The Influence of Uncertainties of Attitude Sensors on Attitude Determination Accuracy by Linear Covariance Analysis

Johan Blomqvist

Luleå University of Technology
Master Thesis, Continuation Courses
Space Science and Technology
Department of Space Science, Kiruna

CRANFIELD UNIVERSITY

JOHAN BLOMQVIST

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Supervisor: Dr Peter Roberts

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Abstract

The attitude sensors for small spacecraft are often selected from commercial components to reduce costs. Uncertainties in these sensors hardware contribute significantly to the accuracy of the attitude solution. The idea that Linear Covariance techniques can be used to predict the accuracy of attitude determination systems and assist in their design is investigated. By using the sensor's parameter estimated accuracy in a model, one could calculate the total standard deviation of the attitude determination by propagate errors through the estimation and use simple Root-Sum-Square. The linearization of the estimation process as a function of error sources can be done automatically using software linearization tools. Generalized Matrix Laboratory (MATLAB) M-functions using this technique are written in order to provide a tool for estimating the attitude determination accuracy of a small spacecraft. This tool were applied to a satellite dynamics truth model developed at Satellite Dynamics Laboratory, UTAH, USA, in order to quantify the effects of sensor uncertainties on this particular spacecraft's attitude determination accuracy. Many error sources, including sensor mounting, bias drift, sensor noise and thermal fluctuations were examined. As expected, primary contributors to the error were identified. Design actions such as other processing methods, sensor calibration and alignment methods to meet the required system specifications were suggested based on the analysis.

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Abbreviations

AD Analog to Digital

ADCS Attitude Determination Control System

ADS Attitude Determination System

BFCS Body Fixed Coordinate System

CLT Central Limit Theorem

DCM Direction Cosine Matrix

dim. less Dimensionless

DOF Degree Of Freedom

ECI Earth Centered Inertial

FOV Field Of View

GPS Global Positioning System

HCI Horizon Crossing Indicator

IGRF International Geomagnetic Reference Field

MATLAB Matrix Laboratory

PQW Perifocal frame

QUEST Quaternion Estimator

RAAN Right Ascension of the Ascending Node

S/C Spacecraft

SDL Satellite Dynamics Laboratory

TAM Three Axes Magnetometer

TRIAD TRI-axial Attitude Determination

UTC Universal Time Coordinated

Chapter 1

Introduction

1.1 The attitude accuracy problem

Small satellites are gaining in popularity, especially among universities who now are designing and building small spacecraft that piggyback onto existing payload, giving students around the world valuable hands on experience in designing and building spacecrafts.

In order to hold cost, power and weight down, the sensors used on these satellites are usually less robust than sensors on larger more complex satellites. This may result in significant errors in the attitude determination solution of the satellite.

The errors induced from measurement are of two types; random and systematic [1]. Systematic errors are induced by inaccuracies in the design of the spacecraft, such as sensor misalignment and calibration offsets. These kinds of errors are usually recognised as biases.

Sensors are subject to calibration during manufacturing where mechanical misalignments and electronic biases are reduced. Laboratory tests with flight hardware further improves the knowledge of the biases. However, inaccuracies still persist and during launch, alignments can change as much as 0.1 degree [2]. Systematic errors such as the sensors mounting relative the spacecraft is one of the most important to consider [3].

Random errors are unpredictable time variations in measurements and are due to the limitations in the sensor. Often these are caused by the surrounding environment, such as electromagnetic effects. Random errors can be reduced by increasing the number of measurements used in the estimator. Systematic errors cannot be reduced in this way. Therefore there will always be errors in the attitude determination solution [3].

Both these kinds of errors arise from many sources, they can be counted as hundreds. The magnitude of many are negligible in comparison with the measured

parameter. One important question is if the variance of the uncertainty changes, how will that affect the variance of the attitude solution?

The spacecraft will know its attitude within specific bounds that are determined by the sensor accuracy. It is crucial for the designer to have an accurate knowledge of these bounds, in order to determine if the requirements for the mission can be met with the current design. One problem to answer is:

Given a set of hardware options with different component accuracy specifications, will this sensor combination meet the mission's requirement of attitude determination accuracy?

The accuracy of the attitude estimation is important. For a spinning satellite, it is important to know the accuracy to which the spin axis orientation can be determined [4], especially during orbit insertions. An error in the attitude at the point of insertion must be corrected by the means of costly fuel use. Thus, the more accurate the attitude pointing is, the less fuel is used and the mission's time span is extended. As an example, an improvement of 0.13 deg accuracy could translate into an lifetime extension of one month [5] for a normal sized satellite.

1.2 Approaching the attitude accuracy problem

The method of approach in this work is a statistical form of analysis called Linear Covariance. The effect of the uncertainties on attitude determination accuracy are determined by propagating the variance of the uncertainties forward into the variance of the attitude solution.

Linear Covariance analysis can be used for estimating the influence of each sensor uncertainty on the final attitude solution accuracy by simple matrix algebra. The result of this study determines the variance of the attitude determination as a function of the sensor uncertainties variance. A designer can use the result as a trade-off tool, by determining what variance of the uncertainty can be tolerated and still meet the mission's requirements.

This method was used in a previous Master's thesis [6] where a set of uncertainties in the spacecraft structure were propagated to the coelevation of the spin axis. A generalized software program AFORCE utilizing Matrix Laboratory (MATLAB) tools was developed. The software linearise a Simulink model, propagates the uncertainties and displays the result in an intuitive manner.

AFORCE could be applied to this problem. However, as the inputs grow in number, AFORCE gets inefficient and it becomes hard to keep track of the inputs which are defined by command lines. This software also neglects feed forward elements and only accounts for errors that affect the states in the system.

Further, AFORCE requires the model to be continuous, which is not suitable for modelling sampled data sensors. Also, problems arise when states in the model

linearises to zero. Although there are similarities between this project and the previous one, Forslund dealt with the dynamics whereas this project is dealing with attitude estimates.

An improvement was needed, and for this work, three generalized MATLAB¹ functions were written. Each with specific tasks that make the process of defining inputs/outputs in a Simulink model, entering the uncertainties variance and inspecting the result straightforward. The functions are also more time efficient in the linearisation and propagation process and can easily be called from an M-file used to initialize the model.

These functions are applied to a Simulink satellite model developed by Dr. Rees Fullmer at Utah State University/Satellite Dynamics Laboratory, Logan, Utah, USA. This model consists of a truth dynamic generator that generates true Earth Centered Inertial (ECI) and Body Fixed Coordinate System (BFCS) vectors of the sun and magnetic field, which are then available for measurements.

1.3 Aims and objectives

The main idea was to investigate whether Linear Covariance techniques can be used to predict the accuracy of attitude determination systems and assist in their design. That is, by using the estimated accuracies of sensor parameters in a model, one would get an estimate of the total error. The error model would combine error estimates from many sources of error to give a cumulative measure of the attitude estimate error.

The specific tasks of this project were to:

- Identify the uncertainties of a spacecraft Attitude Determination System (ADS)
- Create a design tool that can determine the possible attitude determination accuracy based on the uncertainties in the system
- Apply the tool to a satellite model to quantify the design of the ADS and test the tool
- Identify what factors have the most influence on the accuracy of the attitude determination

The main outcome of the study results in a tool which allows the design team of a small spacecraft to determine the contribution of each error source to the overall accuracy of the spacecraft's attitude determination system.

¹Refer to Chapter 4 The M-Functions for an introduction to the programming environment

1.4 Document structure

- *Chapter 1* provides a brief introduction to the attitude accuracy problem and how it is approached in this work.
 - *Chapter 2* introduces the basic theory by an introductory overview.
 - *Chapter 3* clarifies the overview by the use of a hypothetical one degree of freedom illustrative example.
 - *Chapter 4* introduces the programming environment and the functions that were developed for this work by providing information of the arguments they take, the outputs as well as the data storing scheme.
 - *Chapter 5* presents the satellite and sensor models used to investigate the influence of sensor uncertainties on attitude determination accuracy and provides information of the parameters under investigation.
 - *Chapter 6* contains the results of the analysis and discusses the improvements that can be made based on the results of the analysis.
 - *Chapter 7* summarises the work by conclusions and supplying directions for possible further work.
-

Chapter 2

Introductory Overview

In this chapter, basic attitude determination theory is briefly reviewed in order to introduce or remind the reader of the concepts to which this work is related. The mathematical approach is described in this chapter, which is put to the test with an illustrative example in the following chapter, in order to justify further analyses on more complex systems.

2.1 Fundamentals of attitude determination

There are several different ways of representing attitude. Depending on what is to be described one representation is more suitable than another. For attitude representation, the best choice is a system centred on the spacecraft [7]. One such system could be the BFCS which is attached to the spacecraft and rotating with it. The attitude is basically the relation between this system, and a defined reference system. There are several common reference frames (coordinate system) in use. An example is the ECI which is non-rotating and centred at the Earth.

Other coordinate systems exist, not really used for attitude determination, for example the Perifocal frame (PQW) system which is mainly used to describe orbits. All of the above consist of three mutually orthogonal unit vectors with defined directions [4] [7].

The attitude of the spacecraft is determined by finding the rotation(s) that would align a spacecraft fixed coordinate system with a known reference frame. This is really a coordinate transformation between the reference system to the body system.

The rotation could be a single angle around a vector specified axis (Euler's theorem), or several rotations, each around the coordinate frame axis. They can be described by different representations, all with benefits and drawbacks. A rotation is completely described by a minimum of three parameters. However, many

representations use more than that. Three representations in common use are briefly discussed below. A full definition can be found, among others, in [7] and [8].

Rotation matrix

An orientation of one coordinate frame relative to another defines a unique 3×3 matrix describing the rotation (transformation) of a vector's coordinates in one system frame into the coordinates of the other frame.

This rotation matrix is also known as a Direction Cosine Matrix (DCM) because of the structure of the matrix as the cosine of the angle between the frame's unit vectors. It is orthonormal and are not commutative, i.e. several consecutive rotations must take the order into account.

This representation is not easy to visualise. However, the ease of representing a sequence of rotations by right-multiplying the DCM's make it popular. The rotation matrix is denoted by R_1^2 which is interpreted as the rotation of coordinate system 1 that would align it to coordinate system 2. A simple transpose would change the direction of rotation $R_2^1 = R_1^{2'}$.

Euler angles

A set of three angles, describing (in a defined order) the rotations about each of the frame's axis is used to determine the body systems orientation relative the reference system. This is essentially three simply rotations by the DCM, each about one of the reference systems axis.

A different sequence of rotation by the same set of angles result in a different position, as with the DCM not being commutative. Thus, a set of Euler angles must be accompanied by the corresponding sequence in order to have a unique attitude.

There are twelve different sequences of rotation divided in two groups of six, Type 1 and Type 2. Type 1 sequence is defined as where each rotation is around one of the three coordinate axis, e.g. (X, Y, Z). The second type, Type 2 sequence, is defined as where the first and third rotation is about the same axis, e.g. (Z, Y, Z) [8].

Quaternions

Quaternions is a four dimensional quantity $\mathbf{q} = [q_4 \ \mathbf{q}_1 \ \mathbf{q}_2 \ \mathbf{q}_3]$ consisting of a three dimensional vector and a scalar. The vector part has its components in a orthogonal system. It is worth noting that the scalar term q_4 is in some definitions

written last, and it is important to have knowledge of what definition is used¹.

Every rotation matrix is real and orthonormal, and has at least one eigenvector with a corresponding unity eigenvalue. If the rotation matrix acts on this vector, it will remain unchanged as it has the same coordinates in both the body and the reference system. This corresponds to Euler's Theorem, which states that the simplest rotation can be described by a rotation about some specified axis, and it is from this the quaternion has its basic definition [8]. This eigenvector and the corresponding rotation angle can easily be solved from the quaternion entries.

Converting between representations

Conversion between the discussed representations can easily be done. The TRI-axial Attitude Determination (TRIAD) algorithm for example, Eqs. (2.2) - (2.5), returns the attitude as a rotation matrix A . This is not a very convenient representation for this work. However, DCM's can be converted to the more descriptive Euler angle form by using the Rotation matrix (E)uler angles (p)arametrisation A_{Ep} . For the Type 2 sequence (Z, Y, Z) the parametrisation is given in [7] as

$$A_{Ep} = \begin{bmatrix} C\psi C\theta C\phi - S\psi S\phi & C\psi C\theta S\phi + S\psi C\phi & -C\psi S\theta \\ -S\psi C\theta C\phi - C\psi S\phi & -S\psi C\theta S\phi + S\psi C\phi & S\psi S\theta \\ S\theta C\phi & S\theta S\phi & C\theta \end{bmatrix} \quad (2.1)$$

where the Euler angles are in the sequence (ϕ, θ, ψ) . The Euler angles can easily be determined by trigonometry from this. S and C are shorthand for Sine and Cosine operators respectively.

2.1.1 Attitude solution methods

Attitude determination accuracy is a combination of the hardware and the processing methods. This work will only address the hardware part. To be able to solve for a three axis attitude, three independent parameters are needed. These can be generated from only two non-parallel vector measurements.

Hence, two or more measurements are required together with an attitude solution algorithm. A common combination for small satellites is a sun direction vector and a magnetic field measurement [9], and are also the vectors used in this work.

The sensors give information of vectors measured within their body fixed frame of reference; these vectors are then compared to known reference vectors. There are several different methods to do this, and they are divided by the way the algorithm uses the data available. These methods are the single-frame, sequential and batch solutions [2].

¹By experience of this author, where a great deal of trouble could have been avoided.

Single frame solutions, for example the TRIAD method and the q-method, use data from a single point in time to estimate the attitude at that time [2]. These solutions are simple and robust, however; with drawbacks such as missing ways of incorporating fault tolerance and attitude estimation during periods of missing spacecraft dynamics sensor data [10]. The TRIAD algorithm allows for only two vector measurements while the q-method optimises the solution from a set of vector measurements.

The sequential (e.g. Kalman filter) and batch algorithms (e.g. Kalman Smoother) use sensor data from previous solutions that have been propagated up until present time by using rate information. Since more data is used, the random component of the error will be reduced. It is also possible to estimate the attitude at times when no sensor data is available by using rate information.

The TRIAD method is, since using only a single measurement, more sensible to random errors than to systematic. Although not used in this work, it is possible to average a set of single frame solution over time to get a better knowledge of the attitude [2].

If no rate information is available (sensor or dynamic model), a single frame solution is the only way [2]. Sequential and batch algorithms can be used with assumption about rates from dynamic models, but this severely limits the attitude solution accuracy.

TRIAD algorithm

The TRIAD method was one of the first attitude determination methods. It was first published in 1964 [11] and has traditionally [10] been the algorithm of choice for small satellites together with other single frame solutions. It takes two unit vectors², $\vec{w}_1$ and $\vec{w}_2$ as measured in the spacecraft's frame. When compared to the reference unit vectors $\vec{v}_1$ and $\vec{v}_2$ in the reference frame, the vectors satisfy (when no measurement noise is present) the relationships

$$\vec{w}_1 = A\vec{v}_1 \quad \text{and} \quad \vec{w}_2 = A\vec{v}_2 \quad (2.2)$$

A is the orthogonal DCM describing the attitude of the spacecraft with respect to the reference system, and can be found by defining two orthonormal triads. As described in [11], this is done by

$$\vec{r}_1 = \vec{v}_1 \quad \vec{r}_2 = \frac{\vec{v}_1 \times \vec{v}_2}{|\vec{v}_1 \times \vec{v}_2|} \quad \vec{r}_3 = \vec{r}_1 \times \vec{r}_2 \quad (2.3)$$

$$\vec{s}_1 = \vec{w}_1 \quad \vec{s}_2 = \frac{\vec{w}_1 \times \vec{w}_2}{|\vec{w}_1 \times \vec{w}_2|} \quad \vec{s}_3 = \vec{s}_1 \times \vec{s}_2 \quad (2.4)$$

²In this sub-section, to avoid confusion with the coming notation (see Table 3.1) of estimated values by the superscript $\hat{}$ unit vectors are not denoted by this symbol, but simply stated as unit vector.

The attitude is found by defining

$$A = [\vec{s}_1 \quad \vec{s}_2 \quad \vec{s}_3][\vec{r}_1 \quad \vec{r}_2 \quad \vec{r}_3]^T = SR^T \quad (2.5)$$

where A is a orthogonal rotation matrix that will satisfy $\vec{w}_1 = A\vec{v}_1$ exactly. When there is no noise present, $\vec{w}_2 = A\vec{v}_2$ is also satisfied. Since there always will be a certain level of noise, the first equation defined should correspond to the most exact measurement.

The limitations of this algorithm are, according to [11], that

- It assumes that the measurements are unit vectors.
- It can make use of only two unit-vector measurements.

q-method

If more than two vector measurements are available, the q-method can be used [12]. It uses N number of available measurements in order to optimise the solution of the quaternion. Details of the method is described in [12], and involves minimizing a loss-function, which is minimum when the estimated attitude matrix $\hat{A}$ becomes the best estimate. The algorithm is computationally costly as it involves computing eigenvalues.

Another algorithm, the Quaternion Estimator (QUEST), developed by Malcolm D. Shuster in the 1970's, can calculate the quaternion without the obvious drawbacks of eigenvalue decomposition. This makes it a very popular and efficient algorithm and is still in use today [9].

Kalman filter

The Kalman filter is a recursive algorithm, which reuses data in order to reduce measurement noise and estimate the state of a system. It is the most commonly used estimation algorithm [13].

This powerful method uses rate information from a dynamic model of the spacecraft to propagate earlier measurements to present time in order to increase the amount of available data, and thus reduce the random errors in the estimation. Gains are calculated continuously to give an indication on how important a measurement is.

2.2 Baseline spacecraft: PEARL

The Cubesat PEARL is under development by the Satellite Dynamics Laboratory (SDL), Logan, Utah, USA. This spacecraft has been used as a baseline for this project, in order to have real world parameters to analyse. PEARL is a prototype for a larger constellation mission, that will involve up to 100 similar spacecraft. The task is to measure and create a more detailed model of the Earth's magnetic field taking the many irregularities of the field into account.

PEARL is a spin-stabilised spacecraft equipped with a magnetometer and two sun sensors for attitude determination. Other sensors, such as Global Positioning System (GPS) receivers and Earth Horizon Crossing Indicator (HCI) are also involved in the attitude determination process, but are not considered in this work. Temperature sensors are available to estimate the bias and sensitivity thermal drift in the sensors.

Various versions of PEARL has been considered in its development. The actual original PEARL spacecraft design as used in this thesis may not be launched; however, a bias momentum descendent is expected to be launched in 2010. A truth model describing the dynamics of PEARL and a orbit propagator is available for this project, which provides the dynamics that are to be measured and used in the attitude determination process.

2.3 Hardware

Different kinds of sensor hardware are available to get the vector measurements needed for the attitude solution algorithm. These sensors have a limit to the level of accuracy that can be achieved. The type of sensors represent how accurate the measurement can be, and the quality of the sensor determines the range of accuracy achievable. An example of available accuracy is a sun sensor which can achieve between 0.015° to 0.5° [8] accuracy depending on the specific sensor.

No sensor is perfect, e.g. the sensitivity will be known to a certain accuracy, and the accuracy will also vary with the temperature. There will be a bias in the sensor, and an estimate of the bias is used in the reconstruction to the measured property. There will always be a level of uncertainty and the properties will also have an uncertain drift with the temperature. Further, all sensors are also subjected to a certain degree of noise.

All of the uncertainties present in the sensor propagate through the system to the estimate of the attitude. What is interesting to determine is the impact the different sensor uncertainties have on the attitude estimate, which is the goal of this work. To achieve this, a sun sensor is used together with a magnetometer to determine the spacecrafts attitude.

2.3.1 Mounting

The sensors measure the angles to a target relative to the sensor. However; the sensor is mounted relative the BFCS and this must be taken into account when estimating the attitude from sensor data. Accurate knowledge of the sensor mounting is crucial and the analysis of the attitude error resulting from sensor misalignment is important.

The case where the sensor's axes are not a perfect triad is also crucial. Even if this is known to near perfection after calibration at manufacturing, launch vibrations will add to the mounting uncertainty. Hence, assembling of sensors usually causes system errors [14].

Most sensors can provide a unit vector in direction to the target; however, this gives no information about the rotation around this direction vector. For this, two separate vector measurements are needed, which are usually obtained from different sensors [2].

2.3.2 Magnetometer (Three-axis)

Magnetometers are a common sensor used on spacecrafts for attitude determination, mainly because they are reliable and of low weight [7], thus being suitable for small spacecrafts such as Cubesats. Figure 2.1 shows the magnetic sensor hybrid Honeywell HMC2003 used on PEARL, a Cubesat being developed by Satellite Dynamics Laboratory, Logan, Utah, USA, which is used as a baseline for this work.

This magnetometer, which is of three axis configuration using high sensitivity magneto-resistive sensors (HMC1001) from Honeywell, has a very low magnetic material content and a high sensitivity in order to measure magnetic fields of low strengths [15], thus making it suitable for orbits with higher altitudes. As will be discussed later in Chapter 5, a Sun-synchronous orbit is used in this work, which by nature has a high inclination, causing a rapidly changing magnetic field at high latitudes of the orbit.

A regular magnetometer measures the Earth's magnetic field $\vec{B}$ components B_x , B_y , B_z direction and magnitude. A three axis magnetometer is really three single orthogonally mounted sensors that each measures the corresponding magnetic field component. These kind of sensors are very sensitive to residual magnetic fields from the spacecraft's electronics, and are hence usually mounted on a boom as far away as possible from the spacecraft's bus.

The residual field can be mitigated by the choice of the spacecraft's structure materials, however, a bias still persists. The sensors obtain information of the BFCS magnetic field vectors which are compared to models of the Earth's $\vec{B}$ -field. These models are usually a tilted dipole model with various level of complexity

Figure 2.1: PEARL Cubesat Magnetometer: Honeywell HMC2003 [15]

added to it. The advantage of using magnetometers is continuous measurements, while a strong disadvantage is the limited knowledge of the Earth's magnetic field [7].

The magnetometer provides a complete vector measurement that includes the magnetic field's direction and magnitude at the present position. Knowledge of the spacecraft's position in the orbit together with a model of the Earth's magnetic field are required in order to solve for the attitude.

The actual way a magneto-resistive sensor senses the magnetic field applied to it is quite complicated. It can however be explained rather simply as a thin film of nickel-iron that is applied on a wafer made from silicon. When a magnetic field is applied, a change in the resistance causes the voltage output to change, which can be measured. The change of the resistance in the nickel-iron is called a magneto-resistive effect [16].

2.3.3 Sun sensor

Also common for attitude determination is a sun sensor. As an illustration, Fig. 2.2 shows the sun sensor used on the Cubesat SwissCube, since sun sensors for the PEARL spacecraft is still under development.

A sun sensor gives information of the unit direction vector to the sun, as seen by

Figure 2.2: A Cubesat sensor: Sun sensor [17]

the sensor. As with magnetometers, sun sensors are very common, and are used for attitude determination on virtually every spacecraft. This is mainly because of the sun's luminosity (the sensor is not affected by stars) and small angular radius (point source approximation) [7]. The best possible accuracy is about 0.015° [8] for a good quality sensor.

A strong disadvantage is that the sun is not always visible. During the spacecraft's orbit multiple sun sensors are needed to keep the sun in Field Of View (FOV). Also, the Earth may cover the sun causing an eclipse, and if the spacecraft is of a spinning configuration the sun will *rotate* outside the FOV of the sensor.

To get a solution of the attitude, knowledge of the sun's position in the reference system is needed. This is provided by a model of the Earth's orbit around the sun, from which sun vectors in ECI can be calculated.

2.3.4 Other sensors

A rate sensor is not really an attitude sensor, but an attitude rate sensor. However, attitude can be solved by integration from an initial attitude. Rate gyros measure the angular rate of the spacecraft with respect to inertial space. At least three are needed for three-axis measurement, usually, four are used in a special mounting configuration to include a level of redundancy [4].

A star sensor has not been used in the work, but is included in the discussion as a comparison of available accuracy. It provides information about the angular displacement from the sensor's boresight and the target, usually in the form of unit vectors in direction to multiple stars [2].

To solve for attitude information, the star must be identified and the actual position determined from a star catalogue. Some star sensors can track more than one star at a time, providing enough information for a attitude solution from a single

frame method. Star sensors suffer from high weight, computing, and power requirements as they are the most accurate of all the sensors. Accuracies of an arc-second are achievable; however, with increasing cost, and are not very suitable on small a spacecraft.

Another attitude determination sensor is HCI that scans a ring in space and detects the Earth's horizon, accuracies better than 0.1° is possible for low Earth orbits [8].

2.4 Mathematical tools

A work about uncertainties requires some knowledge about statistics and probability. In this section a basic review is presented. This review is based on basic knowledge and can be found elsewhere throughout the literature. The books *An Introduction to Uncertainty in Measurement* [1] and the appendix of *Statistical Orbit Determination* [18] complements each other in a convenient way where the latter provides formal definitions and the former present basic examples.

Consider the function $y = f(u)$ where u is a Gaussian random variable $N(\bar{u}, \sigma_u)$ with the probability density function f_{pd} . The mean $\bar{u}$, or nominal value of u , is given by the expectation operator E

$$E(u) = \int_{-\infty}^{\infty} u f_{pd}(\zeta) d\zeta = \bar{u} \quad (2.6)$$

whereas the variance σ_u^2 of u is

$$\sigma_u^2 = E((u - \bar{u})^2) \quad (2.7)$$

It follows that the mean of y is $\bar{y} = E(f(\bar{u}))$ and the variance σ_y^2 of y is

$$\sigma_y^2 = E((y - \bar{y})^2) \quad (2.8)$$

For the linear function $y(u) = au + b$ it is easy to see that if u follows a distribution about a mean $\bar{u}$, then y will follow the same distribution about $\bar{y}$. When the function becomes a nonlinear system, this is not as easy to conclude.

2.4.1 Monte Carlo analysis

In a complicated nonlinear system the variance matrix P_y can statistically be estimated; however, with a highly computational effort. The Monte Carlo analysis to calculate the variance of the system $y = f(u)$ is described by the pseudo code below.

```

for i=N
select  $u_i$  from  $N(\bar{u}, \sigma_u)$ 
 $y_i = f(u_i)$ 
end
 $\bar{y} = \frac{1}{N} \sum y_i$ 
 $\hat{P} = \frac{1}{N-1} \sum (y_i - \bar{y})^2$ 

```

As can be seen, if y is a complicated system requiring many calculations to compute, it can be very time consuming to calculate a converged value of the Covariance P . An alternative approach is the Linear Covariance Analysis. As the name implies, this is only valid for linear system, thus $y = f(u)$ must be linearised.

2.4.2 Linear Covariance

The derivation of the time propagation of the covariance of a continuous or discrete random process is too long and doesn't belong here. A good derivation (with a slightly different notation) can be found in the first volume of Maybeck's *Stochastic Models, Estimation, and Control* [19]

In this section, it's enough to state the equations for the propagation of the discrete case. The linear discrete state space representation is given by

$$\begin{aligned}\vec{x}_{k+1} &= \phi \vec{x}_k + \Gamma \vec{u}_k \\ \vec{y}_k &= C \vec{x}_k + D \vec{u}_k\end{aligned}\quad (2.9)$$

where x is the state variables, u is the inputs to the system, and y is the outputs. The conversion from a continuous state space model

$$\begin{aligned}\dot{\vec{x}} &= A \vec{x} + B \vec{u} \\ \vec{y} &= C \vec{x} + D \vec{u}\end{aligned}\quad (2.10)$$

to the discrete form is done by Eqn. 3.109 a-b derived in [12]

$$\begin{aligned}\phi &= e^{A\Delta t} \\ \Gamma &= \left[\int_0^{\Delta t} e^{A t} dt \right] B\end{aligned}\quad (2.11)$$

A is the state matrix and B is the input matrix of the state space model. The output matrix C and feed through matrix D are unaffected by the conversion.

Maybeck [19] presents an equation of discrete form to propagate the covariance P of the states

$$P(t_{i+1}) = \phi(t_i)P(t_i)\phi(t_i)^T + \Gamma(t_i)Q\Gamma(t_i)^T \quad (2.12)$$

where P is the state covariance matrix and Q is the noise covariance matrix of a Gaussian white noise process that is uncorrelated in time.

Assumptions during the derivation of the propagation equation includes that the system is driven by Gaussian noise, thus the Q matrix contains Gaussian white

noise only. To propagate the variance of the biases in the system, they must be defined as states.

Converging covariance argument

By making the assumption that the system is approximately time invariant and defining a converged covariance matrix as

$$P_c = \lim_{k \rightarrow \infty} P(t_k) \quad (2.13)$$

$$P_c = \phi P_c \phi^T + \Gamma Q \Gamma^T \quad (2.14)$$

Q is diagonal by definition (Gaussian white noise), then, without proof, the claim that P_c is approximately diagonal is made.

$$y = f(u) = Cu \quad (2.15)$$

$$\text{Cov}(y) \approx E[yy^T] = CE(uu^T)C^T \quad (2.16)$$

$$\approx CP_c C \quad (2.17)$$

$$\approx \Sigma c_i^2 P_{ii} \quad (2.18)$$

General discussion

Linear Covariance has the disadvantage of time consuming set up of the model. An advantage is that after the linear matrices have been set up, the propagation can be repeated with different values of the distributions with simple matrix algebra.

The algorithm for using Linear Covariance involves the linearisation of a non-linear model along a user-defined nominal trajectory (operating points). Errors in constant parameters will have a random distribution. They are constant and unknown and are treated as biases defined as states.

The uncertainties are assumed to be Gaussian distributed. A state covariance matrix P is then propagated forward in time from an initial condition P_0 . Errors in this approach include assumptions of the nominal path and the assumption that uncertainties are normally distributed. However; the Central Limit Theorem (CLT) validates an assumption to treat the uncertainties as a random variable with Gaussian distribution and zero mean.

Central Limit Theorem

A fundamental assumption in this work is that the uncertainties are distributed normally. This is validated by the Central Limit Theorem:

“The tendency for the distribution of sums and means of samples taken from a distribution to become nearly Gaussian as the sample size increases is a prediction of the central limit theorem” [1, p.144]

This means that a large set of various distribution added together produce a Gaussian distribution. Since the uncertainties present in a system consist of various kinds of different distributions added together, the assumption of normal distribution is valid by the Central Limit Theorem.

2.4.3 Linear Covariance vs Monte Carlo

In a traditional Monte Carlo simulation, the covariance is calculated from a large number of simulations of a model, where for each individual simulation the parameters are randomly chosen from a distribution. This can be very time consuming for large complicated system models. Also, if a change to the system is made, for example changing a component to one with different specifications, the entire simulation must be repeated.

For the Linear Covariance method, the covariance of the output is obtained by propagating the covariance from input to output. If the system is linear, the resulting covariance as obtained from the Monte Carlo analysis will converge towards the Linear Covariance result. If the system is non-linear (as most real world systems are), the nonlinear equations describing the dynamics of the system must be linearised, assuming small changes in attitude. If system changes are large, as in re-pointing manoeuvres, one must obtain a set of linear equations at small instances in time over a user defined nominal path.

By using a state space representation, the covariance is easily propagated by the state transition matrix ϕ . Tortora et al. [20] states that it is well known that if the state transition matrix is used only for the propagation of the covariance matrix, the accuracy of ϕ it is not crucial, and a first order approximation is generally sufficient.

This statement motivates the linearisation of the model. The linearisation is still a major disadvantage as it induces errors in the system, however, for orbital dynamics, the deviations from the nominal path is usually very small and a linear approximation has proven to be accurate for orbital dynamics [21].

Chapter 3

Illustrative example: One degree of freedom

To clearly demonstrate the attitude accuracy problem and Linear Covariance as an analysis tool, an illustrative one degree of freedom problem is defined. The spacecraft in the problem has one rate sensor and one attitude sensor measuring the angular velocity and orientation angle of the spacecraft relative to a defined frame. Given a set of uncertainties present in the sensors, the interest lies in how these uncertainties propagate to the resulting attitude determination accuracy of the spacecraft.

As the true property x is unknown, it is denoted by $x = \hat{x} + \delta x$, where $\hat{x}$ is the estimated value of the true property and δx is the unknown error. This error is treated as a Gaussian random variable with zero mean and standard deviation of σ_x and is considered as a small perturbation from the true value.

Measured values will be denoted by $\tilde{x}$ while nominal values during the linearisation are denoted by $\bar{x}$. A summary of the superscript notation is presented in Table 3.1.

Table 3.1: Notation: The superscripts on the property x denotes how the value is obtained.

Value	Superscript
True Value	x
Nominal Value	$\bar{x}$
Estimated Value	$\hat{x}$
Measured Value	$\tilde{x}$
Prefix	
Error (small perturbation)	δx

3.1 Second order nonlinear differential equation

Figure 3.1: One degree of freedom satellite model designed for the illustrative example.

Consider the satellite model shown in Fig. 3.1. This simplified one degree of freedom example can be thought of as a space telescope that wants to point its instruments in a certain direction, for example, to a distant star. The model has a reference angle θ^* as input, and the resulting attitude angle θ as output.

It is straight forward to represent the dynamics of the model by writing it as a differential equation. Models for the dynamics and control law are

$$J\ddot{\theta} = M + \delta M_d \quad (3.1)$$

$$M = (k_p(\theta^* - \hat{\theta}) - k_d\hat{\omega})\hat{f} \quad (3.2)$$

where M is the control torque, J is the principal moment of inertia, and M_d is a random white noise disturbance torque, resulting from various environmental effects or actuator imperfections.

A sensor, S , is included in each feedback loop which measure the spacecraft's angular velocity $\omega = \dot{\theta}$, and the orientation angle θ relative an inertial reference frame. The rate and attitude measurements are given by the sensors S as

$$\tilde{V}_\omega = S_\omega \dot{\theta} + V_{\omega_o} + \delta\eta_\omega \quad (3.3)$$

$$\tilde{V}_\theta = S_\theta \theta + V_{\theta_o} + \delta\eta_\theta \quad (3.4)$$

The sensor models multiply an input with the true sensitivity S resulting in a voltage, and add the true voltage bias, V_o , and noise, $\delta\eta$.

The gains k_p and k_d are known and defined without error by the designer. The bias, sensitivity and inertia have constant errors and are treated as Gaussian random initial conditions with zero mean and standard deviations of σ . The noises are seen as a zero mean Gaussian random white noise process. The uncertain parameters present in the model are summarised in Table 3.2.

The output from the sensors is a voltage, meaning that an estimate of the angular velocity $\hat{\omega}$ and attitude $\hat{\theta}$ is must computed by a reconstructor, R , implemented in the feedback loops.

Table 3.2: Uncertain parameters, the true value x is modelled as the estimated value $\hat{x}$ + the error δ in the estimate.

Property	Symbol
Inertia	$J = \hat{J} + \delta J$
Attitude sensor sensitivity	$S_\theta = \hat{S}_\theta + \delta S_\theta$
Rate sensor sensitivity	$S_\omega = \hat{S}_\omega + \delta S_\omega$
Attitude sensor bias voltage	$V_{\theta_o} = \hat{V}_{\theta_o} + \delta V_{\theta_o}$
Rate sensor bias voltage	$V_{\omega_o} = \hat{V}_{\omega_o} + \delta V_{\omega_o}$
Attitude sensor noise	$N_\theta = \delta \eta_\theta$
Rate sensor noise	$N_\omega = \delta \eta_\omega$
Disturbance noise	$M_d = \delta M_d$

$$\hat{\omega} = \frac{\tilde{V}_\omega - \hat{V}_{\omega_o}}{\hat{S}_\omega} \quad (3.5)$$

$$\hat{\theta} = \frac{\tilde{V}_\theta - \hat{V}_{\theta_o}}{\hat{S}_\theta} \quad (3.6)$$

Equations (3.5) and (3.6) reconstruct rate and attitude angle using the measured voltages $\tilde{V}$, the estimated values of the bias $\hat{V}_0$, and sensitivity $\hat{S}$.

By substituting Eqs. (3.2) - (3.6) into Eqn. (3.1) and replacing the true values $J S_\theta S_\omega V_\theta V_\omega$ with the estimated values and their errors from Table 3.2, the second order non-linear differential equation representing the dynamics of the system is obtained as described by Eqn.(3.7).

$$(\hat{J} + \delta J)\ddot{\theta} + \frac{k_d \hat{J}}{\hat{S}_\omega}(\hat{S}_\omega + \delta S_\omega)\dot{\theta} + \frac{k_p \hat{J}}{\hat{S}_\theta}(\hat{S}_\theta + \delta S_\theta)\theta = k_p \hat{J} \theta^* + \delta M_d - \frac{k_p \hat{J}}{\hat{S}_\theta} \delta V_{\theta_o} - \frac{k_d \hat{J}}{\hat{S}_\omega} \delta V_{\omega_o} - \frac{k_p \hat{J}}{\hat{S}_\theta} \delta \eta_\theta - \frac{k_d \hat{J}}{\hat{S}_\omega} \delta \eta_\omega \quad (3.7)$$

The estimates induce an error in the measurement and a disturbance torque, δM_d , caused by various environmental factors affects the model.

To investigate how much the errors affect the attitude determination, the nonlinear differential equation is linearised and the sensor and actuator uncertainties are propagated to the attitude by Linear Covariance.

3.1.1 Nominal response

By setting all the error terms, δ , in Eqn. (3.7) to zero, the estimated value, $\hat{x}$, equals the true value x . Then the differential equation describing the nominal response

$\bar{\theta}(t)$ to a perfect system is familiar second order equation

$$\ddot{\bar{\theta}} + k_d \dot{\bar{\theta}} + k_p \bar{\theta} = k_p \theta^* \quad (3.8)$$

This nominal response is used to linearise Eqn (3.7) so that perturbations from this path can be considered.

3.2 Linearisation

A linear state space representation can be obtained as follows. Equation (3.7) can be written in the form

$$\ddot{\theta} = \ddot{\theta}(\theta^*, \hat{\theta}, \hat{\omega}) \quad (3.9)$$

$$\hat{\theta} = \hat{\theta}(\theta, \hat{S}_\theta, \hat{V}_\theta, \eta_\theta, M_d, J) \quad (3.10)$$

$$\hat{\omega} = \hat{\omega}(\omega, \hat{S}_\omega, \hat{V}_\omega, \eta_\omega, M_d, J) \quad (3.11)$$

As discussed in Sec. 2.4.2, the bias uncertainty in the system should be defined as states. The state variables are thus

$$\vec{x} = [\theta \quad \dot{\theta} \quad \delta V_{\theta_0} \quad \delta V_{\omega_0} \quad \delta S_\theta \quad \delta S_\omega \quad \delta J]^T \quad (3.12)$$

and the noise inputs are

$$\vec{u} = [\delta M_d \quad \delta \eta_\theta \quad \delta \eta_\omega]^T \quad (3.13)$$

The nonlinear state space representation is given by

$$\dot{\vec{x}} = A\vec{x} + B\vec{u} = F(\vec{x}, \vec{u}) \quad (3.14)$$

$$\vec{y} = C\vec{x} + D\vec{u} \quad (3.15)$$

Linearising about the nominal path $\bar{\theta}(t)$ defined in Sec. 3.1.1 gives

$$\Delta \dot{\vec{x}}(t) = \left. \frac{\partial F}{\partial \vec{x}} \right|_{\substack{\bar{\theta}(t) \\ \dot{\bar{\theta}}(t) \\ \delta=0}} \Delta \vec{x} + \left. \frac{\partial F}{\partial \vec{u}} \right|_{\substack{\bar{\theta}(t) \\ \dot{\bar{\theta}}(t) \\ \delta=0}} \Delta \vec{u} \quad (3.16)$$

$$\Delta x = x - \bar{x} \quad (3.17)$$

$$\Delta u = u - \bar{u} \quad (3.18)$$

where $\Delta \dot{\vec{x}}(t) \approx \dot{\vec{x}}(t)$ is valid for small deviations from the nominal path $\bar{\theta}$. The biases are constant, so their derivatives are zero and the reference value θ^* is assumed to be a constant input. The variable $\bar{\theta}(t)$ in Eq.(3.16) is the solution to the nominal response Eqn. (3.8).

The differentiation results in the linear state space representation

$$A(t) = \frac{\partial F}{\partial \vec{x}} = \begin{bmatrix} \frac{\partial F}{\partial \theta} & \frac{\partial F}{\partial \dot{\theta}} & \frac{\partial F}{\partial \delta V_{\theta_0}} & \frac{\partial F}{\partial \delta V_{\omega_0}} & \frac{\partial F}{\partial \delta S_{\theta}} & \frac{\partial F}{\partial \delta S_{\omega}} & \frac{\partial F}{\partial \delta J} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -k_p & -k_d & -\frac{k_p}{\hat{S}_{\theta}} & -\frac{k_d}{\hat{S}_{\omega}} & -\frac{k_p}{\hat{S}_{\theta}}\bar{\theta}(t) & -\frac{k_d}{\hat{S}_{\omega}}\dot{\bar{\theta}}(t) & \frac{\partial F}{\partial J} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.19)$$

$$\frac{\partial F}{\partial J} = \frac{k_d}{\hat{J}}\dot{\bar{\theta}}(t) + \frac{k_p}{\hat{J}}\bar{\theta}(t) - \frac{k_p}{\hat{J}}\theta^* \quad (3.20)$$

$$B = \frac{\partial F}{\partial \vec{u}} = \begin{bmatrix} \frac{\partial F}{\partial \delta M_d} & \frac{\partial F}{\partial \delta \eta_{\theta}} & \frac{\partial F}{\partial \delta \eta_{\omega}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{\hat{J}} & -\frac{k_p}{\hat{S}_{\theta}} & -\frac{k_d}{\hat{S}_{\omega}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (3.21)$$

If the output is defined as the original state θ , then the output matrix C and the feed through matrix D are given by

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.22)$$

$$D = 0_{1 \times 3}$$

The complete continuous linear state space representation is then given by

$$\begin{aligned} \Delta \vec{x}(t) &= A(t)\Delta \vec{x}(t) + B\Delta \vec{u} \\ \Delta \vec{y}(t) &= C\Delta \vec{x}(t) + D\Delta \vec{u} \end{aligned} \quad (3.23)$$

The states are dependent on the estimated values, also note that the A matrix is time dependent. To get an accurate representation over a period of time, state space models at different points in time should be obtained during the interval considered.

3.3 Linear Covariance Analysis

With the continuous linear state space representation in hand, conversion to discrete representation is given by Eqn. (2.11). Then the variance of the uncertainties can be propagated by Eqn. (2.12).

An initial state covariance matrix P_o is defined by using the variance of the estimates of the states as entries on the diagonal of P where the states are assumed uncorrelated. The Q matrix is the variance of the noise as entries in the diagonal of Q . The symbol σ denotes the standard deviation.

$$P_o = \begin{bmatrix} \sigma_\theta^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{\dot{\theta}}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{V_{\theta_0}}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{V_{\omega_0}}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{S_\theta}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{S_\omega}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_J^2 \end{bmatrix} \quad (3.24)$$

$$Q = \begin{bmatrix} \sigma_{M_d}^2 & 0 & 0 \\ 0 & \sigma_{\eta_\theta}^2 & 0 \\ 0 & 0 & \sigma_{\eta_\omega}^2 \end{bmatrix} \quad (3.25)$$

To get the variance of the attitude, use the output and feed through matrices and take

$$P_\theta = CPC^T + DQD^T \quad (3.26)$$

Since the model is linearised, the covariance propagation can be applied with only one non-zero uncertainty at a time, in order to determine how much influence each parameter has on the attitude covariance. Thus repeated propagation for each uncertainty allows for the determination of the contribution to the total variance.

The total variance is simply the sum of each variance resulting from uncertainty whereas the total standard deviation is the square root of the total variance.

3.4 Numerical example

The parameters used in the model are given in Table 3.3¹ The initial attitude and rate variances is set to zero at the time of simulation start, in order to see how the sensor uncertainties propagate to the attitude determination. As explained in Sec. 2.4.2, the bias uncertainties are treated as Gaussian random variables with zero mean and standard deviation of σ , while the noise is assumed to be a Gaussian white noise process.

The software MATLAB² is used to substitute numerical values to produce a set of discrete state space representations at equal points in time, and to propagate the state covariance matrix for a time period of 15 seconds. The total attitude

¹These values are typical and chosen to demonstrate Linear Covariance Analysis. The gains k_p and k_d have been chosen to give a non-oscillatory response.

²Chapter 4 introduces MATLAB and the Simulink environment.

Table 3.3: Numerical values of the illustrative example model in Fig. 4.1 used in the calculations of output standard deviation.

Parameter	Value	Dimension
Known values		
Controller Gain	$Kd = 3$	Nm·s/deg
Controller Gain	$Kp = 2$	Nm/deg
Input reference value	$\theta^* = 15$	deg
Estimated Values		
Rate sensor bias	$\hat{V}_{\omega_0} = 2.5$	V
Rate sensor sensitivity	$\hat{S}_{\omega} = 5.0$	V/(deg·s)
Attitude sensor bias	$\hat{V}_{\theta_0} = 2.0$	V
Attitude sensor sensitivity	$\hat{S}_{\theta} = 4.0$	V/deg
Inertia	$\hat{J} = 4.0$	kg/m ²
Uncertainties and noise		
Rate sensor bias	$\sigma_{V_{\omega_0}} = 0.1$	V
Rate sensor sensitivity	$\sigma_{S_{\omega}} = 0.1$	V/(deg·s)
Attitude sensor bias	$\sigma_{V_{\theta_0}} = 0.2$	V
Attitude sensor sensitivity	$\sigma_{S_{\theta}} = 0.1$	V/deg
Inertia	$\sigma_J = 0.2$	kg/m ²
Rate sensor noise	$\sigma_{N_{\omega}} = 0.3$	V
Attitude sensor noise	$\sigma_{N_{\theta}} = 0.2$	V
Disturbance torque	$\sigma_{M_d} = 0.4$	N·m

determination accuracy at steady state is calculated to 0.38 deg and plotted in Fig. 3.2 together with validation by Monte Carlo analysis according to Sec. 2.4.1

The question is; what uncertainty has the largest contribution, that is, what factors are more important than others? This can easily and quickly be determined by Linear Covariance.

The result of the Linear covariance analysis on each input is presented in Fig. 3.3 (a-d) and Fig. 3.4 (a-d), also with validation by Monte Carlo analysis. These figures show the resulting attitude determination accuracy resulting from the respective uncertainties present in the model.

As can be seen in Fig. 3.3 (c), a standard deviation from the attitude solution of about 0.375 deg results from the uncertainty in the attitude sensor sensitivity, which certainty is the largest contribution to the total standard deviation of the output.

It can also be noted that the uncertainty in the inertia and the rate sensor sensitivity has no influence on the final attitude variance at steady state. Errors from these uncertainties is only resulting from the repositioning. This results in an overshoot of the attitude and a longer time to reach steady state. If quick manoeuvres is needed, focus should be made on calibration of these.

Figure 3.2: The total RSS (Root-Sum-Squared) standard deviation of the attitude estimate, resulting from all input uncertainties combined. The initial standard deviation is zero.

With this determined, the designer can decide to exchange the sensor, with one with a more accurate knowledge of the sensitivity. If this change were made, the Monte Carlo simulation must be repeated, in contrast to Linear Covariance, where a new Linear Covariance analysis on the already linearised model saves processing time.

Comparison between methods³

The 1000 run Monte Carlo analysis for one input took 390 s which has to be repeated for each input resulting in a total time of 52 min. One thousand runs is enough for the biases to converge to the Linear Covariance result; however, the noise terms require many more runs to get an accurate estimate. A 10 000 run

³The times stated in this section for the Monte Carlo method is not in any way optimised, the simulation has been run using a Simulink model of Eqn. (3.7) with the normal accelerator mode on a laptop equipped with a Intel Dual Core 1.83 GHz processor. This model can be found in Chapter 4 together with an introduction to the Simulink software. It is very possible that by setting up the simulation differently might decrease the time. However, the Linear Covariance approach still has a strong advantage.

Figure 3.3: The standard deviation of the attitude estimate, corresponding from the specific input uncertainty specified by the caption under the figures. The initial standard deviation is zero.

Monte Carlo analysis converges the noise terms slightly more towards the Linear Covariance result, but it is evident that even more would be required to get an accurate result. One case of 10 000 runs took on average about 1.4 hours, and running all cases resulted in a simulation time of more than 10 hours.

The model only has to be linearised once, and after that, repeated Linear Covariance analysis can be performed on the linear model. The results for a simple system demonstrate the power of Linear Covariance as a pre-design tool. The resulting linearisation error may be tolerated at first, and after iterating through the design process and obtaining a proper design that meets the requirements, run a Monte Carlo analysis to verify the design results obtained from the much quicker Linear Covariance analysis.

Regarding the linearisation error, it can be observed from the linearised input matrix (3.21) that the uncertainties in rate and attitude sensor sensitivity as well as the inertia, depend on the nominal response, which was assumed to be *error free*. Since this is not the case, these terms will contribute with the largest linearisation error.

Figure 3.4: The standard deviation of the attitude estimate, corresponding from the specific input uncertainty specified by the caption under the figures. The initial standard deviation is zero.

3.5 Extension to larger systems

Extending the single Degree Of Freedom (DOF) to three DOF, with the inclusion of more sensors and multiple-axis measurements, will lead to a drastic increase in uncertainties (error sources).

Sensor cross-coupling might be an issue. For example; on a spinning spacecraft, the solar panel's varying current output as it rotates in and out of the sun's creates a magnetic field that will disturb the magnetometers.

Coupling between sensors is not treated in this project as the goal is to find a simple pre-design relation of input uncertainty to output uncertainty. When moving to more complex systems, dynamics also enter the problem.

The previous example motivated the use of Linear Covariance analysis as a pre-design tool; however, linearisation by hand for larger, more complex system would rapidly become unmanageable. Matlab's Control System toolbox provides

means of linearising a Simulink model and return it as a discrete state space model. For this project, generalised M-functions are created that automates all steps undertaken in the previous example and presents the result in a way that keeps track of a large number of inputs propagated to every output.

3.6 Summary of the Linear Covariance approach

The Linear Covariance approach has been demonstrated by the use of an illustrative one degree of freedom example. By the use of a model that is subjected to uncertainties, that can be modelled as perturbation from a nominal value, one can linearise the model with the goal to obtain a state space representation. This representation can be used to propagate the input perturbation to the output in order to check the effect this uncertainty has on the system. Once the state space representation is obtained, repeated propagation can be undertaken by simple matrix algebra providing a computational advantage to other methods available such as the Monte Carlo approach.

Chapter 4

The programing environment and the M-functions

MATLAB is a software featuring computing language. It provides powerful methods of computation and visualisation of technical problems. MATLAB is especially good for problems involving matrices and vectors; in fact, the abbreviation MATLAB stands for Matrix Laboratory. This software is thus highly suitable for this project that involves matrix operations.

Collections of specific tools, called M-functions, for specific problems are grouped in tool boxes, which contain MATLAB functions that extend the MATLAB environment for specific problems. One such toolbox is the Linear Analysis toolbox. This toolbox provides M-functions that computes the linear state space representation of a Simulink model.

Simulink is an application that is included in MATLAB and provides a graphical user interface in order to easily build models as block diagrams and also include user specific code by embedded functions. With Simulink it is possible to analyse and model both linear and nonlinear dynamic systems. The Simulink model used for the Monte Carlo validation of the illustrative example in Chapter 3 is shown in Fig. 4.1. Easy cut and paste operations of blocks also provide methods for easily upgrading and changing any model.

MATLAB also allows for the development of user specific M-functions. The M-functions developed for this project use MATLAB functions from the Linear Analyses toolbox to linearise a dynamic model described in Simulink and propagate the variance according to the discussion in 2.4.2. These functions take away the major hurdle of the Linear Covariance approach, i.e. determine the linearised model for a large number of uncertainty based states.

In the Simulink model, one can simply use the graphical *right-click-mark* method of the Linear Analysis toolbox to define the input/output points in the model that is to be analysed, which is extracted and passed as an argument to the developed M-functions.

(a) Satellite model

(b) Attitude sensor model

Figure 4.1: One degree of freedom Simulink satellite model corresponding to Fig.3.1 which was designed for the illustrative example in Chapter 3. The rate sensor works in the same way as the attitude sensor while the reconstructors are embedded MATLAB functions consisting of Eqs. (3.5) and (3.6)

Three major generalized functions are written, to be called in a specific order. The work-flow is pictured in Fig. 4.2. As can be seen, an intuitive way of storing data in structures keeps all information together in one variable, which is then passed to the next function as an argument. One objective of this project was to create a generalized pre-design analysis-tool, so a detailed explanation of the functions is justified.

A typical MATLAB session for applying the functions to the Simulink model shown in Fig.4.1 named `IllustrativeExample` for a time span of 15 seconds using an operating point every 0.01 second is

```
io=getlinio('IllustrativeExample') % MATLAB Linear toolbox function
model=reDefLin('IllustrativeExample',io,15,1500)
variance=LinCov(model)
inspectLinCov(variance,1,0)
```

which provides the same result as the analytical approach in Chapter 3, but removed the work of finding and linearising the differential equation that describes the system's dynamics. The zero order hold, that can be seen in Fig.4.1, is equivalent to the conversion to discrete form in the analytical approach.

Figure 4.2: The M-functions' work flow, arguments and outputs

Observe that the uncertainties are modelled as described in Chapter3, i.e. estimate plus the error. In this form the error is set to zero in the nominal case, to which a perturbation is added. This has been done for clarity. One could use the estimate value as input point and setting that value to the nominal, which Simulink then will add a perturbation to, thus reducing the amount of blocks needed.

The first function `getlinio` is a MATLAB toolbox function that extracts the linearisation input and output points from the Simulink model while the second function `reDefLin` performs the actual linearisation of the model. The task of `LinCov` is to propagate the variances and `inspectLinCov` provides an visualisation of the result. The individual functions developed are described in detail below.

4.1 ReDefLin: Create a Linear State Space representation

`ReDefLin(MODELNAME, IO, SIMULATIONTIME, OPERATINGPOINTS)`

The function `ReDefLin` (`ReDefineLinearisaton`) linearises the model using MATLAB's numerical perturbation algorithm. This method numerically perturbrates the model's inputs and states around specified operating points. This is a quick and simple method normally used on large complicated systems [22] where the algorithm applies a small perturbation to the input. The response is measured and used together with the perturbation to create a linear state space model.

The nominal path required for the linearisation is interpreted as the value in the root-level-input ports of the Simulink model, the value at the input port should therefore be set to nominal. The linearisation process will then perturbate around this value.

For a non-spinning spacecraft, one operating point would be enough¹ as it would be valid for some time in orbit until deviations from the path have grown too big. For a spinning spacecraft, the states from the measurement model of the non-linear system change rapidly, therefore a set of operating points should be defined in order to create a set of discrete linear state space equations.

In the `LinCov` function, one state space representation is used to propagate by the discrete sampling time step, until the next operating point. This is all taken care of automatically, by specifying the time-span `SIMULATIONTIME` and the number of operatingpoints `OPERATINGPOINTS` on that interval.

A high number of operating points will increase the linearisation time; however, the time variation of the results will get more accurate. The model only has to be linearised once, the state space representation is stored which can be used with different variances of the uncertainties.

The input/output points can be defined graphically in the model by right clicking on the Simulink signal and choosing *Linearisation points*. These points can be seen in Fig.4.1 as down arrows for input and up arrow for output. They are then extracted and stored as the `IO` object by the MATLAB function `getlinio`. There are other ways of specifying `IO` objects, e.g. by command lines. Refer to MATLAB's documentation for these. `ReDefMod` takes `IO` as its second argument.

The above arguments together with the name of the Simulink model to be linearised is all that is needed to set up the linearised matrices to prepare for the propagation of the variances. Observe that the model needs to be discrete time, i.e. there should be a zero-order-hold present in the system.

The function returns a two part structure `LINEARMODEL`, where one part consists of

¹Since linearisation for orbital dynamics is proven accurate [21]

an array `model`, containing the linear discrete state space models with one model for each operating point, where the biases have properly² been defined as states.

The other part is a sub-structure `originalModelInfo` which refers to the linearised model before the biases have been defined as states. This substructure contains the information listed below, to provide the other functions with information needed to propagate and present the data in an intuitive way.

- `StateName` The name of the original states in the model
- `InputName` The input points' name, i.e the uncertainties
- `OutputName` The output points' name
- `paramters` (sub structure)
 - `OriginalSystemSize`
 - `discreteSamplingTime`
 - `operatingPointTime`
 - `stateOperatingCondition`
 - `linearisationTime`
- `inputStd`

The names are extracted from the Simulink model. If a signal name is given; this will be used; if no signal name is defined, the name of the block will be used. `InputName` refers to the linearisation points, thus defining the uncertainties in the system. The vector `OriginalSystemSize` contains the number of states, outputs and inputs of the system before biases are defined as states. `operatingPointTime` specifies what operating points where used in the linearisation and `stateOperatingCondition` contains the current state at the corresponding operating point.

The current state is not needed after the linear system is obtained; however, if the designer spots results in the Linear Covariance analysis that needs further investigation he/she can refer to this parameter to see what was going on in the system. `linearisationTime` is the total time of the linearisation process. The last parameter `inputStd` is explained in the following section.

4.2 LinCov: Propagate the variance

`LinCov(LINEARMODEL)`

²Actually, information that the designer specifies in the structure `inputInfo` is needed before the function has knowledge about what terms are bias and what terms are noise

Figure 4.3: The data storing scheme for the LinCov-function

ReDefLin created a structure `LINEARMODEL`, which the function `LinCov` (LinearCovariance) takes as an argument. Before passing `LinearModel` as an argument to `LinCov`, the standard deviations of the uncertainties must be defined.

ReDefMod also created the cell-variable `inputStd` in the sub-structure `originalModelInfo` of `LINEARMODEL` where the standard deviations are to be entered. The first column of `inputStd` contains the names of the uncertainties, the second column is where the standard deviation of the biases are to be entered, and similarly, the third column is for the noise standard deviation. This can easily be done in Matlab's variable editor.

The output of `LinCov` is the structure `VARIANCE` which can be stored for future postprocessing and visualisation. This structure contains a 3D array relating the outputs to the inputs as a function of time, as shown in Fig. 4.3, together with the previous sub-structure `originalModelInfo`. Keeping `originalModelInfo` readily available makes the analysis easy, and the designer can simply check what the input port was, and what standard deviation was used in the propagation.

Each layer of `VARIANCE` corresponds to an input uncertainty, where each row is the time dependent output variance corresponding to the uncertainty defined for that layer. Observe that the values stated are variances, not standard deviations. This is because of the variance's property of adding, which is lost for the standard deviation.

Should one need another set of inputs propagated, this can easily be done by redefining the standard deviations and call `LinCov` again.

4.3 inspectLinCov: Visualise the result

```
inspectLinCov(VARIANCE, TABLE, PLOT)
```

During testing it proved valuable to have a simple and quick way of graphically inspecting `VARIANCE` before deciding on further analysis. This function provides a quick and simple way of describing what input standard deviation has been set and the influence on the outputs by printing a table with input names, *hard mean* output standard deviations, as well as plotting the input/output time dependent

standard deviation of the output.

Hard mean is defined for this work as the overall average, over the entire time, even if no attitude solution is available. Later, in section 5.5.3 Orbital positions and averaging, a *soft mean* will be defined that will exclude the time where no attitude solution available.

If TABLE is set to 1, a table of the mean standard deviation of each output resulting from each input is printed where the names of the input and outputs are clearly specified, together with an index number. If TABLE is set to 0 nothing is printed.

PLOT is an $N \times 2$ row vector, [inputIndex outputIndex], where the index, as specified in the previous table, decides what input/output relation to plot. The input and output names corresponding to the indices are printed in the plots. If PLOT is set to 0 no plot is produced.

Chapter 5

Truth model, Sensors, and Simulation

Up until now, the Linear Covariance approach to the stated attitude accuracy problem has been presented and compared to the more common Monte Carlo technique. Nothing has been said about the models normally used in both of these approaches. Firstly, in order to simulate the design of a spacecraft's ADS, the truth to be measured must be defined.

For this study, the truth will consist of a dynamic model simulation of the spacecraft and its environment such as orbital dynamics, magnetic field and sun vectors calculated from orbital position and attitude, disturbance torques and so forth. Models of sensors and estimators/reconstructors may then be used to provide an estimate of the truth, as it would have been in reality.

The general setup of an Attitude Determination Control System (ADCS) is shown in Fig. 5.1 which will allow the designer to evaluate the accuracy of the solution by comparing it to the truth. This is a four step process as follows

1. Generate the truth
2. Measure the truth, with appropriate uncertainties and noise.
3. Apply a reconstruction algorithm to estimate the attitude.
4. Compare the attitude estimate with the truth to assess the design.

The last point, comparing the estimate to the truth, is not available in real life as the actual truth is never known, only an estimate of it. Of course, assumptions in both the truth model and sensor will cause differences between the simulated result and what one would actually see in reality.

Typically the estimate of the attitude would be feed back to the attitude control system in order to provide the actuators with attitude and rate information. However, actuators have not been used in this work; the satellite is passively spin-stabilized. Magnetic torquers are available in the model under consideration, but would saturate the magnetometers when turned on.

Figure 5.1: The general setup of an ADCS simulation in order to evaluate the accuracy of the attitude estimate and control system performance.

The model in this case has been developed by Dr Rees Fullmer at Utah State University, for SDL, Logan, Utah, USA. A description of the model to introduce an insight in the accuracy of the model is presented in the next section. Also, the parameters used to initialize the model is presented.

5.1 Truth model

The truth model is constructed in the MATLAB language and the application Simulink. This model is a stand-alone simulation that generates motion of the spacecraft assumed as the truth which is stored for further use.

It uses a model of the Earth's orbit around the sun, the spacecraft's orbit around the Earth and a model of the Earth's magnetic field in order to generate data such as sun vectors, magnetic field direction and strength etc. in ECI coordinates. These are transformed to the BFCS by rotation matrices generated from attitude and environmental torque models.

Additional data such as the field of view for the sun sensor and the eclipse period (umbra and penumbra) are also calculated. To give an insight into the construction of the model, a short description of the components is provided without cluttering the text with equations.

Dynamics and attitude

By integrating Euler's nonlinear differential equations, the rate change of attitude is inferred after which the differential quaternion can be derived. From this, a rotation matrix R_{ECI}^{BFCS} , describing the vector transform from inertial to body coordinates is computed that is used to determine properties as seen in the spacecraft's frame of view. Initial rate and quaternion are defined in the initialisation of the

truth model.

Orbit

An accurate orbit is simulated by taking the oblation of the Earth and the gravitational parameter μ into account. The orbit is described in the orbital plane (PQW-system) and translated into a position vector used to calculate accurate B-field magnitude and direction vectors and unit direction vectors of the sun.

The Earth's orbit around the sun is specified by the Julian date of the epoch to get an initial position of the Earth in its orbit around the sun. A state vector of the Earth and the spacecraft's position in the orbit is used to calculate if the spacecraft is located within the umbra, penumbra or out of eclipse.

By the information from the attitude dynamics block of the model, the sun-vector as seen by the spacecraft is calculated which will be sensed by the sun sensors in their mounted frame of view respective the spacecraft.

Environment

The environment takes into account the atmospheric density at corresponding altitude to produce models of aerodynamic disturbance torques acting on the spacecraft due to surface areas and centre of gravity location. The magnetic field of the Earth is generated by a spherical harmonic model accounting for zonal and tesseral harmonics defined by the International Geomagnetic Reference Field (IGRF).

5.1.1 Initialization of the truth

The truth model's orbit is set up by the Keplerian parameters, initial attitude quaternion and epoch time. The latter is needed for the correct sun vector and magnetic field initial values. The spacecraft is specified by supplying a range of parameters such as inertia matrix, area of and number of solar panels, centre of gravity etc.

Orbit under consideration

A sun synchronous orbit is chosen to be analysed because of the availability to simulate different points in the orbit without risking eclipse, also, the high inclination property of such an orbit is interesting, with the rapidly changing magnetic field close to the poles compared to the equatorial region. The eccentricity is set

to zero and the epoch is chosen as the time of vernal equinox, in order to simplify the calculation for the Right Ascension of the Ascending Node (RAAN).

By specifying the semi-major axis, the inclination that provides a sun synchronous orbit is calculated from $\dot{\Omega} = -1.5nJ_2 (R_E/a)^2 (\cos(i)) (1 - e^2)^{-2}$ by setting the nodal precession rate $\dot{\Omega}$ to 0.9856 deg/day in order to match the Earth's orbit around the sun [4]. Here, n (deg/day) is the mean motion of the satellite, J_2 (dim. less) is the zonal coefficient, R_E (km) is Earth's equatorial radius, a (km) is the semi-major axis, e (dim. less) is the eccentricity and i (deg) is the inclination which can be solved for.

For the epoch, time and date of the vernal equinox are provided by the United States Naval Observatory [23], which gives a simple 90 deg solution of the RAAN. The defined orbit is summarised in Table 5.1.

Of course, for a sun synchronous orbit to really have effect, simulations during one full year should be done. In this work, simulation is made only at specific points in the orbit, all of them at the same epoch time. The sun synchronous orbit is mainly chosen for the previously discussed characteristics.

Table 5.1: The orbit under consideration as specified by Kepler's orbit parameters

Parameter	Value	Dimension
Semimajor axis ¹	7278.14	km
Eccentricity	0	-
Inclination	99.03	deg
RAAN	90	deg
Argument of perigee ²	-	-
True anomaly ³	-	-
Epoch	2009/03/20 11:44:21	UTC ⁴

Initial attitude

The spacecraft is set to spin around its Z-axis with the rate one revolution per second, where the spin-axis is tilted by 45 deg about the x-axis of the ECI coordinate system from an initially aligned position of the BFCS and the ECI. Dynamic motion instabilities as well as disturbance torques will cause the spacecraft to nutate.

¹The radius of the Earth is modelled as 6378.14 km which gives an altitude of 900 km.

²The argument of perigee is not defined for a circular orbit.

³The true anomaly will be defined later, as several points in the orbit will be taken into consideration in the analysis.

⁴Universal Time Coordinated

Baseline for the project

The PEARL spacecraft (see Sec. 2.2) has been used as a baseline for the initialisation of the model's physical parameters, but not repeated here.

5.2 Sensor, error, and reconstruction model

The sensors under consideration are a sun-sensor and a magnetometer. These sensors are modelled using traditional error models. Adding complexity to the sensors does not necessarily mean that the model will be more accurate. Geometric and optical issues would arise that would need to be modelled. Instead, the sensors are modelled in the nominal case as perfect linear sensors. Non-linearities will be taken under consideration in the error analysis.

The true magnetic field B and sun vectors S are obtained from the truth model. They are first rotated into the spacecraft frame of view by the true time varying attitude rotation matrix R_{ECI}^{BFCS} . Then they are rotated into the sensor frame of view by the constant R_{BFCS}^{Sens} obtained from the specified sensor box mounting angles relative the spacecraft. The mounting orientation is specified as Euler angles in X Y Z sequence from the BFCS and transformed to the rotation matrix representation.

The result of this set of rotations by $R_{BFCS}^{Sens} R_{ECI}^{BFCS}$ provides the magnetic and sun vectors as seen by the sensors.

5.2.1 Sensor model

Both the magnetometer and the sun sensor are modelled by the same sensor model. Each axis of the sensors is treated as a separate measurement of the property γ by Eq.(5.1). γ is the property that is to be measured, which in this case is the sun vector S or magnetic field B . The use of superscripts is the same as in Chapter 3 where tilde denotes measured value and hat denotes estimated value.

$$\tilde{\gamma} = S_{dn}\gamma + \gamma_d + \eta \quad (5.1)$$

$$S_{dn} = (S_0 + dS(T - T_n)) S_\zeta \quad (5.2)$$

$$\gamma_d = \gamma_0 + d\gamma_0(T - T_n) \quad (5.3)$$

$$(5.4)$$

γ is the property subjected for measurement

S_{dn} is the true sensitivity due to temperature (d)rift and (n)on-linearity.

γ_d is the true bias due to temperature (d)rift

η is the noise seen by the sensor

S_0 is the true linear sensitivity

dS is the true thermal sensitivity drift

γ_0 is the true bias

$d\gamma_0$ is the true thermal bias drift

T is the true ambient temperature

T_n is the true temperature where the bias and sensitivity is at their (n)ominal values

S_ζ is the nonlinear sensitivity factor modelled as

$$\gamma_\theta = \gamma_{min} + (\gamma_{min} + \gamma_{max}) / 2 \quad (5.5)$$

$$\theta = \pi (\gamma - \gamma_\theta) / (\gamma_{max} - \gamma_{min}) \quad (5.6)$$

$$S_\zeta = 1 + c \cdot \cos \theta + s \cdot \sin \theta \quad (5.7)$$

γ_{max} and γ_{min} are the maximum and minimal values that can be sensed

S_ζ is the sensitivity nonlinear coefficient

c and s are the cosine and sine coefficients

5.2.2 Error model

The true value is in reality never known. It is therefore modelled as the estimate plus an error term.

$$S_o = \hat{S}_o + \delta S_o \quad (5.8)$$

$$dS = \hat{dS} + \delta dS \quad (5.9)$$

$$\gamma_o = \hat{\gamma}_o + \delta \gamma_o \quad (5.10)$$

$$d\gamma_0 = \hat{d\gamma}_0 + \delta d\gamma_0 \quad (5.11)$$

$$T_n = \hat{T}_n + \delta T_n \quad (5.12)$$

$$T = \hat{T} + \delta T \quad (5.13)$$

$$c = \hat{c} + \delta c \quad (5.14)$$

$$s = \hat{s} + \delta s \quad (5.15)$$

$$R_{\text{BFCS}}^{\text{Sens}} = \hat{R}_{\text{BFCS}}^{\text{Sens}} + \delta R_{\text{BFCS}}^{\text{Sens}} \quad (5.16)$$

5.2.3 Reconstruction model

A reconstructor is used to estimate the true property γ from the measurement $\tilde{\gamma}$. This is done by using estimated values of the sensitivity and bias. Essentially, this is a reverse of what the sensors did, with small errors resulting from the sensor's

sampling time, quantization and the estimation of the sensors parameters.

$$\hat{\gamma} = (\tilde{\gamma} - \hat{\gamma}_d) \hat{S}_d \quad (5.17)$$

$$\hat{S}_d = (\hat{S}_0 + d\hat{S}(\hat{T} - \hat{T}_n)) \quad (5.18)$$

$$\hat{\gamma}_d = \gamma_0 + d\hat{\gamma}(\hat{T} - \hat{T}_n) \quad (5.19)$$

$\hat{S}_d$ is the estimated sensitivity due to temperature (d)rift, the nonlinearities are not taken into account in the reconstruction since the sensor is assumed to be linear. The other properties are defined in the previous section; however, the properties in the reconstructor are estimates of the true properties.

5.3 Sensors

5.3.1 Magnetometer

The magnetometer is subjected to a residual magnetic field $B_r = \hat{B}_r + \delta B_r$; this field is added to the magnetic field in the sensor's frame of view before it is measured by the sensor. The magnetic field can saturate, typically by firing torquer-coils.

A saturation check is performed by the Three Axes Magnetometer (TAM) axes $i = (x, y, z)$ where a hard saturation at allowable γ_{max} and γ_{min} magnetic field density is used at each axis. However, since the magnetometer's range spans the Earth's magnetic field and the spacecraft is passive, saturation is not a problem in this work.

The field is filtered by an Anti-Aliasing filter with bandwidth ω , after which the measurement is performed and represented as a voltage. All axes i of the magnetometer work in the same way independently of each other.

The output voltage V_i at each axis i is, like the magnetic field density input, subject to a saturation check at V_{max} and V_{min} . The voltage is also quantized by an Analog to Digital (AD) resolution of r_{AD} .

5.3.2 Sun sensor

Two sun sensors are used, where the boresights have been mounted in opposite directions. The FOV is limited as by projecting the sun directional vector on the image plane; if the sun is not within the maximum half angle FOV γ_{max} this is flagged, which tells the attitude estimation that an attitude estimation is not possible.

While the magnetometer output is interpreted as a voltage, the sun sensors output is interpreted as horizontal and vertical pixels from the map bias centroid. Each

sun sensor is thus a two axis sensor measuring the azimuth and elevation of the sun.

5.3.3 Definition of the nominal case

In the nominal case, the error terms are assumed zero, thus the estimated equals the true value in the error model. A discussion of the nominal parameters is given below.

The sensors are set up with nominal values, i.e no noise and assuming perfect calibrated parameters. By doing this, the developed M-functions can easily be applied to create the discrete linear state space representation of the system, in order to investigate small perturbations on the sensor's nominal parameter and the perturbations' effect on the attitude solution.

The magnetometer HMC2003 used by the PEARL spacecraft has been studied. The data sheet [15] of this sensor is included in appendix A.

Finding a sun sensor with the corresponding data sheet including the parameters needed for the model's initialisation turned out to be difficult. Instead, experimental data from UsuSat1 has been used in the derivation of initialisation settings. Error documentation for this is available in the Master's thesis *The use of CMOS Camera Data for Spacecraft* [24].

The nominal values of sensitivity, bias, and centroid drift with temperature that deviates from the nominal temperature that they were calibrated at, which is 25 °C for both the magnetometer and sun sensors. The ambient temperature in this simulation is set to 35°C and assumed constant. This assumption comes from the fact that the sun is always in view and the spacecraft is spinning, so thermal effects will be constant and evenly distributed.

The temperature drift for the sensitivity is specified in the datasheet for the magnetometer. It is estimated to 600 ppm per °C deviation from the nominal temperature. A thermal drift of the bias is not specified but has been assumed to be the same. The sensitivity in the sun sensors is also affected by this drift whereas the centroid drift has an 0.1 pixel/°C in the nominal case.

Information about the ambient temperature is provided by temperature sensors in order to give the reconstructors information about how much the biases and sensitivity have deviated due to temperature. The temperature sensors are perfectly accurate in the nominal case. However, in the Linear Covariance analysis, deviations on the sensed temperature due to inaccuracies in the temperature sensor will cause errors in the reconstruction to BFCS vectors.

The nonlinear terms s and c are set to zero in the nominal case giving a perfectly linear sensor. Thus the magnetometers output voltage V_i from one axis is proportional to the magnetic field strength in that direction and the sun sensors pixel are

activated in a non distortion fashion.

The nominal mounting of the sensors are assumed perfectly calibrated and not changed during launch. The magnetometer is aligned with the BFCS, i.e. $R_{\text{BFCS}} = I_3$ where I is the unity matrix whereas the two sun sensors are mounted as:

- Sun1 z sensor boresight is in BFCS positive x direction
- Sun2 z sensor boresight is in BFCS negative x direction

The magnetometer has a sampling time of 100 Hz, this is also the case for the sun sensor. This is high sampling time for a sun sensor, but not unreasonable. It has been set to this value due to simulation issues where the rates in the model should be equal.

Table 5.2 presents the initialisation settings of the magnetometer whereas Table 5.3 contains the sun sensors initialisation parameters. All axes of the TAM and the sun sensors are initialised with the same parameters.

5.4 Attitude estimation

5.4.1 Selection of algorithm for analysis

The selection of the attitude determination algorithm is done by considering the algorithms in Sec. 2.1.1. In the model used in this work, only two vector measurements are available at every point in time. Thus, the q-method is not used.

Due to strict limited time and risk of setbacks, the Kalman filter is not used but the more simple single-frame TRIAD method is used to solve for the attitude. However, a rate sensor is included in the sensor model used for this work but not explicitly explained, in order to prepare for future analysis including a Kalman filter.

Regarding the limitations of the TRIAD method (see Sec. 2.1.1), the first limitation, that it assumes that the measurements are unit vectors, has no impact on this work, since the sensors used (magnetometer and sun sensor) supply unit vector directions. The second limitation, that it can make use of only two unit-vector measurements, is supported by the fact that the main focus of this work is not to determine the attitude as accurately as possible, but to determine how accurate the attitude estimate is when influenced by uncertainties in the sensors.

Table 5.2: The initialisation parameters of the magnetometer. All stated parameters are nominal values of the magnetometer that are used in the linearisation process. The symbol [D] marks that the value is obtained from the datasheet of HMC2003 [15] which can be found in the appendix. Properties denoted as estimated (with hat) are considered in the Linear Covariance analysis

	Input name	Value	Dimension
t	Sampling time	100	Hz
γ_{max}	Max magnetic flux	$2 \cdot 10^{-4}$ [D]	Tesla
γ_{min}	Min magnetic flux	$-2 \cdot 10^{-4}$ [D]	Tesla
$\hat{S}_o$	Sensitivity	$1 \cdot 10^4$ [D]	V/Tesla
$\hat{dS}$	Thermal sensitivity drift	$-6 \cdot 10^{-4} \hat{S}_o$ [D]	V/(Tesla °C)
$\hat{\gamma}_0$	Bias voltage	2.5[D]	V
$\hat{d\gamma}_0$	Thermal bias drift	$-6 \cdot 10^{-4} \hat{\gamma}$	V/°C
$\hat{T}_n$	Nominal temperature	25 [D]	°C
$\hat{T}$	Ambient temperature	35	°C
$\hat{c}$	Cosine nonlinear coefficient	0	-
$\hat{s}$	Sine nonlinear coefficient	0	-
$\hat{R}_{BFCs}^{Sens}$	Sensor box mounting rel. BFCs	X:0 Y:0 Z:0	degree
$\hat{\eta}$	Voltage noise	0	V
$\hat{B}_r$	S/C residual field	0	Tesla
$\hat{\gamma}_{max}$	Maximum voltage output	4.5 [D]	V
$\hat{\gamma}_{min}$	Minimum voltage output	0.5 [D]	V
r_{AD}	AD resolution	$4 \cdot 10^{-6}$ [D]	-
ω	Anti-Aliasing filter bandwidth	100π	rad/sec

Thus, the relatively simple TRIAD algorithm is sufficient to demonstrate the techniques in this thesis, but can easily be upgraded to a more sophisticated (e.g. Kalman filter) solution method. The attitude is described as the BFCs rotation with respect to ECI.

5.4.2 Attitude estimation by the TRIAD method

With the estimated BFCs vectors, the attitude with respect to ECI can easily be solved by the TRIAD Eqs. (2.2) - (2.5). The TRIAD algorithm gives the attitude as a rotation matrix and after the conversion to Euler angles by Eqn. (2.1), where the rotation order is (Z, Y, Z) of the BFCs, a complete attitude solution is in hand with an intuitive representation.

The true attitude, specifically the Z Y Z Euler angles, of the spinning spacecraft along with the estimated attitude from the sensors when no uncertainties are present, for five revolutions are shown in Fig. 5.2. A closeup of the solution is shown in Fig. 5.3 to demonstrate the accuracy of the solution when no uncertainties are present.

Table 5.3: The initialisation parameters of the two sun sensors. All stated parameters are nominal values of the sensors that are used in the linearisation process. Both sensors have the same values. Values are derived from experimental data from [24]. Properties denoted as estimated (with hat) are considered in the Linear Covariance analysis

	Input name	Value	Dimension
t	Sampling time	100	Hz
γ_{max}	Field of view (half angle)	33.5	degree
$\hat{S}_o$	Sensitivity	$256 / \sin(\frac{33.5\pi}{180})$	pixel/rad
$\hat{dS}$	Thermal sensitivity drift	$-6 \cdot 10^{-04} \hat{S}_o$	pixel/(rad °C)
$\hat{\gamma}_0$	Pixel centroid	centred	pixel
$\hat{d\gamma}_0$	Thermal centroid drift	0.1	pixel/°C
$\hat{T}_n$	Nominal temperature	25	°C
$\hat{T}$	Ambient temperature	35	°C
$\hat{c}$	Cosine nonlinear coefficient	0	-
$\hat{s}$	Sine nonlinear coefficient	0	-
$\hat{R}_{BFCS}^{Sens}$	Sensor box 1 mounting	X:-90 Y:90 Z:0	degree
$\hat{R}_{BFCS}^{Sens}$	Sensor box 2 mounting	X:-90 Y:-90 Z:0	degree
$\hat{\eta}$	Pixel noise	0	pixel
$\tilde{\gamma}_{max}$	Maximum pixel output	256	pixel
$\tilde{\gamma}_{min}$	Minimum pixel output	-256	pixel
r_{AD}	Resolution	512*512	pixel

The solution does not match the true attitude exactly because

- There are quantization and sampling times in the sensor model.
- The spacecraft is spinning and loses view of the sun. When this happens there is no attitude solution available from the TRIAD algorithm, whereas it is interpreted as aligned BFCS and ECI.
- There are bandwidth effects in the sensor model measurement, causing lags in the system.

The zero line of the estimated attitude in Fig. 5.2 corresponds to when the sun is outside the field of view. There is no solution available and the attitude solution is set to zero, i.e. aligned BFCS and ECI.

This assumes that the ECI data from the truth model is perfect, i.e. no errors have been modelled on the reference vectors. This means that the *true* magnetic field that is sensed by the magnetometer is also used in the attitude solution process as the ECI reference vectors in the TRIAD algorithm. Similar arguments hold for the sun directional vectors.

In reality the measured B-field is compared to models, provided by IGRF, of the true magnetic field. This model is not completely accurate and will induce errors

Figure 5.2: The estimated (thicker line) attitude for a perfect sensor with nominal values compared to the true attitude (dash-dotted lines) from the truth model. When the sun-sensor is out of view, all rotations are set to zero for the estimated attitude. The rotation order for the Euler angles are (Z, Y, Z).

in the attitude solution. Also, the ECI sun vector is not perfectly known. These model errors has not been accounted for. This assumption is valid, because it is the uncertainties in the actual sensor that are under investigation.

With this attitude solution in hand, it is interesting to know how much it will fluctuate depending on the standard deviation of the sensor parameters. A designer wants to know this in order to decide if the chosen hardware meets the mission's requirements.

5.5 Linear Covariance analysis

The sensor errors are modelled by the inclusion of uncertainties as perturbations on the nominal values of the sensor and also by including noise terms as input to the system as explained in Chapter 2 and 3 . The errors are modelled as Gaussian distributed variables, and are discussed below. All stated values of the uncertainties are one standard deviation (1σ) unless stated otherwise.

Sensor box mounting misalignment can be known very accurately and significantly reduced [3] after calibration at ground. However, during launch there are changes due to vibrations in the structure, and it is assumed that the direction of change is in a random direction, where the standard deviation is the same in all

Figure 5.3: The figure shows a zoom in of Fig.5.2 in order to show the accuracy of the TRIAD solution when no uncertainties are present. The nutation of the spacecraft can clearly be seen.

directions.

The sensors are also equipped with a temperature sensor so that thermal drifts of the parameters can be accounted for in the reconstructor. These temperature sensors are not perfect; there exists an uncertainty of $0.1\text{ }^{\circ}\text{C}$ standard deviation of the measured value.

There is also an uncertainty with respect to the nominal value of the sensors. However, this is assumed to be more accurate, only $0.01\text{ }^{\circ}\text{C}$ deviation from the nominal value, because more accurate laboratory equipment was probably used during testing.

A number of other uncertainties have been investigated where the standard deviation from the nominal value is obtained or derived from [15] and [24]. An assumption of a certain percentage uncertainty from the nominal value has been used where no deviation has been available.

5.5.1 Magnetometer uncertainties

The uncertainty of the magnetometer's sensitivity is stated in the datasheet of HMC2003[15]. It would be possible to fit a curve to the bar graph showing the sensitivity seen in a number of units. However, since there is not a significant amount of data available, the assumption of a three sigma 0.02 V/Tesla deviation

from nominal has been made from the graph of field sensitivity distribution shown in the datasheet.

Voltage biases can be well calibrated so a deviation of 1mV is assumed. The sensitivity and bias thermal drift uncertainty is not stated but assumed to be 5% of the nominal values. The high percentage is because thermal drift's is not well known.

Both sine and cosine effects with coefficients of 0.005 deviation from nominal are assumed to be present in the magnetometer in order to see their respective effect. However, it is most likely to see sine effects in a magnetometer, whereas the linear sensitivity will fluctuate around zero.

The residual magnetic field will, in the design, have gone through several mitigation techniques using special materials etc. However, there is an uncertainty of how well the field has been mitigated, so an deviation of 100 nTesla is assumed to be present.

The uncertainties present in the magnetometer are summarised in Table 5.4.

5.5.2 Sun sensor uncertainties

The uncertainty of the sensitivity is not available; however, a 1% deviation from the nominal value has been assumed.

The centroid of the sensor can be well calibrated, but launch vibration may cause misalignments so a deviation of 1 pixel is applied. The sensitivity and centroid thermal drift uncertainty is, like the magnetometer, assumed to be 5% of the nominal values.

Both sine and cosine effects are assumed to be present in the sun sensor in order to see their effect. However, it is most likely to see cosine effects in a sun sensor. This corresponds to the distortion of the image.

A Gaussian white noise of 1 pixel of resulting from electric effects, moon, and sun reflections is applied. Both sun sensors in the model are subject to the same uncertainties.

The uncertainties present in the sun sensors are summarised in Table 5.5.

5.5.3 Orbital positions and averaging

The attitude accuracy is also dependent on the actual ECI vectors that will slowly change during the orbit. The BFCS sun and magnetic vectors cycle through with the spinning motion of the spacecraft. The variance will also follow a similar oscillation.

Table 5.4: The standard deviations in the magnetometer's parameters. The symbol [D] marks that the value is obtained from the datasheet of HMC2003 [15] which can be found in the appendix. N denotes nominal value. All axes of the sensor have the same values of the uncertainty.

	Input name	σ	Dimension
δS_o	Sensitivity	$0.02/3 \cdot N$ [D]	V/Tesla
δdS	Thermal sensitivity drift	$0.05N$	V/(Tesla °C)
$\delta \gamma$	Bias voltage	10^{-3}	V
$\delta d\gamma_0$	Thermal bias drift	$0.05N$	V/°C
δT_n	Nominal temperature	0.01	°C
δT	Temperature sensor	0.1	°C
δc	Cosine nonlinear term	0.005	-
δs	Sine nonlinear term	0.005	-
$\delta R_{\text{BFCS}}^{\text{Sens}}$	Sensor box mounting	0.1	degree
$\delta \eta$	Voltage noise	10^{-3}	V
δB_r	S/C residual field	$100 \cdot 10^{-9}$	Tesla

Therefore, only five revolutions of the spacecraft are simulated, but at different positions in the sun synchronous orbit, where the variance of the attitude determination corresponding to each respective uncertainty is averaged according to a *soft mean* algorithm at each point under consideration.

Soft mean is defined as not accounting for times when there is no attitude solution available due to the sun's location outside the FOV. Thus the variance is averaged only when the attitude solution is available. This averaging allows for a clearer view of the uncertainties impact on the attitude determination accuracy.

Five position are under investigation; they are defined by the true anomaly in degrees as presented in Table 5.6.

Since the model is linear, all the specific output variances resulting from the corresponding input variances will add to a total attitude determination variance.

It is possible to find a coefficient c_i for the variance, that relates the uncertainties standard deviation σ_i to the resulting attitude determination accuracy as

$$\sigma_{\text{Attitude}} = \sqrt{\sum_{i=1}^n c_i^2 \sigma_i^2} \quad (5.20)$$

where σ_i is the input standard deviation corresponding to the coefficient c_i and n is the number of uncertainties one wish to include. This approximation is justified in 2.4.2.

This coefficient is simply calculated as $c_i = \sqrt{\sigma_{\text{out}}^2 / \sigma_i^2}$ where σ_{out} is the soft mean standard deviation resulting from the input uncertainty σ_i .

Equation (5.20) allows the designer to quickly combine n different input uncer-

Table 5.5: The standard deviations in the sun sensors' parameters. Both sensors have the same values. Values are derived from experimental data from [24]. N denotes nominal value. All axes of both sensors have the same values of the uncertainty.

σ	Input name	Dimension	
δS_o	Sensitivity	0.01N	pixel/rad
δdS	Thermal sensitivity drift	0.05N	pixel/°C
$\delta \gamma$	Centroid	1	pixel
$\delta d\gamma_0$	Thermal centroid drift	0.05N	pixel/°C
δT_n	Nominal temperature	0.01	°C
δT	Temperature sensor	0.1	°C
δc	Cosine nonlinear term	0.005	-
δs	Sine nonlinear term	0.005	-
$\delta R_{\text{BFCS}}^{\text{Sens}}$	Sensor box mounting	0.1	degree
$\delta \eta$	Pixel noise	1	pixel

Table 5.6: The positions in the sun synchronous orbit that are under investigation. The output variance corresponding to each input is averaged over five revolutions of the S/C at each point

Position	1	2	3	4	5
True anomaly (deg)	0	45	90	135	315

tainties to get an estimate of what the attitude accuracy will be, or the other way around, find out what input uncertainty is tolerable and still meet the requirements of the attitude accuracy.

Chapter 6

Results and Discussion

6.1 Results

The simulation was run for the five orbital positions in Table 5.6 using the error terms in Tables 5.4 and 5.5.

The bar plots in Figs. 6.1 - 6.5 in this chapter show the total variance of the attitude estimate resulting from the specified input, at the respective orbital position. All stated values of the total variances are soft means averaged over 5 revolutions with the unit degree².

The total variance is calculated as the sum-square of the individual rotations mean variance, $\sigma_{Z1}^2 + \sigma_{Y2}^2 + \sigma_{Z3}^2$ where σ is the standard deviation at the respective Euler axis denoted by the subscript. The contribution from a specific axis can be found in appendix B.

A summary over the attitude determination accuracy at the respective position in the sun synchronous orbit is shown in Table 6.1.

Table 6.1: A summary over the achievable attitude determination accuracy at respective point in the orbit

True anomaly (deg)	0	45	90	135	315
Variance (deg ²)	0.65	0.57	0.61	0.56	0.51
Standard deviation (deg)	0.81	0.75	0.78	0.75	0.71

Tables 6.2 - 6.6 are sorted lists over the coefficients c_i that relates the input standard deviation to the output standard deviation according to Eqn. (5.20). The tables show what inputs have the strongest contribution to the deviation of the attitude determination solution.

Figure 6.1: Position 1: True anomaly = 0 deg.
Contribution to the total variance of 0.64551 deg^2

Table 6.2: Position 1: True anomaly = 0 deg. The coefficients c_i relating input variance to output variance sorted in falling order, most influence $\rightarrow$ least influence. When comparing the coefficient's the dimension must be considered. Only when the units are the same, the coefficients can be compared directly. The units of c_i are presented in the appendix C.

Input name	c_i	Input name	c_i
SC residual field Z	1.90E+06	MagZ Nominal temperature	3.04E-01
SC residual field Y	9.09E+05	MagZ Sensed temperature	3.04E-01
SC residual field X	3.10E+05	MagX Cosine nonlinear term	3.01E-01
MagZ Thermal bias drift	1.93E+03	Sun2Y Cosine nonlinear term	2.43E-01
MagY Thermal bias drift	9.21E+02	Sun2Y Noise	2.18E-01
MagX Thermal bias drift	3.19E+02	Sun2Y Centroid	2.18E-01
MagZ Bias voltage	1.93E+02	Sun1Y Noise	2.14E-01
MagZ Voltage noise	1.93E+02	Sun1Y Centroid	2.14E-01
MagY Bias voltage	9.21E+01	MagY Sensed temperature	1.39E-01
MagY Voltage noise	9.21E+01	MagY Nominal temperature	1.39E-01
Sun1 box mounting Y	5.63E+01	Sun1Y Cosine nonlinear term	1.36E-01
Sun2 box box mounting Y	5.63E+01	Sun1X Centroid	1.32E-01
Mag box mounting X	5.39E+01	Sun1X Noise	1.32E-01
Sun2 box box mounting Z	3.29E+01	Sun2X Centroid	1.32E-01
Sun2 box box mounting X	3.29E+01	Sun2X Noise	1.32E-01
Sun1 box mounting Z	3.25E+01	Sun2Y Thermal sensitivity drift	5.85E-02
Sun1 box mounting X	3.25E+01	MagX Nominal temperature	4.76E-02
MagX Bias voltage	3.19E+01	MagX Sensed temperature	4.76E-02
MagX Voltage noise	3.19E+01	Sun1X Sensitivity	4.53E-02
MagZ Sine nonlinear term	2.32E+01	Sun2X Sensitivity	4.52E-02
MagY Sine nonlinear term	2.08E+01	Sun1Y Thermal sensitivity drift	4.09E-02
Sun1X Cosine nonlinear term	1.91E+01	MagZ Thermal sensitivity drift	2.35E-02
Sun2X Cosine nonlinear term	1.91E+01	Sun1Y Nominal temperature	2.24E-02
Mag box mounting Y	1.86E+01	Sun1Y Sensed temperature	2.24E-02
Sun2X Sine nonlinear term	8.49E+00	MagY Thermal sensitivity drift	2.13E-02
Sun1X Sine nonlinear term	8.48E+00	Sun2Y Nominal temperature	2.03E-02
MagY Cosine nonlinear term	3.84E+00	Sun2Y Sensed temperature	2.03E-02
MagX Sine nonlinear term	3.32E+00	Sun2X Sensed temperature	1.82E-02
Sun2Y Sine nonlinear term	2.69E+00	Sun2X Nominal temperature	1.82E-02
MagZ Cosine nonlinear term	2.22E+00	Sun1X Nominal temperature	1.82E-02
Sun2Y Thermal centroid drift	2.18E+00	Sun1X Sensed temperature	1.82E-02
Sun1Y Thermal centroid drift	2.14E+00	Sun2Y Sensitivity	5.85E-03
Sun1Y Sine nonlinear term	1.88E+00	Sun1Y Sensitivity	4.09E-03
Mag box mounting Z	1.32E+00	MagX Thermal sensitivity drift	3.36E-03
Sun1X Thermal centroid drift	1.32E+00	MagZ Sensitivity	2.35E-03
Sun2X Thermal centroid drift	1.32E+00	MagY Sensitivity	2.13E-03
Sun1X Thermal sensitivity drift	4.53E-01	MagX Sensitivity	3.36E-04
Sun2X Thermal sensitivity drift	4.52E-01

Figure 6.2: Position 2: True anomaly = 45 deg.
Contribution to the total variance of 0.56847 deg²

Table 6.3: Position 2: True anomaly = 45 deg. The coefficients c_i relating input variance to output variance sorted in falling order, most influence $\rightarrow$ least influence. When comparing the coefficient's the dimension must be considered. Only when the units are the same, the coefficients can be compared directly. The units of c_i are presented in the appendix C.

Input name	c_i	Input name	c_i
SC residual field Z	1.34E+06	Sun2X Thermal sensitivity drift	4.52E-01
SC residual field Y	7.17E+05	Sun2Y Cosine nonlinear term	2.46E-01
SC residual field X	2.31E+05	Sun2Y Noise	2.21E-01
MagZ Thermal bias drift	1.36E+03	Sun2Y Centroid	2.21E-01
MagY Thermal bias drift	7.27E+02	MagZ Sensed temperature	2.17E-01
MagX Thermal bias drift	2.38E+02	MagZ Nominal temperature	2.17E-01
MagZ Bias voltage	1.36E+02	Sun1Y Noise	2.17E-01
MagZ Voltage noise	1.36E+02	Sun1Y Centroid	2.17E-01
MagY Bias voltage	7.27E+01	Sun1Y Cosine nonlinear term	1.38E-01
MagY Voltage noise	7.27E+01	Sun1X Centroid	1.33E-01
Sun1 box mounting Y	5.64E+01	Sun1X Noise	1.33E-01
Sun2 box box mounting Y	5.63E+01	Sun2X Centroid	1.32E-01
Mag box mounting X	5.15E+01	Sun2X Noise	1.32E-01
Sun2 box box mounting Z	3.32E+01	MagY Sensed temperature	1.09E-01
Sun2 box box mounting X	3.32E+01	MagY Nominal temperature	1.09E-01
Sun1 box mounting Z	3.28E+01	Sun2Y Thermal sensitivity drift	5.93E-02
Sun1 box mounting X	3.28E+01	Sun1X Sensitivity	4.54E-02
MagX Bias voltage	2.38E+01	Sun2X Sensitivity	4.52E-02
MagX Voltage noise	2.38E+01	Sun1Y Thermal sensitivity drift	4.14E-02
MagZ Sine nonlinear term	2.17E+01	MagX Sensed temperature	3.54E-02
MagY Sine nonlinear term	2.16E+01	MagX Nominal temperature	3.54E-02
Sun1X Cosine nonlinear term	1.91E+01	Sun1Y Sensed temperature	2.27E-02
Sun2X Cosine nonlinear term	1.91E+01	Sun1Y Nominal temperature	2.27E-02
Mag box mounting Y	1.89E+01	MagY Thermal sensitivity drift	2.23E-02
Sun1X Sine nonlinear term	8.50E+00	MagZ Thermal sensitivity drift	2.20E-02
Sun2X Sine nonlinear term	8.49E+00	Sun2Y Sensed temperature	2.05E-02
MagY Cosine nonlinear term	5.36E+00	Sun2Y Nominal temperature	2.05E-02
Mag box mounting Z	4.52E+00	Sun1X Sensed temperature	1.82E-02
MagX Sine nonlinear term	3.39E+00	Sun1X Nominal temperature	1.82E-02
MagZ Cosine nonlinear term	2.76E+00	Sun2X Nominal temperature	1.82E-02
Sun2Y Sine nonlinear term	2.72E+00	Sun2X Sensed temperature	1.82E-02
Sun2Y Thermal centroid drift	2.21E+00	Sun2Y Sensitivity	5.93E-03
Sun1Y Thermal centroid drift	2.17E+00	Sun1Y Sensitivity	4.14E-03
Sun1Y Sine nonlinear term	1.90E+00	MagX Thermal sensitivity drift	3.44E-03
Sun1X Thermal centroid drift	1.33E+00	MagY Sensitivity	2.23E-03
Sun2X Thermal centroid drift	1.32E+00	MagZ Sensitivity	2.20E-03
MagX Cosine nonlinear term	4.61E-01	MagX Sensitivity	3.44E-04
Sun1X Thermal sensitivity drift	4.54E-01

Figure 6.3: Position 3: True anomaly = 90 deg.
Contribution to the total variance of 0.60996 deg^2

Table 6.4: Position 3: True anomaly = 90 deg. The coefficients c_i relating input variance to output variance sorted in falling order, most influence $\rightarrow$ least influence. When comparing the coefficient's the dimension must be considered. Only when the units are the same, the coefficients can be compared directly. The units of c_i are presented in the appendix C.

Input name	c_i	Input name	c_i
SC residual field Z	1.15E+06	Sun1X Thermal sensitivity drift	4.58E-01
SC residual field Y	9.81E+05	Sun2Y Cosine nonlinear term	2.40E-01
SC residual field X	3.42E+05	Sun2Y Noise	2.15E-01
MagZ Thermal bias drift	1.17E+03	Sun2Y Centroid	2.15E-01
MagY Thermal bias drift	9.95E+02	Sun1Y Noise	2.14E-01
MagX Thermal bias drift	3.52E+02	Sun1Y Centroid	2.14E-01
MagZ Bias voltage	1.17E+02	MagZ Sensed temperature	1.57E-01
MagZ Voltage noise	1.17E+02	MagZ Nominal temperature	1.57E-01
MagY Bias voltage	9.95E+01	MagY Nominal temperature	1.50E-01
MagY Voltage noise	9.95E+01	MagY Sensed temperature	1.50E-01
Sun2 box box mounting Y	5.72E+01	Sun1Y Cosine nonlinear term	1.36E-01
Sun1 box mounting Y	5.68E+01	Sun2X Centroid	1.34E-01
Mag box mounting X	5.38E+01	Sun2X Noise	1.34E-01
MagX Bias voltage	3.52E+01	Sun1X Centroid	1.34E-01
MagX Voltage noise	3.52E+01	Sun1X Noise	1.34E-01
Sun2 box box mounting Z	3.26E+01	Sun2Y Thermal sensitivity drift	5.78E-02
Sun2 box box mounting X	3.26E+01	MagX Sensed temperature	5.31E-02
Sun1 box mounting Z	3.24E+01	MagX Nominal temperature	5.31E-02
Sun1 box mounting X	3.24E+01	Sun2X Sensitivity	4.60E-02
MagZ Sine nonlinear term	2.95E+01	Sun1X Sensitivity	4.58E-02
MagY Sine nonlinear term	2.49E+01	Sun1Y Thermal sensitivity drift	4.08E-02
Mag box mounting Y	2.15E+01	MagZ Thermal sensitivity drift	3.03E-02
Sun2X Cosine nonlinear term	1.94E+01	MagY Thermal sensitivity drift	2.56E-02
Sun1X Cosine nonlinear term	1.93E+01	Sun1Y Sensed temperature	2.23E-02
Mag box mounting Z	9.69E+00	Sun1Y Nominal temperature	2.23E-02
Sun2X Sine nonlinear term	8.63E+00	Sun2Y Nominal temperature	2.00E-02
Sun1X Sine nonlinear term	8.57E+00	Sun2Y Sensed temperature	2.00E-02
MagZ Cosine nonlinear term	6.08E+00	Sun2X Nominal temperature	1.85E-02
MagY Cosine nonlinear term	5.22E+00	Sun2X Sensed temperature	1.85E-02
MagX Sine nonlinear term	4.86E+00	Sun1X Nominal temperature	1.84E-02
Sun2Y Sine nonlinear term	2.65E+00	Sun1X Sensed temperature	1.84E-02
Sun2Y Thermal centroid drift	2.15E+00	Sun2Y Sensitivity	5.78E-03
Sun1Y Thermal centroid drift	2.14E+00	MagX Thermal sensitivity drift	4.94E-03
Sun1Y Sine nonlinear term	1.88E+00	Sun1Y Sensitivity	4.08E-03
Sun2X Thermal centroid drift	1.34E+00	MagZ Sensitivity	3.03E-03
Sun1X Thermal centroid drift	1.34E+00	MagY Sensitivity	2.56E-03
MagX Cosine nonlinear term	7.35E-01	MagX Sensitivity	4.94E-04
Sun2X Thermal sensitivity drift	4.60E-01

Figure 6.4: Position 4: True anomaly = 135 deg.
Contribution to the total variance of 0.55769 deg^2

Table 6.5: Position 4: True anomaly = 135 deg. The coefficients c_i relating input variance to output variance sorted in falling order, most influence $\rightarrow$ least influence. When comparing the coefficient's the dimension must be considered. Only when the units are the same, the coefficients can be compared directly. The units of c_i are presented in the appendix C.

Input name	c_i	Input name	c_i
SC residual field Y	1.44E+06	MagX Cosine nonlinear term	4.14E-01
SC residual field Z	5.63E+05	Sun2Y Cosine nonlinear term	2.43E-01
SC residual field X	4.93E+05	MagY Sensed temperature	2.20E-01
MagY Thermal bias drift	1.46E+03	MagY Nominal temperature	2.20E-01
MagZ Thermal bias drift	5.72E+02	Sun2Y Noise	2.17E-01
MagX Thermal bias drift	5.08E+02	Sun2Y Centroid	2.17E-01
MagY Bias voltage	1.46E+02	Sun1Y Noise	2.12E-01
MagY Voltage noise	1.46E+02	Sun1Y Centroid	2.12E-01
Sun2 box box mounting Y	5.78E+01	Sun2X Centroid	1.36E-01
Sun1 box mounting Y	5.73E+01	Sun2X Noise	1.36E-01
MagZ Bias voltage	5.72E+01	Sun1Y Cosine nonlinear term	1.35E-01
MagZ Voltage noise	5.72E+01	Sun1X Centroid	1.35E-01
Mag box mounting X	5.40E+01	Sun1X Noise	1.35E-01
MagX Bias voltage	5.08E+01	MagX Nominal temperature	7.61E-02
MagX Voltage noise	5.08E+01	MagX Sensed temperature	7.61E-02
Sun2 box box mounting Z	3.27E+01	MagZ Sensed temperature	7.44E-02
Sun2 box box mounting X	3.27E+01	MagZ Nominal temperature	7.44E-02
Sun1 box mounting Z	3.22E+01	Sun2Y Thermal sensitivity drift	5.84E-02
Sun1 box mounting X	3.22E+01	Sun2X Sensitivity	4.65E-02
Sun2X Cosine nonlinear term	1.96E+01	Sun1X Sensitivity	4.61E-02
Sun1X Cosine nonlinear term	1.94E+01	Sun1Y Thermal sensitivity drift	4.05E-02
Mag box mounting Y	1.87E+01	Sun1Y Nominal temperature	2.22E-02
MagZ Sine nonlinear term	1.81E+01	Sun1Y Sensed temperature	2.22E-02
MagY Sine nonlinear term	1.64E+01	Sun2Y Nominal temperature	2.02E-02
Mag box mounting Z	1.10E+01	Sun2Y Sensed temperature	2.02E-02
Sun2X Sine nonlinear term	8.73E+00	MagZ Thermal sensitivity drift	1.89E-02
Sun1X Sine nonlinear term	8.64E+00	Sun2X Sensed temperature	1.87E-02
MagZ Cosine nonlinear term	4.81E+00	Sun2X Nominal temperature	1.87E-02
MagX Sine nonlinear term	4.40E+00	Sun1X Sensed temperature	1.85E-02
Sun2Y Sine nonlinear term	2.68E+00	Sun1X Nominal temperature	1.85E-02
Sun2Y Thermal centroid drift	2.17E+00	MagY Thermal sensitivity drift	1.65E-02
Sun1Y Thermal centroid drift	2.12E+00	Sun2Y Sensitivity	5.84E-03
Sun1Y Sine nonlinear term	1.86E+00	MagX Thermal sensitivity drift	4.45E-03
MagY Cosine nonlinear term	1.57E+00	Sun1Y Sensitivity	4.05E-03
Sun2X Thermal centroid drift	1.36E+00	MagZ Sensitivity	1.89E-03
Sun1X Thermal centroid drift	1.35E+00	MagY Sensitivity	1.65E-03
Sun2X Thermal sensitivity drift	4.65E-01	MagX Sensitivity	4.45E-04
Sun1X Thermal sensitivity drift	4.61E-01

Figure 6.5: Position 5: True anomaly = 315 deg.
Contribution to the total variance of 0.50817 deg²

Table 6.6: Position 5: True anomaly = 315 deg. The coefficients c_i relating input variance to output variance sorted in falling order, most influence $\rightarrow$ least influence. When comparing the coefficient's the dimension must be considered. Only when the units are the same, the coefficients can be compared directly. The units of c_i are presented in the appendix C.

Input name	c_i	Input name	c_i
SC residual field Y	1.29E+06	Sun2Y Cosine nonlinear term	2.44E-01
SC residual field X	4.43E+05	Sun2Y Noise	2.19E-01
SC residual field Z	3.66E+05	Sun2Y Centroid	2.19E-01
MagY Thermal bias drift	1.31E+03	Sun1Y Noise	2.14E-01
MagX Thermal bias drift	4.57E+02	Sun1Y Centroid	2.14E-01
MagZ Thermal bias drift	3.71E+02	MagY Sensed temperature	1.97E-01
MagY Bias voltage	1.31E+02	MagY Nominal temperature	1.97E-01
MagY Voltage noise	1.31E+02	Sun1Y Cosine nonlinear term	1.36E-01
Sun2 box box mounting Y	5.64E+01	Sun2X Centroid	1.33E-01
Sun1 box mounting Y	5.63E+01	Sun2X Noise	1.33E-01
Mag box mounting X	5.34E+01	Sun1X Centroid	1.32E-01
MagX Bias voltage	4.57E+01	Sun1X Noise	1.32E-01
MagX Voltage noise	4.57E+01	MagX Cosine nonlinear term	1.07E-01
MagZ Bias voltage	3.71E+01	MagX Sensed temperature	6.84E-02
MagZ Voltage noise	3.71E+01	MagX Nominal temperature	6.84E-02
Sun2 box box mounting Z	3.30E+01	Sun2Y Thermal sensitivity drift	5.88E-02
Sun2 box box mounting X	3.30E+01	MagZ Nominal temperature	4.71E-02
Sun1 box mounting Z	3.25E+01	MagZ Sensed temperature	4.71E-02
Sun1 box mounting X	3.25E+01	Sun2X Sensitivity	4.54E-02
Sun2X Cosine nonlinear term	1.91E+01	Sun1X Sensitivity	4.53E-02
Sun1X Cosine nonlinear term	1.91E+01	Sun1Y Thermal sensitivity drift	4.09E-02
Mag box mounting Y	1.83E+01	Sun1Y Sensed temperature	2.24E-02
MagZ Sine nonlinear term	1.36E+01	Sun1Y Nominal temperature	2.24E-02
MagY Sine nonlinear term	1.21E+01	Sun2Y Nominal temperature	2.03E-02
Sun2X Sine nonlinear term	8.51E+00	Sun2Y Sensed temperature	2.03E-02
Sun1X Sine nonlinear term	8.48E+00	Sun2X Nominal temperature	1.82E-02
MagZ Cosine nonlinear term	4.27E+00	Sun2X Sensed temperature	1.82E-02
Sun2Y Sine nonlinear term	2.70E+00	Sun1X Nominal temperature	1.82E-02
Mag box mounting Z	2.52E+00	Sun1X Sensed temperature	1.82E-02
Sun2Y Thermal centroid drift	2.19E+00	MagZ Thermal sensitivity drift	1.44E-02
MagX Sine nonlinear term	2.18E+00	MagY Thermal sensitivity drift	1.22E-02
Sun1Y Thermal centroid drift	2.14E+00	Sun2Y Sensitivity	5.88E-03
Sun1Y Sine nonlinear term	1.88E+00	Sun1Y Sensitivity	4.09E-03
Sun2X Thermal centroid drift	1.33E+00	MagX Thermal sensitivity drift	2.20E-03
Sun1X Thermal centroid drift	1.32E+00	MagZ Sensitivity	1.44E-03
MagY Cosine nonlinear term	9.06E-01	MagY Sensitivity	1.22E-03
Sun2X Thermal sensitivity drift	4.54E-01	MagX Sensitivity	2.20E-04
Sun1X Thermal sensitivity drift	4.53E-01		

6.2 Discussion

By referring to the Figs. 6.1 - 6.5 one can instantly identify specific sensor errors which dominate the total error estimate. However, most of the sensor errors are low during the orbit.

The sun sensors voltage noise and centroid error are certainly the largest contributor to total variance of attitude estimate. The contribution to the total variance from the sun sensors stays approximately constant during the orbit. This is not surprising since the sun vectors change only slightly during the orbit.

During the orbit, the magnetometer's uncertainty terms varies in contribution to the total variance of the attitude estimate. An interesting result regarding the magnetometer's sensitivity is that the variance of the attitude estimate resulting from this error approximately doubles from position 1 at the equator to position 3 at the northern pole.

By looking at Table 6.1, one can conclude that the total variance of the attitude estimate change during the orbit, the standard deviation is at most about 0.1 degree. It was stated in the introduction that an attitude uncertainty of this magnitude could translate to one month shorter life expectancy for a normal size spacecraft.

For a Cubesat, which have a much shorter life expectancy, one may conclude that if an attitude manoeuvre is to be performed, it doesn't matter where in the orbit the spacecraft is located if one only consider the attitude determination accuracy.

The attitude determination accuracy also seems to follow an oscillatory motion as the spacecraft travels in the orbit, other orbits should be simulated to see if a greater difference in attitude accuracy is seen.

Considering the sine and cosine errors, it is the sun sensors x-axis that is primarily contributing to the variance of the attitude estimate. Out of the two, the cosine term has the strongest contribution in the sun sensor while in the magnetometer, the sine terms generally dominate over cosine.

As discussed in Chapter 2, the sensor box mounting is important, as well as the sensor axes not being a perfect triad. Only the rotation of the sensor box has been investigated as the same effect would be seen by rotating the box about one axis, as having one axis of the triad misaligned.

For the magnetometer, alignments about the BFCS x-axis is the largest contribution to error and should thus be calibrated with more accuracy. For the sun sensor, it is the alignment about the y-axis (the co-elevation) that should be given special attention.

The contribution to the variance of the attitude determination differ between the axes. One could argue for that the variance resulting from the x and y axes should

be the same, since the spacecraft is spinning around its z-axis. This will be the case, if the spacecraft's sun sensors each have a 180 degree FOV. Then the average of the total variance would turn out the same for both axis.

Since this is not the case, the variances differ. One could inspect difference to find out how the spacecraft should be initialised, to get the best possible attitude knowledge where it is needed due to mission requirements.

It is important to understand when looking at the data is that there has been no investigation of coupling between uncertainties since the base of this approach is the linearisation of the model.

6.2.1 Result highlights

From Appendix B, which shows the contribution from each Euler axis to the total standard deviation of the attitude estimate, some interesting conclusions can be made regarding each axis contribution to the total attitude standard deviation.

For the magnetometer, the major part of the total standard deviation is resulting from the spin axis angle γ (this is the co-elevation angle of the spacecraft). The conclusion is that measuring the spin axis co-elevation is most sensible to magnetometer uncertainties and if this is an important design requirement, more effort should be spent on the magnetometers calibration.

In contrast, the contribution to the total attitude standard deviation from the sun sensors uncertainties results from the rotation about the spin axis. Thus, measuring the actual rotational position of the spacecraft is primarily subjected for uncertainties in the sun sensors.

The sorted list of coefficients c_i may tell the designer what axis he/she should put most effort in calibrating, as this will influence the output standard deviation mostly. At the equator (Position 1), mitigation of the residual field at the BFCS z-axis, e.g. by 1/10 to 10 nTesla, results in a 0.17 degree improvement in knowledge of the attitude by using Eqn. (5.20)

$$\Delta \sqrt{\sigma^2 c_{33}^2} = \sqrt{(100 \cdot 10^{-9})^2 (1.90 \cdot 10^6)^2} - \sqrt{(10 \cdot 10^{-9})^2 (1.90 \cdot 10^6)^2} = 0.17 \quad (6.1)$$

The subscript 33 refers to appendix B and the corresponding entry of the magnetic residual field.

6.2.2 Design example

The standard deviation of the attitude estimate over the orbit ranges between 0.71 and 0.81 degrees. Assume that the mission design requirements state that

the maximum allowable uncertainty of the attitude is 0.75 degree requiring an improvement in the sensor design.

By looking at the variance bar plots the designer can easily spot areas that would need improvement. For example, by considering the worst case orbital position, a true anomaly of 0 deg, one could decide that all contributions to the total variance should be below 0.03 deg² per uncertainty which would lower the total standard deviation of the attitude by about

$$\sigma_{lowered} = \sqrt{0.01 \cdot 5 + 0.02 \cdot 4} = 0.36 \quad (6.2)$$

deg meaning that the design requirement will be met.

After investigation, the designer proposes the following improvements to the attitude sensor design based on the Linear Covariance Analysis bar plot.

Required improvements

1. Focus on calibration of the magnetometer voltage bias at the z-axis. Since the magnetometer is essentially three sensors, one at each axis, this can be done.
2. Activate the component specific offset strap in the magnetometer to minimise the residual field in the z-direction
3. Further calibration of the magnetometers Z and Y axes sensitivity
4. Further calibration of the sun sensors sensitivity
5. Further calibration of the centroid of the sun sensors. Maybe consider including sub-pixel resolution.
6. Include a filter so that sun sensor and magnetometer noise is reduced

The designer can use the coefficients c_i to calculate to what accuracy (standard deviation) the uncertainties must be reduced to in order to lower the corresponding variance below 0.03 deg² by

$$\sigma_i = \sqrt{0.03/c_i^2} \quad (6.3)$$

The new specifications on the hardware as obtained by Eqn. (6.3) are summarised in Table 6.7.

After the changes are made, and simply redefining the standard deviation in ReDefLin's output substructure `inputStd` or using Eqn. (5.20), the new total standard deviation of the attitude taking all the uncertainties into account is calculated to 0.73 deg for the worst case position meaning that the design requirement is met. Note that the residual field, which was concluded to have the strongest impact

on attitude accuracy, had the lowest decrease.

Table 6.7: The new specifications of the uncertainties in order to meet the design requirement of 0.75 degree attitude determination accuracy as obtained from Eqn. (6.3). The index refer to the respective input in the Appendix B

Index	Input name	σ_i	Dimension	Decreased %
15	MagZ Bias voltage	9.0e-4	V	10.3
22	MagZ Voltage noise	9.0e-4	V	10.3
33	SC residual field Z	9.1e-008	Tesla	8.8
40	Sun1X Sensitivity	3.8e+0	pixel/rad	17.6
46	Sun1Y noise	8.1e-1	pixel	19.1
48	Sun1Y Centroid	8.1e-1	pixel	19.1
60	Sun2X Sensitivity	3.8e+1	pixel/rad	17.4
64	Sun2Y Centroid	7.9e-1	pixel	20.5
66	Sun2Y Noise	7.9e-1	V	20.5

Chapter 7

Conclusions and Further work

This project has successfully developed a software tool to determine the contribution of sensor uncertainties to the final attitude determination accuracy of a small satellite model. Generalized Matlab functions using Linear Covariance techniques were written which can

- predict the total accuracy of the attitude estimation resulting from sensor uncertainties
- find the contribution of each error source and evaluate its specific impact
- highlight deficiencies in hardware or calibration that need attention and re-design

The attainable accuracy for a number of uncertainties in a particular spacecraft's sensor specification in a sun synchronous orbit were calculated. Particular problems were identified and improved, thus increasing the accuracy of the attitude determination. Sensitivities, voltage biases and centroid calibration errors contribute primarily to the final attitude determination accuracy.

By looking at the calculated coefficients c_i , one can identify areas where the effort to lower the uncertainty would give greatest payback. One could also use a trial and error method to find what uncertainties are tolerable in the sensors and still meet mission requirements on attitude accuracy.

The TRIAD method was used to solve for the attitude. The selection of a more sophisticated solution method such as a Kalman filter would improve the overall accuracy and make it possible to estimate the attitude during periods where no sun measurements are available.

A Master's thesis [25] considering the attitude determination using only B-field and Kalman filtering recently is completed at Utah State University. The merge of this project with the current work is planned for the future. A rate sensor and its reconstructor have been included in the model to prepare for the future use of a Kalman filters. Since the model is constructed in Simulink, simple copy and paste operations can upgrade the solution method.

Future analysis using other sensors such as star cameras, infrared sensors etc, should be performed in order to provide a wider baseline for attitude determination accuracy.

As a final statement, it is important to point out that all the statements and conclusions are based on the particular set of data presented in Chapter 6. There is no guarantee that these results are general. More simulations and analysis should be made on more complex systems.

The availability to divide attitude determination accuracy into its components to point out areas that need special attention is promising, and is worth further research.

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Appendix A

HMC2003 Data sheet

This appendix contains the datasheet of HMC2003[15] which is used as a baseline for the initialisation of the magnetometer under consideration.

THREE-AXIS MAGNETIC SENSOR HYBRID

Features

- 20-pin Wide DIP Footprint (1" by 0.75")
- Precision 3-axis Capability
- Factory Calibrated Analog Outputs
- 40 micro-gauss to ± 2 gauss Dynamic Range
- Analog Output at 1 Volt/gauss (2.5V @ 0 gauss)
- Onboard +2.5 Volt Reference
- +6 to +15 Volt DC Single Supply Operation
- Very Low Magnetic Material Content
- -40° to 85°C Operating Temperature Range

General Description

The Honeywell HMC2003 is a high sensitivity, three-axis magnetic sensor hybrid assembly used to measure low magnetic field strengths. Honeywell's most sensitive magneto-resistive sensors (HMC1001 and HMC1002) are utilized to provide the reliability and precision of this magnetometer design. The HMC2003 interface is all analog with critical nodes brought out to the pin interfaces for maximum user flexibility. The internal excitation current source and selected gain and offset resistors, reduces temperature errors plus gain and offset drift. Three precision low-noise instrumentation amplifiers with 1kHz low pass filters provide accurate measurements while rejecting unwanted noise.

APPLICATIONS

- Precision Compassing
- Navigation Systems
- Attitude Reference
- Traffic Detection
- Proximity Detection
- Medical Devices

BLOCK DIAGRAM

SPECIFICATIONS

Characteristics	Conditions ⁽¹⁾	Min	Typ	Max	Units ⁽²⁾
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Magnetic Field

Sensitivity		0.98	1	1.02	V/gauss
Null Field Output		2.3	2.5	2.7	V
Resolution			40		μgauss
Field Range	Maximum Magnetic Flux Density	-2		2	gauss
Output Voltage	Each Magnetometer Axis Output	0.5		4.5	
Bandwidth			1		kHz

Errors

Linearity Error	±1 gauss Applied Field Sweep ±2 gauss Applied Field Sweep		0.5 1	2 2	%FS
Hysteresis Error	3 Sweeps across ±2 gauss		0.05	0.1	%FS
Repeatability Error	3 Sweeps across ±2 gauss		0.05	0.1	%FS
Power Supply Effect	PS Varied from 6 to 15V With ±1 gauss Applied Field Sweep			0.1	%FS

Offset Strap

Resistance				10.5	ohms
Sensitivity		46.5	47.5	48.5	mA/gauss
Current				200	mA

Set/Reset Strap

Resistance			4.5	6	ohms
Current	2msec pulse, 1% duty cycle	3.0	3.2	5	amps

Tempcos

Field Sensitivity			-600		ppm/°C
Null Field	Set/Reset Not Used Set/Reset Used		±400 ±100		ppm/°C

Environments

Temperature	Operating Storage	-40 -55	- -	+85 +125	°C °C
Shock			100		g
Vibration			2.2		g rms

Electrical

Supply Voltage ⁽³⁾		6		15	VDC
Supply Current				20	mA

(1) Unless otherwise stated, test conditions are as follows: Power Supply = 12VDC, Ambient Temp = 25°C,
Set/Reset switching is active

(2) Units: 1 gauss = 1 Oersted (in air) = 79.58 A/m = 10E5 gamma

(3) Transient protection circuitry should be added across V+ and Gnd if an unregulated power supply is used.

General Description

Honeywell's three axis magnetic sensor hybrid uses three permalloy magneto-resistive sensors and custom interface electronics to measure the strength and direction of an incident magnetic field. These sensors are sensitive to magnetic fields along the length, width, and height (X, Y, Z axis) of the 20-pin dual-in-line hybrid. Fields can be detected less than 40 microgauss and up to ± 2 gauss. Analog outputs are available for each X, Y and Z axis from the hybrid. With the sensitivity and linearity of this hybrid, changes can be detected in the earth's magnetic field to provide compass headings or attitude sensing. The high bandwidth of this hybrid allows for anomaly detection of vehicles, planes, and other ferrous objects at high speeds.

The hybrid is packaged on a small printed circuit board (1" by 0.75") and has an on-chip +2.5 voltage reference that operates from a single 6 to 15V supply. The hybrid is ideal for applications that require two- or three-axis magnetic sensing and have size constraints and need a magnetic transducer (magnetometer) front-end. Note that the hybrid's resistor values will vary, or an absence of some resistor components, is likely due to individual factory calibration.

Integrated with the sensor elements composed of wheatstone bridge circuits, are magnetically coupled straps that replace the need for external field coils and provide various modes of operation. The Honeywell patented integrated field offset straps (Xoff+ and Xoff-, etc.) can be used electrically to apply local magnetic fields to the bridges to buck, or offset an applied incident field. This technique can be used to cancel unwanted ambient magnetic fields (e.g. hard-iron magnetism) or in a closed loop field nulling measurement circuit. The offset straps nominally provide 1 gauss fields along the sensitive axis per 48mA of offset current through each strap.

The HMC2003's magnetic sensors can be affected by high momentary magnetic fields that may lead to output signal degradation. In order to eliminate this effect, and maximize the signal output, a magnetic switching technique can be applied to the bridge using set/reset pins (SR+ and SR-) that eliminates the effect of past magnetic history. Refer to the application notes that provide information on set/reset circuits and operation.

Pinout Diagram and Package Drawing

Symbol	Millimeters		Inches	
	Min	Max	Min	Max
A	10.92	11.94	0.43	0.47
A1	2.92	3.42	0.115	0.135
D	25.91	27.30	1.02	1.075
e	2.41	2.67	0.095	0.105
H	18.03	19.69	0.71	0.775

Ordering Information

Ordering Number	Product
HMC2003	Three-Axis Magnetic Sensor Hybrid

Honeywell reserves the right to make changes to improve reliability, function or design. Honeywell does not assume any liability arising out of the application or use of any product or circuit described herein; neither does it convey any license under its patent rights nor the rights of others.

Appendix B

Raw data

In this appendix, the soft mean raw data is presented. The contributions from each output axis' standard deviation to the total standard deviation from respective input uncertainty can be identified.

The Z Y Z are Euler angles with the specified rotation order. The total standard deviation is calculated as $\sqrt{\sigma_{Z1}^2 + \sigma_{Y2}^2 + \sigma_{Z3}^2}$ where σ is the standard deviation at the respective axis. The subscripts refer to the rotation axial sequence of the Euler angles.

For the mounting uncertainties, the stated axis is about the BFCS.

All stated values of the outputs are soft means averaged over 5 revolutions with the unit degree.

For the units of the inputs uncertainties refer to the main body of the text, chapter 5

Table B.1: Raw data: Position 1: True anomaly = 0 deg

Input Index/Name	Input Std	Coefficient c_i	Total Std	1:Rotation Z	2:Rotation Y	3:Rotation Z
1:MagX Sensed temperature	1.00e-001	4.76e-002	4.76e-003	1.61e-004	4.75e-003	2.23e-004
2:MagY Sensed temperature	1.00e-001	1.39e-001	1.39e-002	5.00e-004	1.38e-002	6.69e-004
3:MagZ Sensed temperature	1.00e-001	3.04e-001	3.04e-002	9.18e-004	3.04e-002	1.36e-003
4:Mag box mounting X	1.70e-003	5.39e+001	9.16e-002	2.91e-003	9.15e-002	4.20e-003
5:Mag box mounting Y	1.70e-003	1.86e+001	3.16e-002	9.44e-004	3.15e-002	1.37e-003
6:Mag box mounting Z	1.70e-003	1.32e+000	2.25e-003	9.33e-005	2.24e-003	1.24e-004
7:MagY Bias voltage	1.00e-003	9.21e+001	9.21e-002	3.44e-003	9.20e-002	4.59e-003
8:MagY Cosine nonlinear term	5.00e-003	3.84e+000	1.92e-002	7.21e-004	1.91e-002	9.64e-004
9:MagY Nominal temperature	1.00e-002	1.39e-001	1.39e-003	5.00e-005	1.38e-003	6.69e-005
10:MagY Sensitivity	6.67e+001	2.13e-003	1.42e-001	5.31e-003	1.42e-001	7.10e-003
11:MagY Sine nonlinear term	5.00e-003	2.08e+001	1.04e-001	3.89e-003	1.04e-001	5.21e-003
12:MagY Thermal bias drift	7.50e-005	9.21e+002	6.91e-002	2.58e-003	6.90e-002	3.45e-003
13:MagY Thermal sensitivity drift	3.00e-001	2.13e-002	6.38e-003	2.39e-004	6.37e-003	3.20e-004
14:MagY Voltage noise	1.00e-003	9.21e+001	9.21e-002	3.44e-003	9.20e-002	4.59e-003
15:MagZ Bias voltage	1.00e-003	1.93e+002	1.93e-001	5.83e-003	1.93e-001	8.63e-003
16:MagZ Sine nonlinear term	5.00e-003	2.32e+001	1.16e-001	3.52e-003	1.16e-001	5.21e-003
17:MagZ Cosine nonlinear term	5.00e-003	2.22e+000	1.11e-002	3.39e-004	1.11e-002	5.02e-004
18:MagZ Nominal temperature	1.00e-002	3.04e-001	3.04e-003	9.18e-005	3.04e-003	1.36e-004
19:MagZ Sensitivity	6.67e+001	2.35e-003	1.56e-001	4.75e-003	1.56e-001	7.03e-003
20:MagZ Thermal bias drift	7.50e-005	1.93e+003	1.45e-001	4.38e-003	1.45e-001	6.48e-003
21:MagZ Thermal sensitivity drift	3.00e-001	2.35e-002	7.04e-003	2.14e-004	7.03e-003	3.16e-004
22:MagZ Voltage noise	1.00e-003	1.93e+002	1.93e-001	5.83e-003	1.93e-001	8.63e-003
23:MagX Bias voltage	1.00e-003	3.19e+001	3.19e-002	1.09e-003	3.19e-002	1.50e-003
24:MagX Cosine nonlinear term	5.00e-003	3.01e-001	1.51e-003	5.00e-005	1.50e-003	6.92e-005
25:MagX Nominal temperature	1.00e-002	4.76e-002	4.76e-004	1.61e-005	4.75e-004	2.23e-005
26:MagX Sensitivity	6.67e+001	3.36e-004	2.24e-002	7.50e-004	2.23e-002	1.04e-003
27:MagX Sine nonlinear term	5.00e-003	3.32e+000	1.66e-002	5.57e-004	1.66e-002	7.69e-004
28:MagX Thermal bias drift	7.50e-005	3.19e+002	2.39e-002	8.14e-004	2.39e-002	1.13e-003
29:MagX Thermal sensitivity drift	3.00e-001	3.36e-003	1.01e-003	3.37e-005	1.01e-003	4.66e-005
30:MagX Voltage noise	1.00e-003	3.19e+001	3.19e-002	1.09e-003	3.19e-002	1.50e-003
31:SC residual field X	1.00e-007	3.10e+005	3.10e-002	1.05e-003	3.09e-002	1.45e-003
32:SC residual field Y	1.00e-007	9.09e+005	9.09e-002	3.39e-003	9.07e-002	4.53e-003
33:SC residual field Z	1.00e-007	1.90e+006	1.90e-001	5.74e-003	1.90e-001	8.49e-003
34:Sun1 box mounting X	1.70e-003	3.25e+001	5.53e-002	4.50e-002	9.44e-004	3.22e-002
35:Sun1 box mounting Y	1.70e-003	5.63e+001	9.57e-002	5.20e-005	1.41e-003	9.56e-002
36:Sun1 box mounting Z	1.70e-003	3.25e+001	5.53e-002	4.50e-002	9.44e-004	3.22e-002
37:Sun1X Cosine nonlinear term	5.00e-003	1.91e+001	9.54e-002	5.10e-005	1.36e-003	9.54e-002
38:Sun1X Nominal temperature	1.00e-002	1.82e-002	1.82e-004	6.97e-008	2.26e-006	1.82e-004
39:Sun1X Centroid	1.00e+000	1.32e-001	1.32e-001	7.18e-005	1.94e-003	1.32e-001
40:Sun1X Sensitivity	4.64e+000	4.53e-002	2.10e-001	1.13e-004	3.02e-003	2.10e-001
41:Sun1X Sine nonlinear term	5.00e-003	8.48e+000	4.24e-002	2.34e-005	6.31e-004	4.24e-002
42:Sun1X Thermal centroid drift	5.00e-003	1.32e+000	6.61e-003	3.59e-006	9.71e-005	6.61e-003
43:Sun1X Thermal sensitivity drift	1.39e-002	4.53e-001	6.29e-003	3.38e-006	9.04e-005	6.29e-003
44:Sun1X Noise	1.00e+000	1.32e-001	1.32e-001	7.18e-005	1.94e-003	1.32e-001
45:Sun1Y Cosine nonlinear term	5.00e-003	1.36e-001	6.82e-004	5.58e-004	1.53e-005	3.93e-004
46:Sun1Y Noise	1.00e+000	2.14e-001	2.14e-001	1.75e-001	3.65e-003	1.23e-001
47:Sun1Y Nominal temperature	1.00e-002	2.24e-002	2.24e-004	1.83e-004	3.88e-006	1.29e-004
48:Sun1Y Centroid	1.00e+000	2.14e-001	2.14e-001	1.75e-001	3.65e-003	1.23e-001
49:Sun1Y Sensitivity	4.64e+000	4.09e-003	1.90e-002	1.55e-002	4.04e-004	1.09e-002
50:Sun1Y Sine nonlinear term	5.00e-003	1.88e+000	9.41e-003	7.69e-003	2.00e-004	5.41e-003
51:Sun1Y Thermal centroid drift	5.00e-003	2.14e+000	1.07e-002	8.76e-003	1.83e-004	6.17e-003
52:Sun1Y Thermal sensitivity drift	1.39e-002	4.09e-002	5.69e-004	4.65e-004	1.21e-005	3.27e-004
53:Sun2 box box mounting X	1.70e-003	3.29e+001	5.60e-002	4.55e-002	5.35e-004	3.26e-002
54:Sun2 box box mounting Y	1.70e-003	5.63e+001	9.56e-002	1.42e-004	3.19e-003	9.56e-002
55:Sun2 box box mounting Z	1.70e-003	3.29e+001	5.60e-002	4.55e-002	5.35e-004	3.26e-002
56:Sun2X Cosine nonlinear term	5.00e-003	1.91e+001	9.53e-002	1.32e-004	3.08e-003	9.52e-002
57:Sun2X Noise	1.00e+000	1.32e-001	1.32e-001	1.95e-004	4.40e-003	1.32e-001
58:Sun2X Nominal temperature	1.00e-002	1.82e-002	1.82e-004	2.38e-007	5.38e-006	1.82e-004
59:Sun2X Centroid	1.00e+000	1.32e-001	1.32e-001	1.95e-004	4.40e-003	1.32e-001
60:Sun2X Sensitivity	4.64e+000	4.52e-002	2.10e-001	2.94e-004	6.81e-003	2.10e-001
61:Sun2X Sine nonlinear term	5.00e-003	8.49e+000	4.24e-002	6.25e-005	1.41e-003	4.24e-002
62:Sun2X Thermal centroid drift	5.00e-003	1.32e+000	6.61e-003	9.76e-006	2.20e-004	6.60e-003
63:Sun2X Thermal sensitivity drift	1.39e-002	4.52e-001	6.29e-003	8.83e-006	2.04e-004	6.29e-003
64:Sun2Y Centroid	1.00e+000	2.18e-001	2.18e-001	1.77e-001	2.05e-003	1.27e-001
65:Sun2Y Cosine nonlinear term	5.00e-003	2.43e-001	1.21e-003	9.88e-004	1.54e-005	7.05e-004
66:Sun2Y Noise	1.00e+000	2.18e-001	2.18e-001	1.77e-001	2.05e-003	1.27e-001
67:Sun2Y Nominal temperature	1.00e-002	2.03e-002	2.03e-004	1.65e-004	1.87e-006	1.18e-004
68:Sun2Y Sensitivity	4.64e+000	5.85e-003	2.71e-002	2.21e-002	3.16e-004	1.58e-002
69:Sun2Y Thermal sensitivity drift	1.39e-002	5.85e-002	8.13e-004	6.62e-004	9.46e-006	4.72e-004
70:Sun2Y Sine nonlinear term	5.00e-003	2.69e+000	1.34e-002	1.09e-002	1.56e-004	7.80e-003
71:Sun2Y Thermal centroid drift	5.00e-003	2.18e+000	1.09e-002	8.87e-003	1.03e-004	6.33e-003
72:Sun1X Sensed temperature	1.00e-002	1.82e-002	1.82e-004	6.97e-008	2.26e-006	1.82e-004
73:Sun2X Sensed temperature	1.00e-001	1.82e-002	1.82e-003	2.38e-006	5.38e-005	1.82e-003
74:Sun1Y Sensed temperature	1.00e-001	2.24e-002	2.24e-003	1.83e-003	3.88e-005	1.29e-003
75:Sun2Y Sensed temperature	1.00e-001	2.03e-002	2.03e-003	1.65e-003	1.87e-005	1.18e-003

Figure B.1: Bar plots visualising data in Table B.1

Table B.2: Raw data: Position 2: True anomaly = 45 deg

Input Index/Name	Input Std	Coefficient c_i	Total Std	1:Rotation Z	2:Rotation Y	3:Rotation Z
1:MagX Sensed temperature	1.00e-001	3.54e-002	3.54e-003	1.16e-004	3.54e-003	1.63e-004
2:MagY Sensed temperature	1.00e-001	1.09e-001	1.09e-002	3.69e-004	1.09e-002	5.28e-004
3:MagZ Sensed temperature	1.00e-001	2.17e-001	2.17e-002	7.91e-004	2.17e-002	1.07e-003
4:Mag box mounting X	1.70e-003	5.15e+001	8.75e-002	3.13e-003	8.74e-002	4.30e-003
5:Mag box mounting Y	1.70e-003	1.89e+001	3.22e-002	1.07e-003	3.22e-002	1.45e-003
6:Mag box mounting Z	1.70e-003	4.52e+000	7.68e-003	2.77e-004	7.66e-003	3.94e-004
7:MagY Bias voltage	1.00e-003	7.27e+001	7.27e-002	2.34e-003	7.25e-002	3.36e-003
8:MagY Cosine nonlinear term	5.00e-003	5.36e+000	2.68e-002	8.87e-004	2.68e-002	1.27e-003
9:MagY Nominal temperature	1.00e-002	1.09e-001	1.09e-003	3.69e-005	1.09e-003	5.28e-005
10:MagY Sensitivity	6.67e+001	2.23e-003	1.49e-001	4.87e-003	1.49e-001	6.99e-003
11:MagY Sine nonlinear term	5.00e-003	2.16e+001	1.08e-001	3.52e-003	1.08e-001	5.05e-003
12:MagY Thermal bias drift	7.50e-005	7.27e+002	5.45e-002	1.76e-003	5.44e-002	2.52e-003
13:MagY Thermal sensitivity drift	3.00e-001	2.23e-002	6.70e-003	2.19e-004	6.69e-003	3.15e-004
14:MagY Voltage noise	1.00e-003	7.27e+001	7.27e-002	2.34e-003	7.25e-002	3.36e-003
15:MagZ Bias voltage	1.00e-003	1.36e+002	1.36e-001	4.95e-003	1.36e-001	6.72e-003
16:MagZ Sine nonlinear term	5.00e-003	2.17e+001	1.08e-001	3.90e-003	1.08e-001	5.30e-003
17:MagZ Cosine nonlinear term	5.00e-003	2.76e+000	1.38e-002	4.94e-004	1.38e-002	6.70e-004
18:MagZ Nominal temperature	1.00e-002	2.17e-001	2.17e-003	7.91e-005	2.17e-003	1.07e-004
19:MagZ Sensitivity	6.67e+001	2.20e-003	1.46e-001	5.28e-003	1.46e-001	7.16e-003
20:MagZ Thermal bias drift	7.50e-005	1.36e+003	1.02e-001	3.72e-003	1.02e-001	5.04e-003
21:MagZ Thermal sensitivity drift	3.00e-001	2.20e-002	6.59e-003	2.38e-004	6.58e-003	3.22e-004
22:MagZ Voltage noise	1.00e-003	1.36e+002	1.36e-001	4.95e-003	1.36e-001	6.72e-003
23:MagX Bias voltage	1.00e-003	2.38e+001	2.38e-002	7.72e-004	2.38e-002	1.09e-003
24:MagX Cosine nonlinear term	5.00e-003	4.61e-001	2.31e-003	6.55e-005	2.30e-003	9.17e-005
25:MagX Nominal temperature	1.00e-002	3.54e-002	3.54e-004	1.16e-005	3.54e-004	1.63e-005
26:MagX Sensitivity	6.67e+001	3.44e-004	2.30e-002	6.79e-004	2.29e-002	9.54e-004
27:MagX Sine nonlinear term	5.00e-003	3.39e+000	1.70e-002	5.02e-004	1.69e-002	7.05e-004
28:MagX Thermal bias drift	7.50e-005	2.38e+002	1.79e-002	5.79e-004	1.78e-002	8.14e-004
29:MagX Thermal sensitivity drift	3.00e-001	3.44e-003	1.03e-003	3.06e-005	1.03e-003	4.29e-005
30:MagX Voltage noise	1.00e-003	2.38e+001	2.38e-002	7.72e-004	2.38e-002	1.09e-003
31:SC residual field X	1.00e-007	2.31e+005	2.31e-002	7.45e-004	2.30e-002	1.05e-003
32:SC residual field Y	1.00e-007	7.17e+005	7.17e-002	2.31e-003	7.16e-002	3.32e-003
33:SC residual field Z	1.00e-007	1.34e+006	1.34e-001	4.88e-003	1.34e-001	6.61e-003
34:Sun1 box mounting X	1.70e-003	3.28e+001	5.58e-002	4.53e-002	2.93e-003	3.25e-002
35:Sun1 box mounting Y	1.70e-003	5.64e+001	9.59e-002	7.65e-005	3.75e-003	9.58e-002
36:Sun1 box mounting Z	1.70e-003	3.28e+001	5.58e-002	4.53e-002	2.93e-003	3.25e-002
37:Sun1X Cosine nonlinear term	5.00e-003	1.91e+001	9.56e-002	7.26e-005	3.92e-003	9.55e-002
38:Sun1X Nominal temperature	1.00e-002	1.82e-002	1.82e-004	1.35e-007	8.42e-006	1.82e-004
39:Sun1X Centroid	1.00e+000	1.33e-001	1.33e-001	1.05e-004	5.22e-003	1.32e-001
40:Sun1X Sensitivity	4.64e+000	4.54e-002	2.10e-001	1.61e-004	8.57e-003	2.10e-001
41:Sun1X Sine nonlinear term	5.00e-003	8.50e+000	4.25e-002	3.41e-005	1.67e-003	4.25e-002
42:Sun1X Thermal centroid drift	5.00e-003	1.33e+000	6.63e-003	5.26e-006	2.61e-004	6.62e-003
43:Sun1X Thermal sensitivity drift	1.39e-002	4.54e-001	6.31e-003	4.84e-006	2.57e-004	6.30e-003
44:Sun1X Noise	1.00e+000	1.33e-001	1.33e-001	1.05e-004	5.22e-003	1.32e-001
45:Sun1Y Cosine nonlinear term	5.00e-003	1.38e-001	6.89e-004	5.62e-004	3.07e-005	3.99e-004
46:Sun1Y Noise	1.00e+000	2.17e-001	2.17e-001	1.77e-001	1.13e-002	1.25e-001
47:Sun1Y Nominal temperature	1.00e-002	2.27e-002	2.27e-004	1.85e-004	1.17e-005	1.31e-004
48:Sun1Y Centroid	1.00e+000	2.17e-001	2.17e-001	1.77e-001	1.13e-002	1.25e-001
49:Sun1Y Sensitivity	4.64e+000	4.14e-003	1.92e-002	1.56e-002	8.85e-004	1.11e-002
50:Sun1Y Sine nonlinear term	5.00e-003	1.90e+000	9.51e-003	7.75e-003	4.39e-004	5.50e-003
51:Sun1Y Thermal centroid drift	5.00e-003	2.17e+000	1.08e-002	8.83e-003	5.63e-004	6.26e-003
52:Sun1Y Thermal sensitivity drift	1.39e-002	4.14e-002	5.75e-004	4.69e-004	2.65e-005	3.33e-004
53:Sun2 box box mounting X	1.70e-003	3.32e+001	5.64e-002	4.57e-002	4.16e-003	3.28e-002
54:Sun2 box box mounting Y	1.70e-003	5.63e+001	9.56e-002	3.19e-004	7.40e-003	9.54e-002
55:Sun2 box box mounting Z	1.70e-003	3.32e+001	5.64e-002	4.57e-002	4.16e-003	3.28e-002
56:Sun2X Cosine nonlinear term	5.00e-003	1.91e+001	9.53e-002	2.96e-004	7.24e-003	9.50e-002
57:Sun2X Noise	1.00e+000	1.32e-001	1.32e-001	4.37e-004	1.02e-002	1.32e-001
58:Sun2X Nominal temperature	1.00e-002	1.82e-002	1.82e-004	5.84e-007	1.41e-005	1.81e-004
59:Sun2X Centroid	1.00e+000	1.32e-001	1.32e-001	4.37e-004	1.02e-002	1.32e-001
60:Sun2X Sensitivity	4.64e+000	4.52e-002	2.10e-001	6.60e-004	1.60e-002	2.09e-001
61:Sun2X Sine nonlinear term	5.00e-003	8.49e+000	4.24e-002	1.42e-004	3.29e-003	4.23e-002
62:Sun2X Thermal centroid drift	5.00e-003	1.32e+000	6.61e-003	2.19e-005	5.10e-004	6.59e-003
63:Sun2X Thermal sensitivity drift	1.39e-002	4.52e-001	6.29e-003	1.98e-005	4.80e-004	6.27e-003
64:Sun2Y Centroid	1.00e+000	2.21e-001	2.21e-001	1.79e-001	1.64e-002	1.28e-001
65:Sun2Y Cosine nonlinear term	5.00e-003	2.46e-001	1.23e-003	9.97e-004	9.75e-005	7.17e-004
66:Sun2Y Noise	1.00e+000	2.21e-001	2.21e-001	1.79e-001	1.64e-002	1.28e-001
67:Sun2Y Nominal temperature	1.00e-002	2.05e-002	2.05e-004	1.66e-004	1.52e-005	1.19e-004
68:Sun2Y Sensitivity	4.64e+000	5.93e-003	2.75e-002	2.23e-002	2.13e-003	1.60e-002
69:Sun2Y Thermal sensitivity drift	1.39e-002	5.93e-002	8.24e-004	6.68e-004	6.40e-005	4.80e-004
70:Sun2Y Sine nonlinear term	5.00e-003	2.72e+000	1.36e-002	1.10e-002	1.06e-003	7.92e-003
71:Sun2Y Thermal centroid drift	5.00e-003	2.21e+000	1.10e-002	8.93e-003	8.18e-004	6.42e-003
72:Sun1X Sensed temperature	1.00e-002	1.82e-002	1.82e-004	1.35e-007	8.42e-006	1.82e-004
73:Sun2X Sensed temperature	1.00e-001	1.82e-002	1.82e-003	5.84e-006	1.41e-004	1.81e-003
74:Sun1Y Sensed temperature	1.00e-001	2.27e-002	2.27e-003	1.85e-003	1.17e-004	1.31e-003
75:Sun2Y Sensed temperature	1.00e-001	2.05e-002	2.05e-003	1.66e-003	1.52e-004	1.19e-003

Figure B.2: Bar plots visualising data in Table B.2

Table B.3: Raw data: Position 3: True anomaly = 90 deg

Input Index/Name	Input Std	Coefficient c_i	Total Std	1:Rotation Z	2:Rotation Y	3:Rotation Z
1:MagX Sensed temperature	1.00e-001	5.31e-002	5.31e-003	1.75e-004	5.30e-003	2.46e-004
2:MagY Sensed temperature	1.00e-001	1.50e-001	1.50e-002	5.40e-004	1.50e-002	7.43e-004
3:MagZ Sensed temperature	1.00e-001	1.57e-001	1.57e-002	4.74e-004	1.57e-002	6.98e-004
4:Mag box mounting X	1.70e-003	5.38e+001	9.14e-002	2.99e-003	9.13e-002	4.26e-003
5:Mag box mounting Y	1.70e-003	2.15e+001	3.65e-002	1.05e-003	3.64e-002	1.51e-003
6:Mag box mounting Z	1.70e-003	9.69e+000	1.65e-002	5.42e-004	1.64e-002	7.48e-004
7:MagY Bias voltage	1.00e-003	9.95e+001	9.95e-002	3.46e-003	9.93e-002	4.76e-003
8:MagY Cosine nonlinear term	5.00e-003	5.22e+000	2.61e-002	9.32e-004	2.61e-002	1.28e-003
9:MagY Nominal temperature	1.00e-002	1.50e-001	1.50e-003	5.40e-005	1.50e-003	7.43e-005
10:MagY Sensitivity	6.67e+001	2.56e-003	1.71e-001	6.02e-003	1.70e-001	8.29e-003
11:MagY Sine nonlinear term	5.00e-003	2.49e+001	1.24e-001	4.39e-003	1.24e-001	6.05e-003
12:MagY Thermal bias drift	7.50e-005	9.95e+002	7.46e-002	2.60e-003	7.45e-002	3.57e-003
13:MagY Thermal sensitivity drift	3.00e-001	2.56e-002	7.67e-003	2.71e-004	7.66e-003	3.73e-004
14:MagY Voltage noise	1.00e-003	9.95e+001	9.95e-002	3.46e-003	9.93e-002	4.76e-003
15:MagZ Bias voltage	1.00e-003	1.17e+002	1.17e-001	3.53e-003	1.17e-001	5.20e-003
16:MagZ Sine nonlinear term	5.00e-003	2.95e+001	1.47e-001	4.47e-003	1.47e-001	6.59e-003
17:MagZ Cosine nonlinear term	5.00e-003	6.08e+000	3.04e-002	9.29e-004	3.04e-002	1.37e-003
18:MagZ Nominal temperature	1.00e-002	1.57e-001	1.57e-003	4.74e-005	1.57e-003	6.98e-005
19:MagZ Sensitivity	6.67e+001	3.03e-003	2.02e-001	6.13e-003	2.01e-001	9.03e-003
20:MagZ Thermal bias drift	7.50e-005	1.17e+003	8.77e-002	2.65e-003	8.75e-002	3.90e-003
21:MagZ Thermal sensitivity drift	3.00e-001	3.03e-002	9.08e-003	2.76e-004	9.07e-003	4.06e-004
22:MagZ Voltage noise	1.00e-003	1.17e+002	1.17e-001	3.53e-003	1.17e-001	5.20e-003
23:MagX Bias voltage	1.00e-003	3.52e+001	3.52e-002	1.15e-003	3.52e-002	1.61e-003
24:MagX Cosine nonlinear term	5.00e-003	7.35e-001	3.67e-003	1.04e-004	3.67e-003	1.45e-004
25:MagX Nominal temperature	1.00e-002	5.31e-002	5.31e-004	1.75e-005	5.30e-004	2.46e-005
26:MagX Sensitivity	6.67e+001	4.94e-004	3.29e-002	9.75e-004	3.29e-002	1.36e-003
27:MagX Sine nonlinear term	5.00e-003	4.86e+000	2.43e-002	7.19e-004	2.43e-002	1.00e-003
28:MagX Thermal bias drift	7.50e-005	3.52e+002	2.64e-002	8.59e-004	2.64e-002	1.21e-003
29:MagX Thermal sensitivity drift	3.00e-001	4.94e-003	1.48e-003	4.39e-005	1.48e-003	6.12e-005
30:MagX Voltage noise	1.00e-003	3.52e+001	3.52e-002	1.15e-003	3.52e-002	1.61e-003
31:SC residual field X	1.00e-007	3.42e+005	3.42e-002	1.11e-003	3.41e-002	1.56e-003
32:SC residual field Y	1.00e-007	9.81e+005	9.81e-002	3.41e-003	9.80e-002	4.70e-003
33:SC residual field Z	1.00e-007	1.15e+006	1.15e-001	3.47e-003	1.15e-001	5.11e-003
34:Sun1 box mounting X	1.70e-003	3.24e+001	5.51e-002	4.47e-002	4.88e-003	3.18e-002
35:Sun1 box mounting Y	1.70e-003	5.68e+001	9.66e-002	4.66e-004	1.70e-002	9.51e-002
36:Sun1 box mounting Z	1.70e-003	3.24e+001	5.51e-002	4.47e-002	4.88e-003	3.18e-002
37:Sun1X Cosine nonlinear term	5.00e-003	1.93e+001	9.64e-002	4.61e-004	1.69e-002	9.49e-002
38:Sun1X Nominal temperature	1.00e-002	1.84e-002	1.84e-004	6.74e-007	3.18e-005	1.81e-004
39:Sun1X Centroid	1.00e+000	1.34e-001	1.34e-001	6.44e-004	2.35e-002	1.32e-001
40:Sun1X Sensitivity	4.64e+000	4.58e-002	2.12e-001	1.02e-003	3.72e-002	2.09e-001
41:Sun1X Sine nonlinear term	5.00e-003	8.57e+000	4.28e-002	2.06e-004	7.53e-003	4.22e-002
42:Sun1X Thermal centroid drift	5.00e-003	1.34e+000	6.68e-003	3.22e-005	1.17e-003	6.58e-003
43:Sun1X Thermal sensitivity drift	1.39e-002	4.58e-001	6.36e-003	3.05e-005	1.11e-003	6.26e-003
44:Sun1X Noise	1.00e+000	1.34e-001	1.34e-001	6.44e-004	2.35e-002	1.32e-001
45:Sun1Y Cosine nonlinear term	5.00e-003	1.36e-001	6.80e-004	5.55e-004	5.61e-005	3.89e-004
46:Sun1Y Noise	1.00e+000	2.14e-001	2.14e-001	1.74e-001	1.91e-002	1.22e-001
47:Sun1Y Nominal temperature	1.00e-002	2.23e-002	2.23e-004	1.82e-004	1.99e-005	1.28e-004
48:Sun1Y Centroid	1.00e+000	2.14e-001	2.14e-001	1.74e-001	1.91e-002	1.22e-001
49:Sun1Y Sensitivity	4.64e+000	4.08e-003	1.89e-002	1.54e-002	1.59e-003	1.08e-002
50:Sun1Y Sine nonlinear term	5.00e-003	1.88e+000	9.38e-003	7.65e-003	7.90e-004	5.36e-003
51:Sun1Y Thermal centroid drift	5.00e-003	2.14e+000	1.07e-002	8.71e-003	9.57e-004	6.10e-003
52:Sun1Y Thermal sensitivity drift	1.39e-002	4.08e-002	5.67e-004	4.63e-004	4.77e-005	3.24e-004
53:Sun2 box box mounting X	1.70e-003	3.26e+001	5.54e-002	4.50e-002	6.06e-003	3.17e-002
54:Sun2 box box mounting Y	1.70e-003	5.72e+001	9.73e-002	4.74e-004	1.30e-002	9.64e-002
55:Sun2 box box mounting Z	1.70e-003	3.26e+001	5.54e-002	4.50e-002	6.06e-003	3.17e-002
56:Sun2X Cosine nonlinear term	5.00e-003	1.94e+001	9.69e-002	4.56e-004	1.31e-002	9.60e-002
57:Sun2X Noise	1.00e+000	1.34e-001	1.34e-001	6.53e-004	1.81e-002	1.33e-001
58:Sun2X Nominal temperature	1.00e-002	1.85e-002	1.85e-004	9.15e-007	2.60e-005	1.83e-004
59:Sun2X Centroid	1.00e+000	1.34e-001	1.34e-001	6.53e-004	1.81e-002	1.33e-001
60:Sun2X Sensitivity	4.64e+000	4.60e-002	2.13e-001	1.01e-003	2.89e-002	2.11e-001
61:Sun2X Sine nonlinear term	5.00e-003	8.63e+000	4.32e-002	2.11e-004	5.80e-003	4.28e-002
62:Sun2X Thermal centroid drift	5.00e-003	1.34e+000	6.72e-003	3.26e-005	9.03e-004	6.66e-003
63:Sun2X Thermal sensitivity drift	1.39e-002	4.60e-001	6.40e-003	3.03e-005	8.66e-004	6.34e-003
64:Sun2Y Centroid	1.00e+000	2.15e-001	2.15e-001	1.75e-001	2.38e-002	1.23e-001
65:Sun2Y Cosine nonlinear term	5.00e-003	2.40e-001	1.20e-003	9.74e-004	1.39e-004	6.84e-004
66:Sun2Y Noise	1.00e+000	2.15e-001	2.15e-001	1.75e-001	2.38e-002	1.23e-001
67:Sun2Y Nominal temperature	1.00e-002	2.00e-002	2.00e-004	1.63e-004	2.21e-005	1.15e-004
68:Sun2Y Sensitivity	4.64e+000	5.78e-003	2.68e-002	2.18e-002	3.06e-003	1.53e-002
69:Sun2Y Thermal sensitivity drift	1.39e-002	5.78e-002	8.03e-004	6.53e-004	9.16e-005	4.59e-004
70:Sun2Y Sine nonlinear term	5.00e-003	2.65e+000	1.33e-002	1.08e-002	1.51e-003	7.58e-003
71:Sun2Y Thermal centroid drift	5.00e-003	2.15e+000	1.08e-002	8.75e-003	1.19e-003	6.16e-003
72:Sun1X Sensed temperature	1.00e-002	1.84e-002	1.84e-004	6.74e-007	3.18e-005	1.81e-004
73:Sun2X Sensed temperature	1.00e-001	1.85e-002	1.85e-003	9.15e-006	2.60e-004	1.83e-003
74:Sun1Y Sensed temperature	1.00e-001	2.23e-002	2.23e-003	1.82e-003	1.99e-004	1.28e-003
75:Sun2Y Sensed temperature	1.00e-001	2.00e-002	2.00e-003	1.63e-003	2.21e-004	1.15e-003

Figure B.3: Bar plots visualising data in Table B.3

Table B.4: Raw data: Position 4: True anomaly = 135 deg

Input Index/Name	Input Std	Coefficient c_i	Total Std	1:Rotation Z	2:Rotation Y	3:Rotation Z
1:MagX Sensed temperature	1.00e-001	7.61e-002	7.61e-003	2.41e-004	7.60e-003	3.42e-004
2:MagY Sensed temperature	1.00e-001	2.20e-001	2.20e-002	6.83e-004	2.19e-002	9.88e-004
3:MagZ Sensed temperature	1.00e-001	7.44e-002	7.44e-003	2.62e-004	7.43e-003	3.60e-004
4:Mag box mounting X	1.70e-003	5.40e+001	9.17e-002	2.93e-003	9.16e-002	4.21e-003
5:Mag box mounting Y	1.70e-003	1.87e+001	3.19e-002	1.04e-003	3.18e-002	1.47e-003
6:Mag box mounting Z	1.70e-003	1.10e+001	1.86e-002	5.63e-004	1.86e-002	8.15e-004
7:MagY Bias voltage	1.00e-003	1.46e+002	1.46e-001	4.63e-003	1.46e-001	6.69e-003
8:MagY Cosine nonlinear term	5.00e-003	1.57e+000	7.87e-003	2.43e-004	7.86e-003	3.50e-004
9:MagY Nominal temperature	1.00e-002	2.20e-001	2.20e-003	6.83e-005	2.19e-003	9.88e-005
10:MagY Sensitivity	6.67e+001	1.65e-003	1.10e-001	3.44e-003	1.10e-001	4.96e-003
11:MagY Sine nonlinear term	5.00e-003	1.64e+001	8.18e-002	2.55e-003	8.17e-002	3.68e-003
12:MagY Thermal bias drift	7.50e-005	1.46e+003	1.10e-001	3.47e-003	1.10e-001	5.02e-003
13:MagY Thermal sensitivity drift	3.00e-001	1.65e-002	4.96e-003	1.55e-004	4.95e-003	2.23e-004
14:MagY Voltage noise	1.00e-003	1.46e+002	1.46e-001	4.63e-003	1.46e-001	6.69e-003
15:MagZ Bias voltage	1.00e-003	5.72e+001	5.72e-002	2.01e-003	5.71e-002	2.76e-003
16:MagZ Sine nonlinear term	5.00e-003	1.81e+001	9.07e-002	3.19e-003	9.05e-002	4.39e-003
17:MagZ Cosine nonlinear term	5.00e-003	4.81e+000	2.41e-002	8.47e-004	2.40e-002	1.16e-003
18:MagZ Nominal temperature	1.00e-002	7.44e-002	7.44e-004	2.62e-005	7.43e-004	3.60e-005
19:MagZ Sensitivity	6.67e+001	1.89e-003	1.26e-001	4.43e-003	1.26e-001	6.09e-003
20:MagZ Thermal bias drift	7.50e-005	5.72e+002	4.29e-002	1.51e-003	4.28e-002	2.07e-003
21:MagZ Thermal sensitivity drift	3.00e-001	1.89e-002	5.66e-003	1.99e-004	5.65e-003	2.74e-004
22:MagZ Voltage noise	1.00e-003	5.72e+001	5.72e-002	2.01e-003	5.71e-002	2.76e-003
23:MagX Bias voltage	1.00e-003	5.08e+001	5.08e-002	1.60e-003	5.07e-002	2.27e-003
24:MagX Cosine nonlinear term	5.00e-003	4.14e-001	2.07e-003	6.89e-005	2.07e-003	9.83e-005
25:MagX Nominal temperature	1.00e-002	7.61e-002	7.61e-004	2.41e-005	7.60e-004	3.42e-005
26:MagX Sensitivity	6.67e+001	4.45e-004	2.97e-002	9.90e-004	2.96e-002	1.41e-003
27:MagX Sine nonlinear term	5.00e-003	4.40e+000	2.20e-002	7.35e-004	2.20e-002	1.05e-003
28:MagX Thermal bias drift	7.50e-005	5.08e+002	3.81e-002	1.20e-003	3.80e-002	1.70e-003
29:MagX Thermal sensitivity drift	3.00e-001	4.45e-003	1.33e-003	4.45e-005	1.33e-003	6.36e-005
30:MagX Voltage noise	1.00e-003	5.08e+001	5.08e-002	1.60e-003	5.07e-002	2.27e-003
31:SC residual field X	1.00e-007	4.93e+005	4.93e-002	1.54e-003	4.92e-002	2.19e-003
32:SC residual field Y	1.00e-007	1.44e+006	1.44e-001	4.57e-003	1.44e-001	6.60e-003
33:SC residual field Z	1.00e-007	5.63e+005	5.63e-002	1.98e-003	5.62e-002	2.72e-003
34:Sun1 box mounting X	1.70e-003	3.22e+001	5.47e-002	4.46e-002	2.98e-003	3.16e-002
35:Sun1 box mounting Y	1.70e-003	5.73e+001	9.75e-002	6.00e-004	2.20e-002	9.50e-002
36:Sun1 box mounting Z	1.70e-003	3.22e+001	5.47e-002	4.46e-002	2.98e-003	3.16e-002
37:Sun1X Cosine nonlinear term	5.00e-003	1.94e+001	9.72e-002	5.78e-004	2.17e-002	9.47e-002
38:Sun1X Nominal temperature	1.00e-002	1.85e-002	1.85e-004	8.64e-007	4.07e-005	1.81e-004
39:Sun1X Centroid	1.00e+000	1.35e-001	1.35e-001	8.27e-004	3.03e-002	1.31e-001
40:Sun1X Sensitivity	4.64e+000	4.61e-002	2.14e-001	1.28e-003	4.79e-002	2.09e-001
41:Sun1X Sine nonlinear term	5.00e-003	8.64e+000	4.32e-002	2.65e-004	9.73e-003	4.21e-002
42:Sun1X Thermal centroid drift	5.00e-003	1.35e+000	6.74e-003	4.13e-005	1.52e-003	6.57e-003
43:Sun1X Thermal sensitivity drift	1.39e-002	4.61e-001	6.41e-003	3.84e-005	1.44e-003	6.25e-003
44:Sun1X Noise	1.00e+000	1.35e-001	1.35e-001	8.27e-004	3.03e-002	1.31e-001
45:Sun1Y Cosine nonlinear term	5.00e-003	1.35e-001	6.74e-004	5.52e-004	3.93e-005	3.85e-004
46:Sun1Y Noise	1.00e+000	2.12e-001	2.12e-001	1.74e-001	1.12e-002	1.21e-001
47:Sun1Y Nominal temperature	1.00e-002	2.22e-002	2.22e-004	1.82e-004	1.17e-005	1.27e-004
48:Sun1Y Centroid	1.00e+000	2.12e-001	2.12e-001	1.74e-001	1.12e-002	1.21e-001
49:Sun1Y Sensitivity	4.64e+000	4.05e-003	1.88e-002	1.54e-002	1.07e-003	1.07e-002
50:Sun1Y Sine nonlinear term	5.00e-003	1.86e+000	9.30e-003	7.62e-003	5.31e-004	5.31e-003
51:Sun1Y Thermal centroid drift	5.00e-003	2.12e+000	1.06e-002	8.69e-003	5.58e-004	6.07e-003
52:Sun1Y Thermal sensitivity drift	1.39e-002	4.05e-002	5.62e-004	4.61e-004	3.21e-005	3.21e-004
53:Sun2 box box mounting X	1.70e-003	3.27e+001	5.56e-002	4.53e-002	1.72e-003	3.22e-002
54:Sun2 box box mounting Y	1.70e-003	5.78e+001	9.83e-002	6.72e-004	1.80e-002	9.67e-002
55:Sun2 box box mounting Z	1.70e-003	3.27e+001	5.56e-002	4.53e-002	1.72e-003	3.22e-002
56:Sun2X Cosine nonlinear term	5.00e-003	1.96e+001	9.79e-002	6.36e-004	1.80e-002	9.62e-002
57:Sun2X Noise	1.00e+000	1.36e-001	1.36e-001	9.24e-004	2.48e-002	1.34e-001
58:Sun2X Nominal temperature	1.00e-002	1.87e-002	1.87e-004	1.22e-006	3.39e-005	1.84e-004
59:Sun2X Centroid	1.00e+000	1.36e-001	1.36e-001	9.24e-004	2.48e-002	1.34e-001
60:Sun2X Sensitivity	4.64e+000	4.65e-002	2.16e-001	1.41e-003	3.95e-002	2.12e-001
61:Sun2X Sine nonlinear term	5.00e-003	8.73e+000	4.36e-002	2.98e-004	7.98e-003	4.29e-002
62:Sun2X Thermal centroid drift	5.00e-003	1.36e+000	6.79e-003	4.62e-005	1.24e-003	6.68e-003
63:Sun2X Thermal sensitivity drift	1.39e-002	4.65e-001	6.46e-003	4.24e-005	1.18e-003	6.35e-003
64:Sun2Y Centroid	1.00e+000	2.17e-001	2.17e-001	1.77e-001	6.48e-003	1.26e-001
65:Sun2Y Cosine nonlinear term	5.00e-003	2.43e-001	1.21e-003	9.88e-004	3.02e-005	7.04e-004
66:Sun2Y Noise	1.00e+000	2.17e-001	2.17e-001	1.77e-001	6.48e-003	1.26e-001
67:Sun2Y Nominal temperature	1.00e-002	2.02e-002	2.02e-004	1.64e-004	6.08e-006	1.17e-004
68:Sun2Y Sensitivity	4.64e+000	5.84e-003	2.71e-002	2.21e-002	7.18e-004	1.57e-002
69:Sun2Y Thermal sensitivity drift	1.39e-002	5.84e-002	8.12e-004	6.61e-004	2.15e-005	4.71e-004
70:Sun2Y Sine nonlinear term	5.00e-003	2.68e+000	1.34e-002	1.09e-002	3.55e-004	7.78e-003
71:Sun2Y Thermal centroid drift	5.00e-003	2.17e+000	1.09e-002	8.84e-003	3.24e-004	6.29e-003
72:Sun1X Sensed temperature	1.00e-002	1.85e-002	1.85e-004	8.64e-007	4.07e-005	1.81e-004
73:Sun2X Sensed temperature	1.00e-001	1.87e-002	1.87e-003	1.22e-005	3.39e-004	1.84e-003
74:Sun1Y Sensed temperature	1.00e-001	2.22e-002	2.22e-003	1.82e-003	1.17e-004	1.27e-003
75:Sun2Y Sensed temperature	1.00e-001	2.02e-002	2.02e-003	1.64e-003	6.08e-005	1.17e-003

Figure B.4: Bar plots visualising data in Table B.4

Table B.5: Raw data: Position 5: True anomaly = 315 deg

Input Index/Name	Input Std	Coefficient c_i	Total Std	1:Rotation Z	2:Rotation Y	3:Rotation Z
1:MagX Sensed temperature	1.00e-001	6.84e-002	6.84e-003	2.21e-004	6.83e-003	3.11e-004
2:MagY Sensed temperature	1.00e-001	1.97e-001	1.97e-002	6.38e-004	1.96e-002	9.07e-004
3:MagZ Sensed temperature	1.00e-001	4.71e-002	4.71e-003	1.66e-004	4.70e-003	2.29e-004
4:Mag box mounting X	1.70e-003	5.34e+001	9.07e-002	3.00e-003	9.06e-002	4.25e-003
5:Mag box mounting Y	1.70e-003	1.83e+001	3.11e-002	1.01e-003	3.11e-002	1.42e-003
6:Mag box mounting Z	1.70e-003	2.52e+000	4.28e-003	1.25e-004	4.28e-003	1.80e-004
7:MagY Bias voltage	1.00e-003	1.31e+002	1.31e-001	4.31e-003	1.31e-001	6.14e-003
8:MagY Cosine nonlinear term	5.00e-003	9.06e-001	4.53e-003	1.52e-004	4.52e-003	2.16e-004
9:MagY Nominal temperature	1.00e-002	1.97e-001	1.97e-003	6.38e-005	1.96e-003	9.07e-005
10:MagY Sensitivity	6.67e+001	1.22e-003	8.15e-002	2.71e-003	8.14e-002	3.84e-003
11:MagY Sine nonlinear term	5.00e-003	1.21e+001	6.06e-002	2.01e-003	6.05e-002	2.86e-003
12:MagY Thermal bias drift	7.50e-005	1.31e+003	9.83e-002	3.24e-003	9.82e-002	4.60e-003
13:MagY Thermal sensitivity drift	3.00e-001	1.22e-002	3.67e-003	1.22e-004	3.66e-003	1.73e-004
14:MagY Voltage noise	1.00e-003	1.31e+002	1.31e-001	4.31e-003	1.31e-001	6.14e-003
15:MagZ Bias voltage	1.00e-003	3.71e+001	3.71e-002	1.31e-003	3.71e-002	1.81e-003
16:MagZ Sine nonlinear term	5.00e-003	1.36e+001	6.81e-002	2.40e-003	6.80e-002	3.31e-003
17:MagZ Cosine nonlinear term	5.00e-003	4.27e+000	2.13e-002	7.52e-004	2.13e-002	1.04e-003
18:MagZ Nominal temperature	1.00e-002	4.71e-002	4.71e-004	1.66e-005	4.70e-004	2.29e-005
19:MagZ Sensitivity	6.67e+001	1.44e-003	9.57e-002	3.38e-003	9.56e-002	4.66e-003
20:MagZ Thermal bias drift	7.50e-005	3.71e+002	2.79e-002	9.83e-004	2.78e-002	1.36e-003
21:MagZ Thermal sensitivity drift	3.00e-001	1.44e-002	4.31e-003	1.52e-004	4.30e-003	2.10e-004
22:MagZ Voltage noise	1.00e-003	3.71e+001	3.71e-002	1.31e-003	3.71e-002	1.81e-003
23:MagX Bias voltage	1.00e-003	4.57e+001	4.57e-002	1.48e-003	4.56e-002	2.08e-003
24:MagX Cosine nonlinear term	5.00e-003	1.07e-001	5.36e-004	1.83e-005	5.35e-004	2.57e-005
25:MagX Nominal temperature	1.00e-002	6.84e-002	6.84e-004	2.21e-005	6.83e-004	3.11e-005
26:MagX Sensitivity	6.67e+001	2.20e-004	1.47e-002	5.00e-004	1.46e-002	7.04e-004
27:MagX Sine nonlinear term	5.00e-003	2.18e+000	1.09e-002	3.72e-004	1.09e-002	5.24e-004
28:MagX Thermal bias drift	7.50e-005	4.57e+002	3.43e-002	1.11e-003	3.42e-002	1.56e-003
29:MagX Thermal sensitivity drift	3.00e-001	2.20e-003	6.60e-004	2.25e-005	6.59e-004	3.17e-005
30:MagX Voltage noise	1.00e-003	4.57e+001	4.57e-002	1.48e-003	4.56e-002	2.08e-003
31:SC residual field X	1.00e-007	4.43e+005	4.43e-002	1.43e-003	4.43e-002	2.01e-003
32:SC residual field Y	1.00e-007	1.29e+006	1.29e-001	4.25e-003	1.29e-001	6.05e-003
33:SC residual field Z	1.00e-007	3.66e+005	3.66e-002	1.29e-003	3.65e-002	1.78e-003
34:Sun1 box mounting X	1.70e-003	3.25e+001	5.52e-002	4.49e-002	1.02e-003	3.21e-002
35:Sun1 box mounting Y	1.70e-003	5.63e+001	9.57e-002	2.02e-004	6.69e-003	9.55e-002
36:Sun1 box mounting Z	1.70e-003	3.25e+001	5.52e-002	4.49e-002	1.02e-003	3.21e-002
37:Sun1X Cosine nonlinear term	5.00e-003	1.91e+001	9.54e-002	1.96e-004	6.54e-003	9.52e-002
38:Sun1X Nominal temperature	1.00e-002	1.82e-002	1.82e-004	2.79e-007	1.17e-005	1.82e-004
39:Sun1X Centroid	1.00e+000	1.32e-001	1.32e-001	2.78e-004	9.23e-003	1.32e-001
40:Sun1X Sensitivity	4.64e+000	4.53e-002	2.10e-001	4.32e-004	1.45e-002	2.10e-001
41:Sun1X Sine nonlinear term	5.00e-003	8.48e+000	4.24e-002	8.89e-005	2.96e-003	4.23e-002
42:Sun1X Thermal centroid drift	5.00e-003	1.32e+000	6.61e-003	1.39e-005	4.62e-004	6.60e-003
43:Sun1X Thermal sensitivity drift	1.39e-002	4.53e-001	6.30e-003	1.30e-005	4.33e-004	6.28e-003
44:Sun1X Noise	1.00e+000	1.32e-001	1.32e-001	2.78e-004	9.23e-003	1.32e-001
45:Sun1Y Cosine nonlinear term	5.00e-003	1.36e-001	6.81e-004	5.57e-004	1.58e-005	3.92e-004
46:Sun1Y Noise	1.00e+000	2.14e-001	2.14e-001	1.75e-001	3.83e-003	1.23e-001
47:Sun1Y Nominal temperature	1.00e-002	2.24e-002	2.24e-004	1.83e-004	4.07e-006	1.29e-004
48:Sun1Y Centroid	1.00e+000	2.14e-001	2.14e-001	1.75e-001	3.83e-003	1.23e-001
49:Sun1Y Sensitivity	4.64e+000	4.09e-003	1.90e-002	1.55e-002	4.18e-004	1.09e-002
50:Sun1Y Sine nonlinear term	5.00e-003	1.88e+000	9.40e-003	7.68e-003	2.07e-004	5.41e-003
51:Sun1Y Thermal centroid drift	5.00e-003	2.14e+000	1.07e-002	8.76e-003	1.92e-004	6.17e-003
52:Sun1Y Thermal sensitivity drift	1.39e-002	4.09e-002	5.68e-004	4.65e-004	1.25e-005	3.27e-004
53:Sun2 box box mounting X	1.70e-003	3.30e+001	5.60e-002	4.56e-002	4.71e-004	3.26e-002
54:Sun2 box box mounting Y	1.70e-003	5.64e+001	9.59e-002	8.61e-005	2.77e-003	9.59e-002
55:Sun2 box box mounting Z	1.70e-003	3.30e+001	5.60e-002	4.56e-002	4.71e-004	3.26e-002
56:Sun2X Cosine nonlinear term	5.00e-003	1.91e+001	9.55e-002	8.53e-005	2.86e-003	9.55e-002
57:Sun2X Noise	1.00e+000	1.33e-001	1.33e-001	1.19e-004	3.85e-003	1.32e-001
58:Sun2X Nominal temperature	1.00e-002	1.82e-002	1.82e-004	1.54e-007	5.28e-006	1.82e-004
59:Sun2X Centroid	1.00e+000	1.33e-001	1.33e-001	1.19e-004	3.85e-003	1.32e-001
60:Sun2X Sensitivity	4.64e+000	4.54e-002	2.10e-001	1.88e-004	6.27e-003	2.10e-001
61:Sun2X Sine nonlinear term	5.00e-003	8.51e+000	4.26e-002	3.81e-005	1.23e-003	4.25e-002
62:Sun2X Thermal centroid drift	5.00e-003	1.33e+000	6.63e-003	5.94e-006	1.92e-004	6.62e-003
63:Sun2X Thermal sensitivity drift	1.39e-002	4.54e-001	6.31e-003	5.63e-006	1.88e-004	6.30e-003
64:Sun2Y Centroid	1.00e+000	2.19e-001	2.19e-001	1.78e-001	1.91e-003	1.27e-001
65:Sun2Y Cosine nonlinear term	5.00e-003	2.44e-001	1.22e-003	9.93e-004	1.48e-005	7.11e-004
66:Sun2Y Noise	1.00e+000	2.19e-001	2.19e-001	1.78e-001	1.91e-003	1.27e-001
67:Sun2Y Nominal temperature	1.00e-002	2.03e-002	2.03e-004	1.65e-004	1.73e-006	1.18e-004
68:Sun2Y Sensitivity	4.64e+000	5.88e-003	2.73e-002	2.22e-002	3.00e-004	1.59e-002
69:Sun2Y Thermal sensitivity drift	1.39e-002	5.88e-002	8.18e-004	6.65e-004	8.98e-006	4.76e-004
70:Sun2Y Sine nonlinear term	5.00e-003	2.70e+000	1.35e-002	1.10e-002	1.48e-004	7.86e-003
71:Sun2Y Thermal centroid drift	5.00e-003	2.19e+000	1.09e-002	8.89e-003	9.56e-005	6.37e-003
72:Sun1X Sensed temperature	1.00e-002	1.82e-002	1.82e-004	2.79e-007	1.17e-005	1.82e-004
73:Sun2X Sensed temperature	1.00e-001	1.82e-002	1.82e-003	1.54e-006	5.28e-005	1.82e-003
74:Sun1Y Sensed temperature	1.00e-001	2.24e-002	2.24e-003	1.83e-003	4.07e-005	1.29e-003
75:Sun2Y Sensed temperature	1.00e-001	2.03e-002	2.03e-003	1.65e-003	1.73e-005	1.18e-003

Figure B.5: Bar plots visualising data in Table B.5

Appendix C

The units of the coefficient c_i

Table C.1: The units of c_i . When comparing the coefficient's the dimension must be considered. Only when the units are the same, the coefficients can be compared directly.

Input name	Dimension
Magnetometer	
Sensitivity	deg·Tesla/V
Thermal sensitivity drift	deg·Tesla·°C/V
Bias voltage	deg/V
Thermal bias drift	deg·°C/V
Nominal temperature	deg/°C
Temperature sensor	deg/°C
Cosine nonlinear term	deg
Sine nonlinear term	deg
Sensor box mounting	-
Voltage noise	deg/V
S/C residual field	deg/Tesla
Sun sensor	
Sensitivity	deg·rad/pixel
Thermal sensitivity drift	deg·°C/pixel
Centroid	deg/pixel
Thermal bias drift	deg·°C/pixel
Nominal temperature	deg/°C
Temperature sensor	deg/°C
Cosine nonlinear term	deg
Sine nonlinear term	deg
Sensor box mounting	-
Pixel noise	deg/pixel