Abstract

This MSc diploma thesis in Space Engineering at Luleå University of Technology, was carried out at Monash University in Melbourne, Australia. It is within the area of solar physics with focus on the magnetic field of the Sun.

Within solar physics, the process generating the magnetic field of the Sun is normally referred to as the solar dynamo. An important tool when it comes to understanding the magnetic field of the Sun and its interactions with the whole solar system, is to construct solar dynamo models. The aim is to obtain a model, which produces output corresponding to the solar observations and which preferably also could be used to predict the solar magnetic activity.

A solar flux transport dynamo model code, which was to be used for the project, was provided by Dr. M. Dikpati. It simulates the magnetic field of the Sun for a half-sphere solution and the output can be compared to solar observations. The initial focus of the project was set on varying a specific magnetic field parameter in the code and the effects the variations would have on the solar dynamo model. Interaction of two different kind of these parameters could possibly lead to destructive interference, which could terminate the solar dynamo or lead to a significant decrease of its magnitude. If such a relation is found, it could possibly be related to the grand minima which has been observed in sunspot occurrences. Some additional numerical surveys were also made, plus two subprojects of trying to extend the provided code to a full-sphere solution and to reproduce output for a low-order dynamo model.

Considering the main focus of the project, the differences that could be identified for varying the magnetic field parameter, were related to the field strength and locations of the different kind of solar magnetic fields. Regarding some of the flow parameters connected to the magnetic field, some conclusions could be made related to their effect on the solar dynamo. For example, the surface-flow velocity is an efficient parameter for regulating the period of the solar cycle. No major conclusions regarding constructive or destructive interference between the magnetic field parameters in question could be made and as a consequence of this, no conclusions regarding connections to grand minima or maxima could be made. Regarding the above issues, a reliable full-sphere solution, which could be run for simulations over longer periods of time, would have been to prefer.
Preface

As a final step toward my Master of Science degree in Space Engineering at Luleå University of Technology, a project in solar physics was carried out at Monash University in Melbourne, Australia. The project was carried out from October 3, 2005 to February 24, 2006 under supervision by Professor Paul Cally at Centre of Stellar and Planetary Astrophysics. Examiner at my home university was Professor Sverker Fredriksson at the Division of Physics and the project, including this thesis, is to be followed by a final presentation at the Division of Physics at Luleå University of Technology on May 24, 2006.

Throughout this project, time has also been spent learning and obtaining experience within the software required for the different parts of the project. For example, basic knowledge of Fortran77 was necessary since the dynamo code, which was used for the main parts of this project, was written in Fortran77. All plots and animations, for analysing the output of the simulations, have been made in Interactive Data Language (IDL). The initial code for the butterfly plots was provided together with the dynamo code, and was a very useful tool when learning the IDL software. For the low-order dynamo simulations, Mathematica was used, which also took some time to learn. Furthermore, this report has been written in \LaTeX{} which was a totally new experience.

Finally, I would like to thank Professor Paul Cally, for all his time and support throughout the project, and everyone in his solar physics group at CSPA, for their never-ending patience in helping me with my numerous software related problems. Also many thanks to Professor Sverker Fredriksson for his encouragement and professional comments during my time with this project.

Caroline Beijersten
## CONTENTS

3.7 Dynamo waves ............................................. 20

4 Solar dynamo models ........................................ 23
   4.1 Location of the $\alpha$-effect ........................... 23
   4.2 Babcock-Leighton flux transport dynamo ............... 24
   4.3 Low-order dynamo models .............................. 24
      4.3.1 Low-order simulations ........................... 25
      4.3.2 Low-order models and solar characteristics ....... 26
      4.3.3 Transition to chaos ............................ 27
      4.3.4 Low-order models versus numerical models ....... 27
   4.4 Characteristics required for a realistic solar dynamo ... 28

5 Half-sphere solution ......................................... 33
   5.1 The code ............................................... 33
      5.1.1 Boundary conditions ............................. 33
      5.1.2 Conversion to non-dimensional units .............. 34
      5.1.3 Changes made to the half-sphere code ............. 34
   5.2 The tools to analyse the output ....................... 34
      5.2.1 Butterfly plots ................................... 35
      5.2.2 Animations ....................................... 35
   5.3 A reference solution ................................... 37
      5.3.1 Butterfly plot analysis .......................... 38
      5.3.2 Animation analysis ............................... 38
   5.4 Varying the $\alpha$-parameters ......................... 39
      5.4.1 Butterfly plot analysis .......................... 40
      5.4.2 Animation analysis and images from animations ... 42
   5.5 Varying meridional circulation parameters ............ 43
      5.5.1 Surface velocity ................................... 43
      5.5.2 Radial dependence ................................ 45
      5.5.3 Latitudinal dependence .......................... 45
   5.6 Numerical solution of a reference article ............ 46

6 Full-sphere solution .......................................... 53
   6.1 Extending the code ..................................... 53
   6.2 The tools to analyse the output ....................... 54
   6.3 Analysis of full-sphere solution ...................... 54
      6.3.1 Butterfly plot analysis .......................... 55
      6.3.2 Animation analysis ............................... 56
   6.4 Numerical solution of a reference article ............ 56

7 Discussion and conclusions .................................. 61
   7.1 The half-sphere solution ............................... 61
      7.1.1 The $\alpha$-parameters ............................ 62
      7.1.2 The meridional flow parameters .................... 62
      7.1.3 Numerical solution of a reference article ......... 62
   7.2 The full-sphere solution ............................... 62
   7.3 The poloidal field strength ............................ 63
   7.4 Future research ........................................ 63
      7.4.1 Meridional flow parameters ....................... 63
      7.4.2 Polarity of full-sphere solution ................... 64
CONTENTS

7.4.3 Tachocline thickness ........................................ 64
A request of carrying out a final project within solar physics at Monash University was sent to Professor Paul Cally at the Centre of Stellar and Planetary Astrophysics in the beginning of 2005. After a few months of e-mail conversation the project and dates were set.

Within solar physics, the process generating the magnetic field of the Sun is normally referred to as a solar dynamo. An important tool when it comes to trying to understand the magnetic field of the Sun and its interactions with the whole solar system, is to construct solar dynamo models. The aim is of course to obtain a model that produces output corresponding to observations made of the Sun and also can be used to predict the solar magnetic activity. The project consisted mainly of running simulations with a solar dynamo code provided by M. Dikpati at the High Altitude Observatory in Boulder, Colorado, USA. In general, the solar dynamo code in question simulates the magnetic field of the Sun for a half-sphere solution and the output can be compared to solar observations. The initial focus was set on varying one of the $\alpha$-effect parameters in the code and the effects the variations would have on the solar dynamo model. According to Dikpati, interaction of two $\alpha$-effects could lead to destructive interference, which could kill the solar dynamo or lead to a significant decrease of its magnitude. If such a relation is found, it could possibly be related to the grand minima observed. Since the grand minima are not understood, it would be important to study whether the two $\alpha$-effects could have a destructive interaction for certain values and conditions. As the project proceeded, some additional surveys were also made, plus two subprojects of trying to extend the provided code to a full-sphere solution and to reproduce output for a low-order dynamo model.

To start off with, Chapter 2 contains mainly some magnetohydrodynamic theory and equations. In Chapter 3 these will be applied to the magnetic field of the Sun and the profiles used in the dynamo model will be presented. Some general information about the Sun, the solar cycle and sunspots is also given in the first parts of Chapter 3. As a more thorough introduction to solar dynamos, some dynamo models and the main theories behind them will be presented in Chapter 4. Some extra time is spent on low-order models, and their applications within solar physics, in Section 4.3. In Chapter 5 the main part of the project is presented, including some of the obtained plots and images from animations made. The task of trying to extend the project to a full-sphere solution of the
dynamo code is discussed in Chapter 6. Finally, conclusions and discussion regarding the project are presented in Chapter 7.

1.1 Monash University

Monash University was founded in 1958 and now it includes eight campuses in three countries; six campuses in Australia, one in Malaysia and one in South Africa. Already in 1967 more than 21,700 students were enrolled, and in 2005 the university had more than 52,400 students. The same year, about 23,500 students were enrolled at the Clayton Campus, which is the original Monash Campus and also the campus where the Centre of Stellar and Planetary Astrophysics is located. (Monash University website, 2006-03-28) Since 2005 Luleå University of Technology is one of Monash University’s alliance universities, which is a unique opportunity and a great honour for this Swedish university.

1.1.1 Clayton Campus

Clayton Campus gives an impression of being a dynamic campus with a great atmosphere, in spite the fact that many of the university buildings are quite old and well used. A lot of effort has been put into keeping green areas with trees and plants all over the campus. In close connection to the student resident halls at campus, there is a park with a small lake and the University Sports and Recreation facilities also play an important role at campus with e.g. various sports fields etcetera.

Figure 1.1: The Clayton Campus is the oldest and largest campus of Monash University. The campus gives an impression of being dynamic, and a lot of effort has been put into keeping green areas within the campus area. Image collected from Monash University website (2006-03-28).
1.1. MONASH UNIVERSITY

Thanks to the number of students and the size of Clayton Campus, it is also possible to provide most services needed for the students at the campus centre. Restaurants, bank branches, hairdresser, cinema, post office, optometrist and travel agent are examples of services provided in addition to the normal student services, such as book shops and student administration offices.

1.1.2 The Centre of Stellar and Planetary Astrophysics

The project, which this thesis is based on, was carried out at the Centre of Stellar and Planetary Astrophysics (from here on referred to as CSPA) at Monash University. The project was supervised by Professor Paul Cally, who’s main fields of research are within magnetohydrodynamics. For example, he has carried out research within transition region physics, coronal loops and tachocline instabilities. At the moment Dr. Alina Donea and four PhD students are connected to his solar physics research group, and examples of fields of studies are local helioseismology and far-side imaging. CSPA is housed under the School of Mathematical Sciences and is one of seven science research centres at Monash University. Except for solar physics, research is also being carried out at CSPA within areas such as stellar evolution and dynamics, planetary dynamics and general relativity.
Chapter 2
Basic MHD for plasma

The interaction between a plasma and a magnetic field can be described by magnetohydrodynamic equations (from here on referred to as MHD). If assuming that the plasma in the convection zone of the Sun is a perfectly conductive fluid, Maxwell’s equations and gas dynamic equations can be combined to obtain a set of ideal MHD equations. (Goedbloed & Poedts, 2004)

2.1 MHD equations

2.1.1 Faraday’s law
A changing magnetic field, $\mathbf{B}$, generates an electric field, $\mathbf{E}$, while a stationary magnetic field, $\mathbf{B}$, does not. This can be expressed as:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

(2.1)

which is referred to as Faraday’s law.

2.1.2 Ampere’s law
A magnetic field, $\mathbf{B}$, can be generated by a current, $\mathbf{j}$, and/or a changing electric field, $\mathbf{E}$, as of:

$$\nabla \times \mathbf{B} = \mu \mathbf{j} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t},$$

(2.2)

where $\mu$ is the permeability, $c$ the speed of light and $t$ the time. The expression above is referred to as Ampere’s law. $\nabla$ is of order:

$$\nabla \sim \frac{1}{L},$$

(2.3)

where $L$ is the typical length scale, and $\partial t$ is of order:

$$\partial t \sim T,$$

(2.4)
CHAPTER 2. BASIC MHD FOR PLASMA

where \( t \) is the time and \( T \) is the typical time scale. The velocity, \( v \), is significantly smaller than the speed of light \( c \). In other words \( v < < c \Rightarrow v^2 < < c^2 \), and the following approximations can be made from Equations 2.1, 2.3 and 2.4:

\[
(2.3) \Rightarrow \nabla \times E \sim \frac{E}{L} \Rightarrow \frac{E}{L} \sim \frac{B}{T} \Rightarrow E \sim \frac{BL}{T}.
\]

\[
(2.4) \Rightarrow \frac{\partial B}{\partial t} \sim \frac{B}{T}
\]

where \( B \) is the typical magnetic field scale. \( L \) is also here the typical length scale. From Equation 2.5, Ampere’s law can be approximated and rearranged as:

\[
\mu \sim O\left(\frac{B}{L}\right) + O\left(\frac{BL}{c^2T^2}\right) \Rightarrow \frac{B}{L} + \frac{B}{L} = \frac{B}{L} + \frac{B}{L} \approx \frac{B}{L}
\]

and finally rewritten as:

\[
\nabla \times B = \mu j \Rightarrow j = \frac{1}{\mu} \nabla \times B.
\]

where \( \mu \) still represents the permeability.

2.1.3 Coulomb’s law

Monopoles can exist in an electric field, \( E \), in other words, electrons and protons can exist on their own. An electric field, \( E \), is linear to the charge density, \( q_e \):

\[
\nabla \cdot E = \frac{q_e}{\varepsilon},
\]

where \( \varepsilon = 4\pi k_C \) and \( k_C \) is Coulomb’s constant.

2.1.4 Maxwell’s additional equation

Monopoles cannot exist in a magnetic field, \( B \), which means that a positive or negative pole cannot exist alone without the other one. Thus:

\[
\nabla \cdot B = 0,
\]

which, for example, can be used to explain the flux tube structure of a magnetic field in combination with a plasma.

2.1.5 Gas dynamic equations

If Maxwell’s equations describe the electric and magnetic fields as of the current and the charge densities, the gas dynamic equations describe the density, \( \rho \), and the pressure, \( p \), as:

\[
\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} = \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,
\]

\(^{1}\)For most plasmas non-relativistic velocities can be assumed.
2.2. DIFFUSION

which represents mass conservation and:

\[
\frac{D\rho}{Dt} + \gamma \rho \nabla \cdot \mathbf{v} = \frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho + \gamma \rho \nabla \cdot \mathbf{v} = 0 ,
\] (2.11)

which represents conservation of entropy. \( \frac{D\rho}{Dt} \) corresponds to the Lagrangian time derivative, which is evaluated while moving with the fluid instead of at a fixed position. The ratio \( \gamma \) of specific heats at constant pressure and volume is \( \gamma = \frac{C_p}{C_v} = \frac{5}{3} \) for an ideal plasma. The vector \( \mathbf{v} \) represents the velocity. Thus, combined with Maxwell’s equations and using the same assumptions as for Equation 2.7, the basic equations for an ideal plasma can be expressed as:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 ,
\] (2.12)

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) + \nabla \rho - \rho \mathbf{g} - \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} = 0 ,
\] (2.13)

\[
\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p + \gamma p \nabla \cdot \mathbf{v} = 0
\] (2.14)

and

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) = 0 .
\] (2.15)

where \( \mathbf{g} \) is gravity.

2.2 Diffusion

According to Ohm’s law a current, \( \mathbf{j} \), is linear to an electric field, \( \mathbf{E} \). However, since this is not true for all materials and examples, it cannot be considered a fundamental law. Ohm’s law is applicable for fluids within MHD though, and can be written as:

\[
\mathbf{j} = \sigma \mathbf{E}
\] (2.16)

for the stationary case and as:

\[
\mathbf{j} = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B})
\] (2.17)

for the non-stationary case. Combining Ohm’s law as in Equation 2.17 and Faraday’s law as in Equation 2.1, the changing magnetic field, \( \mathbf{B} \), can be rewritten as:

\[
\begin{align*}
(2.17) \Rightarrow \mathbf{E} &= \frac{\mathbf{j}}{\sigma} - \mathbf{v} \times \mathbf{B} \\
(2.1) \Rightarrow \frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times \frac{\mathbf{j}}{\sigma}
\end{align*}
\] (2.18)

The diffusion, \( \eta \), can be expressed as:

\[
\eta = \frac{1}{\mu \sigma} ,
\] (2.19)
and together with Ampere’s law as in Equation 2.7, Equation 2.18 can be rewritten as:

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B}) = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B},
\]

(2.20)

where \(\mathbf{v} \times \mathbf{B}\) corresponds to the advection part and \(\eta \nabla^2 \mathbf{B}\) to the diffusion part. Equation 2.20 is also referred to as the *Induction equation*. To obtain the relation between the two parts generating the change of the magnetic field, \(\mathbf{B}\), the following expression can be set up:

\[
\frac{\nabla \times (\mathbf{v} \times \mathbf{B})}{\eta \nabla^2 \mathbf{B}} \sim \frac{VB/L}{\eta B/L^2} = \frac{V}{\eta} \equiv R_m,
\]

(2.21)

where \(R_m\) is also referred to as the *magnetic Reynold number*. It can be noticed that for \(R_m >> 1\), the diffusion contribution is not significant, while for \(R_m << 1\), diffusion is the dominating part of generating changes in the magnetic field, \(\mathbf{B}\). If considering the diffusion part only for \(R_m << 1\), a diffusion time scale, \(\tau\), can be expressed as:

\[
\frac{\partial \mathbf{B}}{\partial t} \simeq \eta \nabla^2 \mathbf{B} \Rightarrow \frac{B}{\tau} \sim \frac{B}{L^2} \Rightarrow \tau \sim \frac{L^2}{\eta},
\]

(2.22)

from where a conclusion whether the diffusion is relevant for the case in question or not can be formulated.

- The diffusion time scale, \(\tau\), is small for a small length scale, \(L\), or a large diffusion parameter, \(\eta\), and the diffusion could be important for the case in question.
- For a large length scale, \(L\), or a small diffusion parameter, \(\eta\), the diffusion time scale, \(\tau\), becomes large and is too big for diffusion to be relevant for the case.

However, the diffusion time scale, \(\tau\), always has to be compared with the total time range and length scale considered. For example, for the whole solar convection zone, \(\tau\) might be too large and diffusion is therefore irrelevant. However, in a thin layer of the convection zone, \(L\) is small, and the diffusion time scale, \(\tau\), might get small enough to make diffusion relevant.

### 2.3 Magnetic pressure

The *Lorentz force*, \(\mathbf{F}\), can be expressed as:

\[
\mathbf{F} = \mathbf{j} \times \mathbf{B} = \frac{1}{\mu} (\nabla \times \mathbf{B}) \times \mathbf{B},
\]

(2.23)

where

\[
(\nabla \times \mathbf{B})_m = \varepsilon_{mpq} \partial_p B_q
\]

(2.24)

in tensor form. To express also the Lorentz force, \(F_r\), in tensor form the following expression can be set up:

\[
(2.24) \Rightarrow F_r = \frac{1}{\mu} \varepsilon_{rmn} (\varepsilon_{mpq} \partial_p B_q) B_n = \frac{1}{\mu} \varepsilon_{mnr} \varepsilon_{mpq} (\partial_p B_q) B_n
\]
2.3. MAGNETIC PRESSURE

\[ F_r = \frac{1}{\mu} \left[ (\partial_p B_r) B_p - \frac{1}{\mu} (\partial_r B_n) B_n \right] \]

(2.25)

where \( \varepsilon_{ijk} \) is 1 if cyclic, -1 if anticyclic and 0 otherwise. \( \delta_{ij} \) is Dirac’s delta function, which has the value 1 if \( i = j \) and 0 otherwise. If converting this back from tensor form, the Lorentz force, \( F \) can be written as:

\[ F = \frac{1}{\mu} (B \cdot \nabla) B - \frac{1}{2\mu} \nabla (B^2) , \]

(2.26)

where \( \frac{1}{\mu} (B \cdot \nabla) B \) corresponds to the force generated by magnetic tension and \( \frac{1}{2\mu} \nabla (B^2) \) to the force generated by magnetic pressure. The Lorentz force is, for example, applied in the momentum equation, which describes how the fluid is affected by the magnetic field. It and can be written as:

\[ \rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \rho \mathbf{g} + \mathbf{j} \times \mathbf{B} \]

\( \text{L.F.} \)

(2.27)

where \( \rho \) is the density, \( \mathbf{v} \) is the velocity, \( p \) is the pressure and \( \mathbf{g} \) is gravity.

2.3.1 The plasma \( \beta \)

The plasma \( \beta \) is a ratio used to conclude whether the plasma pressure, \( p_p \), or the magnetic pressure, \( p_m \), is the dominating one in the fluid, and can be expressed as:

\[ \beta = \frac{p_p}{p_m} . \]

(2.28)

If \( \beta \) is significantly larger than 1, the plasma pressure is the dominating pressure and the magnetic field lines are considered to be frozen in the plasma. If \( \beta \) is smaller than 1, the magnetic pressure is the dominating pressure and the plasma is controlled by the magnetic field lines. If \( \beta \approx 1 \), very complex relations occur and this will not be further discussed here. Considering the solar characteristics, the following two examples are representative applications of the plasma \( \beta \):

- In the solar interior, the plasma pressure is significantly higher than the magnetic pressure, i.e., \( p_p >> p_m \) and \( \beta >> 1 \). The magnetic field lines can be considered frozen in the plasma since the fluid is the dominating one.

- In the solar wind, the magnetic pressure is significantly higher than the plasma pressure, i.e., \( p_m >> p_p \) and \( \beta << 1 \). The magnetic field lines are then the dominating part, and the fluid is controlled by the magnetic field lines.
2.3.2 Magnetic buoyancy

A magnetic flux tube with a density, \( \rho_{in} \), and a total internal pressure of \( p_{in} + p_{mag} \) can be assumed. The parameter \( p_{in} \) is the internal gas pressure and \( p_{mag} \) is the magnetic pressure in the flux tube. It can also be assumed that the flux tube is located in the convection zone, which has a density, \( \rho_{ext} \) and a pressure \( p_{ext} \), a pressure equilibrium can be expressed as:

\[
p_{ext} = p_{in} + p_{mag} .
\]  

(2.29)

For simplicity, the temperature in the flux tube, \( T_{in} \), is approximated as the temperature in the convection zone, \( T_{ext} \):

\[
T_{in} \approx T_{ext} = T .
\]  

(2.30)

For a gas, the pressure is proportional to the density and temperature, so that the following expression can be written:

\[
p_{gas} \propto \rho_{gas} T_{gas} \Rightarrow T_{gas} \propto \frac{p_{gas}}{\rho_{gas}} .
\]  

(2.31)

Together with Equation 2.30 this gives:

\[
\frac{p_{in}}{\rho_{in}} = \frac{p_{ext}}{\rho_{ext}} .
\]  

(2.32)

Rearranging Equation 2.32 gives:

\[
\frac{p_{ext}}{\rho_{ext}} \Rightarrow \frac{p_{in}}{\rho_{ext}} = 1 - \frac{p_{mag}}{p_{ext}} \Rightarrow \rho_{in} < \rho_{ext} .
\]  

(2.33)

Since the density in the flux tube is lower than the density in the surrounding convection zone, a buoyancy effect is generated and the magnetic flux tube will rise toward the outer layers of the convection zone.
Chapter 3

The magnetic field of the Sun

The solar magnetic field is believed to have its origin in the convection zone of the Sun, which is the layer from 0.7 of the solar radius to the surface of the Sun, i.e., to 1.0 of the solar radius. (From here on the solar radius will also be referred to as $R_\odot$.) Most theories agree that the magnetic field of the Sun does not penetrate into the radiative core of the Sun. However, there is no doubt that the magnetic field affects and interacts with, for example, planets at great distances from the Sun. Just beneath the interface of the radiative core and the convection zone of the Sun, a layer with very strong rotation shear is located, which is called the tachocline. It plays a major role in the solar dynamo models discussed in this thesis. An approximation has been made that the thickness of the tachocline is roughly $0.025R_\odot$.

The magnetic field of the Sun and its activity definitely affect life on Earth and are of highest importance regarding various technologies used in the everyday life. For example, instruments on satellites are exposed to the solar wind and may be sensitive to its fluctuations. Occurrences where telecommunication systems have been significantly affected by drastic increases of solar activity are also quite wellknown facts.

3.1 The solar cycle and sunspots

Sunspots were mentioned in Chinese chronicles as far back as 800 BC, and the sunspot pattern in general was observed already in the 17th century. Between 1826 and 1851 Heinrich Schwabe discovered and recorded the sunspot cycle while he was looking for a planet with an orbit inside that of Mercury. However, he did not find a planet but thanks to his sunspot records he concluded that the sunspots occurred periodically over a cycle of approximately 11 years. According to Goedbloed & Poedts (2004), it was not until the 20th century that G.E. Hale discovered that the sunspots somehow are connected to the magnetic field of the Sun. (Goedbloed & Poedts, 2004)
3.1.1 Sunspot characteristics

Sunspots are generated by magnetic flux ropes rising up through the convection zone due to magnetic buoyancy (see Section 2.3.2). When they reach the surface of the Sun, a visible dark spot is produced. Sunspots can have a diameter of up to 60,000 km and the magnetic field in the centre of the sunspot, the umbra, can reach values of up to 4 kG. The umbra has an approximate temperature of 3700 K, which is considerably cooler than the surrounding surface of the Sun and also the reason why the sunspots appear as dark spots. (Goeblued & Poedts, 2004) If the number of sunspots is plotted as a function of time and latitude, a characteristic butterfly-like pattern is normally observed. Therefore, such plots are normally referred to as butterfly diagrams or butterfly plots. In Figure 3.5 the butterfly pattern is shown in the top part of the image. The butterfly-like relation is referred to as Spörer’s law. The general butterfly pattern is caused by the differential rotation of the Sun and the oscillations in field strength between the poloidal and toroidal fields, which will be further discussed in Sections 3.2 and 3.3.1.

The sunspots always occur in pairs, with one leading sunspot on a slightly lower latitude, and a trailing one on a slightly higher latitude. This relation is caused by the Coriolis force, and the angle between the leading and the trailing sunspots are approximately $10^\circ$ on higher latitudes and about $4^\circ$ closer to the equator. Hence the sunspot tilt increases with increasing latitude, which is referred to as Joy’s law. There have also been theories that the tilt could correspond to the subsurface poloidal field. However, no such tilts have been observed and this would also result in a decreasing tilt for increasing latitude, which is the opposite to what has been observed. The sunspots in a sunspot pair have different polarities and as of Hale’s polarity law, the leading sunspots in one hemisphere always have the same polarity. Due to the anti-symmetry of the magnetic field of the Sun (see Chapter 3) the leading sunspots in the other hemisphere will have the opposite polarity to the leading ones in the first hemisphere. Since the magnetic field of the Sun swaps polarity approximately every 11 years, as of the solar cycle, the polarities in the sunspot patterns also swap, and the polarity of the trailing sunspots in a hemisphere then becomes the polarity of the leading ones for the next solar cycle. (Dikpati & Charbonneau, 1999)

3.1.2 Grand minima and maxima

Except for the well-recorded solar cycle of approximately 11 years, there are also records of a possible longer cycle of solar activity. As can be seen in the lower part of Figure 3.5, the amplitude of the solar activity changes over a longer time range, and periods of both lower and higher activity than average have been observed. These are normally referred to as grand minima and grand maxima.

The most obvious grand minimum is the Maunder minimum, which occurred during the time period of 1645-1715, when the number of observed sunspots decreased drastically. This is shown in Figure 3.1. It has been agreed on that this was not due to lack of observational data, but to an obvious decrease in solar activity during that time. By studying the remnants of $^{10}$Be in the polar ice caps and $^{14}$C in trees, it is possible to determine the production rates of
these isotopes throughout the years. They are produced by galactic cosmic rays, so that variations in solar activity should affect the production rate and abundance of the isotopes. According to the data obtained, the production rates for both isotopes increased during the time of the Maunder minimum, and it could be concluded that a solar minimum actually took place. The $^{14}$C records have been extended back for approximately 9000 years, and similar increases of the isotope, as the one during the Maunder minimum, have been recorded for several periods of time. For example, grand minima have been recorded for the approximate time periods of 1282-1342 (the Wolf minimum) and 1416-1534 (the Spörer minimum). (Tobias, Weiss & Kirk, 1995) However, the $^{10}$Be records do not show a total interruption of the solar cycle, so that it is most probable that the solar cycle did not change its characteristics, but only its amplitude during the minima. A likely explanation is that the toroidal flux ropes did not exceed the threshold strength and would not rise through the convection zone during these periods, is a likely explanation. (Charbonneau, 2005)

Furthermore, the Maunder minimum coincides with the little ice age on Earth. During the second half of the 17th century, both northern Europe and North America experienced extraordinary cold winters. For example, there are historical records of Swedish army troops walking across the ice to Denmark in 1658, and in 1696 and 1708-1709 Finland and Sweden had some extremely poor years due to early frost and cold winters.

Also worth mentioning is that the solar activity seems to contribute to the climate changes on Earth. In other words, the observed global warming might not be due to the greenhouse effect only, but more likely to a combination of the same and a general increase in solar activity over the last few solar cycles.

One of the main aims within the research of the magnetic field of the Sun is to explain and simulate these grand minima and maxima.

### 3.2 Poloidal and toroidal fields

The magnetic field of the Sun has two main components, a poloidal magnetic field and a toroidal magnetic field. The poloidal field can be compared to a dipole field, which to some extent can be compared to the structure of the dipole field of the Earth. According to Yoshimura (1975), the poloidal field
consists of a latitudinal field and a radial field, and according to Charbonneau, St-Jean & Zacharias (2005) the areas of maximum poloidal field strength have been observed close to the poles. The toroidal field, on the other hand, has a torus shaped structure and is mainly located in subsurface layers. Thanks to the tachocline characteristics, theories that the toroidal field can be stored in the tachocline for longer periods of time exist. The interaction between the two magnetic fields, and their oscillations, are the main causes of the solar cycle. Solar maxima, i.e., when the maximum number of sunspots are produced, occur when the toroidal field has its maximum, which also coincides with the reversal of the poloidal field polarity.

3.2.1 The magnetic field profile

In the dynamo model used for this thesis, the magnetic field, \( B \), can be expressed as a function of the flow, \( \mathbf{U} \), by the induction equation (Equation 2.20) as:

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B} - \eta \nabla \times \mathbf{B}),
\]

where \( \eta \) is the magnetic diffusivity (see Section 3.5). Assuming axisymmetry and that the flow, \( \mathbf{U} \), is given, the poloidal field component can be written as:

\[
\frac{\partial A}{\partial t} + \frac{1}{r \sin \theta} (\mathbf{u} \cdot \nabla)(r \sin \theta A) = \eta \left( \nabla^2 - \frac{1}{r^2 \sin^2 \theta} \right) A + S
\]

and the toroidal field component as:

\[
\frac{\partial B_\phi}{\partial t} + \frac{1}{r} \left[ \frac{\partial}{\partial r}(ru_r B_\phi) + \frac{\partial}{\partial \theta}(u_\theta B_\phi) \right] =
\]

\[
v \sin \theta (\mathbf{B}_p \cdot \nabla) \Omega - \nabla \eta \times \nabla \times B_\phi \mathbf{e}_\phi + \eta \left( \nabla^2 - \frac{1}{r^2 \sin^2 \theta} \right) B_\phi,
\]

where the poloidal magnetic field, \( \mathbf{B}_p \equiv \nabla \times (A \mathbf{e}_\phi) \) and \( B_\phi \) is the toroidal component. \( \mathbf{u} \) is the meridional circulation (see Section 3.6.1), \( S \) is the source term corresponding to the \( \alpha \)-effect (see Section 3.4.1) and \( \Omega \) is the differential rotation of the model (see Section 3.3.1). (Dikpati & Charbonneau, 1999)

3.2.2 Field strength limits

Thanks to helioseismology, the field strength of the surface poloidal field has been set to the most probable and confirmed values. As of observations made, the surface poloidal field has a field strength of approximately 10 G. (Dikpati & Charbonneau, 1999)

For the MHD calculations and estimations made considering the strength of the toroidal magnetic field, the sunspot tilts are the most useful tool. If the field is too weak, the flux ropes would rise too slowly towards the surface and the Coriolis force would cause the flux ropes to drift too far pole-ward while rising radially outwards. This would result in sunspots occurring on significantly higher latitudes than observed. As of estimations made, even sunspots originating from flux ropes at the equator would appear on too high latitudes if the toroidal field is weaker than approximately 60 kG. If the field would be too strong on the other hand, the flux ropes would rise too quickly towards the
3.3 Differential Rotation

3.3.1 Differential rotation in general

The differential rotation of the Sun was first discovered in the 17th century, thanks to sunspot observations. In general, the differential rotation means that the rotation period of the solar equator is a few days shorter than the rotation period of the poles. The plasma in the convection zone rotates one lap around the rotation axis in approximately 25 days at the equator and in approximately 33 days at the poles. Today, the differential rotation of the Sun is one of the very few processes that actually are quite well known, mainly thanks to helioseismology.

Since the poloidal magnetic field lines can be considered frozen in the plasma of the convection zone, they are bound to follow the differential rotation. This leads to the field lines being dragged around the Sun and the previously straight dipole field, is now twisted around the Sun, as shown in Figure 3.2. The more the field lines are twisted around the solar equator, the stronger the toroidal magnetic field grows, until it becomes the dominant part of the magnetic field. Instead of having field lines parallel to the rotation axis, the generated field is now located as a torus around the Sun. The transition from a poloidal field to a toroidal one is normally referred to as the Ω-effect. The strongest shear due to differential rotation is believed to take place in the tachocline.

3.3.2 The Ω-effect profile

The differential rotation profile used for the simulations, is almost identical to the one in Dikpati & Charbonneau (1999) and can be expressed as:

$$\Omega(r, \theta) = \Omega_c + \frac{1}{2} \left[ 1 + erf \left( \frac{2r - r_T}{d} \right) \right] \cdot \left[ \Omega_{ eq} + a_2 \cos^2 \theta + a_4 \cos^4 \theta - \Omega_c \right], \quad (3.4)$$
where $\Omega_c$ is the core rotation, $\Omega_s$ the surface latitudinal differential rotation and $\Omega_{eq}$ the differential rotation at the equator. The radius, $r_T$ is set to $0.7R_\odot$, to correspond to a central radius of the tachocline, and the tachocline thickness, $d$, is set to $0.025R_\odot$. According to Dikpati & Charbonneau (1999) the rotation parameters can be expressed numerically as $\Omega_c/2\pi = 432.8$ nHz, $\Omega_{eq}/2\pi = 460.7$ nHz and the constants as $a_2 = -62.69$ nHz and $a_4 = -67.13$ nHz, to match helioseismic observations. These numerical values have been converted into the non-dimensional units used in the dynamo code, which is further discussed in Section 5.1.2. Finally, the expression $erf$, represents the error function to give a smooth transition throughout the convection zone and will also occur in future equations.

### 3.4 The $\alpha$-effect

#### 3.4.1 The $\alpha$-effect in general

In the previous chapter, it was explained how the poloidal field generates a toroidal field due to differential rotation. However, to keep an oscillating solar dynamo going, the toroidal field also has to regenerate a poloidal field to make the whole process start over again. This process is normally referred to as the $\alpha$-effect and if the $\Omega$-effect is relatively straight-forward and well known within solar dynamo research, the $\alpha$-effect is definitely open to a lot of questions still.
3.4. THE $\alpha$-EFFECT

Figure 3.3: The $\alpha$-effect can be described as a lifting and twisting movement of the toroidal flux ropes. When the flux ropes reach the surface of the Sun, they reconnect, and a poloidal magnetic field is regenerated. Image collected from Science@NASA’s website (2006-03-28).

Magnetic buoyancy causes the toroidal flux ropes to rise through the convection zone towards the surface. Due to cyclonic turbulence, caused by the Coriolis effect, a twisting effect acts upon the flux loops during the rise time. This lifting and twisting movement is illustrated by Figure 3.3 and can be compared to an $\alpha$-like shape of the flux ropes\(^1\). When the twisted flux ropes reach the surface they reconnect, and a new poloidal field, with the polarity opposite to the previous one, is generated. In other words, the dynamo process can start over again from the new dipole field. Where in the convection zone the $\alpha$-effect takes place though, is one of the main areas of solar dynamo research. Theories of $\alpha$-effects at various locations throughout the convection zone have been published, and some different ones, as well as the dynamo models connected to them, will be discussed in Section 4.1.

3.4.2 The $\alpha$-effect profiles

In the solar dynamo code used for this thesis, the two $\alpha$-effects are defined separately. For the $\alpha$-effect at the tachocline the following profile has been used:

$$\alpha_T = \frac{\alpha_T 0}{4} \sin \left[ 6(\theta - \frac{\pi}{2}) \right] e^{-\beta (\theta - \frac{\pi}{4})^2} \left[ 1 + \text{erf} \left( \frac{r - r_T}{d} \right) \right] \left[ 1 - \text{erf} \left( \frac{r - r_t}{d} \right) \right],$$

where $\alpha_T 0$ is the amplitude of the $\alpha$-effect at the tachocline, $r_T = 0.7R_\odot$, $r_t = 0.72R_\odot$, the thickness, $d = 0.0125R_\odot$ and the relative amplitude, $\beta$, is set to

\(^1\)The name $\alpha$-effect is not referring to the shape of the flux loops, but to the constant $\alpha$ in $\nabla \times (\alpha \mathbf{B})$, which was used for the first additional source term referring to this lifting and twisting effect.
50 in non-dimensional units. For the $\alpha$-effect at the surface, which also can be referred to as a Babcock-Leighton $\alpha$-effect, the profile can be expressed as:

$$\alpha_{BL} = \frac{\alpha_{BL0}}{2} \cos \theta \sin \theta \left[ 1 + \text{erf} \left( \frac{r - r_{surf}}{d} \right) \right]$$

$$\times \left[ 1 - \text{erf} \left( \frac{r - r_s}{d} \right) \right] \left[ 1 + e^{\gamma \left( \frac{\pi}{4} - \theta \right)} \right]^{-1}, \quad (3.6)$$

where $\alpha_{BL0}$ is the amplitude of the surface $\alpha$-effect, $r_{surf} = 0.96 R_\odot$, $r_s = 0.99 R_\odot$ and the relative amplitude, $\gamma$, is set to 20.

In Dikpati & Charbonneau (1999) the $\alpha$-effect has been expressed as an additional source term, $S$, in the poloidal component of the magnetic field.

### 3.5 Diffusivity

The diffusivity is assumed to be dominated by the diffusivity contribution due to turbulence, so that the diffusivity in the core of the Sun is considered to be very low, and the diffusivity in the convection zone is set to significantly larger magnitudes.

The diffusivity profile used in the dynamo code for this thesis is an extended version of the one discussed by Dikpati & Charbonneau (1999). Their original profile can be expressed as:

$$\eta(r) = \eta_c + \frac{\eta_T}{2} \left[ 1 + \text{erf} \left( \frac{r - r_T}{d_1} \right) \right], \quad (3.7)$$

where $\eta_c$ is the core diffusivity at 0.6$R_\odot$, $\eta_T$ is the bulk diffusivity in the convection zone, $r$ is the radius, $r_T = 0.7 R_\odot$ for the tachocline radius and $d_1 = 0.05 R_\odot$ for the thickness of the layer in question.

In the extended version, a surface diffusivity has been also included in the profile as of:

$$\eta(r) = \eta_c + \frac{\eta_T}{2} \left[ 1 + \text{erf} \left( \frac{r - r_T}{d_1} \right) \right] + \frac{\eta_{surf}}{2} \left[ 1 + \text{erf} \left( \frac{r - r_{surf}}{d_{surf}} \right) \right], \quad (3.8)$$

where $\eta_{surf}$ is the surface diffusivity, $r_{surf} = 0.96 R_\odot$, $d_T = 0.0125 R_\odot$ and $d_{surf} = 0.025 R_\odot$. For the simulations in this project, the diffusivity parameters $\eta$ have been set, and will not be varied throughout the simulations. The values of the different $\eta$ can be found in Table 5.1.

### 3.6 Meridional circulation

#### 3.6.1 Meridional circulation in general

One of the more recent discoveries, of highest importance considering the solar dynamo models, is the meridional circulation of the magnetic flux. This flux transport can be seen as a coupling between the poloidal field flux at the surface and the subsurface toroidal field flux, and can be illustrated as a circulating single cell in each hemisphere, as in Figure 3.4. (There are also some models
showing that the meridional flow should consist of more than one flow cell per hemisphere.) By including meridional flow in a solar dynamo, it is possible to place the $\alpha$-effect closer to the surface than for models without a meridional flow, since the meridional circulation will take care of the flux transport back to the tachocline and the location of the $\Omega$-effect. Furthermore, the meridional circulation is the most likely cause of the observed pole-ward migration of diffuse magnetic field on the surface of the Sun. (Choudhouri, Schüssler & Dikpati, 1995)

Similar to most of the other physical processes in the Sun, the meridional circulation is yet to be fully explored. A surface flow of approximately 20 ms$^{-1}$ has been observed, thanks to helioseismology and various other techniques. (Gilman & Miesch, 2004) However, the magnitude of the surface flow seems to fluctuate significantly and values between 10 ms$^{-1}$ and 25 ms$^{-1}$ are likely to match with observations. The surface meridional flow has a pole-ward direction at all latitudes in both hemispheres, so that an equator-ward flow, which transports the flux back towards the equator from the poles, is believed to be present, but yet to be observed. According to Dikpati, De Toma & Gilman (2004) the pole-ward flow occupies the top half of the convection zone and should therefore be present down to approximately 0.85$R_\odot$. The equator-ward flow is assumed to take place in the overshooting tachocline, which would give quite a thin layer of the equator-ward flow, compared to the pole-ward flow. Due to the significant difference in density within the convection zone, it is also most likely that the equator-ward flow is occupying a thin layer compared to the pole-ward flow. The theory that the equator-ward flow does not penetrate deeper than to the overshoot tachocline is based on theoretical assumptions and calculations. One example, that contradicts the theory of a deeper penetrating meridional flow, is the lithium-burning processes that take place for $r < 0.7R_\odot$. No observations show any sign of such elemental destruction, which would be present if the meridional circulation transported convection zone elements into deeper radiative layers of the Sun. (Gilman & Miesch, 2004)
3.6.2 The meridional flow profile

The meridional flow profile used in the dynamo model in question is the same as in Dikpati, De Toma & Gilman (2004) and is a modified and extended flow profile compared to the initial one used by Dikpati & Charbonneau (1999). The meridional flow profile can be expressed by the following equations:

\[
\begin{align*}
    u_r(r, \theta) &= u_0 \left( \frac{R_\odot}{r} \right) \left( - \frac{1}{m+1} + \frac{c_1}{2m+1} \xi_m - \frac{c_2}{2m+p+1} \xi^{m+p} \right) \\
    &\times \xi \sin^q \theta \left[ (q+2) \cos^2 \theta - \sin^2 \theta \right] \\
    &\times \left( - \frac{1}{m+1} + c_1 \xi_m - c_2 \xi^{m+p} \right) \sin^{q+1} \theta \cos \theta ,
\end{align*}
\]

and

\[
\begin{align*}
    u_\theta(r, \theta) &= u_0 \left( \frac{R_\odot}{r} \right)^3 \left( - \frac{1}{m+1} + c_1 \xi_m - c_2 \xi^{m+p} \right) \sin^{q+1} \theta \cos \theta ,
\end{align*}
\]

where \( u_r(r, \theta) \) represents the radial part of the meridional circulation and \( u_\theta(r, \theta) \) the angular part. Powers \( m, p, \) and \( q \) are all parameters ruling the characteristics of the meridional flow, \( m \) corresponds to an exponent in a convection zone density profile and has been set to a value of 1.5, while \( p \) is related to the relative amplitude of the surface and subsurface flows. Hence \( p \) sets the relative thickness of the two opposite flows. The parameter \( q \) controls the radial flow of rising flux at the equator and sinking flux at the poles. The initial velocity amplitude, \( u_0 \) has been set to 12 ms\(^{-1}\), but will be subject to variation throughout the project. The parameters \( c_1, c_2, \xi(r) \) and \( \xi_0 \) can be expressed as:

\[
\begin{align*}
    c_1 &= \frac{(2m+1)(m+p)}{(m+1)p} \xi_m^{-m} ,
\end{align*}
\]

\[
\begin{align*}
    c_2 &= \frac{(2m+p+1)m}{(m+1)p} \xi_m^{-m+p} ,
\end{align*}
\]

\[
\begin{align*}
    \xi(r) &= \frac{R_\odot}{r} - 1 ,
\end{align*}
\]

and

\[
\begin{align*}
    \xi_0 &= \frac{R_\odot}{r_0} - 1 .
\end{align*}
\]

The initial numerical values for the parameters defining the characteristics of the meridional flow have been provided together with the dynamo code by Dikpati. In Section 5.5 it is shown how varying meridional flow parameters affect the solar dynamo in general.

3.7 Dynamo waves

According to Dikpati & Charbonneau (1999) the dynamo code does not take any dynamo waves in consideration, but the dynamo wave number, \( k \), is used when converting all units into non-dimensional ones; see Section 5.1.2. The theory about dynamo waves has its origin from when the differential rotation of the Sun was not yet discovered. Instead, the approximation of the rotation of the Sun was illustrated by cylindrical shells. The rotation of the shells would
generate dynamo waves migrating through the convection zone to the surface of the Sun, where the waves are related to the magnetic activity of the Sun. According to Parker (1955) the convection zone could be considered a thin shell, only occupying approximately 15% of the Solar radius. Due to the very small thickness of the layer, the curvature could be ignored and cartesian coordinates could be used. Later on the dynamo wave theory was shown to hold also for spherical coordinates. These dynamo models, based on dynamo waves, were actually capable of producing solar like output, and the butterfly plots obtained corresponded very well to observations made. (Parker, 1955) However, when helioseismology was introduced, it was discovered that these waves were actually moving in the wrong direction compared to what was predicted by the scientists, and the model based on cylindrical shells had to be abandoned. Even though this early model based on dynamo waves did not hold, the dynamo wave solution was also used to explain the equator-ward drift of sunspots throughout the solar cycle in linear $\alpha \Omega$-dynamos (see Chapter 4 for further reading about solar dynamos). Later on, it was also applied on non-linear dynamo solutions. (Charbonneau, 2005)
Figure 3.5: Sunspot diagram for the 20\textsuperscript{th} century. The image to the right shows the area of the solar surface, with the number of sunspots as a function of time and latitude. The characteristic butterfly shape is clearly shown. The left image shows the solar activity measured in average visible sunspot area visible, numerically measured in percent of visible solar surface. The solar activity is plotted as a function of time and the labels on the horizontal axis correspond to the years of registration. Image collected from Science@NASA’s website (2006-03-28).
Chapter 4

Solar dynamo models

As mentioned in Chapter 1, the main aim with solar dynamo models is to simulate the magnetic field of the Sun, both in order to match output with solar observations and to predict future solar cycles. In general, all solar dynamo models include a solar structural model, a differential rotation profile and a diffusivity profile. Since the meridional circulation is basically confirmed by helioseismology, most models also include a meridional circulation profile. Some models based on the physical processes in the Sun are mean field dynamos, which means that they are based on a generation of a mean toroidal current due to the twist of the toroidal magnetic field, in other words a small scale $\alpha$-effect. There are also models focusing on a more large scale structure considering the $\alpha$-effect, and the common name for those dynamos is $\alpha\Omega$-dynamos. The $\alpha\Omega$-dynamos try to model the two main magnetic processes and produce solar like output. They differ somewhat though, especially regarding where the shear profiles and $\alpha$-effects are located. (Charbonneau, 2005) Most dynamos are kinematic models due to computational limits. In other words, the models cannot take all physical processes and their mutual interactions into consideration, and are therefore not dynamic models.

4.1 Location of the $\alpha$-effect

Within $\alpha\Omega$-dynamo theory there are a few main ideas about where the $\alpha$-effect is located. In convection zone dynamos the $\alpha$-effect is assumed to be located in the bulk of the convection zone and acting throughout the whole zone. In thin-layer dynamos the $\alpha$-effect is located at the bottom of the convection zone together with the $\Omega$-effect. In interface dynamos the two effects are separated slightly but still located near the bottom of the convection zone, so that the $\alpha$-effect is assumed to be located at a slightly larger solar radius than the $\Omega$-effect. The Babcock-Leighton dynamos focus on an $\alpha$-effect located close to the surface of the Sun.

Magnetic buoyancy has been used also as an argument considering the location of the solar dynamo. As of these theories, if the solar dynamo was located in the bulk of the convection zone, the magnetic field would never become strong enough to generate sunspots, before it rises to the surface due to magnetic buoyancy. Of course, there are also combinations of these different theories of
α-effect locations and the code used for the main part of this thesis is based on both a Babcock-Leighton α-effect and an α-effect located at the overshoot tachocline. The model is called a Babcock-Leighton flux transport dynamo by Dikpati & Charbonneau (1999).

4.2 Babcock-Leighton flux transport dynamo

After first being discussed some 40 years ago, the Babcock-Leighton dynamos were set aside in favour of mean field dynamos until about 20 years ago. After observations showing that the regeneration of the poloidal field might be connected to a decay of active regions on the solar surface, the Babcock-Leighton theory was once again considered as one of the more likely dynamo models.

Previous simulations show that the Babcock-Leighton surface α-effect itself cannot produce a self-excited solar dynamo, which supports the theory that another additional α-effect is required in the model to match the grand minima theories. With a surface α-effect only, the solar dynamo would not have been able to revive after the Maunder minima, for example, while a tachocline α-effect could have fulfilled that aspect. This is one argument supporting that Dikpati & Charbonneau (1999) and Dikpati & Gilman (2001) use a combination of the two α-effects. For further reading about flux transport dynamos, Dikpati (2005) is recommended as an excellent overview.

4.3 Low-order dynamo models

In general, theory is lagging observations at present. Thanks to the evolution within observational techniques, including both satellites and earth-based devices, the theory behind the observed structure and dynamics is yet to catch up with the observations. Therefore, simple models and illustrations can sometimes be useful tools to increase the general understanding and exchange ideas within the field. Examples are low-order models. While the aim for the more complex models is to consider a full set of magnetohydrodynamic equations, believed to govern the dynamo, the simpler models are heavily generalized and use simple relations and approximations to produce a solar like output. Most of the low-order models are based on theories that the Sun might be subject to a chaotic system, and not a periodic one, as most of the solar dynamo models assume. Due to the fact that grand minima and maxima have been observed, the solar cycle could possibly match with characteristics of a chaotic system. In general, the solutions to a chaotic dynamical system are characterised by a periodic solution, periods of instability and the onset of chaos.

However, the present sunspot record of approximately 300 years, is not long enough to conclude that the solar dynamo system chaotic (Tobías, Weiss & Kirk, 1995). The solar dynamo could also match with a stochastic dynamo model, as well as with a deterministic chaotic one, as assumed in this case. Regarding a possible grand minima cycle, the $^{14}$C records of approximately 9000 years are not sufficient to set the characteristics of the system. See Section 3.1.2 for further reading about grand minima.

The low-order dynamo used by Wilmot-Smith et al. (2005) is based on a non-linear dynamical system, which has been derived from the system used in
4.3. LOW-ORDER DYNAMO MODELS

It is assumed that a bifurcation structure might be present in stellar dynamo systems, including the system of the solar dynamo. Wilmot-Smith et al. (2005) show that the non-linear system produces output varying from being constant to periodic and eventually chaotic. The output has been possible to match with the observed aperiodic solar cycle. The low-order model according to Wilmot-Smith et al. (2005) can be set up as three non-linear equations:

\[
\begin{align*}
\dot{x} &= \lambda x - \omega y + azx + d(x^3 - 3xy^2) , \\
\dot{y} &= \lambda y + \omega x + azy + d(3x^2y - y^3) , \\
\dot{z} &= \mu - z^2 - (x^2 + y^2) + cz^3 ,
\end{align*}
\]

where \(x\) represents the toroidal component of the magnetic field, \(y\) the poloidal component and \(z\) the hydrodynamics in the model. Compared to the dynamo model, which has been used for the main part of this thesis, this way of defining the dynamo system is quite generalized. Since \(z\) is supposed to represent all the hydrodynamics of the dynamo, it covers, for example, the differential rotation and other MHD parameters mentioned earlier, and therefore also a major part of Dikpati’s dynamo model. \(\lambda\) gives the growth rate and \(\omega\) the basic linear frequency of the system, i.e., \(\omega\) controls the period of the system, \(\mu\) controls the hydrodynamics and \(d\) makes the system three-dimensional, while \(a\) and \(c\) have no physical interpretation. \(x^2\) can be seen as representing the total activity of the magnetic field. The parameters \(a, c, d,\) and \(\omega\) have been set to \(a = 3, c = -0.4, d = 0.4\) and \(\omega = 10.25\).

For most of the simulations, Wilmot-Smith chooses a one-parameter path in the \(\lambda - \mu\) plane, along which the model is studied. The path has been set both considering bifurcation points in the system and to show the stellar behaviour as a function of the rotation rate. The parametrisation used in Wilmot-Smith et al. (2005) can be expressed as:

\[
\mu = \sqrt{\Omega}
\]

and

\[
\lambda = \frac{1}{4} \left\{ \left[ \ln (\Omega) + \frac{1}{3} \right] \exp \left( - \frac{1}{100} \Omega \right) \right\} ,
\]

where \(\Omega\) is the effect of the rotation of the system. This causes an increase of the radius of the periodic orbit when \(\Omega\) is increased. Hence, the amplitude of the magnetic field grows when \(\Omega\) is increased. All images referring to images from Wilmot-Smith et al. (2005) below have been kindly provided by Wilmot-Smith at the Institute of Mathematics at the University of St Andrews, United Kingdom.

4.3.1 Low-order simulations

In Wilmot-Smith et al. (2005) the Runge-Kutta Fehlberg method in Maple was used for the simulations. In Tobias, Weiss & Kirk (1995) the same numerical method was used in Matlab. For comparison, the numerical methods, accuracy characteristics and step-lengths used to produce the figures in this chapter, have been varied to investigate if that might cause any differences in output. Quite
naturally, that should not make any major differences when solving a non-linear system. However, due to the characteristics of a chaotic system, the slightest change or variation along the line of numerical calculations, including factors such as accuracy and methods, could cause significant differences in the output obtained. For example, a simulation with an accuracy not high enough, might produce a chaotic-like output, while the same system might seem periodic when using the appropriate accuracy and vice versa.

In this work Mathematica was used to simulate the same low-order model as in Wilmot-Smith et al. (2005). The numerical method has not been specified for the different simulations, but has been relied on the powerful tool of the software.

The output was not always identical to that of Wilmot-Smith et al. (2005), even if the same parameter values were used. As will be shown, some outputs were similar to the ones in Wilmot-Smith et al. (2005), while others did not show the same characteristics at all. However, this could to some extent be adjusted by changing some of the input parameters, such as the rotation parameter $\Omega$. It should be mentioned that the exact initial values of $x$, $y$ and $z$ were not known for the simulations. However, based on random trials with varying initial conditions this seemed to be of minor importance for the output in general.

### 4.3.2 Low-order models and solar characteristics

One of the more illustrative ways to show how these low-order models can be related to the solar cycle would be to plot the toroidal field, $x$, as a function of time. For a chosen time interval, both periodic and chaotic behaviour can be observed. This is shown in Figure 4.1 for the parametrized path, close to a frequency locked region. Since low-order models simulate the common behaviour of the star, a specific time scale is irrelevant and the magnitude of time will be referred to a time unit. The periodic parts in this case, for example for $280 \lesssim t \lesssim 360$ time units, would represent the general solar cycle, while the chaotic parts (for example at $t \approx 400$ time units) could possibly represent grand minima or maxima for example.

When trying to reproduce these plots some very interesting outputs were obtained. For exactly the same set-up as in Wilmot-Smith et al. (2005), a totally periodic solution was obtained (see Figure 4.2a). By varying the rotation parameter, a similar plot to Figure 4.1 was obtained for $\Omega = 6$, which is shown in Figure 4.2b.

Furthermore, for these plots, also the accuracy of the numerical method seems to play a role for the output, which is worrying. See Figure 4.3a for an example with an increased accuracy. This could show that the chaotic and quasi-periodic behaviour, might not be due to the system, but to the numerical method and its settings, and could possibly be applicable for most simulations for this system. For the increased accuracy, the quasi-periodic behaviour within the time interval seems to disappear, and the toroidal field seems to have non-periodic characteristics. However, if studying a longer period of time, the toroidal field seems to stabilize in a periodic behaviour for $t > 650$ time units, which is illustrated in Figure 4.3(b).
4.3. LOW-ORDER DYNAMO MODELS

4.3.3 Transition to chaos

The transition to chaos normally occurs in four different ways for a chaotic systems. It could either be through a *subcritical instability*, a *sequence of bifurcations*, *period doubling* or through *intermittent transition*. (Drazin, 1992) In this case the transition is based on a set of Hopf bifurcations (Wilmot-Smith et al., 2005; Tobias, Weiss & Kirk, 1995). The transition to chaos for the low-order dynamo can be illustrated by Poincaré sections through the $y = 0$ plane. As $\Omega$ is increased the initially smooth torus-like solution slowly starts to show wrinkles before a transition to chaos. However, the transition did not take place for the same set-up values for the reproduced simulations as for the ones in Wilmot-Smith et al. (2005). For the same values of the rotation parameter, $\Omega$, the solution stayed non-chaotic, and no transition took place. However, by increasing $\Omega$ further, a somewhat similar transition could be obtained. An increase of $\omega$ seemed to give more stable solutions, while a decreased $\omega$ produced chaotic-like behavior for lower values of $\Omega$. The plots from the reproduced solution can be seen in Figure 4.4.

4.3.4 Low-order models versus numerical models

The low-order dynamos are useful mainly for understanding the general structure of the solar dynamo, and solar like star dynamos, and their stellar behavior. Even with such simple mathematical models as the low-order dynamos, output matching stellar cycles can be produced. Also grand minima have been represented in those simulations, in a pattern that could correspond to the Sun. It should also be noted that these low-order models could not be of any use for predicting the solar dynamo, but more for studying the common properties of the system.

Using the most advanced numerical models to simulate the magnetic field of the Sun is still not possible due to computational limits. Therefore, there are also
dynamo models based on the physical processes, but where some parameters and processes have been set to constants or have been simplified to make numerical simulations possible. The flux transport dynamo used for this thesis, is one example of such a model.

4.4 Characteristics required for a realistic solar dynamo

As a summary of Chapter 3 and closure of this chapter, the main characteristics of a solar dynamo model, to correspond to solar observations, are:

- A correct solar cycle period time of approximately 11 years between polarity shifts, which gives a total magnetic cycle of approximately 22 years.
- A butterfly like output at lower latitudes, which can be matched with butterfly diagrams, when the subsurface toroidal field is plotted as a function of time and latitude.
- A magnetic field strength of the subsurface toroidal field within the range of approximately 60-160 kG.
- A surface poloidal field with extreme value areas located at the higher latitudes.
- A magnetic field strength of the surface poloidal field of approximately 10 G.
- A correct shift of polarity between the poloidal and toroidal fields, i.e., the onset of one of them occurs when the other one has a maximum.
4.4. CHARACTERISTICS REQUIRED FOR A REALISTIC SOLAR DYNAMO

(a) The toroidal field, $x$, as a function of time, $t$, reproduced with the same parameter values as of Wilmut-Smith et al. (2005), $\Omega = 3.35$. No chaotic behaviour can be seen, but a periodic solution.

(b) The toroidal field, $x$, as a function of time, $t$, reproduced with $\Omega = 6.0$. The solution shows a periodic solution for $170 \lesssim t \lesssim 340$ time units and chaotic for the rest of the displayed time range.

Figure 4.2: A comparison of output of the toroidal field, $x$, as a function of time $t$, for varying $\Omega$ values and accuracy. Another two plots are shown in Figure 4.3. The time is expressed in time units.
(a) The toroidal field, $x$, as a function of time, $t$, reproduced with $\Omega = 6.0$ and with an increased accuracy. When the accuracy is increased, the solution seems to behave chaotically for the whole simulated time. The differences from Figure 4.2b are obvious.

(b) The toroidal field, $x$, as a function of time, $t$, reproduced with $\Omega = 6.0$ and with an increased accuracy, simulated for a longer period of time. If the simulation is left for a longer period of time, the solution turns periodic at $t \approx 650$ time units.

Figure 4.3: A comparison of output of the toroidal field, $x$, as a function of time, $t$, for varying $\Omega$ values and accuracy. Another two plots are shown in Figure 4.2. The time is expressed in time units.
4.4. Characteristics Required for a Realistic Solar Dynamo

Figure 4.4: Poincaré sections through $y = 0$. The plots show the corresponding transition to chaos as $\Omega$ is increased, for simulations made with Mathematica. Different values of $\Omega$ than in Wilmot-Smith et al. (2005) have been used, and the transition can be seen as the initially smooth cut starts to wrinkle to finally give a chaotic solution. The horizontal axis represents the toroidal component, $x$, of the magnetic field and the vertical axis represents the hydrodynamic parameter, $z$. 

(a) A smooth solution for $\Omega = 2.0$ as of the reproduced set-up.

(b) The transition has started for $\Omega = 2.5$ as of the reproduced set-up.

(c) The transition has moved even closer to the onset of chaos for $\Omega = 2.6$ as of the reproduced set-up.
Chapter 5

Half-sphere solution

The solar dynamo model code provided by Dikpati is a Babcock-Leighton flux transport dynamo code, which includes both the differential rotation of the Sun and meridional circulation. The code is simulating one hemisphere of the Sun and will therefore from here on be referred to as a half-sphere solution. According to Dikpati & Charbonneau (1999) and Dikpati & Gilman (2001) it is likely that both a Babcock-Leighton surface $\alpha$-effect and a tachocline $\alpha$-effect exist. Their simulations show that the tachocline $\alpha$-effect is likely to have more influence on the dynamo than the surface $\alpha$-effect, since the surface effect seems to choose the wrong parity of the magnetic field. The tachocline $\alpha$-effect, on the other hand, chooses the odd parity that corresponds to the observed one.

5.1 The code

During the time evolution in the code, the radius, $r$, and the polar angle, $\theta$, are treated implicitly and explicitly in the first half of the time step. In the second half of the time step, the parameters treated implicitly/explicitly are swapped. The linear equations, set up for the different parameters in the code, are set in a tridiagonal system. Hence only the diagonal elements of the matrix (and elements in plus/minus one column from the diagonal) are non-zero elements. Some additional subroutines were used, mainly for calculating the upper boundary conditions, i.e., where $r = R_\odot$ and one for calculating the Legendre polynomials, which are used by the two other subroutines. Some of the subroutines used in the code can be found in Press et al. (1992).

The initial values of the magnetic fields used were provided by Dikpati and have originally been obtained from a converged solution (Dikpati, private communication).

5.1.1 Boundary conditions

Except for the boundary conditions at $r = R_\odot$, which are calculated for a smooth transition to outer layers, both the poloidal and toroidal fields are assumed to be equal to zero at the bottom boundary, $r = 0.7R_\odot$. In other words, no field lines penetrate the radiative core of the Sun. At the pole, the poloidal and the toroidal fields are both set to zero. For the half-sphere solution the toroidal field
is set to zero at the equator and the poloidal field is estimated to have the same value as at the first time step from the equator.

5.1.2 Conversion to non-dimensional units

The conversion from dimensional units to non-dimensional units is based on the dynamo wave number, $k$, (see Section 3.7 for dynamo waves) which has a value of $9.2 \cdot 10^{-11}$ cm$^{-1}$. For the simulations, the value of $k$ is set to 1, so that the non-dimensional length unit, $1/k$, becomes $1.09 \cdot 10^{10}$ cm. Thus, the solar radius, $R_\odot = 6.96 \cdot 10^{10}$ cm can be set to 6.39 in the non-dimensional units.

For the output values of the magnetic field, the unit energy is set to approximately $0.5 \cdot 10^8$ erg in CGS units (equal to $0.5 \cdot 10^{-3}$ J in SI units). The unit magnetic energy can be expressed as $\frac{B^2}{8\pi}$, so that the dimensional magnetic field, $B$, is approximately 400 G. Thus, if the output value is multiplied by 0.4, the field strength will be obtained in kG. (Dikpati, private communication) However, to express the magnetic field values in the SI unit T, the obtained field values (in G) must be multiplied by $1 \cdot 10^{-4}$.

5.1.3 Changes made to the half-sphere code

Initially, there were some efforts to make the dynamo code work properly. Most likely, this was due to using compilers and systems different from what Dikpati previously had been using. The changes made to the half-sphere code were mainly of a rearranging character. For example, moving some general commands to the very beginning or to the very end of the code, instead of having them at various locations throughout the code. Also other adjustments were made to make the code slightly easier to read, and therefore also easier to improve or adjust at later stages. At a couple of places, the code was performing unnecessary extensive calculations, which had been left in the code since previous modifications by Dikpati. These were inactivated for the simulations made.

However, the change which made the larger difference regarding using the code throughout the project, was probably to remove all the hard-wired parameters from the code and use an input file, which is read by the programme instead. In the original version, the actual code would have to be modified and compiled for every parameter change made. By creating an input file consisting of all parameters that might be subject to change at any time, and calling that file from the programme, the parameters could easily be modified without recompiling the actual code.

5.2 The tools to analyse the output

To analyse the output obtained from the simulations, both butterfly plots and animations were used. Plotting the magnetic fields in a kind of butterfly diagram is a very efficient way of comparing the output to solar observations, while animations could be used to illustrate the magnetic field structure throughout the convection zone.
5.2. THE TOOLS TO ANALYSE THE OUTPUT

5.2.1 Butterfly plots

An Interactive Data Language (from here on referred to as IDL) programme for producing butterfly plots, was also provided by Dikpati. Just like regarding the dynamo code, some minor changes had to be made to make the code work according to the systems used and requirements. Once again, a lot of parameters were hard-wired into the code, which instead were changed to input variables.

The butterfly plots illustrate the radial poloidal field strength at the surface of the Sun as shades and the toroidal field strength at the overshoot tachocline as contour lines. The range of the shades covers field strengths of both negative and positive values, and they are clarified by vertical colour bars in the butterfly plot images. The colour bars show the corresponding values of the poloidal field strength in non-dimensional units. To clarify, the values of the colour bars therefore need to be multiplied by 0.4 to obtain the field strength in kG. For the toroidal field strength the change of polarity is instead illustrated with solid or dashed contour lines. The dashed ones correspond to negative field values and the solid lines to positive ones. The contour lines can be compared to topographic contours, in the sense that the field strength increases/decreases for each contour, and the extreme value areas of the toroidal field can be found inside the innermost contours in the circular patterns. Also for the toroidal field output, the values need to be multiplied by 0.4 to obtain the field strength in kG. The horizontal axes in the butterfly plots correspond to the simulated time and the vertical axes to the solar latitude in degrees. For the butterfly plots, four different output files were used; time, latitude, poloidal field strength at the surface and toroidal field strength at the tachocline respectively.

According to Charbonneau, St-Jean & Zacharias (2005) the order of growth rates for the toroidal field, to rise from the subsurface layer to the surface producing sunspots, is only a few months. Thus, the difference in radial location for the two fields plotted are of minor importance.

5.2.2 Animations

Since the butterfly images show only field strength on the surface of the Sun and at the tachocline, for the poloidal and toroidal fields, another IDL programme was written to animate the magnetic field throughout the whole convection zone. The programme plots the field lines as a function of the solar radius and latitude, and animate over the simulated time. For the animations a combined output file containing radius, latitude, toroidal field and poloidal field values was used, together with an appropriate time vector. The time steps chosen in the animations correspond to the time steps used for the butterfly plots. The horizontal axis in the animations correspond to the solar radius, and the vertical axis can be seen as approximations of the solar latitude, if a horistontal line is estimated from the axis to the surface of the Sun, \( r=1R_\odot \). Of course, the vertical axis can also be seen as the solar radius, with \( r=0R_\odot \) at the origin and \( r=1R_\odot \) at the top of the vertical axis. If nothing else is noted, the poloidal field is represented by dashed, red or brighter lines and the toroidal field by solid, black lines in the images of the animations. Thanks to the animations it is possible to study the interaction between the poloidal and toroidal field in a way completely different from that through the butterfly plots.
CHAPTER 5. HALF-SPHERE SOLUTION

Plotting the field lines for the animations

For the toroidal field, the contours of the field lines could be plotted. This illustrates a 'cut' through the field lines. The field lines are perpendicular to the plane of the plot. Regarding plotting the poloidal field lines, this could not be performed in such a straight-forward way as for the toroidal field lines. Since the output produced by the code is the vector potential, \( A \), of the poloidal field, some kind of conversion was needed to obtain a vector corresponding to the poloidal field. In general, the vector potential does not have to be constant on a field line, however, it can be, so that it was worth looking into. If the vector potential turned out to be constant on the field lines in question, the vector potential would correspond to the field lines of the poloidal field and could be used for the animations. The poloidal field can be expressed as:

\[
B_p = \nabla \times [A(r, \theta) \hat{e}_\phi] . \tag{5.1}
\]

If \( B_p \cdot \nabla A = 0 \), plotting the contour of \( A \) would correspond to the magnetic field lines. Also, \( B_p \cdot \nabla \) corresponds to the derivative along the field line, so that if \( A \) is constant, the derivative is equal to 0. For spherical coordinates:

\[
\nabla A = \frac{\partial A}{\partial r} \hat{e}_r + \frac{1}{r} \frac{\partial A}{\partial \theta} \hat{e}_\theta + \frac{1}{r \sin \theta} \frac{\partial A}{\partial \phi} \hat{e}_\phi . \tag{5.2}
\]

Equations 5.1 and 5.2 give:

\[
B_p = \nabla \times A \hat{e}_\phi = \frac{1}{r \sin \theta} \left[ \frac{\partial (A \sin \theta)}{\partial \theta} - \frac{\partial A}{\partial \phi} \right] \hat{e}_r + \frac{1}{r \sin \theta} \left[ \frac{\partial A}{\partial \phi} - \sin \theta \frac{\partial (rA)}{\partial r} \right] \hat{e}_\theta + \frac{1}{r} \left[ \frac{\partial (rA)}{\partial r} - \frac{\partial A}{\partial \theta} \right] \hat{e}_\phi = 0 \text{ due to symmetry}.
\]

\[
= \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} A \sin \theta \hat{e}_r - \frac{1}{r} \frac{\partial}{\partial r} (rA) \hat{e}_\theta = B_p . \tag{5.3}
\]

In other words:

\[
\nabla A = \frac{\partial A}{\partial r} \hat{e}_r + \frac{1}{r} \frac{\partial A}{\partial \theta} \hat{e}_\theta
\]

and since there is no \( \hat{e}_\phi \), \( B_p \nabla A \) can be expressed as:
5.3 A REFERENCE SOLUTION

\[ B_p \nabla A = B_p \hat{e}_r \cdot \nabla A \hat{e}_r + B_p \hat{e}_\theta \cdot \nabla A \hat{e}_\theta = \]

\[ = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} [A \sin \theta] \frac{\partial A}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} (rA) \cdot \frac{1}{r} \frac{\partial A}{\partial \theta} = \]

\[ = \frac{1}{r \sin \theta} \frac{\partial A}{\partial r} \frac{\partial}{\partial \theta} [A \sin \theta] = \frac{1}{r} \frac{\partial}{\partial r} (rA) \cdot \frac{\partial A}{\partial \theta} = \]

\[ = \frac{1}{r \sin \theta} \frac{\partial A}{\partial r} \left[ \sin \theta \frac{\partial A}{\partial \theta} + \cos \theta \frac{\partial A}{\partial \theta} \right] = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r \frac{\partial A}{\partial r} + A \right) = \]

\[ = \frac{\cos \theta}{r \sin \theta} \frac{\partial A}{\partial r} - \frac{1}{r^2} \frac{\partial A}{\partial \theta} \neq 0 . \quad (5.5) \]

In other words, a contour of \( A \) only could not be used to plot the poloidal field lines. Instead, finding a product of \( A \) that is constant on the field lines could be the solution. Therefore, once again the poloidal field was set to:

\[ B_p = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} A \sin \theta \hat{e}_r = \frac{1}{r} \frac{\partial}{\partial r} (rA) \hat{e}_\theta . \quad (5.6) \]

To find a product of \( A \), a \( \psi \) was set, where \( \nabla \psi = \frac{\partial \psi}{\partial r} \hat{e}_r + \frac{1}{r} \frac{\partial \psi}{\partial \theta} \hat{e}_\theta \) and \( B_p \cdot \nabla \psi = 0 \) is wanted. \( \psi \) was chosen as \( \psi = rA \sin \theta \) which gives:

\[ \nabla \psi = \frac{\partial rA}{\partial r} \sin \theta \hat{e}_r + \frac{r}{\theta} \frac{\partial (rA \sin \theta)}{\partial \theta} \hat{e}_\theta \Rightarrow \]

\[ \Rightarrow B_p \cdot \nabla \psi = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left[ \sin \theta \right] \frac{\partial (rA)}{\partial r} \sin \theta - \frac{1}{r} \frac{\partial}{\partial r} (rA) \frac{\partial (A \sin \theta)}{\partial \theta} = \]

\[ = \frac{1}{r} \left[ A \cos \theta + \sin \theta \frac{\partial A}{\partial \theta} \right] \left( A + \frac{\partial A}{\partial r} \right) - \frac{1}{r} \left( \frac{\partial A}{\partial r} \right) \left[ A \cos \theta + \sin \theta \frac{\partial A}{\partial \theta} \right] = 0 . \quad (5.7) \]

Hence, it could be concluded that a contour of \( rA \sin \theta \) would give a plot corresponding to the poloidal field lines.

5.3 A reference solution

For the reference parameters, which were provided by Dikpati together with the dynamo code, the model produces quite a nice butterfly-like plot, shown in Figure 5.1. The set-up of reference parameters can be found in Table 5.1, and further information about the parameters and equations in Chapter 3.
CHAPTER 5. HALF-SPHERE SOLUTION

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value [SI units]</th>
<th>Value [non-dim. units]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meridional flow, surface velocity, $u_0$</td>
<td>12 m/s</td>
<td>24.0</td>
</tr>
<tr>
<td>Mer. flow, radial dependence, $p$</td>
<td></td>
<td>1.0</td>
</tr>
<tr>
<td>Mer. flow, latitudinal dependence, $q$</td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td>Mer. flow, density profile exponent, $m$</td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td>Diffusivity in convection zone, $\eta_T$</td>
<td>$0.5 \cdot 10^{12}$ cm$^2$/s</td>
<td>0.05</td>
</tr>
<tr>
<td>Diffusivity near surface, $\eta_{surf}$</td>
<td>$20 \cdot 10^{12}$ cm$^2$/s</td>
<td>2.0</td>
</tr>
<tr>
<td>Diffusivity in core, $\eta_c$</td>
<td>$5 \cdot 10^9$ cm$^2$/s</td>
<td>0.0005</td>
</tr>
<tr>
<td>Amplitude of tachocline $\alpha$-effect, $\alpha_T$</td>
<td>0.2 m/s</td>
<td>0.2</td>
</tr>
<tr>
<td>Amplitude of surface $\alpha$-effect, $\alpha_{surf}$</td>
<td>2.0 m/s</td>
<td>2.0</td>
</tr>
<tr>
<td>Differential rotation, in core, $\Omega_c$</td>
<td>$\sim 432$ nHz</td>
<td>300.0</td>
</tr>
<tr>
<td>Differential rotation, at equator, $\Omega_{eq}$</td>
<td>$\sim 460$ nHz</td>
<td>319.9</td>
</tr>
<tr>
<td>Radius for bottom boundary, $r_c$</td>
<td>$0.6R_\odot$</td>
<td>3.83</td>
</tr>
<tr>
<td>Radius at centre of tachocline, $r_T$</td>
<td>$0.7R_\odot$</td>
<td>4.47</td>
</tr>
<tr>
<td>Radius at surface, $r_{surf}$</td>
<td>$1.0R_\odot$</td>
<td>6.39</td>
</tr>
</tbody>
</table>

Table 5.1: Parameter values for the reference solution.

5.3.1 Butterfly plot analysis

As of the butterfly plot in Figure 5.1, the solar cycle period can be estimated to about 10 years and the field strength variations for the two plotted fields seem to correspond well with each other. The poloidal field, which is illustrated by shades, reaches its maxima on high latitudes, coinciding with the time of onset for the toroidal field polarity shifts. The toroidal field has its maxima on latitudes of approximately $55^\circ$, which can be considered a bit high compared to the latitudes where sunspots have been observed (see Section 3.1).

There is one main aspect of the reference solution though, which does not match the required solar observations as of Section 4.4. It is the poloidal field strength. Extreme values are set to approximately $\pm 2.6$ kG for this set-up of parameters, which is significantly higher than the observed values of 10 G.

5.3.2 Animation analysis

When studying the animations for the reference solution, a clear pattern for the toroidal field can be observed. A new toroidal field is generated on a latitude of approximately $60^\circ$ just below the surface. The field is then moving inwards until it reaches the overshoot tachocline, where the meridional flow causes the transportation of the field toward the equator. Once at a lower latitude, the field is stretching in a radial direction, and gets closer to the surface again where sunspots are produced. At this point of time, the generation of a toroidal field with opposite polarity can be observed already at higher latitudes, which is
5.4 VARYING THE $\alpha$-PARAMETERS

Figure 5.1: Butterfly plot for the reference case. The radial poloidal field on the solar surface is represented by shades and the subsurface toroidal field by contours. The simulated time in years is represented on the horizontal axis and the solar latitude in degrees on the vertical axis. The field strength values in the plot and represented by the colourbar, should be multiplied by 0.4 to obtain the field strengths in kG. The poloidal field maximum values are approximately $\pm 2.6$ kG and the toroidal field maximum values approximately $\pm 67$ kG.

shown in Figure 5.2.

The maximum field strength of the toroidal field can be observed at a radius of about $0.75R_\odot$ and at latitudes of approximately 55-60°. An increase of the poloidal field can be observed just after the toroidal field has started to move radially outwards close to the equator, which corresponds well with polarity shift observations. After the onset of the new toroidal field on higher latitudes, the poloidal field decreases as expected. A set of images for both the poloidal and toroidal fields are shown in Figure 5.10. However, these are not from the animation of the reference case, but the same pattern as mentioned above can be observed in general.

5.4 Varying the $\alpha$-parameters

Initially, the main focus for the project was set on changing the surface $\alpha$-effect, $\alpha_{\text{surf}}$, to see what effect that would have on the dynamo model. According to previous studies (Dikpati & Gilman, 2001; Mason, Hughes & Tobias, 2002) the tachocline $\alpha$-effect, $\alpha_T$, is to be the more important one within the solar dynamo. However, the surface $\alpha$-effect cannot be ruled out, and it is likely that both $\alpha$-effects are required to produce a solar like output. For example, the surface based Babcock-Leighton dynamo is required to reproduce the poloidal field at the surface.
Figure 5.2: Image from an animation of the reference solution. As a toroidal field of one polarity is decreasing at lower latitudes (dark, dashed contours), the onset of a field with the opposite polarity can be observed on higher latitudes (dark, solid contours). The dark contours represent a cut through the toroidal field, i.e., the toroidal field lines are perpendicular to the plane of the image. The brighter, dashed contours correspond to the poloidal field. The horizontal and vertical axes both correspond to the solar radius and the origin represents the solar centre. The semi-circles in the image represent the solar surface and the radii of $0.7R_\odot$ and $0.6R_\odot$.

5.4.1 Butterfly plot analysis

For increasing values of the surface $\alpha$-effect, the extreme values for both the toroidal and the poloidal fields increase, and when the surface $\alpha$-parameter is increased to a value larger than a factor 5 of the reference value, the toroidal field values start to exceed, or at least get close to the maximum field strength levels of approximately 160 kG. The cycle period and patterns in general remain the same for an increasing surface $\alpha$-effect. Some variation in the poloidal field pattern at the surface of the Sun can be observed though. The areas with the extreme values for the different cases, seem to get smaller for an increasing $\alpha$-effect. These areas also move closer to the equator for an increasing surface $\alpha$-parameter. For the reference case, the extreme values of the poloidal field can be found at latitudes of approximately 80° (see Figure 5.1), while for a surface $\alpha$-parameter of a factor 5 higher than the reference case, the extreme areas can be found at approximately 20-40° as seen in Figure 5.3. The shape of these areas have also gone from being quite large and being present for most part of a cycle, to become more short-lived with more distinct edges. It can be noticed also that for the reference case, the extreme values of the poloidal field overlap and only slightly trail the extreme toroidal field areas, while for an increasing surface $\alpha$-effect the poloidal extreme values trail the toroidal field a bit more.
5.4. VARYING THE $\alpha$-PARAMETERS

Figure 5.3: Butterfly plot for a dynamo case with the surface $\alpha$-effect increased with a factor five compared to the reference case. Field values have changed and also the shape of the extreme value areas of the poloidal field. The poloidal field maximum values are approximately $\pm 10$ kG and the toroidal field maximum values approximately $+180/-236$ kG. The same notation and scaling are used as for previous butterfly plots.

However, the locations of the extreme values of the poloidal field still coincides with the onset of a new toroidal field, which is according to the required dynamo characteristics.

If the surface $\alpha$-effect is totally removed from the model, no butterfly pattern at all will be produced, and the magnetic oscillations never take place; see Figure 5.4. However, removing the tachocline $\alpha$-effect does not seem to change the output for the reference solution at all, which is shown in Figure 5.5.

That the dynamo fails to continue, when the surface $\alpha$-effect is removed, does not have to be of major importance. It basically proves only that this model requires a Babcock-Leighton dynamo to be able to keep a solar like oscillation. According to some theories regarding grand minima the dynamo is still active during the minima (see Section 3.1.2), even though hardly any sunspots are produced. This could possibly be explained by a tachocline dynamo that is active throughout the minimum, without causing toroidal fields strong enough to exceed the threshold limit for magnetic buoyancy. When the surface dynamo is reactivated, the field strength increases and sunspots can once again be observed on the surface. For such a case a totally correct solar dynamo model would still show a toroidal field at the overshoot tachocline though, but with insufficient field strength to produce sunspots, when the surface $\alpha$-effect is turned off.

The fact that drastic changes of the tachocline $\alpha$-effect do not seem to affect the output at all is a bit confusing at first. According to Dikpati (private communication) a slow-down of the solar cycle should be possible to observe for distinct increases of the tachocline $\alpha$-effect (approaching $\frac{\alpha_T}{\alpha_{surf}} \sim 1$).
CHAPTER 5. HALF-SPHERE SOLUTION

Figure 5.4: Butterfly plot for a dynamo case where the surface $\alpha$-effect is removed. The oscillations will not proceed and no butterfly pattern at all will be produced. The notation and scaling are the same as for the previous butterfly plots.

This is due to the fact that the tachocline $\alpha$-effect causes a pole-ward flow component of the magnetic fields at the tachocline. This will counteract with the meridional flow, and therefore slow the cycle down. Further reading about how the meridional circulation affects the period time of the solar dynamo can be found in Section 5.5.

For the half-sphere solutions, no consideration has been taken regarding the full-sphere set-up and its polarity structure. Thus, the main controlling effects of the tachocline $\alpha$-parameter for this dynamo model are for full-sphere characteristics only. According to Dikpati & Gilman (2001) and private communication with Dikpati, the tachocline $\alpha$-effect is the controlling parameter for an odd parity output. In other words, for a full-sphere solution the polarity aspect would have been possible to take into consideration, and the dynamo model used would possibly change from an odd parity solution to a quadrupole solution for a ratio of $\frac{\alpha_T}{\alpha_{surf}} \leq 0.05$ for simulations over a longer period of time. At least exceeding some 500 years of simulated time. (Dikpati, private communication). For our reference value of $\alpha_T$ the ratio would be exceeded for a surface $\alpha$ greater than 2 times the reference value. Quite naturally, this shift in polarity would also appear if the tachocline $\alpha$-effect is turned off completely. (See Chapter 6 for further discussion regarding a full-sphere solution.)

5.4.2 Animation analysis and images from animations

The animations for cases with an increased surface $\alpha$-effect are almost identical to the reference solutions. Combined with the butterfly plots this shows that the only mechanism that is significantly affected by an increased Babcock-Leighton effect within these ranges is the magnitude of the field strengths.
5.5. VARYING MERIDIONAL CIRCULATION PARAMETERS

5.5.1 Surface velocity

When increasing the velocity, $u_0$, of the pole-ward meridional flow on the surface, it is clearly shown that the period of the magnetic cycle is decreased, which is illustrated by Figure 5.6.
In the reference solution, the surface velocity has been set to 12 m s\(^{-1}\), which gives a solar cycle period of approximately 10 years. By increasing the surface velocity to 15 and 20 m s\(^{-1}\), the period time is shortened to approximately 8 and 6.5 years. The general butterfly pattern stays the same for an increasing surface velocity. The extreme values for the poloidal field remain approximately the same, while a decrease of the extreme values for the toroidal field can be noticed, from ±67 kG for the reference solution (see Table 5.1) to ±60 kG and ±52 kG for increasing velocities.

![Figure 5.6: Butterfly plot for a dynamo case with a meridional surface flow velocity of 20 m s\(^{-1}\). It is shown that the solar cycle period is decreased when the meridional surface flow velocity is increased. The solar cycle period is now approximately 6.5 years, in comparison to approximately 10 years for the reference solution. The numerical values of the magnetic fields are almost identical to the reference case (see Figure 5.1). The same notation and scaling are used as for previous butterfly plots.](image)

Thus, it can be concluded that for this dynamo model, the surface velocity of the meridional flow can be used to adjust the cycle period, without causing major variations in the rest of the output. This goes well with previous conclusions for Babcock-Leighton flux transport dynamos.

**Animation analysis**

Also for these cases, the patterns observed in the animations are similar to the reference case. As mentioned in Section 5.5.1, the only variation that can be observed for an increasing meridional surface flow velocity is the period time of the solar cycle.
5.5. VARYING MERIDIONAL CIRCULATION PARAMETERS

5.5.2 Radial dependence

The parameter $p$ describes the radial dependence of the meridional flow. For an increasing $p$, the thickness of the pole-ward meridional flow is increased. Correspondingly, the thickness of the subsurface layer for the equator-ward flow decreases. A similar pattern to the output for varying surface velocity of the meridional flow can be observed for an increasing value of the radial dependence of the meridional flow. Furthermore, for an increasing value of $p$, the cycle period decreases, the extreme values for the toroidal field decrease and the poloidal field remains basically unchanged. No change in latitude can be noticed for the appearance of the extreme values for the toroidal field.

Unfortunately, the pole-ward flow and equator-ward flow thicknesses are not separated in this model, and it is assumed that some kind of meridional flow occupies the whole convection zone. Thus, the thickness of the layers cannot be changed separately, and an increase of thickness for one layer of flow is connected with a decrease of thickness of the layer for the opposite flow. This could be an interesting subject for further modifications of the dynamo model, since it is yet unknown both where the subsurface layers of the meridional flow are located, and how the structure of the same is set up considering, for example, thickness of the layers and flow velocities.

Animation analysis

Also for the radii not illustrated in the butterfly plots, the output for an increasing radial dependence parameter is similar to the reference case. Both considering the toroidal and poloidal field structures.

5.5.3 Latitudinal dependence

The parameter $q$ controls the latitudinal dependence of the meridional flow and significant changes in output can be noticed for a varying value of $q$. For drastic changes, such as 0.1 to 10 times the reference value, some major effects are shown in the butterfly pattern. For simulations run with an increased value for the latitudinal dependence, the butterfly pattern loses its characteristic shape. To begin with, only at high latitudes, but for a big enough $q$, also the lower latitudes are affected, which is shown in Figure 5.7. The cycle time is also increased for an increasing value of $q$. An increase of the extreme values of the poloidal field can be noticed and the locations of the poloidal field move closer to the equator. For an extremely small value of $q$ (about a factor of 0.1 of the reference value), the toroidal field gets weaker, while the poloidal field increases significantly. The poloidal field is almost isolated to the very highest latitudes, while the toroidal field has still got quite a butterfly shape to it. The cycle period has decreased to approximately 7 years for the decreased value of $q$.

Animation analysis

For a increased latitudinal dependence parameter, the field patterns loose their buttefly characteristics. For a $q$ of 10 times the reference value, most of the pattern of circulating movement has been replaced by field interactions on a more limited latitude. The poloidal field is no longer covering the higher latitudes, but has its maxima on approximately 35°. Also the toroidal field has lost its
solar like pattern and is now active on higher latitudes than for the reference solution. Only a weak part of the toroidal field reaches latitudes close to the equator and the field lines no longer move outward on those latitudes.

5.6 Numerical solution of a reference article

Since the reference values, provided by Dikpati together with the dynamo code, turned out to be quite different to the values of the reference solution in the main published article based on the code, Dikpati & Charbonneau (1999), a solution with parameter values more similar to the ones in the article was also run. The main differences are the values of the meridional flow parameters, $u_0$, $p$, $q$ and $m$, so that these were the only parameters changed as to correspond to the values of the solution in the article. Except for the surface flow velocity, they were all set to significantly lower values for the simulations in Dikpati & Charbonneau (1999) than for the reference case, and the latitudinal dependence, $q$, was even set to 0. The surface velocity of the pole-ward meridional flow was set to 20 ms$^{-1}$, instead of 12 ms$^{-1}$ as in the reference solution. The toroidal field reaches maximum and minimum values of approximately 62 and -54 kG for this solution, and the poloidal field output is about a factor two higher than for the reference solution. For the solution as of the article, the poloidal field is more concentrated to higher latitudes than for the reference solution. Furthermore, the toroidal field has its extreme value areas on lower latitudes than for the previous reference solution and the curves of the toroidal field on high latitudes are no longer as distinct, which both correspond better with solar observations.
5.6. NUMERICAL SOLUTION OF A REFERENCE ARTICLE

Figure 5.8: Butterfly plot for a solution with meridional circulation parameters set up similar to as in Dikpati & Charbonneau (1999). The general butterfly pattern of this set-up corresponds better to solar observations than the butterfly plot of the reference case. The poloidal field maximum values are approximately +6.4/-6.8 kG and the toroidal field maximum values approximately +62/-54 kG. The same notation and scaling are used as for previous butterfly plots.

The only thing that did not improve regarding the output for this set-up is the phase difference between the toroidal and poloidal field. The polarity shifts, of the two fields, are now slightly shifted compared to the field strength maximum of the other fields. Even though the shifts might not be as aligned as for the reference case, the output from this set-up can still be considered to follow the requirement regarding the polarity shifts. Since not all parameters were changed to correspond to the values in Dikpati & Charbonneau (1999), the output differs slightly from that obtained by Dikpati & Charbonneau (1999).

Animation analysis

Also when analysing the animations, the numerical solution similar to the one in Dikpati & Charbonneau (1999), seems to correspond better to solar observations than the reference solution. Especially the toroidal field seems to follow a more solar like pattern for this set-up. For example, the toroidal field now continues further equator-ward than in the reference solution, and it also stretched further toward the surface at latitudes corresponding to sunspot generation latitudes.

In Figures 5.9, 5.10 and 5.11 a solar cycle of approximately 11 years, represented by six images, is shown. The evolution of the toroidal field is clearly shown. The field cycle of the toroidal field starts with a field generation at high latitudes, (see Figure 5.9 a), which continues to grow and move inward and equator-ward throughout the cycle, (see Figures 5.9 b, 5.10 and 5.11 a). In the
final stage of the cycle, (see Figure 5.11 b), the toroidal field is about to produce sunspots at lower latitudes. Since the images have been selected from an early stage of the simulation, the toroidal field closest to the equator, represented by dashed contours, in Figures 5.9 a and b, has not yet fully developed and should not be given too much attention. Instead it is recommended to study the corresponding stage for the proceeding toroidal field, represented by solid contours, in Figures 5.11 a and b.
(a) A new toroidal field (dark, dashed contours) is generated at high latitudes. The previous one (solid contours) is moving equator-ward but has not yet started to produce sunspots.

(b) After a couple of years the new toroidal field (dark, dashed contours) has grown and starts to move inwards. Also the previous one (solid contours) is moving equator-ward, while the old toroidal field, represented by dashed contours close to the equator, is at its final stage of producing sunspots.

Figure 5.9: A set of six images in total, over a solar cycle of approximately 11 years. The animation is of a solution with a similar set-up as in Dikpati & Charbonneau (1999). The dark contours correspond to a cut through the toroidal field (solid and dashed for different polarities respectively), i.e., the toroidal field lines are perpendicular to the plane of the animation. The brighter, dashed ones correspond to the poloidal field lines. The times of the images are shown at the top of the images. The horizontal and vertical axes both correspond to the solar radius and the origin represents the solar centre. The semi-circles in the images are different radii of the Sun and the contours follow field strengths of approximately the same values. Figure 5.10 and 5.11 show the following four images.
(a) The new toroidal field (dark, dashed contours) is growing, while the previous one (solid contours) starts to produce sunspots at this stage.

(b) The new toroidal field (dark, dashed contours) still increases and moves equator-ward. The previous toroidal field (solid contours) has now reached its maximum activity and produces sunspots.

Figure 5.10: Images number three and four in the series of six of a solar cycle. The same notation and scaling are used as in Figure 5.9. The two previous images are shown in Figure 5.9 and the following two in Figure 5.11.
(a) The toroidal field represented by dark dashed contours has increased even more and still moves inwards and equatorward due to the meridional circulation. The toroidal field producing sunspots (solid contours) has also moved further equatorward.

(b) Another toroidal field (solid contours), with the opposite polarity to the recent one, is generated at high latitudes. The previous one (dark, dashed contours) is now at lower latitudes and will start to produce sunspots. The toroidal field represented by solid lines close to the equator has reached its final stage of producing sunspots.

Figure 5.11: Images number five and six in the series of six of a solar cycle. The same notation and scaling are used as in Figures 5.9 and 5.10. The four preceding images are shown in Figures 5.9 and 5.10.
Chapter 6

Full-sphere solution

One of the crucial parts regarding solar dynamo models is to produce a solar like polarity for a full-sphere solution. As long as only a half-sphere solution is simulated, the polarity issue for the whole sphere cannot be taken into consideration. As described in Chapter 3, the magnetic field of the Sun is antisymmetric about the equator with a changing polarity for every 11 year solar cycle. Thus, an appropriate solar dynamo model should produce such an output when the whole sphere is simulated. As a part of the project, the aim was to extend the half-sphere dynamo code provided by Dikpati to a full-sphere code. For the record, this has been done previously by Dikpati & Gilman (2001). However, to increase the understanding of the dynamo model and to improve some Fortran programming skills, it was decided to make an extended version of the original dynamo code, in spite of the fact that this had already been done.

6.1 Extending the code

When extending the code, the radial dimensions could be kept the same since the same layer and thickness were considered regarding the convection zone. The polar angle dimensions, however, had to be extended to include also a bottom hemisphere with a maximum polar angle of $\theta = \pi$ instead of $\frac{\pi}{2}$. The differential rotation, $\alpha$-effect, diffusivity and meridional circulation profiles were all assumed to be symmetrical about the equator for a full-sphere solution, which should have been quite a straight-forward extension. The initial values from Dikpati’s converged solution were also read in for the bottom hemisphere, with variables added to be able to choose whether a symmetrical or anti-symmetrical initial set-up of the poloidal and toroidal fields were to be selected for the simulations.

Regarding the boundary conditions for the full-sphere solution, the toroidal field is once again set to zero at the equator, and the poloidal field is zero for an initial anti-symmetrical set-up for the poloidal fields, and nonzero for an initial symmetrical set-up. The bottom pole boundaries are set to zero, and the two boundaries in radial direction are both kept the same as for the half-sphere solution.
6.2 The tools to analyse the output

In general, the same IDL codes were used to plot and animate the output from the full-sphere solution. However, some changes had to be made, mainly regarding how to read the output files and adapt the format of the output to match the required format of the IDL functions. Of course both the butterfly plot and animation codes had to be extended with a bottom hemisphere as well. For the full-sphere animations two additional lines at latitudes ± 45° were added for clarity. For the vertical axes in the full-sphere animations, no units have been added. However, they correspond to the solar radius symmetrical about the equator, with the same scaling as represented by the horizontal axis through the centre of the animations. The top of the vertical axes therefore corresponds to the top pole of the Sun and the bottom of the vertical axes to the bottom pole of the Sun. The origin represents the centre of the Sun. The shades and contours in the butterfly plots of the full-sphere solution correspond to the same fields and ranges as for the half-sphere solution. The same scaling for the field strength has also been used.

6.3 Analysis of full-sphere solution

![Butterfly plot](image)

Figure 6.1: Butterfly plot for a full-sphere solution with a numerical set-up as of the reference solution in Section 5.3. While the toroidal field keeps a correct anti-symmetry, the poloidal field stabilises for a symmetric solution, which does not correspond to solar observations. It can also be noted that the poloidal field is approximately a factor two stronger than for the half-sphere solution. The toroidal field keeps roughly the same values in the northern hemisphere, but are lower in the southern one. The poloidal field maximum values are approximately ± 5.6 kG and the toroidal field maximum values approximately ± 67 kG. The same notation and scaling are used as for previous butterfly plots.
6.3.1 Butterfly plot analysis

In general, the full-sphere model seems to produce solar like output, the shapes and cycle periods correspond well with the half-sphere solution for the top hemisphere, and similar output for the bottom one. Also the relation between the locations of the poloidal extreme values and the toroidal extreme ones seem reasonable at a first look. However, as can be seen in Figure 6.1, the model fails to retain the correct polarity for the bottom hemisphere. For an initial condition with a symmetric poloidal field potential, $A$, the model is not able to retain the set polarity. Already after approximately 5 years, the potential has become antisymmetric about the equator. This is clearly shown in Figure 6.2, where the poloidal field potential, $A$, is plotted instead of the poloidal field strength. In fact, Figure 6.2 corresponds to the kind of plot that would have been expected when plotting the poloidal field, if the full-sphere solution had been considered accurate. Thus, the poloidal field becomes symmetric about the equator, and the observed dipole structure of the magnetic field has been replaced by a quadrupole field. For an initial condition with an antisymmetric poloidal field potential, the potential remains antisymmetric throughout the simulations. This kind of behaviour, where the magnetic field shifts to a quadrupole set-up, has been observed for dynamo models where the surface-$\alpha$ dynamo is the dominant one. However, in those cases the change from an odd parity to an even one occurs after hundreds of simulated years of solar life time. (Dikpati
Furthermore, the poloidal field strength is approximately a factor two higher for the full-sphere solution compared to the same parameter set-up for a half-sphere solution. At the same time, the toroidal field keeps roughly the same values in the northern hemisphere as for the half-sphere solution, while the toroidal field strength in the southern hemisphere is considerably weaker.

### 6.3.2 Animation analysis

The quadrupole effect is shown in a more illustrative way in Figure 6.4, where an image from the animation is shown. The ticks on the field lines correspond to the direction of increasing field strength, so that the circulating movement of the field can be found by relating the direction of movement perpendicular to the direction of the increasing field strength. In Figure 6.4 the ticks are directed ‘inwards’ in one hemisphere and ‘outwards’ in the other, so that the hemispheres can be considered having a circulating movement of opposite directions to each other. Hence a quadrupole field is generated if the whole sphere is taken into consideration.

### 6.4 Numerical solution of a reference article

![Butterfly plot of a full-sphere solution with meridional circulation parameter set-up as by Dikpati & Charbonneau (1999). The poloidal field now shows an antisymmetric pattern, while the toroidal field does not. The poloidal field maximum values are approximately +22/-26 kG and the toroidal field maximum values approximately ± 70 kG. The same notation and scaling are used as in Figure 6.1.](image)

Interestingly enough, the general characteristics of the full-sphere output turned out to be quite different for a set-up with meridional flow parameters as
in the reference article by Dikpati & Charbonneau (1999). As can be seen in Figure 6.3, the poloidal field now follows an antisymmetric pattern throughout the simulations, while the toroidal field has changed into an even parity mode. Once again, this does not correspond to solar observations, but instead of the poloidal field not matching observations, the toroidal field is now the faulty one.

To illustrate this with an image from an animation of the full-sphere solution with a parameter setup as of the article, the poloidal field lines have been plotted with ticks showing the direction of increasing field strength, as of Figure 6.5. First of all, this proves even more that the full-sphere code cannot be considered reliable, but it also gives a hint that the faulty part of the code might be connected to the meridional circulation profile for the extended code.
Figure 6.4: Image from the animation of a full-sphere solution as of the reference case. Only the poloidal field is shown. By taking the ticks on the plotted field lines into consideration, it can be seen that the poloidal field generates a quadrupole field instead of a dipole field for this set-up.
Figure 6.5: Image from the animation of a full-sphere solution as of the article set-up. Only the poloidal fields are shown and by taking the ticks into consideration, it is seen that the poloidal field now generates a dipole field instead of a quadrupole one, which corresponds to solar observations.
Chapter 7

Discussion and conclusions

Regarding the initial focus of the project, the main conclusion that is made, is that the dynamo model depends on both a tachocline $\alpha$-effect and a surface $\alpha$-effect. The major differences that were identified, were related to the toroidal field strength and the location of the maximum poloidal field strength areas. The toroidal field strength increased with an increasing surface $\alpha$-effect and the poloidal field strength areas in question moved towards the equator and lost their characteristic shape. Therefore, a possible conclusion is that an increasing surface $\alpha$-effect causes constructive interference affecting the toroidal field. Except for that, no conclusions regarding constructive or destructive interference between the two $\alpha$-effect parameters could be made. As a consequence of this, no conclusions regarding grand minima or maxima could be made. Also the polarity characteristics of the dynamo output, could not be studied in any extent. This is due to the fact that a half-sphere solution does not show any shifts to a quadropole magnetic field for the whole Sun, since only one hemisphere is simulated and analysed. Regarding both these issues, a reliable full-sphere solution, which can be used for simulations over a longer period of time, would be to prefer.

7.1 The half-sphere solution

In general, the butterfly plot and animation obtained for the reference solution correspond quite well with solar observations. However, the pattern of the toroidal magnetic field should be subject to some comments. Primarily, the latitudes of the maximum field strength areas for the toroidal field can be considered too high to agree with solar observations. If the sunspot generation was to originate at the latitudes as of the half-sphere reference solution only, the sunspots would most likely appear on higher latitudes than observed. Therefore, having the maximum toroidal fields at such high latitudes, requires that they somehow do not produce many sunspots. There are theories though, that toroidal flux ropes occurring at higher latitudes are more stable than flux ropes at lower latitudes, and therefore do not rise to the surface. This effect would be caused by the increased magnetic tension due to the greater curvature at higher latitudes. If so, it is possible that the subsurface toroidal field can have maxima at latitudes where sunspots will not be produced, while the flux ropes
at lower latitudes are the ones rising to the surface and generating sunspots.  
(Charbonneau, St-Jean & Zacharias, 2005)

### 7.1.1 The $\alpha$-parameters

From the analysis in Section 5.4 it can be concluded that it is unlikely with a surface $\alpha$-effect parameter larger than a factor two compared to the reference solution. This is not only because the subsurface toroidal field exceeds the well-accepted limits for an overshoot tachocline toroidal field, and becomes too strong. It is also because the observed changes in shape and location of the surface radial poloidal field.

Furthermore, the surface $\alpha$-effect is likely to have a magnitude of at least the reference value for the toroidal field to exceed the minimum values of about 60 kG. However, a surface $\alpha$-parameter of 1.5 to 2 times the reference value might be a better option, regarding the toroidal field values, than the set reference value. If the arguments regarding the high latitudes of the location of the toroidal field maxima are taken into consideration, an even higher surface $\alpha$-effect should be evaluated. In that case, the actual extreme values are no longer the most interesting ones, but the field strengths at lower latitudes, where the sunspots are believed to have their origin.

### 7.1.2 The meridional flow parameters

It is concluded that the magnitude of the surface velocity for the meridional flow can be used to regulate the solar cycle period efficiently. This has been concluded also by, for example, Dikpati & Charbonneau (1999).

Both increasing and decreasing values of the latitudinal dependence produce an output that does not correspond to solar observations. Furthermore, it can be concluded that the meridional flow parameter $m$, corresponding to an exponent in the convection zone density profile, should have been subject to variations.

### 7.1.3 Numerical solution of a reference article

Overall, the solution based on a parameter set-up similar to the one in Dikpati & Charbonneau (1999) produces a more solar like output than the provided reference solution. The major improvement, compared to the reference solution, is the areas with the extreme values of the toroidal field, which are now at significantly lower latitudes. As could be seen in Figure 5.8, they now appear on an approximate latitude of 30°, which corresponds better to latitudes of sunspot observations. Also, the poloidal field is more concentrated to the poles, and the peak values are still located just before onset of toroidal field polarity shifts. The meridional surface flow velocity of this solution corresponds better also to observations than for the reference solution.

### 7.2 The full-sphere solution

As was shown in Chapter 6, the full-sphere solution does not correspond to the expected output or to solar observations made. The most obvious disagreement is the failure of keeping the correct polarity and antisymmetry for the poloidal
7.3. THE POLOIDAL FIELD STRENGTH

and toroidal fields. Furthermore, the output does not correspond to the output obtained for the half-sphere solution, or to the solution in Dikpati & Gilman (2001).

Compared to the half-sphere solution, the northern hemisphere in the full-sphere solution corresponds well to the half-sphere solution for the same set of parameters. However, a variation in magnitude of the poloidal field strength can be observed. The magnitude of the poloidal field strength in the full-sphere case is roughly twice that of the half-sphere solution, where they should be more or less identical. As mentioned in Section 6.3.1, variations in the toroidal field were identified between the two hemispheres. These values should also be roughly the same for a correct model.

Finally, if comparing the full-sphere output of the reference case (see Figure 6.1) and the output of a simulation with a parameter set-up similar to the one in Dikpati & Charbonneau (1999) (see Figure 6.3), it can be seen that the polarity and antisymmetry patterns are different. Since the main differences between the two parameter set-ups are the values of the meridional circulation parameters, the conclusion is that the fault might be connected, fully or partly, to the meridional circulation profile of the full-sphere solution. In other words, the full-sphere dynamo model discussed here should not be considered accurate or reliable when it comes to output and solar like simulations.

7.3 The poloidal field strength

For all simulations, the poloidal field values are definitely exceeding the maximum values from observations. No parameters or processes in the dynamo model, which seem to affect mainly the poloidal field strength, have been identified. This is also the case for the simulations discussed in Dikpati & Charbonneau (1999), where no specific solution has been suggested to adjust this behaviour of the model. According to these authors the poloidal field strength depends on the meridional flow model. Since the meridional circulation model used is of geometrically converging character, some flux conservation of the poloidal field is unavoidable according to the authors. They also write that a possible solution to adjust this would be to change the meridional flow pattern at high latitudes. Even though this is one of the main observations of the solar characteristics, the authors did not find it critical enough at the time to adjust for further simulations.

7.4 Future research

Instead of the provided reference parameter set-up, the set-up similar to the one in Dikpati & Charbonneau (1999) should be used for future research. This due to the fact that the output of the latter corresponds better to solar observations, than the output of the set-up used for the reference solution in this project.

7.4.1 Meridional flow parameters

All variations of the latitudinal dependence parameter caused output that did not correspond to solar observations. However, this was when varying the latitudinal dependence parameter only. Therefore, it cannot be excluded that the
parameter can be subject to variation, if the combination of all meridional flow
parameters are considered to cause possible set-ups.

Regarding the meridional flow parameter $m$, it was concluded that also this
parameter should have been subject to variations. This due to the fact that it
was not until the simulations with a parameter set-up similar to the the one in
Dikpati & Charbonneau (1999) were run, that it was realised that the value of $m$
varied significantly between the provided reference solution and the solution
of the article. For future research additional simulations with various values of
the parameter in question, should be run to give a more complete overview.

After completing the project it is concluded that an interesting future task
would be to spend more time changing the meridional circulation parameters
and make a thorough survey of how even smaller changes, regarding the merid-
ional circulation parameters, might affect the dynamo model. The combination
of the meridional flow parameters should also be considered to a larger extent
than during this project.

7.4.2 Polarity of full-sphere solution

There is a possibility, that the polarity and antisymmetry of the full-sphere
output, is the correct solution for this solar dynamo model. If so, this would
rule out that the model produces output corresponding to the observational
data. However, this is most likely not the case. More time should therefore
be spent checking the methods of extending the original code, including more
thorough checks of the set-ups for the numerical methods used. This would be
to make sure that they are applicable also for a full-sphere case with non-square
matrices. Due to the fact that the previously squared matrices became non-
square after changing the polar angle vector dimensions, some issues regarding
the mathematical routines in the code were raised. These were mainly adjusted
to make the code working in general and to obtain a solar-like output, but the
routines have not been investigated properly enough to be considered reliable.
Checking and correcting these routines, and the rest of the full-sphere code, are
the primary tasks for any future research based on this project.

7.4.3 Tachocline thickness

During the time working on this project, it was noticed that at a couple of places
in the dynamo code, different values were used for thicknesses that seem to refer
to the same layers or thicknesses in the Sun. The most obvious occurrence is
the tachocline thickness, which is referred to as $d_T$ throughout the thesis. For
the differential rotation profile, for example, the thickness was set to $0.025R_\odot$,
while for the other physical profiles, it was set to $0.0125R_\odot$. Whether this would
have had any major influence on the output or not is not known, and no further
analysis of it has been carried out, mainly since both values are reasonable
for the tachocline thickness. No obvious reason for this choice of values has
been found. For future research this should be be looked into and corrected if
necessary.
Bibliography


Science@NASA Website: [http://science.nasa.gov/](http://science.nasa.gov/) (2006-03-28)

