frequency of 4-wave mode with increased stiffener height from left to right.
5.2. PSD analysis

The results from the PSD analysis are given in 1-sigma RMS values and not the maximum values given in a transient analysis. To compare the results, a good match would be to use the 3-sigma values from the PSD analysis, by multiple the 1-sigma values with a factor three, ref.[13]. Figure 5-8 show the two measurement points used on the last stiffener.

![Diagram showing measurement points for radial displacement of the nozzle wall and stiffener tip displacement.](image)

Figure 5-8 Measurement points for radial displacement of the nozzle wall and stiffener tip displacement.
5.2.1. Radial displacement (RMS) of the nozzle contour

As one would expect the largest displacement is found towards the nozzle exit; furthest from the fixed boundary and with the largest surface area. It is clearly shown in Figure 5-9 to Figure 5-12 that there is great potential to lower the radial displacement up to a stiffener height of 60 mm. Above 60 mm the contour deformation (waviness) between each stiffener starts to occur, which is due to the requirement to have a fixed distance relation between the stiffener height and the distance between each stiffener, see section 3.4.

![Figure 5-9 - Contour displacement for 20 mm stiffeners.](image)

![Figure 5-10 - Contour displacement for 40 mm stiffeners.](image)
5.2.2. Radial displacement (RMS) vs. Stiffener height

Comparing the highest response, located at the nozzle exit, for different stiffener height will give an indication of the contribution the stiffener configuration has on the structural stiffness. Figure 5-13 shows an exponentially decreasing nozzle wall displacement for increased stiffener height. Where two of the most commonly used stiffener sheet metal thicknesses was compared. Above a stiffener height of 80 mm the displacement levels out and it gets less beneficial of increasing the stiffener height.
Figure 5-13 - Radial displacement at nozzle exit at zero degrees orientation.

Figure 5-14 - Axial displacement at nozzle exit at zero degrees orientation.
5.2.3. Radial displacement (RMS) vs. Orientation

The nozzle wall displacement peaks with a 180 degree interval, due to the configuration of the Ariane 5 rocket; the two boosters will during buffeting phase protect the nozzle from acoustic loads, at 90 and 270 degrees. However the stiffener displacement depends solely on which Eigen mode that are excited by the load spectrum. The displacement in the stiffeners peaks with a 90 degrees interval for stiffener heights from 20-70 mm. At around 80 mm a lower mode is excited by the load spectrum and the stiffener displacement peaks with 45 degrees interval.

![Circumferential displacement for stiffener height 20 mm.](image)

**Figure 5-15 – Circumferential displacement for stiffener height 20 mm.**

![Circumferential displacement for stiffener height 40 mm.](image)

**Figure 5-16 – Circumferential displacement for stiffener height 40 mm.**
Figure 5-17 – Circumferential displacement for stiffener height of 60 mm.

Figure 5-18 – Circumferential displacement for stiffener height 80 mm.
5.2.4. Radial displacement (RMS) vs. stiffener distance

To optimize the reinforcement structure to make it as light and strong as possible one needs to understand how stiffeners should be distributed. The stiffener height is kept constant throughout the simulation, 50 mm, varying only the distance between stiffeners, see Figure 5-19.

![Figure 5-19 Distance between stiffeners.](image)

No clear optimum could be found in the trade between deformation and mass, when increasing the distance between stiffeners. A clear linear relationship between displacement and distance between stiffeners can be seen in Figure 5-20.

![Figure 5-20 – Displacement vs. stiffener distance, with 50 mm stiffeners.](image)
5.2.5. Radial displacement (RMS) vs. stiffener position (z-direction)

To get a deeper understanding of the structural contribute of the stiffeners in axial direction, a study was performed by lowering the axial location of the first stiffener until it reaches the nozzle exit, see Figure 5-21.

![Figure 5-21 Axial location of the first stiffener.](image)

It is possible to lower the stiffener layout rather close to the nozzle exit without danger the structural integrity, see Figure 5-22. It is recommended to use a shorter distance between stiffeners close to the nozzle exit.

![Figure 5-22 - Displacement at nozzle exit, with 50 mm stiffeners.](image)
5.2.6. Comparison of fixture constraints

In this section the parametric shell model used throughout this thesis, see Figure 3-1, is extended with an arbitrary combustion chamber, see Figure 3-2, to study the impacts of using a fixed boundary condition at the inlet of the nozzle extension or at the top of the thrust chamber when the combustion chamber is added to the nozzle extension. The nozzle extension is constrained in axial and tangential direction whereas the deformation in radial direction is left free, see section 3.2. Introducing an arbitrary combustion chamber, see Figure 5-23, shows a more rapid increase of the radial displacement for lower stiffeners i.e. shows the importance of defining a correct correlate boundary condition.

Figure 5-23 Radial displacement for two different boundary conditions.
5.3. Response PSD

As described in the theory section, the eigenfrequency is dependent on the geometry of the model and in this case the stiffener dimensions and configuration. A comparison has been done between four different stiffener heights 20, 40, 60 and 80 mm.

The stiffener thickness has minor response on the eigenfrequency and the damping ratio only affect the amplitude of the response; these parameters are kept constant. When evaluating the RPSD plots it is important to remember that its not the peaks values that are important but where the energy is located and the energy content (width of the peak). The critical modes is clearly visible in the RPSD plots. The first peak is the ovalisation mode, followed by the 4-wave modes.

Note: due to rot-reflective symmetry of the defined load distribution of the PSD spectrum, see Figure 4-12, the load case does not activate the 3-wave mode or equivalent modes, see the RPSD results, Figure 5-24 to Figure 5-27. Introducing an asymmetry in the load distribution would activate the 3-wave mode and a response would be detected in the RPSD. However optimization of the load distribution is outside the scope of this thesis. Therefore the 3-wave mode is not included in the following RPSD results.

![Figure 5-24 - PSD Response spectrum for 20 mm stiffeners.](image-url)
Figure 5-25 - PSD Response spectrum for 40 mm stiffeners.

Figure 5-26- PSD Response spectrum for 60 mm stiffeners.
Figure 5-27- PSD Response spectrum for 80 mm stiffeners.
6. DISCUSSION AND CONCLUSION

Analysis of the critical mode frequencies indicated a linear relationship, up to 60 mm stiffener height, between the eigenfrequency of the critical modes and the stiffener height. Where modes like 3-wave and 4-wave more rapidly increases for higher stiffeners than for the ovalisation mode, see section 5.1.1. The pendulum and torsion mode will have nearly constant frequencies. Both the torsion and pendulum mode are only affected by the stiffness of the upper part of the nozzle extension which is affected by the thickness of the MD layer. Connecting these linear relations to the energy distribution of the PSD spectrum shows an increased response for the ovalisation mode when it reaches frequencies with higher energy and a decreased response for both 3-wave and 4-wave modes as they reach frequencies with lower energy, see section 5.1.3. The same trends are shown in the PSD plots see section 5.3. The response of the ovalisation and 4-wave mode are magnified by a load distribution with symmetric peak loads at +/-90 degree angular location from the booster’s axel of symmetry, due to the booster setup of the Ariane 5 rocket, see Figure 4-12. It is important to remember that the Ariane 5 PSD spectrum was replaced by a spectrum from a slightly different nozzle extension, i.e. different setup of the launcher. This affects both the magnitude of the response and how the energy is distributed in the PSD spectra. An increase or decrease in frequency can have a large effect on the way the critical modes are excited. An expected change would be that the Ariane 5 spectrum is located at lower frequencies. If this would be the case we would see a more dramatic decrease in response for higher stiffeners. This would be caused by a decrease of the ovalisation response as it goes toward higher frequencies with lower energy. A further study of how the reinforcement structure affects the eigenfrequencies and the magnitude of the critical modes states that the stiffener height has the largest contribution. Figure 5-13 shows that the radial displacement of the nozzle wall does exponentially decrease with increased stiffener height. The stiffener thickness has a minor effect on the mode frequencies and contributes only to an increased structural stiffness of the reinforcement structure. This state it is better to use higher stiffeners than an increased number of lower stiffeners. At the same time the allowed stiffener height is limited by an increased stiffness of the wall at each stiffener due to increased radial displacement between stiffeners.

Finally, a problem that was stated early on was the lack of support to handle nonlinear behaviors such as plasticity in ANSYS. This meant that large temperature changes occurring during flight could not be accounted for in this PSD analysis. However LS-DYNA has recently implemented a non-linear solver for the aerospace industry, ref.[14], to solve these problems and might be an alternative to perform time history analysis.
7. REFERENCES


[12] ANSYS element Reference for release 12.0, ANSYS Inc


8. APPENDIX A

A short comparison between the PSD analysis and the time history analysis to see possible pros and cons with each method.

PSD Analysis vs. Time history Analysis

PSD:

(+)

- Response spectrum analysis is useful for design decision making because it relates structural type selection to dynamic performance.
- Gives a good overview of which eigenfrequencies that have the largest contribution.
- Short process time and memory efficient. In general faster than time history analysis.

(-)

- The response is restricted to the number of modes specified by the engineer.
- No phase information. Lost in the process of generating the response spectrum.
- Only linear behaviors in ANSYS, do not account for plasticity etc. However, now available in LS-DYNA, ref.[14].

Time history:

(+)

- Capable of producing results with relatively low uncertainty.
- Non-linear behaviors.

(-)

- Long process time and memory demanding.
- In contrast to PSD analysis, time history analysis generally requires hundreds or thousands of time steps to be analyzed in order to evaluate the response of a structure.
- Subsequently, the results must be scanned to identify the maximum absolute response of the system.