Low–voltage External Discharge Plasma Thruster and Hollow Cathodes Plasma Plume Diagnostics Utilising Electrostatic Probes and Retarding Potential Analyser

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Space Engineering, masters level
2016

Luleå University of Technology
Department of Computer Science, Electrical and Space Engineering
LOW-VOLTAGE EXTERNAL DISCHARGE PLASMA THRUSTER
AND HOLLOW CATHODES PLASMA PLUME DIAGNOSTICS
UTILISING ELECTROSTATIC PROBES AND RETARDING
POTENTIAL ANALYSER

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Co – funded by the Erasmus Programme of the European Union
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LOW-VOLTAGE EXTERNAL DISCHARGE PLASMA THRUSTER AND HOLLOW CATHODES PLASMA PLUME DIAGNOSTICS UTILISING ELECTROSTATIC PROBES AND RETARDING POTENTIAL ANALYSER

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B.Eng. Aerospace Engineering, Military Technical Academy, Bucharest 2014

A thesis submitted to the Department of Computer Science, Electrical and Space Engineering of Luleå University of Technology, Sweden in partial fulfilment of the requirements of the degree of Master of Science in Space Technology

And to the Faculty of Sciences and Engineering of University Toulouse III Paul Sabatier, France in partial fulfilment of the requirements of the degree of Master of Science in Space Technology and Instrumentation

September 2016

ABSTRACT

Ever since their appearance in the realm of science fiction, the advanced space propulsion systems fascinated researchers and the dream of interplanetary journeys became almost certitude. Back in 1906, the American physicist Robert Goddard considered the idea of using eclectically powered propulsion systems [11], and therefore the field of space electric propulsion was born, exhibiting a formidable growth in the later decades.

The present thesis is the result of a research period at the Institute of Space and Astronautical Science of the Japanese Aerospace Exploration Agency, ISAS/JAXA within Funaki Laboratory of the Department of Space Flight Systems that followed the path of plume plasma diagnostics for space electric propulsion drives. During the experimental studies two high-current hollow cathodes and an innovative prototype of a low-voltage fully external discharge plasma thruster (XPT) had their plasma plumes diagnosed using electrostatic probes and retarding potential analyser (RPA).

A Hall thruster and hollow cathode plume is defined as an unmagnetised quasi-neutral plasma which is mainly formed of neutral particles, electrons, singly and doubly charged ions. Plasma diagnostic techniques provide information through practical observations in order to fully understand the dynamics of the aforementioned plume components, the physical processes taking place within the plume and their effects on the spacecraft, for instance. Mastering these aspects of the plasma plume of space electric propulsion drives bolster the design processes, leading to highly efficient devices.

Firstly, the introduction provides insights on the fundamental principles of hollow cathodes and Hall thrusters and a brief presentation of the plasma diagnostic techniques used during the
research: single and double Langmuir probes, emissive probes and retarding potential analyser. Then, the fundamental plume diagnostics principles are depicted in an exhaustive way, departing from classical plasma kinetic theory, energy distribution functions and ending with an overview on the theory of charge collection by cylindrical probes. Subsequently, peculiarities of various analysis techniques are exposed for the Langmuir probes, emissive probes and RPA, with an emphasis on their strengths and demerits.

The experimental setups for the cathodes and XPT plume diagnostic procedures are then outlined. The experimental logic, setup and electrical diagrams as well as a presentation of each probe design and manufacturing details are extensively discussed.

The hollow cathodes experimental results are exposed with a discourse that aims of overviewing the difference between the various data analysis methods applied for the raw data. A discussion ensued based on the results in order to effectively identify mechanisms that produced the observed plasma parameters distributions.

For the first time, the plume of a fully external discharge plasma thruster was diagnosed utilising double Langmuir probes. The thesis highlights the main results obtained for the XPT far-field plume plasma diagnostics. The experimental findings for both thruster centreline positions and 2D plume maps for several axial distances away from the anode plate offer a ground basis for future measurements, a comparison term and a database to support ongoing computational codes. The results are discussed and related to the thruster performances data obtained during previous experiments.

The thesis includes consistency analyses between the experimental data and the numerical simulation results and the uncertainties in measured plasma parameters associated with each data analysis procedure are evaluated for each data set. Last, the conclusions underline the main aspects of the research and further work on the previously mentioned plasma diagnostic techniques for hollow cathodes and XPT is suggested.

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ABSTRACT (version française)

La présente thèse est le résultat d'une période de recherche à l'Institut des Sciences Spatiales et Astronautiques de l'Agence Spatiale Japonaise, ISAS / JAXA qui a suivi la voie des diagnostics du plasma de la plume de propulseurs électriques spatiaux. Au cours des études expérimentales, deux cathodes creuses à fort courant et un prototype innovant d'un propulseur basse tension à décharge externe de plasma (XPT) avaient leurs faisceaux de plasma diagnostiqués en utilisant des sondes électrostatiques et un analyseur à potentiel retardé. La plume d'un propulseur à effet Hall et d'une cathode creuse est définie comme un plasma quasi-neutre non-magnétisé qui est principalement formé de particules neutres, d'électrons, d'ions monovalents et bivalents. Les techniques de diagnostic du plasma fournissent des informations, via des observations pratiques, afin de bien comprendre la dynamique des composants de la plume mentionnés ci-dessus, les processus physiques qui se déroulent dans la plume et leurs effets sur une sonde spatiale, par exemple. La maîtrise de ces aspects du plasma de la plume généré par les propulseurs électriques spatiaux renforce les processus de conception de ce type de propulsion, ce qui conduit à des dispositifs hautement efficaces.

Tout d'abord, l'introduction donne un aperçu sur les principes fondamentaux de cathodes creuses et de propulseurs à effet Hall, et une brève présentation des techniques de diagnostic du plasma utilisées lors de la recherche : sondes de Langmuir simples et doubles, des sondes émissives et d'analyseur à potentiel retardé. Ensuite, les principes fondamentaux de diagnostic de la plume sont représentés de manière exhaustive, d'abord la théorie cinétique classique du plasma, les fonctions de distribution en énergie et pour terminer une vue d'ensemble de la théorie de la collecte de charge par des sondes cylindriques. Par la suite, les particularités des diverses techniques d'analyse sont exposées pour les sondes de Langmuir, les sondes émissives et RPA, en mettant l'accent sur leurs avantages et leurs inconvénients. Les montages expérimentaux pour les procédures de diagnostic de la plume-plasma de cathodes et du XPT sont ensuite décrits. La logique expérimentale, les schémas électriques ainsi qu'une présentation de la conception et de la fabrication de chaque sonde sont largement discutés. Les résultats expérimentaux pour les cathodes creuses sont exposés de façon à présenter la différence entre plusieurs méthodes d'analyse de données appliquées aux données brutes. Une discussion s'ensuit, basée sur les résultats afin d'identifier efficacement les mécanismes qui ont produits les propriétés électroniques observées.

Pour la première fois, la plume d'un propulseur à décharge externe de plasma a été diagnostiquée en utilisant des sondes de Langmuir doubles. La thèse met en évidence les principaux résultats obtenus pour le diagnostic en champ lointain de la plume-plasma du XPT. Les résultats expérimentaux pour les positions sur l’axe du propulseur et le cartes 2D de la plume pour plusieurs distances axiales loin de l'anode offrent une base pour de futures mesures, un terme de comparaison et une base de données pour appuyer les codes numériques. Les résultats sont discutés et sont rapportés aux données de performances du propulseur obtenus lors des essais précédents. La thèse comprend des analyses de la cohérence entre les données expérimentales et les résultats de simulation numérique, et les incertitudes des paramètres mesurés du plasma associées à chaque procédure d'analyse des données sont évaluées pour chaque ensemble de données. Enfin, les conclusions soulignent les principaux aspects de la recherche et une poursuite des travaux sur les techniques de diagnostic de plasma pour les cathodes creuses et le XPT est suggérée.
ACKNOWLEDGMENTS

First and foremost, I would like to thank my thesis supervisor from the Institute of Space and Astronautical Science of the Japanese Aerospace Exploration Agency, Professor Ikkoh Funaki. Thanks to him my research period at ISAS was possible, giving me the great opportunity to continue my quest in understanding space electric propulsion. His priceless advice and guidance throughout my entire period at ISAS has led me to acquire consistent knowledge on Hall thrusters and hollow cathode operation and above all to gain valuable hands-on experience in plasma plume diagnostics for those electric drives. Knowing that this is a very important asset for someone that just starts his career in this highly competitive domain, I am deeply thankful for the chance Professor Ikkoh Funaki gave me!

The research herein would not have come to this present form without the tremendous support of Dr. Akira Kawasaki, research fellow of the Japan Society for the Promotion of Science. I am deeply thankful for all the encouragements and inspirational discussions that we had as well as for supervising my entire research at ISAS. Moreover Dr. Kawasaki’s recommendations were extremely valuable during the correcting phase of the thesis. I really hope that this is just the start of a strong friendship and I look forward with impatience for future collaborations!

Almost my entire research envisaged the mapping of the low-voltage external discharge plasma thruster (XPT) designed and manufactured by Burak Karadag, PhD candidate at The Graduate University for Advanced Studies, Japan. We worked side by side during the probing of the XPT plume and I would like to thank him for all the priceless lessons he thought me about how to think as a real engineer and for all the inspirational discussions we had!

Another important part of my research involved collaboration with Dr. Kinichi Kubota for the plasma plume diagnostics of his laboratory model hollow cathode. Dr. Kubota, a researcher at the Aeronautical Technology Directorate of JAXA, thought me precious lessons on hollow cathode operation techniques as well as on how to consider my further steps in my career in space electric propulsion. Dr. Kiyoshi Kinefuchi, associate senior researcher at the R&D Directorate of JAXA, also helped us with the experiments for the hollow cathode, for which I am thankful.

During my experiments I had the luck to be advised by Dr. Yuya Oshio, Associate Assistant Professor at Tokyo University of Agriculture and Technology, Japan. With a vast experience on electrostatic probes operation for electric drives plume plasma diagnostics, Dr. Oshio’s advice was priceless. I am truly and deeply thankful for all his support during my research period at ISAS! Important advice, especially when it came to data interpretation was given by Dr. Shinatora Cho, researcher at the Institute of Aeronautical Technology of JAXA. His extensive experience on Hall thruster operation supported my quest in properly describing the results of the XPT plasma plume mapping, for which I am deeply grateful!

Dedication

The work for this thesis is dedicated to my beloved grandmother who departed for the stars far too early and has guided my steps ever since!

George – Cristian Potrivitu
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August 2016
NOMENCLATURE

$A_c$ collector area, m$^2$
$A_g$ open area fraction of the ion repeller grid
$A_p$ probe area, m$^2$
$B$ magnetic field strength, T
$C$ capacitance, C
$e$ elementary charge, C
$E$ electric field, V/m
$(f_j Q)$ average charge of the exhausted gas/plasma mix
$F$ Faraday constant, C/mol
$g_0$ gravitational constant, m/s$^2$
$I$ current, A
$I_{beam}$ beam current, A
$I_{cap}$ capacitive current, A
$I_d$ discharge current, A
$I_{em}$ emitted current, A
$I_e$ electron current, A
$I_{es}$ electron saturation current, A
$I_h$ emissive probe heating current/cathode heater current, A
$I_i$ ion current, A
$I_{is}$ ion saturation current, A
$I_k$ cathode keeper current, A
$I_{sp}$ specific impulse, s
$j$ current density, A/m$^2$
$k_B$ Boltzmann constant, J/kg·K
$K_n$ Knudsen number
$l_p$ probe length, m
$L$ characteristic length, m
$m$ electron mass, kg
$m$ mass flow rate, mg/s
$m_a$ anode mass flow rate, mg/s
$m_c$ cathode mass flow rate, mg/s
$M$ xenon ion mass, kg
$M$ xenon molecular mass
$n$ number density, m$^{-3}$
$n_e$ electron number density, m$^{-3}$
$n_i$ ion number density, m$^{-3}$
$N_A$ Avogadro number, mol$^{-1}$
$P$ pressure, Pa
$P_c$ background pressure, Pa
$P_d$ discharge power, W
$P_k$ kinetic power, W
$q$ electric charge, C
$Q$ heat, J
$r_p$ probe radius, m
R resistance, Ω
Rₔ specific gas constant, J/K·mol
Rₓe Xenon specific constant, J/K·mol
T temperature, K or eV
Tₐ neutral atom temperature, K
Tₑ electron temperature, K or eV
Tᵢ ion temperature, K
Tₖ gas temperature, K
T₉ filament temperature, K or eV
U ion/plasma drift velocity, m/s
v velocity, m/s
vₑx exhaust velocity, m/s
vᵢ ion velocity, m/s
vᵣn thermal speed, m/s
V voltage, V
Vₐ local ion average acceleration potential, V
Vₖ probe bias voltage, V
Vₑm beam voltage, V
Vₑ collector bias voltage, V
Vₑ discharge voltage, V
Vₑ electron repeller bias voltage, V
Vₑ floating potential, V
Vₑ loss potential loss, V
Vₑ mp most probable potential, V
Vₑp plasma potential, V
Z ion charge state

α propellant utilization efficiency correction factor
β ion acceleration potential loss fraction
γₔ gas specific heat ratio
γₓe Xenon specific heat ratio
δ plasma sheath thickness, m
Δv change in velocity, m/s
ε₀ permittivity of free space, H/m
ε energy, J
ηₑ beam divergence efficiency
ηₑ current efficiency
ηₑ mass utilization efficiency
ηₑ propellant efficiency
ηₑ voltage efficiency
θ beam divergence, degree
λ mean free path, m
λₑ Debye length, m
λₑi electron-ion mean free path, m
λₑn electron-neutral mean free path, m
ν collision frequency, s⁻¹
ξ ratio of probe radius to Debye length
σ  collision cross section, m²
τₑ  end effect parameter
Ψ  work function
ωₓ  cyclotron frequency, s⁻¹
Ωₑ  Hall parameter, m⁻²
Φ  potential, V
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CHAPTER 1: Introduction

1.1 Space electric propulsion

As the mankind’s dream of interplanetary travel is closer to reality as never before, the electric propulsion drives are regarded as good candidates for the job since they can offer a reduction to minimum of the propellant mass needed for such a mission and being powered by the endless power source found in the outer space, the electrical power. Back in 2003, the European Space Agency proved that one can reach the Moon by using just 82 kg of propellant [16]. Successful were also the interplanetary Hayabusa missions of the Japanese Aerospace Exploration Agency (JAXA) or Dawn and Deep Space 1 missions of NASA, proving the excellent versatility of electric propulsion thrusters and forging the path of a more intensive research in this field.

The simplest classification of spacecraft propulsion systems takes into consideration the existence of an internal reaction mass. Thus, one can identify rocket engines and electric propulsion systems in the first category and solar sails, for example, or the gravitational slingshot technique, in the second category. Functionally, the inability of chemical propulsion systems to achieve higher exhaust velocities is due to the limitation in the maximum tolerable temperature in the combustion chamber, to avoid excessive heat transfer to the walls. Fundamentally, there is also an intrinsic limitation on the maximum energy that is available from the chemical reactions [25, 31]. Both these limitations can be overcome by the use of electric propulsion (EP). A working definition for electric propulsion could be: the acceleration of gases for propulsion by electrical heating and/or by electric and magnetic body forces [25].

Perhaps the most important parameter that characterise a space mission is the total change in velocity or the delta-v (Δv) which defines the change in velocity of a spacecraft or launching vehicle due to the utilization of a thruster. Tsiolkovsky equation [66] defines best the total change in velocity as a function of the propellant mass and the total mass of the spacecraft:

\[
\Delta v = v_{ex} \ln \left( \frac{m_d + m_p}{m_d} \right),
\]

(1.1)

where the spacecraft total mass \( M_d = m_d + m_p \), with \( m_p \) the propellant mass and \( m_d \) the payload mass or delivered mass and \( v_{ex} \) is the effective exhaust velocity. The same equation written with the respect to the specific impulse is:

\[
\Delta v = I_{sp} g_0 \ln \left( \frac{m_d + m_p}{m_d} \right).
\]

(1.2)

The specific impulse, \( I_{sp} \), is a property of a particular propellant and equals the ratio between the thrust obtained due to the usage of the propellant and the product between the propellant mass flow rate and the gravitational constant at sea-level (regarding Tsioloksky equation, equals the ratio between the effective exhaust velocity and the gravitational constant) [25, 31]:

\[
I_{sp} = \frac{\text{Thrust}}{m_p g_o} = \frac{v_{ex}}{g_o},
\]

(1.3)

The specific impulse equation reveals that the only way to obtain more impulse from a certain quantity of ejected propellant is to increase its exhaust velocity. As previously said, the chemical rockets are offering limited exhaust velocities, and consequently specific impulses, due
to the limitations imposed by the physics of thermodynamics. On the other hand the electric propulsion devices, having and energy source that is independent of the propellant are capable to eject the ionized propellant with velocities up to 45000 m/s and hence achieving specific impulses up to 10000s [1, 25]. This reduces the propellant mass needed for a mission and hence the entire mass and size of the spacecraft can be reduced as well. This comes with a downside as the increase in specific impulse implies an additional electrical power and in some cases the additional mass of the power supply can become an important fraction of the total spacecraft mass [25]. An optimum $I_{sp}$ can be defined for a specific configuration of an electric propulsion spacecraft.

The electric propulsion thrusters can be classified regarding the different techniques used in accelerating the propellant and producing the thrust. Three different methods are used today: electrothermal (resistojet, arcjet), electrostatic (gridded ion thrusters, Hall Effect thrusters, electroospray, field emission electric propulsion) and electromagnetic (magnetoplasmadynamic thrusters, pulsed plasma thrusters).

The electrostatic propulsion systems use plasma generation techniques to ionize a large fraction of the propellant and to accelerate them electrostatically [25]. The ion thrusters, known also as gridded ion thrusters (GIT), use biased grids to electrostatically extract ions from plasma and to accelerate them with high efficiency (the highest from all the existing electric propulsion systems: 60% to 80%) and with higher specific impulses then in the case of electrothermal devices: from 2000s up to 10000s [1, 7, 25]. In the same category are also included the Hall Effect thrusters (HET), which are one of the subject of the present thesis. A Hall Effect thruster uses a cross-field discharge configuration to produce ions which are accelerated due to the voltage drop between the anode and an external cathode. A Hall current is created at the exit plate of the thruster channel due to the existence of the magnetic field, perpendicular to the primary electric field, which traps and inhibits electron motion that would tend to short out the electric field [25]. At a given power a HET has a specific impulse and efficiency lower than an ion thruster, but the produced thrust is higher and the constructive geometry is simpler. Both the ion and Hall thrusters can deliver very high specific impulses but their thrust output spans from $10^{-3}$N up to several newtons [1, 7, 25]. As a consequence, the required thrust time in order to achieve a certain $\Delta v$ is usually large, up to several months.

1.2 Hall Effect Thrusters (HET)

Although the idea of electric propulsion drives was coined by the American physicist Robert Goddard at the beginning of twentieth century, the concept of Hall thrusters also known as Stationary Plasma Thrusters or Closed Electron Drift Thrusters appeared just in the 1960's. The research of this type of propulsion quickly lost interest in United States since the efficient operation of those devices was doubtful in comparison with the ion thrusters, mainly because of the electron back flowing that rose above classical diffusion models for high discharge voltage operations [57]. However the research continued in the Soviet Union focused on a lower discharge voltage range and offering specific impulses of up to 2000s [57]. Those devices proved to be perfect for low- Earth orbit (LEO) missions.

Capable of delivering specific impulses in a rage from 1200 to 3500s [1, 25] with efficiencies up to 60% and thrust to power ratios up to 85mN/kW [25, 41, 57], the Hall Effect thrusters have a relatively simple design based on four main components: the anode, the cathode, the discharge channel and the magnetic system.
The mechanism of plasma formation inside a Hall thruster can be explained as follows. The anode of the device has three functions: maintaining the electric field (together with the cathode), working as an inlet for the propellant neutrals and being a sink for the back-streaming electrons (free electrons resulting from ionization events and incident electrons emitted by the cathode). Since all the energy of those electrons is deposited in the anode, this part is usually forged from a refractory metal to withstand the joule heating as well as heating due to ion bombardment and electron absorption. Except for the loss mechanisms, the anode potential is the only one that ascertains for the velocity at which the propellant is accelerated.

As the external or internal cathode emits electrons, they tend to reach the positively charged anode positioned upstream the thruster exit plane, as presented in Figure 1.1. On their way to the anode a part of the electrons are trapped into the radial magnetic field employed near the exit plane of the thruster. This radial magnetic field is produced by the means of electromagnets or permanent magnets, perpendicular to the electric field created between the anode and cathode and having a specific strength and field lines topology. For the electrons to remain trapped and spiral the magnetic field lines, their Larmor radius should be small when compared to the dimensions of the discharge channel. By employing the general equation for the Larmor radius, the equation for the cyclotron frequency, \( \omega_c \) and assuming that the thermal velocity of the electrons, \( v_{th} \) can be substituted for the perpendicular velocity, \( v_\perp \) (since in the absence of any magnetic field the electrons depict an axial motion), the electron Larmor radius can be written as follows [25]:

\[
r_e = \frac{v_\perp}{\omega_c} = \frac{v_{th}}{\omega_c} = \frac{m v_{th}}{eB} = m \frac{e}{eB} \frac{8k_B T_e}{nm} \approx \frac{1}{B} \frac{8m}{ne} T_e \ll L,
\]

where \( L \) is the characteristic length or the plasma length inside the discharge channel, \( T_e \) is the electron temperature in eV. In the same time the electrons should remain magnetised, in other
words the magnetic field should confine the electrons. For this to be satisfied the Hall parameter should be much larger than unity [25]:

\[ \Omega_e^2 = \frac{\omega_e^2}{v^2} \gg 1, \]

where \( v \) is the collision frequency. The purpose of the electron confinement is to reduce the axial mobility of the electrons and force them to create a local gradient in the plasma potential that leads to the violation of the plasma quasi-neutrality. This potential hill accelerates the ions downstream the thruster exit plane. Moreover, in the cross-field region an azimuthal drift is imparted to the electrons and their movement produces a high induced current, the Hall current. As the axial electron mobility is minimized the applied discharge between the cathode and the anode aligns with the channel axis.

Ions are produced from electron-neutral collisions near the exit plane. The propellant neutrals are injected upstream through the anode plate and the pressure gradients cause the diffusion of the neutral molecules toward the exit plane. Reaching the closed-drift electrons cloud, the neutral molecules are ionized if the electrons possess an energy larger than the ionization potential. When choosing the propellant three main aspects are of concern: the mass should be as high as possible, the electron-neutral cross-section coefficient should be high and low ionization energy is desired [3, 57]. The only source of thrust lies in the electrostatic force acting on the newly formed ions, repelled by the anode and accelerated though the potential hill [57]. The newly formed electrons are in the end collected by the anode and inserted in the power supply circuit. Furthermore the drifting electrons are not affected by the electrostatic pull of the anode since this is counterbalanced by the magnetic pressure [3].

The ions trajectories should not be influenced by the magnetic field, so the latter should be weak enough not to interfere with the ions. This can be translated in the ions Larmor radius being much larger than the characteristic plasma length [25]:

\[ r_i = \frac{v_i}{\Omega} = \frac{M}{eB} \sqrt{\frac{2eV_{beam}}{M}} = \frac{1}{B} \sqrt{\frac{2M}{e}} V_{beam} \gg L, \]

where \( V_{beam} \) is the beam potential.

The accumulation of ions downstream the Hall thruster can lead to the creation of a space charge situation and some of the ions can be drawn back towards the spacecraft. To avoid those situations the cathode serves as beam neutraliser, as in the case of ion thrusters.

### 1.3 Hollow cathodes

Hall thrusters utilise an electron discharge to ionize the propellant neutrals in order to create plasma, hence a cathode has to be employed in correlation with the anode in order to establish the discharge bridge. The cathode has two important functions: emitting electrons in order to maintain the discharge between anode-cathode and emitting electrons in order to neutralize the ions created by the thruster and expelled into the plume. This last function has a crucial importance avoiding the contamination of the spacecraft onboarded by the thruster with ions back-flowing from the plume.

The cathode consists of a hollow tube made of a refractory material that can withstand very high temperatures and ended with an orifice plate on the downstream. Inside the tube an insert is placed, being the active source that emits the electrons [25], as depicted in Figure1.2. The
electrons are created via thermionic emission which can be predicted by using the Richardson-Dushman equation (or Schotty-Richardson) \([12, 28]\):

\[
I(T_{\text{emitter}}) = AT_{\text{emitter}}^{2}\exp\left(\frac{-e\psi}{k_B T}\right).
\]  

(1.7)

where \(I(T)\) is the emitted current density, \(T\) is the temperature of the emitter, \(\psi\) is the work function of the material and \(A\) is a theoretical constant and equals to \(6.02 \cdot 10^5\) A/m\(^2\)/K\(^2\) \([12]\). It can be clearly seen that as the work function increases, higher temperatures are needed to have sufficient electron emission.

The insert pushes against the orifice plate and it is made of materials that offer a low work function surface on the inside diameter where it is in contact with the cathode plasma \([25]\). The cathode insert can be made of molybdenum, tantalum, tungsten, lanthanum hexaboride (LaB\(_6\)), or barium oxide-tungsten alloy (BaO – W) \([25]\).

To raise the temperature of the insert up to the emissive temperature, the tube is wrapped with a heater. Through the tube it is inserted some fraction of the thruster propellant which is ionized by the electrons emitted by the heated insert and form the cathode plasma, a cold high-density plasma. From this plasma the discharge current electrons are extracted through the orifice into the thruster plasma. The extraction happens in two steps. First the electrons from the internal plasma are extracted by the keeper electrode that surrounds the cathode and which is positively biased with respect to the cathode tube. Hence, the electric field between the keeper and the cathode tube accelerates the electrons through the cathode orifice towards the keeper and creates the secondary discharge. Since the anode of the thruster is biased at a potential above the cathode potential, the space potential outside the keeper accelerates the electrons through the keeper orifice into the thruster plume and towards the anode forming the primary discharge.

### 1.4 Plasma diagnostics

A Hall thruster and hollow cathode plume is defines as an unmagnetised quasi-neutral plasma which is mainly formed of neutral particles, electrons, singly and doubly charged ions \([3]\). The

![Figure 1.2: Schematic of a hollow cathode.](image-url)
main objective of the plasma diagnostics is to provide enough information via practical observations in order to fully understand the dynamics of the plume components, the physical processes and their effects. Several plasma diagnostic methods are available and can be categorised, first of all, taking into account the type of plasma they are used for: cold or hot plasma ($T_e \geq \text{few keV}$) [55]. A second characterisation criterion divides the plasma diagnostic techniques into intrusive (the device employed makes contact with the plasma being diagnosed) and non-intrusive. Maybe the most cited classification accounts for the physics behind the working principle of the different diagnostics.

![Classification of the main plasma diagnostic techniques.](image)

The diagnostic techniques presented in Figure 1.3 can provide valuable information about some plasma parameters as the electron temperature, plasma number density, plasma kinetic energy, ion and electron energy distributions, plasma potential, plasma currents, radiation intensity, plasma spectra (line and continuum spectra). In this research two types of devices were used: single and double Langmuir probes and emissive probes (electron temperature, plasma number density, plasma potential and electron energy distribution) and a retarding potential analyser (ion energy distribution) which are described hereinafter.

### 1.4.1 Langmuir probes

Since its invention by Irving Langmuir [39, 40], back in the early 1920’s, the Langmuir probe, part of the electrostatic probes family, is a constructively simple device but with a working...
principle based on complex plasma physics mechanisms. A Langmuir probe is formed of one or more electrodes under potential that are inserted into the plasma. Due to the small amount of material that is inserted inside the plasma only minor perturbations are induced during the probing process. By varying the electrodes potential, both more positively and more negatively than the plasma potential, the current drawn by the probe can be measured against the probe’s bias potential variation. This leads to a characteristic current-voltage (I-V) curve which, upon post processing using classical Langmuir probe theory which assumes a stationary plasma and a thin sheath, gives information about several plasma parameters as the electron temperature, plasma number density and plasma potential.

Figure 1.4 shows a schematic of a single Langmuir probe and its electric circuit as well as an ideal I-V curve provided by such a device. The current collected during the operation of the probe is the sum of electron and ion currents. When the probe’s bias voltage, $V_b$, is sufficiently negative, all the electrons are repelled and the ions are accelerated towards the metallic electrode giving birth to the ion saturation current. As the voltage is swept towards more positive values, at a certain point the current is null. This voltage is by definition the floating potential, $V_f$. As the bias voltage is increased even further above the floating potential both ions and high energy electrons, situated in the tail of the electron energy distribution function, are collected. As more and more electrons are able to travel through the potential gradient of the probe’s sheath and get collected by the probe, a positive current is recorded and this region is known as the retarding field region or the electron retarding regime. The inverse slope of the natural logarithm plot of this region is proportional with the electron temperature $T_e$:

$$\frac{d\ln(|I(V_b)|)}{dV_b} = \frac{e}{k_B T_e}$$

The more electrons collected the more the probe’s potential is approaching the plasma potential, $V_p$. At this moment all the ions are repelled and the electrons are accelerated towards the electrode giving rise to the electron saturation current. In reality the electron saturation is never reached and the knee of the curve (inflection point that marks the plasma potential position) is
difficult to be isolated. The impossibility of reaching the electron saturation is due to two phenomena: the growth of the sheath with the increase in voltage and the presence of flowing plasma instead of a stationary one which implies the fact that the wake region of the probe does not collect current, hence assessing for the collecting area, \( A_p \) is difficult. However, utilising the electron saturation current, \( I_{es} \) read at the inflection point of the natural logarithm plot of the I-V trace, the plasma number density can be computed [42]:

\[
n_e = \frac{I_{es}}{eA_p \frac{k_B T_e}{2 n m}}
\]

(1.9)

The necessity of high voltage sweeps in order to catch both ion and electron saturation regions in the case of a single Langmuir probe can create strong plasma perturbations leading to difficulties in interpreting correctly the results. To overcome this situation a double Langmuir probe can be used.

A double Langmuir probe is formed of two electrodes of areas that can be equal and is most of the time used for plasmas where no reference point is available. Moreover, the probe does not disturb the plasma as much as a single probe since the system “floats” and follows any change in the plasma potential, having no plasma ground [55]. However, besides the precise results given by the probe, this comes also with a decrease in spatial resolution, dictated by the size of the probe. A schematic of a double Langmuir probe and the ideal I-V curve of such a device are depicted in Figure 1.5. The double probe is used in the same way as the single probe. The two electrodes are simultaneously immersed into the plasma and the biasing voltage of the probe is varied from negative to positive, whilst the probe’s current is recorded. When the probe voltage is sufficiently negative, the ion saturation current in the first electrode is reached \( I_{is1} \). As the voltage goes towards positive values the curve passes through the origin \( (I, V_b = 0) \) where the current varies linearly with the probe’s voltage. As the probe’s voltage becomes more and more positive, the ion saturation current in the second electrode is reached \( I_{is2} \) (the current in the first electrode cannot exceed this second current).

![Figure 1.5: Left: Schematic of a double Langmuir probe. Right: Ideal I-V double Langmuir probe trace.](image)
In the present research both single and double Langmuir probes were used in order to
determine the plasma characteristics as number density, electron temperature and plasma
potential. The single probes were used mainly for plasma potential determination, while in the
case of the external discharge plasma thruster (XPT) a method based on the dependency
between the floating potential and the plasma potential was used, as described in Chapter 2.

1.4.2 Emissive probes
It is well known that many negative effects can affect the measurements with the Langmuir
probes and delicate information as the plasma potential can be computed erroneous. The
geometry of the probe is an important factor that may bring many unknowns in the equations
and influences the formation and the thickness of the plasma sheath around the collecting
electrode. Magnetic fields, collisional effects or the flowing nature of the probed plasma may also
influence the results especially in the region of the I-V knee that should predict the plasma
potential. To overcome those effects and to provide accurate measurements for the plasma
potential an emissive probe should be used. Such a device is formed of a hairpin loop filament
passed through a double-bore ceramic tube. A schematic of an emissive probe is presented in
Figure 1.6 as well as the ideal I-V curve recorded by an emissive probe. The filament is heated
by an external current source and this initiates the thermionic emission of electrons which
neutralise the sheath of ions around the probe. The emitted electrons can just travel up potential
gradients, hence emission occurs only when the biasing voltage of the probe is under the plasma
potential. As the voltage is increased more negatively with respect to the plasma potential, for
each electron emitted an ion is collected. The electron emission occurs proportionally with the
filament temperature and this can lead to a very sharp drop in the current collected in the
retarding field region. This phenomenon put better in evidence the knee of the I-V trace, the
inflection point between the electron saturation region and the electron retarding region. Thus,
plasma potential can be easily identified from the curve, as presented in Figure 1.6. In the case of
a Langmuir probe, also called a “cold” probe the phenomenon abovementioned does not take
place since the filament does not reach thermionic emission temperatures. For bias voltages
higher than plasma potential the emissive probe behaves as a regular Langmuir probe since no
electron emission occurs in the electron saturation region.

Several methods exist in order to best predict the plasma potential using an emissive probe.
The most common methods are the inflection point method and the floating emissive probe
method. There are also several other methods as the differential emissive probes, droop method, self-emitting probes and double-cross method. The methods can be used depending mainly on the characteristics of the plasma to be probed (number density, electron temperature). Since the emissive probes are incapable to deliver such information, they are usually used complementary to Langmuir probes.

The inflection point method is the most straight-forward method and most of the time implies none or light data processing. The plasma potential can be determined directly from the I-V curve delivered by the probe. Another technique would imply the calculation of the first derivative of the emissive probe trace, the maximum of this corresponding to the plasma potential. The zero of the second derivative gives the same information, the location of the plasma potential. The floating emissive probe technique implies the measurement of the floating potential at different heating currents. As the heating current in increased, the probe’s floating potential increases until reaches a saturation level equal to the plasma potential. The method is less precise since the floating potential does not occur exactly at the plasma potential. In the present research the inflection point technique was used in order to determine the plasma potential in the plume of a hollow cathode at the same positions where retarding potential analyser results were available.

1.4.3 Retarding Potential Analyser (RPA)

In order to exhaustively characterise the plasma properties in the plume of a Hall thruster or hollow cathode information about the ion energy distributions is needed. Since the main acceleration potential for the newly formed ions in the ionization region is the discharge potential or anode-cathode potential (beside the loss mechanisms), knowing the most probable potential, \( V_{mp} \) of the ion energy per charge distribution helps in computing the voltage utilisation efficiency, important parameter that defines the overall efficiency of an electric propulsion device. Voltage utilisation efficiency is defined as follows [57]:

\[
\eta_v = \frac{(1/2)M\langle v^2 \rangle}{eV_a} \frac{1}{(f_iQ)},
\]

where \((1/2)M\langle v^2 \rangle\) is the average energy of the expelled ions and \((f_iQ)\) defines the average charge of the exhausted gas/plasma mix, accounting for both neutral and all multiple ionization species [40]. By measuring the ions velocity, one may find the average portion of the electric potential in which the ions have fallen [57]:

\[
\frac{1}{2}MU^2 = qV_a,
\]

where \(U\) is the ion velocity and \(V_a\) is the acceleration potential.

In order to measure the current density of the ions and the ion energy a retarding potential analyser (RPA) can be used. Such a device has a simple design based on a series of biased grids, insulated by each other using phenolic sleeves, and a collector. Figure 1.7 presents a simple configuration of an RPA together with a graphic depicting the voltage variation along the probe and the trace recorded with the analyser. The first grid of the RPA, \( f_i \) is usually floating in order to minimize the perturbations that the probe may induce to the surrounding plasma. Moreover this first grid has the task to attenuate the plasma flow towards the probe, reducing the number density and increasing the internal Debye length [6]. The second grid is called the electron
repeller, $e^-$ and it is biased at $V_e = -50V$ under the ground potential and has the main purpose to repel the electrons from the plume. The third grid, known as the ion repeller, $i^+$ and its potential, $V_b$ is swept from 0 to 120 V. This grid acts as a high pass filter for the beam ions allowing just the ions with energies above the grid threshold to pass towards the collector. While sweeping the ion repeller grid voltage a current is collected by the collector and measured. A typical RPA I-V trace is presented in Figure 1.6 – upper right (green line). The collector, $c$ may be a grid in order to reduce even more the risk of increasing plasma density within the probe and hence a reduction in the Debye length, or a solid material. In the present research a grid collector RPA was used. Moreover the collector grid was biased at $V_c = -50V$ under the ground potential in order to reduce the possibility of secondary electrons emission due to ion impacts with the collector or with the previous grids. The grids, the collector and the insulators are cased into the probe’s body. The resulting I-V curve represents, upon derivation, the distribution function of ion energies per charge (Figure 1.6 – upper right, blue line). This leads to the identification of the most probable voltage and hence to the calculation of the acceleration voltage. This last parameter can be find if the plasma potential, previously computed using Langmuir probes or emissive probes, is subtracted from the most probable voltage [6].

Although the relatively simple design and ease of use of a RPA the device fails is providing a direct measure of the ions energy since it employs grids in order to filter the incoming plasma flux. Such a device cannot make a difference between a triple-charge ion and three single charged ions, the former being detected as the latter [5, 57]. This issue might be of less importance especially in the study of low voltage discharges, since the fraction of multiple charged ions is directly proportional with the discharge voltage [57]. However, the internal pressure inside the probe may force the incoming ions to collide before arriving at the collector [3]. In this way the energy peak is decreased and the distribution function becomes wider in the lower ion energy region, leading to erroneous interpretations [3]. In the present research an RPA was used to characterise the ion energy in radial direction of a laboratory hallow cathode. The probe was manufactured at Tsukizaki Laboratory of ISAS/JAXA and used in a previous study [29].

![Figure 1.7: Left: Schematic of a retarding potential analyser (RPA). Right-top: RPA trace showing the data processing techniques used in finding the most probable potential, $V_{mp}$. Right-bottom: variation of potential inside the RPA.](image)
1.5 Motivation

Once existing only in the realm of science fiction, electric propulsion has proven to be an excellent option for the future of space exploration. In more than sixty years of research on Hall thrusters and hollow cathodes the researchers proved the reliability of these relatively simple devices whose operation mechanism is based on complicated and still not very well explained plasma formation physics. At this moment the Earth is orbited by many satellites equipped with HET employed for orbital stationkeeping, especially for large GEO telecommunication satellites or for other low to medium delta-v missions. HETs suit well a mission with a trip time fixed, because they can deliver higher payload fractions, due to their higher specific power than other electric propulsion devices [25].

Research on plasma plume diagnostics went in parallel with the research in developing new electric thrusters, providing a better understanding of the plume dynamics and on the overall operation of an electric drive. In the later years intensive research was conducted on low voltage Hall thrusters and hollow cathodes mainly because those devices are targeted to onboard microsatellites on low Earth orbit missions. Langmuir probes, emissive probes and the retarding potential analysers were between the diagnostic techniques most commonly used to characterise the plume plasma of such devices mainly because of their simple design and ease of operation. A better understanding of the plume dynamics may contribute to better numerical codes by providing the necessary boundary and initial conditions and offer priceless information about how the plume plasma may interact with the spacecraft in an in-space situation. Since within the plume plasma reside electrons, high energy source ions and low energy charge exchange ions, these particles may produce damage to the spacecraft [3]. For example, the high energy ions can cause sputtering of the surfaces, hence erosion may occur, and the sputtered material may contaminate the sensitive surfaces or the solar panels, leading to a decrease in the energy delivered to the spacecraft [3, 26]. The low energy ions can build up charges on the different materials that cover the spacecraft and may induce changes in the spacecraft potential, leading to dielectric or metallic discharges [26]. The spacecraft may sense thrust loss and torque perturbations due to the plume impingement on the solar arrays [26]. Plume optical emission can affect the sensitive optical instruments onboarded, while the electromagnetic signature of the plume can induce attenuation and phase changes to the telecommunication signals [26].

At the Institute of Space and Astronautical Science (ISAS) of the Japanese Aerospace Exploration Agency (JAXA) a continuous research work is done in the field of electric propulsion at Funaki Laboratory, the place where the research presented in the current thesis was conducted. The plumes of two hollow cathodes, a commercial one and a laboratory model, were never characterised before using plasma diagnostics. By using Langmuir probes, emissive probes and a retarding potential analyser this was done for the first time. Moreover, the plume plasma of an innovative, proof-of-concept prototype of a low-voltage external discharge plasma thruster (XPT) was diagnosed for the first time utilising double Langmuir probes. Hence, the current research brings into discussions the results of plume plasma diagnostics of new devices never tested before together with an analysis that involves a merging process between the plasma plume parameters and the numerical simulations results and operational characteristics and I-V curves, thrust and efficiency data in the case of the XPT and the laboratory model hollow cathode.
1.6 Research objectives and thesis outline

The objectives of the present thesis are threefold. The first objective was to develop and test plume plasma diagnostic instruments based on specific design requirements dictated by the physics of plasma-probe interactions. The main goal was to build instruments that cause minimum perturbations to the plasma and provide high enough resolution, which is a very important factor especially for small devices. Single and double Langmuir probes were developed, manufactured and used to measure electron temperature, plasma number density and plasma potential as well as derivations of electron energy distribution functions. Emissive probes were also designed and built in order to provide consistency measurements for the plasma potential in the case of the laboratory model high-current hollow cathode. A retarding potential analyser, previously used in other study [29], was customised in order to match the plume plasma characteristics of the laboratory model hollow cathode, and used to compute the ion energy per charge distribution function for a radial position in the cathode plume.

Chapter 1 and Chapter 2 of the thesis present an overview of the electric propulsion systems and of the main plasma diagnostic instruments as well as a review of the main physics principles that underpin the operation of the electrostatic probes and RPA (plasma interactions with intrusive probing instruments). Chapter 3 provides a detailed presentation of the experimental setup during the three experimental sessions, including descriptions of the vacuum facilities, presentations of the devices under test as well as exhaustive descriptions of the Langmuir probes, emissive probes and RPA design.

The second objective of the research was to conduct preliminary testing for the Langmuir probes for a commercial hollow cathode plume. Single and double Langmuir probe traces were recorded in order to provide information about the plasma number density, the electron temperature and plasma potential. It was for the first time when such diagnostic devices were used to characterise the plume plasma of this particular hollow cathode. Those preliminary results, although carrying valuable information, were conducting in order to better understand the physics of operation of the Langmuir probes and to get acquainted with their utilisation for plume plasma diagnostics for electric propulsion devices. The results are included in Chapter 4.

A second cathode plasma plume was diagnosed using Langmuir probe, emissive probe and RPA as a transition from the testing experimental sessions towards the XPT tests. The laboratory model high-current hollow cathode had its plasma characterised for the first time by using diagnostic instruments. Emissive probes were used in both “cold” and “hot” modes. When in “cold” mode they behaved as regular single Langmuir probes providing information about the plasma number density, the electron temperature, plasma potential and electron energy distribution functions. Data about plasma number density and electron temperature was subtracted from double Langmuir probe traces as well. Consistency measurements were taken for the plasma potential using the emissive probes in “hot” mode. In this way it was possible to double check the plasma potential value for the position where RPA data were available. RPA was deployed in radial direction measuring the ion energy per charge distribution. Consistency analysis between the information of the three types of instruments as well as comparisons with the numerical simulations data are provided in Chapter 4.

The third objective was to map the plasma plume of the erosion-free low-voltage external discharge plasma thruster (XPT). The thruster is a proof-of-concept prototype and the results offer the first glance upon the characteristics of the plasma plume developed by such a device. Double Langmuir probes mounted on a three-axis positioning system via a custom-made
interface provided a 2D tomography of the central region of the plume. Information about plasma number density, electron temperature and plasma potential were computed, increasing the physical understanding of the thruster operation and providing a database of measurements to support the ongoing computational codes. In the end comparisons and discussions with regards to the thrust and efficiency data as well as numerical simulations results are also included and ascertained in Chapter 5.

Chapter 6 highlights the general conclusions of the herein research, the personal accomplishments and contributions to the research of plasma plume diagnostics of hollow cathodes and low-voltage external discharge plasma thrusters and gives an idea about the general and personal perspectives in this field of electric propulsion research.
CHAPTER 2: Fundamental principles of plasma diagnostics and thruster propulsion

As stated before the electrostatic probes (Langmuir, emissive) and the retarding potential analysers are useful instruments for plasma diagnostics with relatively simple constructive designs and characterised by an ease of use. However, constructing those devices and implementing them in the plasma it is a relatively easy task but the extraction of accurate values for the plasma parameters is a much more challenging task. Although the literature of electrostatic probes and energy collectors’ theory is extensive, there is no unified theory explaining the post-processing and interpretation of the data delivered by such instruments. In the absence of a general model relating the I-V curves to the actual plasma properties under all possible physical conditions, the intricate theoretical and especially practical issues involved in the charge collection mechanism from plasma makes the interpretation of data difficult and obscure outside the narrow limits of some simplified theories. This chapter aims to present a brief description of the main principles of plasma physics and plasma kinetic theory as well as the different theories involved in data post-processing for single and double Langmuir probes, emissive probes and RPA, departing from the basics of charge collection processes from plasma. Notions as collisionality, plasma sheaths, distribution functions, orbital motion limit collection theory are explained with an emphasis on their impact on I-V traces processing. Furthermore, some concepts related to the electric thrusters operation as efficiency, thrust and specific impulse are also depicted.

2.1 Physics of electrostatic probes

A plasma can be described as a medium which contains various charged particles and neutrals that are free to move in response to fields they generate or fields that are applied to the medium and, on the average, is almost electrically neutral [25]. This means that the ion and electron densities are nearly equal, \( n_i \approx n_e \approx n \), a condition commonly named “quasi-neutrality” [25]. This condition implies that the electrons and the ions are moving freely in the plasma. The electrons and the ions from plasma are characterized by their temperatures, which are not necessarily equal. Moreover, a plasma exhibits collective behaviour related to the long-range effects of the Coulomb force. Thus, an element of plasma can influence not just its immediate neighbours. The study of kinetic theory for plasmas is based on the concept of velocity distribution functions. Those functions characterise equilibrium plasmas in which the particles can, theoretically, have any velocity with the additional information that statistically some particles would have a certain velocity. Hereinafter the concept of Maxwellian and Druyvesteyn distribution functions is outlined.

2.1.1 Plasma kinetic theory and distribution functions

The idea of statistical probability of some specific velocities for a group of particles within plasma is mirrored by the concept of velocity space. A differential element of particle density in such a space is proportional to the distribution function, \( f(v_a) \), where the index \( \alpha \) denotes the velocity space coordinates \( v_x, v_y, v_z \) (velocities) and the total number of particles, \( N \), within the differential volume element in this space, \( dV \) [2, 4, 5]:
\[ dn = N f(\nu_a). \]  

(2.1)

Hence, one may define the non-normalised distribution function as follows [2, 4]:

\[ F(\nu_a) = n f(\nu_a). \] 

(2.2)

Of high importance in the study of plasma properties is the concept of energy distribution functions, \( F(\epsilon) \) which define the total number of particles with a particular energy \( \epsilon \). If this function is known, the number density and the temperature of the plasma can be solved by taking moments of the distribution.

For gases in equilibrium the velocity distribution is well described by the Maxwellian distribution function (MDF). This distribution function is considered to be uniform and isotropic in the configuration space (spatial-Cartesian-coordinates) and isotropic in the velocity space and equals the product of three distinct distribution functions, one for each direction of the velocity space [4]:

\[ f(\nu_a) = \left( \frac{m}{2\pi k_B T} \right)^{3/2} \exp \left[ -\frac{m}{2\pi k_B T} (\nu_x^2 + \nu_y^2 + \nu_z^2) \right]. \] 

(2.3)

When it comes to Maxwellian energy distribution function, since the energy is related to the magnitude of velocity, hence the speed, one may define the speed distribution function [4]:

\[ S(\nu) = 4\pi \left( \frac{m}{2\pi k_B T} \right)^{3/2} \nu^2 \exp \left[ -\frac{m}{2\pi k_B T} \nu^2 \right]. \] 

(2.4)

Departing from Equation (2.4) the normalised energy distribution function can be stated as follows [4]:

\[ f(\epsilon) = S(\nu) \frac{d\nu}{d\epsilon} = \frac{1}{\sqrt{2\pi m\epsilon}} S(\nu). \] 

(2.5)

Further development of Equation (2.5) leads to the following expression for the normalised Maxwellian energy distribution function [4, 5]:

\[ f_M(\epsilon) = \frac{2}{(k_B T)^{3/2}} \sqrt{\frac{\epsilon}{\pi}} \exp \left( -\frac{\epsilon}{k_B T} \right). \] 

(2.6)

Figure 2.1: Equilibrium velocity distribution function for species \( \alpha \).

If the plasma is characterised by Maxwellian equilibrium velocity distribution functions for all the species (electrons, ions), then a first approximation of the current drawn by a biased electrode with a specific potential, \( V_b \) is given as follows [42]:

34
\[
I(v) = n q_\alpha A_p \left[ \int_{-\infty}^{\infty} dv_y \left( \frac{2\pi k_B T_\alpha}{m_\alpha} \right)^{-\frac{1}{2}} \exp\left( -\frac{m_\alpha v_y^2}{2k_B T_\alpha} \right) \cdot \int_{-\infty}^{\infty} dv_z \left( \frac{2\pi k_B T_\alpha}{m_\alpha} \right)^{-\frac{1}{2}} \exp\left( -\frac{m_\alpha v_z^2}{2k_B T_\alpha} \right) \cdot \int_{v_{\min}}^{\infty} dv_x v_x \left( \frac{2\pi k_B T_\alpha}{m_\alpha} \right)^{-\frac{1}{2}} \exp\left( -\frac{m_\alpha v_x^2}{2k_B T_\alpha} \right) \right].
\] (2.7)

where \(m_\alpha, q_\alpha, T_\alpha\) are the mass, charge and temperature of the species and \(n\) is the total number density within the plasma. In the previous equation \(v_{\min}\), as limit in the distribution function integral, is the minimum velocity of the species in order to overcome the sheath potential barrier and be collected by the probe. A qualitative interpretation of this velocity is presented in Figure 2.1. The equilibrium velocity distribution function is given as follows [42]:

\[
f(v) = n \left( \frac{2\pi k_B T_\alpha}{m_\alpha} \right)^{-\frac{3}{2}} \exp\left( -\frac{m_\alpha v^2}{2k_B T_\alpha} \right).
\] (2.8)

Hence, the species \(\alpha\) is collected if its minimum velocity satisfies the following condition [42]:

\[
v \geq v_{\min} = \frac{2q_\alpha V_b}{m_\alpha}.
\] (2.9)

In Equation (2.9) \(V_b\) is the potential of the electrode immersed in the plasma. Moreover, since it can be considered that the collected current depends only on \(v_x\), Equation (2.7) becomes:

\[
I(v_x) = n q_\alpha A_p \int_{v_{\min}}^{\infty} dv_x v_x \left( \frac{2\pi k_B T_\alpha}{m_\alpha} \right)^{-\frac{1}{2}} \exp\left( -\frac{m_\alpha v_x^2}{2k_B T_\alpha} \right).
\] (2.10)

It is well known that in the case of gas discharges, thus the case of the Hall thrusters and hollow cathodes plumes, the electron temperature is much greater than the ion and neutral molecules temperature. This means that the energy distribution function is not described the best by a Maxwellian assumption. Thus, non-Maxwellian, non-equilibrium distribution functions should be used to better characterise the situation and one example is the Druyvesteyn distribution [2, 4]. This distribution function is a perfect match for the electron distribution in gas discharges and its derivation is based on elastic collisions between the electrons of energy \(\epsilon\) and much slower ions and neutrals in an electric field \(E\).

In order to derive the Druyvesteyn energy distribution function one may commence with equating the energy taken from the electric field with the total kinetic energy loss for all electrons per plasma unit volume, per unit time due to elastic collisions with neutrals and ions of comparatively low energies. The later term depends on the mean free path, \(\lambda\) and electron thermal velocity \(v_{th}\) [15]:

\[
J(\epsilon) eE = F(\epsilon) \frac{2m \epsilon}{M \lambda} \sqrt{\frac{2\epsilon}{m}}.
\] (2.11)

By defining the electron current density \(J(\epsilon)\) by the use of mobility equation it is possible to state a first approach of a working definition for the Druyvesteyn energy distribution function [15]:

35
\[ F(\epsilon) = \alpha \sqrt{\epsilon} \exp \left( \frac{3me^2}{M\lambda^2E^2} \right). \] (2.12)

The variable $\alpha$ is an undetermined coefficient. Thus Equation (2.12) cannot be easily used. By further derivation, the electric field dependence can be eliminated and the equation can be stated in terms of electron temperature. The specific expression for the Druyvesteyn electron energy distribution function was derived by Ming Li et al. as suggested in Behlam [4]:

\[ f_D(\epsilon) = \frac{0.5648n_e}{(k_BT)^{3/2}} \sqrt{\epsilon} \exp \left( -0.243 \left( \frac{\epsilon}{k_B T} \right)^2 \right). \] (2.13)

### 2.1.2 Plasma sheath and Debye shielding

It is well known that a body immersed into the plasma disturbs locally the quasi-neutrality. In order to cope with the difference in potential between the electrode and itself, the plasma forms a potential sheath next to the surface of the electrode where the quasi-neutrality is violated, since the electron number density drops faster than the ions number density, as qualitatively presented in Figure 2.2. This effect is called Debye shielding.

Within the sheath one can notice a drop in the electrostatic potential as well as an increase in the plasma flow velocity compared to the species velocity at the edge of the sheath. It might be of interest to state that, upon conservation of energy and particle conservation, the ion number density within the sheath is as follows [18]:

\[ n_i = n_{sheath \ edge} \left( \frac{V_{sheath \ edge}}{V} \right)^{1/2}. \] (2.14)

$V$ is the plasma potential at a specific distance from the electrode, within the sheath. Moreover, it can be stated that an electrode biased at a potential near the plasma potential is not surrounded by a sheath, hence the quasi-neutrality is conserved until the surface of the electrode. The condition for this to happen is as follows [18]:

\[ V_{sheath \ edge} > \frac{T_e}{2}. \] (2.15)

$T_e$ is the plasma electron temperature.

**Figure 2.2:** Plasma sheath formation at the contact between the quasi-neutral bulk region of the plasma and the surface of the electrodes.
The thickness of the sheath is approximated to several Debye lengths. In order to find this characteristic length, it is important to know that within the sheath only Maxwell equations are valid since the quasi-neutrality is violated. Thus, one may use a 1D Poisson equation:

\[ \nabla \cdot E = \frac{e}{\varepsilon_0}(n_i - n_e) \]  

(2.16)

Furthermore, based on the ion number density variation within the sheath, described by Equation (2.14) and assuming that the electron number density within the sheath falls off according to a Boltzmann factor as in the following equation [18]:

\[ n_e = n_{\text{sheath edge}} \exp \left( \frac{e(V - V_{\text{sheath edge}})}{k_B T_e} \right). \]  

(2.17)

Now Equation (2.16) can be re-written as [18]:

\[ \frac{d^2V(x)}{dx^2} = -\frac{e}{\varepsilon_0} n_{\text{sheath edge}} \left[ \left( \frac{V_{\text{sheath edge}}}{V(x)} \right)^{1/2} - \exp \left( \frac{e(V(x) - V_{\text{sheath edge}})}{k_B T_e} \right) \right]. \]  

(2.18)

For a specific difference in potential defined as \( \Delta = V_{\text{sheath edge}} - V > 0 \) the following approximations can be made [18]:

\[ \left( \frac{V_{\text{sheath edge}}}{V} \right)^{1/2} \approx 1 + \frac{\Delta}{2V_{\text{sheath edge}}} = 1 - \frac{\Delta}{2|V_{\text{sheath edge}}|}, \]

\[ \exp \left( \frac{e(V - V_{\text{sheath edge}})}{k_B T_e} \right) \approx 1 - \frac{e\Delta}{k_B T_e}. \]  

(2.19)

Now, Equation (2.18) becomes:

\[ \frac{d^2\Delta}{dx^2} = \frac{e\Delta}{\varepsilon_0} n_{\text{sheath edge}} \left[ \frac{e}{k_B T_e} - \frac{1}{2|V_{\text{sheath edge}}|} \right]. \]  

(2.20)

The Debye length can now be computed by assuming that at this distance away from the electrode the potential equals the sheath edge potential. Hence, Equation (2.20) becomes:

\[ \frac{\Delta}{\lambda_D^2} \approx \frac{e\Delta}{\varepsilon_0 n_{\text{sheath edge}} k_B T_e}. \]  

(2.21)

The Debye length is defined as:

\[ \lambda_D \approx \sqrt{\frac{\varepsilon_0 k_B T_e}{e^2 n_{\text{sheath edge}}}}. \]  

(2.22)

### 2.1.3 Collisionality in plasmas

Knowing the bulk behaviour of the plasma that is to be diagnosed, it is of high importance in order to properly size the electrodes of the electrostatic probes or the grids of the retarding potential analyser. This behaviour is mainly dependent on the collision regime at microscopic level. An extensive discussion on how a RPA design is influenced by the plasma parameters to be probed is presented later in this chapter. Now, the interactions between the electrodes of an electrostatic probe and the surrounding plasma are ensued. Three parameters have to be defined in order to properly characterise those interactions.

The previously defined Debye length is an instruments used to appreciate the thickness of the plasma sheath that forms around an electrode immersed into the plasma. The ratio between the probe’s radius, \( r_p \) and the Debye length, \( \lambda_D \) is defined as follows:
Two regimes can be defined based on Equation (2.23): if $\xi < 3$ [8, 9], the probe works in the orbital motion limited regime (OML) where the edge effects may be important. The regime is known as thick sheath collection regime and may be defined sometimes by $\xi \ll 1$ [57]; if $\xi > 10$ [8, 9] or $\xi \gg 1$ [57], the probe works in the thin sheath limited regime (TSL) and the edge effects can be neglected. Further discussion about the two working regimes is presented later in the chapter.

The previous two regimes are part of the classical Langmuir probe theory and are true only in the case of collisionless plasmas. Hence, another important parameter, the Knudsen number, $Kn$ is used to categorise the flow regime of the plasma, thus the collisionality. Knudsen number is generally defined as the ration between mean free path $\lambda$ and the probe’s radius:

$$Kn = \frac{\lambda}{r_p} \quad (2.24)$$

The validity of the following probe theories is highly dependent on the flow regime that best describes the plume region of the thruster or hollow cathodes. Several flow regimes can be categorised by the means of Knudsen number: for $Kn > 10$ or simply $Kn \gg 1$ collisionless regime is established, known as free molecular flow [4]. A Knudsen number $Kn \ll 1$ defines a continuum flow [4] and in between the two limit cases transitional and slip flow may arise for $10 > Kn > 0.01$ and $1 > Kn > 0.01$, respectively [4]. In the case of plasmas several Knudsen numbers can be defined with respect to the different collision mean free paths between the plasma constituents. The mean free path is defined as the distance that a certain plasma constituent travels before a collision arise with another particle and can be written as follows [2, 4]:

$$\lambda = \frac{1}{n \sigma} \quad (2.25)$$

where $\sigma$ is the collision cross-section.

Several mean free paths can be calculated and for the plasma plume to be collisionless the different mean free paths should be much larger than the probe’s radius. The most important collision mean free paths are the electro-ion mean free path and the electron-neutral mean free path. The former is usually the shortest, since the attraction force between the species is high. Less probable interactions are the electron-electron and ion-ion due to the Coulomb forces. One may assess also for ionization collisions, neutral-neutral collisions and ions-neutral collision (elastic scattering and charge exchange interactions), but in the plume plasma those collisions are characterise by small cross-sections, hence the probability to occur is lower compared to the first two types. The electron-neutral mean free path is defined as follows [25]:

$$\lambda_{en} = \frac{1}{n_a \sigma_{en}} \quad (2.26)$$

where $n_a$ is the neutral density which may be approximated as $n_a = P / k_B T_g$, where $P$ is the pressure inside the vacuum chamber and $T_g$ is the propellant temperature. The total scattering electron-neutral collision cross-section for electron temperatures higher than 0.4 eV is defined as [25]:

$$\xi = \frac{r_p}{\lambda_p} \quad (2.23)$$
The electron-ion mean free path, can be defined as follows [4]:

\[ \lambda_{ei} = \frac{1}{4\pi n_e b_0^2 \ln(\frac{\lambda_D}{b_0^2})} \text{ and } b_0 = \frac{-e^2}{12\pi \varepsilon_0 k_B T_e}, \]

(2.28)

In some conditions, especially when probing flowing plasmas, the alignment of the probe with the direction of flow may influence the collection of ions. Since the length of a cylindrical probe is usually much bigger than its radius the issue of alignment and the nature of the plasma sheath that forms at the tip of the electrode may cause deviations from the orbital motion limited theory. Thus, a probe that is aligned with its longitudinal axis parallel to the direction of flow may collect more ions than the OML theory predicts, since the ions that would have normally been repelled and orbit the probe are collected as well. This mismatch between the OML theory and the actual collected current in such configuration had to do with the definition of particles impact parameters. Usually, when entering the sheath of the probe, the particles are characterised by a specific impact parameter. For a zero angle of attack of the probe the ions that approach the sheath surrounding the probe do not have an impact parameter, as needed in the derivation of OML theory, and they are most probably collected by the probe instead of having orbital trajectories. The effect is denominated as the "end effect" and it is of high concern when the probe's longitudinal axis is not perpendicular to the plasma flow. The end effect parameter is defined as follows [4]:

\[ \tau_l = \left( \frac{l_p}{\lambda_D} \right)^{1/2} \left( \frac{k_B T_e}{M} \right)^{1/2} \frac{1}{U}. \]

(2.29)

In the previous equation \( l_p \) is the length of the probe and \( U \) is the ion drift velocity. Assuming a neutral continuum fluid and that the flow is choked, the ion drift velocity can be approximated as follows [4]:

\[ U = \sqrt{\frac{\gamma R T_l}{m}}, \]

(2.30)

When \( \tau_l \gg 1 \), or sometimes referred as \( \tau_l > 50 \) [4], the end effect can be neglected and a cylindrical probe working in a flowing plasma behaves as the probe works in a stationary plasma [4]. In all the following results the probe had an angle of attack of 90° in order to avoid any farther complication of the data analysis due to those effects. Moreover, it worth adding the fact that the end effect affects just the ion collection since the electrons, having low mass and high mobility, are not affected by the relative position between the probe and the direction of plasma flow [4].

### 2.1.4 Theory of charge collection by cylindrical probes

In order to apply the simple collection model relating the plasma properties with the current-voltage curves, as formulated by Langmuir and Mott-Smith [50], some assumptions have to be stated. First, the plasma has to be unmagnetised and collisionless. The latter condition is satisfied if the Knudsen number defined by Equation (2.24) is large enough. Moreover the theory holds for thin plasma sheath which means that \( \xi \) should be as large as possible. If this last condition is satisfied, end effects may be neglected as well. Besides these assumptions several others have to be satisfied [12, 58]:

\[ \sigma_{en} = 6.6 \cdot 10^{-19} \left[ \frac{T_e}{4} - 0.1 \frac{T_e^{1.6}}{1 + \left( \frac{T_e}{4} \right)} \right] (m^2). \]

(2.27)
a) Firstly the bulk plasma is considered to have an infinite, stationary and homogeneous volume and compliance with quasi-neutrality, hence \( n_e \approx n_i = n \)

b) In order to avoid the complications brought by the existence of drift motions, the electrons and the ions within the plasma should have Maxwellian velocity distributions and in compliance with \( k_B T_e \gg k_B T_i \approx k_B T_a \), where \( T_a \) is the neutral temperature

c) It is important as well to assume that the charges reaching the probe surface do not chemically interact with the probe’s material and they are always collected by the probe and contributes to the current \( I(V_b) \).

d) The probe do not perturb the surrounding plasma outside a well defined space charge sheath; the space plasma potential remains uniform outside the sheath’s boundary

e) Around the probe’s surface the potential preserves the symmetry and is described by a monotonically increasing or decreasing function between \( r_p \) and \( r_s \)

The electrons and ions have the charges \( q_\alpha \), where \( \alpha = e, i \) and masses \( m_\alpha \). The charges have a certain velocity \( v \) at the distance \( r \) away from the centre of the cylindrical probe with the radius \( r_p \). All the potentials are defined with respect to the plasma potential of the bulk, unperturbed plasma \( V_p \). Hence, the potential at a certain distance \( r \) away from the probes centre is \( \Phi(r) = V(r) - V_p \), while the potential at the probe’s surface is \( \Phi_p = \Phi(r_p) = V_b - V_p \). The first relation remains undetermined but it has to be characterised by a monotonically increasing or decreasing function inside the plasma sheath. At the edge of the sheath the potential is \( \Phi_s = \Phi(r_s) \). The bulk plasma is characterised by the number density \( n_\alpha \).

Two cases can be identified. When \( q_\alpha \Phi(r) > 0 \) (ions and positive potential and electrons and negative potential) the particles are repelled by the retarding electric field formed around the probe. When \( q_\alpha \Phi(r) < 0 \) (ions and negative potential and electrons and positive potential) the particles are attracted by the means of the accelerating electric field.

Figure 2.3 depicts the situation when a charged particle approaches a cylindrical probe. Since the bulk plasma is being considered collisionless, the energy conservation as well as the angular momentum conservation can be applied at the distance \( r \) from the probe and at probe’s surface, \( r_p \).
In the case of repelled particles the following is valid \( q_a \Phi(r) > 0 \). If isotropic energy distribution function is assumed, the current density though a surface situated at the distance \( r \) from the probe’s surface is [12]:

\[
j(r) = 2q_a n_a \frac{r_p}{r} \int_{v_{\perp min}}^{\infty} v_{\perp}^2 \sqrt{1 - \frac{2q_a \Phi_p}{m_a v_{\perp}^2}} f_{1,a}(v_{\perp}) \, dv_{\perp}.
\] (2.31)

In order to reach the probe’s surface and be collected, the particle should have a minimum tangential velocity defined as [12]:

\[
v_{\perp min} = \frac{2q_a \Phi_p}{m_a}.
\] (2.32)

Moreover, the maximum allowed approaching angle, \( \varphi_{max} \) is defined as [12]:

\[
\sin(\varphi_{max}) = \pm r_p \frac{2q_a \Phi_p}{r m_a v_{\perp}^2}.
\] (2.33)

If the energy distribution function is considered to be Maxwellian and introducing the thermal velocity of the particle, \( v_{th} \), Equation (2.31) becomes [12]:

\[
j(r) = q_a n_a \frac{r_p}{4} \sqrt{v_{th}} \exp \left( \frac{q_a \Phi_p}{k_B T_a} \right).
\] (2.34)

Now, the total collected current by the probe can be defined as:

\[
I(r_p) = A_p j(r_p)
\]

\[
I(r_p) = \frac{q_a n_a}{4} A_p v_{th} \exp \left( \frac{q_a \Phi_p}{k_B T_a} \right)
\] (2.35)

Expressing Equation (2.36) in terms of electron and ion currents, one may get:

\[
I(r_p) = \frac{-e n_e}{4} A_p \sqrt{\frac{8k_B T_e}{\pi m_e}} \exp \left( \frac{e \Phi_p}{k_B T_e} \right) + e n_i A_s \sqrt{\frac{8k_B T_i}{\pi M}} \exp \left( \frac{e \Phi_s}{k_B T_i} \right).
\] (2.36)

As it may be seen, the electron current is proportional with a saturation term, equal to the classical one-sided flux to a surface, and an exponential term depending on the probe’s potential with respect to the bulk plasma potential. On another hand, the ion current is proportional with the full flux to the plasma sheath surface, defined by the plasma sheath radius \( r_s \) and the exponential term depends on the plasma potential at the edge of the sheath, \( \Phi_s \). Later in the chapter the theory of a positive space charge formation around the probe is explained. For now it must be stated that due to the acceleration of ions inside the sheath, when the probe’s potential is negative, the ions acquire a supplementary velocity equivalent to a potential drop defined as:

\[
\Phi_s = \Phi(r_s) = V_s - V_p = -\frac{k_B T_e}{2e}
\] (2.37)

Moreover, since \( k_B T_e > k_B T_i \) and due to this acceleration, the ions are characterised using Bohm velocity instead of their thermal velocity [12,48]:
Using Equation (2.37) and Equation (2.38), assuming thin sheath theory, hence \( A_p \approx A_s \), and quasi-neutrality, Equation (2.36) becomes:

\[
I(r_p) = \frac{-en}{4} A_p \sqrt{\frac{8k_B T_e}{\pi n m_e}} \exp \left( \frac{e \Phi_p}{k_B T_e} \right) + enA_p \sqrt{\frac{k_B T_e}{M}} \exp \left( -\frac{1}{2} \right).
\]  

(2.39)

The electron and ion saturation currents are defined as follows:

\[
I_{es} = \frac{-enA_p}{4} \sqrt{\frac{8k_B T_e}{\pi n m_e}},
\]  

(2.40)

\[
I_{is} = 0.6enA_p \sqrt{\frac{k_B T_e}{M}}.
\]  

(2.41)

In the case of attracted particles, when \( q_a \Phi(r) < 0 \) the situation is more complex, since anisotropic velocity distribution should be assumed. The current density at the sheath edge, \( r_s \), is a sum of both sheath limited and orbit limited parts of the current density. Hence, the attracted particles current depends on the thickness of the sheath. This is mainly because the behaviour of the particle orbits depends on whether the initial velocity is smaller or larger than a certain velocity defined as \( v_s \). The sheath radius is defined as follows [12]:

\[
r_s = r_p \sqrt{1 + \left| \frac{q_a \Phi_p}{\epsilon_a} \right|}.
\]  

(2.42)

From Equation (2.42) the velocity of the particle when entering the sheath can be obtained via the energy of the particle [12]:

\[
v_s^2 = 2 \frac{\left| q_a \Phi_p \right|}{m_a \left( r_p^2 r_p^2 - 1 \right)}.
\]  

(2.43)

Assuming a two dimensional Maxwellian velocity distribution function due to the cylindrical symmetry, the current density at the sheath edge can be defined as follows [12]:

\[
j(r_s) = \frac{q_a n_a}{4} v_{th} \left[ \text{Erf}(\tilde{u}_s) + \frac{r_p}{r} e^{\tilde{\phi}_p} \text{Erfc} \left( \sqrt{\tilde{u}_s^2 + |\tilde{\phi}_p|} \right) \right],
\]  

(2.44)

where \( \text{Erf}() \) and \( \text{Erfc}() \) are the error and complementary error functions and the dimensionless velocity and potential are defined as:

\[
\tilde{u}_s = \frac{v}{2k_B T_a}, \quad \tilde{\phi}_p = \frac{q_a \Phi_p}{k_B T_a}.
\]  

(2.45)

Thus, the current collected by the probe is as follows [12]:

\[
I(r_p) = A_s j(r_s) = \frac{q_a n_a}{4} A_p v_{th} \left[ \frac{r_s}{r_p} \text{Erf}(\tilde{u}_s) + e^{\tilde{\phi}_p} \text{Erfc} \left( \sqrt{\tilde{u}_s^2 + |\tilde{\phi}_p|} \right) \right].
\]  

(2.46)
As it can be clearly seen, for the case of repelled particles the collected current does not depend on the geometry of the probe or on the thickness of the plasma sheath that surrounds the probe. For the case of attracted particles the collected current depends on the thickness of the plasma sheath, $r_s$. Two cases can be identified in this last situation.

When $r_s - r_p > r_p$ thin sheath situation is identified. The collected current for attracted particles in Equation (2.46) can be written as the product between the saturation current $I_{as}$ and a function $f(\tilde{u}_s, |\tilde{\Phi}_p|)$:

$$I(r_p) = I_{as} f(\tilde{u}_s, |\tilde{\Phi}_p|). \quad (2.47)$$

In the case of thin sheath limit collection $f(\tilde{u}_s, |\tilde{\Phi}_p|) = 1$ and the collected currents are the ones expressed in Equations (2.40) and (2.41).

When $r_s/r_p > 1$ thick sheath limit is observed, also known as orbital motion limited collection. In this case the function identified in Equation (2.47) equals the following [12]:

$$f(\tilde{u}_s, |\tilde{\Phi}_p|) \approx \frac{2}{\sqrt{\pi}} \sqrt{1 + \frac{q_a \Phi_p}{k_B T_a}}. \quad (2.48)$$

In this case the collected current in the case of attracted particles is defined as follows [12]:

$$I(r_p) = I_{as} \frac{2}{\sqrt{\pi}} \sqrt{1 + \frac{q_a \Phi_p}{k_B T_a}}. \quad (2.49)$$

This brief exposition is important to understand the main analysis methods for experimental results obtained using Langmuir probes and emissive probes as well as retarding potential analysers. Next, the main analysis methods used for single and double Langmuir probe data processing are outlined.

### 2.2 Physics of Langmuir probes

Based on the previously explained theories, several methods for experimental data analysis are explained hereinafter. An important source of information is the current-voltage trace shape that may give a first glance upon the theory that should be used for data analysis. In the case of a cylindrical single Langmuir probe several possibilities may arise, as depicted by Figure 2.4. The ideal thin sheath curve (black curve) allows for a direct identification of the plasma potential by its sharp "knee" between the retarding field region and electron saturation region [68]. The current variation with the bias voltage is defined by Equation (2.39) for the retarding field region and by Equations (2.40) and (2.41) for the saturation regions. Hence, the ion current is given as [48]:

$$I_i(V_b) = \begin{cases} 
-I_{es} \exp \left[ \frac{e(V_p - V_b)}{k_B T_i} \right], & V_b \geq V_p, \\
-I_{es}, & V_b < V_p. 
\end{cases} \quad (2.50)$$

The electron current is given as [48]:

$$I_e(V_b) = \begin{cases} 
-I_{es} \exp \left[ \frac{-e(V_p - V_b)}{k_B T_e} \right], & V_b \leq V_p, \\
-I_{es}, & V_b > V_p. 
\end{cases} \quad (2.51)$$
The floating potential of the probe is now easily defined as the bias voltage for which no net current is collected \[48\]:

\[
I_{es}\exp\left(-\frac{e(V_p - V_f)}{k_B T_e}\right) - I_{is} = 0
\]  

(2.52)

Further derivation leads to the following expression \[48\]:

\[
V_f = V_p + \frac{k_B T_e}{e}\ln\left(\frac{0.6}{\sqrt{\frac{2\pi m}{M}}}\right).
\]  

(2.53)

In most of the cases, in real conditions, the shape of a single Langmuir probe current-voltage trace is rather characterised by the other graphics presented in Figure 2.4. This is mainly because of the formation of the plasma sheath around the probe’s electrode. The size of the sheath influences the appearance of the trace and implies different analysis methods. For example, the green trace can be analysed using the thin sheath theory (TSL). The other two traces correspond to orbital motion limited (OML) operation regimes which do not allow for graphical identification of the plasma potential. In the case of a 2D OML (red trace) the square of the collected current has a linear dependence with the bias voltage, while for the 3D OML (blue trace) there is a linear dependence between the bias voltage and the collected current \[68\].

2.2.1 Plasma sheath perspective

As formerly stated, Debye shielding occurs when an electrode is immersed into the plasma. Any particle that is in a collision course with the probe’s surface may strike the probe directly and in these conditions the plasma sheath allow for less change in the particle’s trajectory. Otherwise the particle can be deviated. In the same way, particles that are not in a collision course with the probe before entering the plasma sheath may be deviated by the strong potential gradient within the sheath and eventually be collected by the probe. Thus, knowing the dimensions of the plasma sheath is an important aspect for the further data analysis.

When the electrode potential, \(V_b\), is much negatively than the plasma potential, \(V_p\), the electrons are repelled and the ions are attracted by the probe and an ion current is drained from the plasma. This current decreases asymptotically for \(V_b << V_p\) and is known as the ion saturation current, \(I_{is}\). The spatial variation of the plasma potential next to a negatively biased electrode is presented in Figure 2.5-left. In the vicinity of the negatively charged electrode, the
electron and ion densities decrease, but not at the same rate. The drop in electron density is due to the repelling mechanism, while the ions are accelerated towards the probe and due to the continuity of the current density the ion density decreases. Around the electrode a positive space charge, known as positive sheath, is formed with a thickness \( \delta = r_5 - r_p \). In order to have such a situation, the ion density should exceed the electron density at the edge of the plasma sheath. This is possible only if the ions approach the sheath with a much larger velocity than their thermal velocity. In fact the ions approach with a speed exceeding the Bohm velocity. Achieving this speed is possible if the ions acquire a supplementary energy corresponding to the potential drop defined by Equation (2.37). This acceleration mechanism takes place in the presheath over a long distance in the plasma. This explains the factor of 0.6 in Equation (2.41) which defines the reduction in the ion density in the presheath acceleration to reach the Bohm velocity.

A second case is presented in Figure 2.5-right. In this case the biasing becomes more positive and \( V_b \gg V_p \) and the ions are repelled, while the electrons are attracted. The current collected by the probe is responsible for the electrical shielding of the probe and is known as the electron saturation current, \( I_{es} \). In this case a negative sheath is created for \( r < r_s \), connecting the space potential within the sheath with the unperturbed bulk plasma potential.

The dimension of the plasma sheath can be calculated as follows [4]:

\[
\delta = 1.02 \lambda_p \sqrt{\left(-\frac{1}{2} \ln \left(\frac{m_e}{M}\right)^{1/2} - \frac{1}{\sqrt{2}}\right) \left(-\frac{1}{2} \ln \left(\frac{m_e}{M}\right) + \sqrt{2}\right)}. \quad (2.54)
\]

Moreover, there can be formulated the dependence between the thickness of the sheath and the bias voltage of the probe [28]:

\[
\delta = 1.1 \lambda_p \left(\frac{e(V_p - V_b)}{T_e}\right)^{3/4}. \quad (2.55)
\]

The expansion of the sheath with the increasing bias voltage influences the collected current. The phenomenon can be modelled as a linear function of the bias voltage [48]:

![Figure 2.5: Left: Potential profile as a function of radial distance from a positively biased electrode (electron collecting surface) [12]. Right: Potential profile as a function of radial distance from a negatively biased electrode (ion collecting surface).](image-url)
\[
\begin{align*}
I_{ls}(V_b) &= -[0.2(V_p - V_b) + I_{ls}], \quad V_b < V_p \\
I_{es}(V_b) &= 0.7(V_b - V_p) + I_{es}, \quad V_b > V_p
\end{align*}
\]

The expansion of the sheath is usually seen in the lack of electron saturation and such probe traces should be analysed using OML theory or an electron energy distribution function.

2.2.2 Thin Sheath Limited theory (TSL)

A single Langmuir probe trace allows, in good conditions, for a graphical direct identification of the plasma potential. As presented in Figure 2.6, the plasma potential position is marked by the "knee" of the curve, when the retarding field region gives place to the electron saturation region. In practice this "knee" is very hard to be identified directly from the raw or smoothed data and post-processing analysis should be involved. The electron saturation region is rarely attained mainly due to the expansion of the plasma sheath as the potential of the probe increase. Moreover, the electrode contamination leads to a decrease in the collected current and to a round off in the region of the "knee". Thus, although the probe may work in a thin sheath limited condition, the contamination of the electrode drives a lack of saturation implying a false orbital motion limited working regime. Usually, for Hall thrusters the problem of contamination is limited by the possibility of cleaning the probe by the energetic ions present in the plume. On the other hand, for the case of hollow cathodes the cleaning of the probe tip is hard to be achieved due to the smaller population of energetic ions. Figure 2.6 depicts two more regions characteristic for a Langmuir probe working at high bias voltages, electron emitting and secondary ionization regions. At very negative bias voltages the ions gain high accelerations within the sheath and due to bombardment on the surface of the probe the latter starts emitting electrons. The probe starts to behave as an emissive probe and the ion collected current increases. When the bias potential becomes very positive, the accelerated electrons produce secondary ionizations in the sheath leading to an increase in the collected electron current. Both situations should be avoided since the electrode can be damaged and contaminated.

**Figure 2.6:** Single Langmuir probe trace highlighting the different collection regions.
Figure 2.7: Typical example of raw current data for a single Langmuir probe trace.

Figure 2.8: Top: Example of a single Langmuir probe smoothed trace. Bottom: Enlarged view of ion saturation region of a single Langmuir probe trace. The ion saturation current magnitude increases as the probe voltage is decreased because of plasma sheath expansion.
In order to extract the plasma characteristics from a single Langmuir probe trace using the thin sheath theory, several steps have to be completed. A raw current data for a single Langmuir probe trace for hollow cathode xenon plume plasma is presented in Figure 2.7. The raw data is then smoothed, as depicted in Figure 2.8. As previously described, the current collected by the probe is the sum of electron and ion currents (Equation (2.39), Equations (2.50) and (2.51)). In order to find the electron temperature and the electron number density, the ion saturation current is extracted from the total collected current leading to an identification of the electron current. Figure 2.8-bottom presents a close-up view on the ion current region. By fitting this region using a linear function the ion saturation current can be identified.

The electron current in the retarding field region has an exponential variation until saturation is reached. The inflexion point between the two regions gives the plasma potential. Thus, a natural logarithm plot of the electron current reveals a linear dependency between the current and voltage in the retarding field region. The situation is presented in Figure 2.9. Since the electron current in this region is defined as follows [42]:

$$I(V_b) \approx -\frac{en_e}{4A_p} \frac{8k_BT_e}{\pi m} \exp \left( \frac{eV_b}{k_BT_e} \right) = I_{es} \exp \left( \frac{eV_b}{k_BT_e} \right),$$  \hspace{1cm} (2.57)

the electron temperature can be easily subtracted using the inverse of the slope of the retarding field region linear fit:

$$\left( \frac{d\ln(I(V_b))}{dV_b} \right)^{-1} = \frac{k_BT_e}{e}. \hspace{1cm} (2.58)$$

The second linear fit on the electron saturation region provides the electron saturation current which subsequently leads to the derivation of the electron number density by using Equation (2.40):

$$n_e = \frac{-4I_{es}}{eA_p \frac{8k_BT_e}{\pi m}}. \hspace{1cm} (2.59)$$

![Figure 2.9: Plot of the natural logarithm of the electron current for a single Langmuir probe trace. Linear fits of the retarding field and electron saturation regions reveal the position of the plasma potential and the electron saturation current.](image)
Assuming quasi-neutrality, the electron number density above mentioned would reflect the plasma number density. However, the ion number density can be also computed by the means of the ion saturation current found through the linear fit presented in Figure 2.8-bottom. From Equation (2.41), the ion number density using the thin sheath assumption is as follows:

$$n_{i,TSL} = \frac{I_s}{0.6eA_p} \sqrt{\frac{M}{k_B T_e}}$$

(2.60)

The electron and ion number densities should, in theory, be equal. However, there is a difference between the two, the ion number density being in general larger than the electron number density \([48]\). The difference is a typical occurrence with Langmuir probes measurements.

The floating potential can be directly subtracted from the trace as the bias voltage for which zero net current is collected. A cross check for the plasma potential can be computed by using Equation (2.53), as the floating potential and the electron temperature are known at this stage.

At this stage it is possible to compute the Debye length of the plasma and check if the thin sheath assumption it truly valid. Moreover, computation of the mean free path and check for the plasma collisionality is also possible. A simple check for the single probe traces obtained for the hollow cathodes revealed that the ratio \(r_p/\lambda_D\) is in most of the cases smaller than 10 and just a bit bigger than 1. Hence, it was difficult to assess if the probe worked in thin sheath limited regime or the orbital motion regime. However, both theories were applied in order to compare the results for the different plasma characteristics.

### 2.2.3 Thick Sheath or Orbital Motion Limited theory (OML)

If the thickness of the plasma sheath becomes large compared to the probe’s radius, the orbital motion limited theory should be applied for data analysis. This theory states that the probe’s squared ion and electron currents vary linearly with the bias voltage. Figure 2.10 presents the probe’s squared current plotted against the bias voltage and Figure 2.11 depicts an enlarged view only for the ion current region. The collected current in the case of a thick sheath formed around the probe’s electrode is approximated by Equation (2.49). The saturation currents are given by Equations (2.40) and (2.41). A linear fit to the electron collection region helps finding the electron number density as follows \([4]\):

$$n_{e,OML} = \frac{\pi}{A_p} \sqrt{\frac{m}{2e^3} \left( \frac{dI_e^2}{dV_b} \right)}.$$  

(2.61)

The intercept of the linear fit together with the slope reveal the electron temperature:

$$k_B T_e \approx \frac{(I_e^2)_{\text{intercept}}}{\frac{dI_e^2}{dV_b}}.$$  

(2.62)

The ion number density can be as well computed by using the slope of the linear fit to the ion collection current region \([4, 57]\):

$$n_{i,OML} = \frac{1}{A_p} \sqrt{\frac{\pi M}{1.44e^3} \left( \frac{dI_i^2}{dV_b} \right)}.$$  

(2.63)

The OML theory is less precise for the electron temperature approximation than the thin sheath theory, but the approximations for the number densities are in general trustful \([4, 57]\).
The number density computed using OML theory is used again to find the Debye length in order to verify which theory is valid and can produce reliable results. Since the results for the electron temperature obtained with OML theory are not to be trusted, for the computation of the Debye length and mean free paths the electron temperature found with the thin sheath approximation is to be used.

Figure 2.10: Squared collected current for a single Langmuir probe trace and linear fit to the electron collection region.

Figure 2.11: Linear fit to the squared ion collected current by a single Langmuir probe.
2.2.4 Double probe thin sheath theory

A double probe is formed of two electrodes usually with the same collecting area with no plasma ground connection. Since the electrodes are floating, the Kirchhoff's current law states that the total current through the probe's circuit is the sum of the currents collected by each electrode [52]:

$$\sum I_p = I_{p1} + I_{p2} = 0.$$ (2.64)

The current collected by each electrode is mainly formed by the ion saturation current and an electron current with an exponential variation, similar to the current presented by Equation (3.39) [52]:

$$I_{is} + I_{es} \exp \left(\frac{-eV_1}{k_BT_e}\right) + I_{is} + I_{es} \exp \left(\frac{-eV_2}{k_BT_e}\right) = 0.$$ (2.65)

For a Maxwellian distribution and assuming the thin sheath approximation the ion saturation current equals the electron saturation current $-I_{is} = I_{es}$. In this case Equation (2.65) becomes:

$$\left[ 1 - \exp \left(\frac{-eV_1}{k_BT_e}\right) \right] + \left[ 1 - \exp \left(\frac{-eV_2}{k_BT_e}\right) \right] = 0.$$ (2.66)

The probe's circuit zero net current can be written as follows [52]:

$$I_p = I_{p1} = -I_{p2}.$$ (2.67)

The following can be stated:

$$I = I_{is} \left[ 1 - \exp \left(\frac{-eV_1}{k_BT_e}\right) \right] = -I_{is} \left[ 1 - \exp \left(\frac{-eV_2}{k_BT_e}\right) \right].$$ (2.68)

Baring that $\tanh(x) = (e^{2x} - 1)/(e^{2x} + 1)$, the total current collected by a double Langmuir probe can be stated as follows [52]:

$$I = I_{is} \tanh \left[ \frac{-eV_b}{2k_BT_e} \right],$$ (2.69)

where $V_b = V_2 - V_1$.

![Double probe I-V characteristic](image)

Figure 2.12: Typical example of raw current data for a double Langmuir probe trace.
The raw current data measured with a double Langmuir probe with identical cylindrical electrodes for hollow cathode xenon plume plasma is presented in Figure 2.12. In order to analyse the data using the thin sheath theory, the raw data is smoothed and then the two ion saturation regions as well as the retarding field region are fitted using linear functions. This procedure is depicted in Figure 2.13.

Departing from Equation (2.68) the electron temperature can be easily computed. The rates of change of current with respect to voltage in the retarding field region and the ones in the ion saturation regions are used to compute the electron temperature as follows [14]:

$$\frac{k_B T_e}{e} = -\frac{\sum I_i}{A_p \left( \frac{dI}{dV_B} \bigg|_{V_B=0} - \frac{dI}{dV_B} \bigg|_{V_B=I_{is}} \right)} \quad (2.70)$$

The slope corresponding to the ion saturation \( (dI/dV_B)_{I_{is}} \) is computed as the average between the slopes of the ion saturation regions of the two electrodes.

Once having the electron temperature it is possible to compute the plasma number density using the ion saturation current \( I_{is} \) as an average of the two ion saturation current recorded. Assuming that the probe works in the thin sheath regime, Equation (2.60) can be used to compute the ion number density.

A second method for double probe trace analysis implies the direct fitting of the smoothed raw data with a custom curve based on Equation (2.69) [13, 57]:

![Figure 2.13: Linear fits to the different regions of a double Langmuir probe smoothed trace used in the thin sheath approximation.](image-url)
where \(a, b, c\) and \(d\) and \(h\) are the fitting coefficients that have to be found. The linear part of the fitting curve \(c x\) takes into account the growing of the plasma sheath around the electrodes as the bias voltage is increased [13, 57] and \(h\) accounts for plasma non-uniformities [13]. In order to find the electron temperature and the ion number density, equivalence between Equations (2.69) and (2.71) is done. Thus, the electron number density is given by the coefficient \(b\):

\[
T_e = \frac{1}{2b}
\]

(2.72)

while the ion number density can be computed using Equation (2.60) where the ion saturation current, or Bohm current, is exactly the coefficient \(a\):

\[
n_i = \frac{a}{0.6eA_p \sqrt{k_B T_e}}
\]

(2.73)

Figure 2.13 presents a typical example of a smoothed raw current data measured with a double Langmuir probe and the fitting curve generated using Equation (2.71).

![Double probe I-V characteristic](image)

**Figure 2.14:** Smoothed double Langmuir trace data and the fitting curve.

Since the probe’s electrodes are floating and no plasma ground connection is provided, the plasma potential cannot be calculated directly from the probe’s recorded trace. However, by recording the voltage half way between the two electrodes, since the supply is floating, the floating potential can be acquired and if used in correlation with the electron temperature the plasma potential can be computed [57]:

\[
f(x) = a \cdot \tanh(b \cdot x + d) + c \cdot x + h,
\]

(2.71)
Although this method is a reliable way of computing the plasma potential from using double Langmuir probe data, the errors in computing the electron temperature propagate through the process.

2.2.5 Druyvesteyn electron energy distribution function theory

The thin sheath limited theory and the orbital motion limited theory as well as the double probe analysis methods are highly dependent on the assumptions of collisionless plasma with a Maxwellian energy distribution function. Furthermore, the boundary between the spheres of influence of each theory is sometimes vague and is influenced by the thickness of the plasma sheath around the probe. This quantity is most of the time difficult to assess for and in some cases none of the previous theories are fully valid. Those problems can partly be circumvented by a method that is based on the electron energy distribution functions (EEDF). As it was presented in Chapter 1, in the case of low pressure plasmas the electron energy distribution function can be non-Maxwellian [4, 20, 21, 22], since more than one distinct population of electrons can exist [4, 22]. In this case the electron temperature cannot anymore be defined as the mean energy. Druyvesteyn energy distribution function can be used in collisionless plasmas regardless the sheath thickness and also in some collisional plasmas. If the electron energy distribution function is known, the electron temperature and the plasma number density can be calculated by taking moments of the distribution, thus no assumptions about the energy distribution should be made.

The electrons that have a certain minimum amount of velocity, expressed by Equation (2.9), in the direction of the probe are collected and the total electron current is [4, 12]:

\[
I_e = eA_p \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{v_{\min}}^{\infty} dv_z v_z f_e(v),
\]  

(2.75)

where \( f_e(v) \) is the electron velocity distribution function.

If the energy is expressed in electron volts using \( \epsilon = \frac{mv^2}{2e} \) and integrating Equation (2.75) expressed in spherical coordinates, the following is obtained [2, 4]:

\[
I_e = \frac{2\pi e^3}{m^2} A_p \int_{V}^\infty \epsilon \left[ \left( 1 - \frac{V}{\epsilon} \right) f_e(\epsilon(V)) \right] d\epsilon.
\]

(2.76)

A double derivation of the previous expression gives [2, 4]:

\[
\frac{d^2 I_e}{dV^2} = \frac{2\pi e^3}{m^2} A_p f_e(V(V)).
\]

(2.77)

The relation between the electron velocity distribution function and the electron energy distribution function is given as follows [2, 4]:

\[
F(\epsilon) d\epsilon = 4\pi v^2 f_e(v) dv.
\]

(2.78)

Utilising the relation between the velocity and the energy and by retrieving the electron velocity distribution form Equation (2.77) as a function of the second derivative of the collected electron current, the electron energy distribution function is as follows [2, 4]:

\[
F(\epsilon) = \frac{4}{e^2 A_p} \frac{e^2 m d^2 I_e}{2e^2 dV^2}.
\]

(2.79)
Druyvesteijn formula for the EEDF is obtained if the energy is substituted with \( \varepsilon = -eV \) where \( V = V_p - V_b \) is the potential of the probe with respect to the plasma, since the probe bias voltage, \( V_b \), is measured with respect to facility ground in the case of the single Langmuir probe [2, 4]:

![Figure 2.15: Top: First derivative of the current collected by a single Langmuir probe. The position of the plasma potential is inferred by the position of the peak of the first derivative and of the zero of the second derivative. Bottom: second derivative.](image)
This method provides a straightforward tool for the analysis of single Langmuir probe traces. Foremost, the first derivative of the probe current is computed using a second order central difference approximation:

\[
\frac{dI(i)}{dV} = \frac{I(i-1) - I(i+1)}{2dV},
\]

where \(dV = \text{mean}(V(i-1) - V(i+1))\). The second derivative is computed departing from the smoothed data of the first derivative using the same algorithm. Figure 2.15 shows the first and the second derivatives of the probe's current (the raw data is the one presented in Figure 2.7). Since this method implies a beforehand knowledge of the plasma potential, this parameter can be found by searching for the maximum of the first derivative or the zero of the second derivative of the probe's collected current, as illustrated in Figure 2.15. Before computing Equation (2.79), the second derivative data was smoothed. Thereby, the EEDF constructed by the means of Druyvesteyn method is presented in Figure 2.16.

\[
F(\epsilon) = \frac{4}{e^2 A_p} \sqrt{\frac{V_m d^2 I_e}{2e^2 dV^2}} \quad [eV \cdot m^3].
\]  

At this point the electron temperature and the electron number density can be easily subtracted by applying moments to the EEDF. Thus, the electron number density is [2, 4]:

\[
n_e = \int_0^\infty F(\epsilon) d\epsilon \quad [/m^3].
\]

While the electron temperature is as follows:

\[
T_e = \frac{2}{3} \epsilon(\epsilon) = \frac{2}{3} \int_0^\infty \epsilon F(\epsilon) d\epsilon \quad [\text{Joule}],
\]

Or, in electron volt:
In order to evaluate the plasma potential, several methods were hitherto described. Both the thin sheath theory fitting method and the first derivative method of a single Langmuir probe are prone to errors because of the round-off in the region of the “knee” in real current-voltage curves. The precision of the first derivative method for a single Langmuir probe is in some cases decreased by the round-off or the total absence of the “knee” which leads to a less distinctive peak of the derivative. Furthermore, since the derivative has to be smoothed, errors can add up during the procedure. The emissive probes also known as “hot” Langmuir probes are simple devices that can provide reliable measurements of the plasma potential. As explained in Chapter 1, the thin filament of the probe is heated by a DC or AC current till the point when thermionic electron emission occurs, described by Equation (1.7). In the same time the voltage of the probe is swept as in the case of a regular single Langmuir probe, known also as a “cold” probe. Thus, an emissive probe without a heating current behaves as a classical single Langmuir probe. The emitting behaviour occurs only for probe potentials under the plasma potential, since the electrons can only travel up potential gradients.

Under a heating current the emissive probe emits a certain current proportional to its surface and its temperature $T_w$ via the current density defined in Equation (1.7):

$$ I_{em} = A_p J(T_w) $$

(2.84)

Depending on this emitted current two different working regimes arise. The strong emission regime appears when the emitted current overpasses the electron saturation current collected by the “cold” probe. In those cases the electrons emitted are cooler than the surrounding plasma electrons, therefore space charge effects are induced around the probe [12]. One direct output of this situation should be the fact the plasma potential sensed by the probe is lower than the true plasma potential [12]. The space charge effects can be neglected in the low emission mode when the emitted current is under the electron saturation current of the “cold” probe. Several methods are available for the interpretation of data collected with an emissive probe. The two most common methods are the floating emissive probe method and the inflection point method.

The floating emissive probe method relies on the theoretical working principles of such a device. When operating in low emission mode, the probe’s current is the sum of an emitted and collected current. The “hot” wire emitted current can be described as follows [3, 12]:

$$ I_{ew} = \begin{cases} 
- I_{em} & V_b \leq V_p \\
- I_{em} \exp \left[ -\frac{e(V_b - V_p)}{k_BT_w} \right] \psi & V_b > V_p 
\end{cases} $$

(2.85)

The emitted current, $I_{em}$ is considered to be constant for bias voltages under the potential of plasma. For more positive bias potentials compared to the plasma potential, the emitted current decreases exponentially. The term $\psi$ accounts for the orbital motion of electrons and can be approximated by using the thick sheath theory [3]:

$$ T_e = \frac{2}{3} \frac{1}{en_e} \int_0^\infty F(e) de \quad [\text{eV}]. $$

(2.83)

### 2.3 Physics of emissive probes

In order to evaluate the plasma potential, several methods were hitherto described. Both the thin sheath theory fitting method and the first derivative method of a single Langmuir probe are prone to errors because of the round-off in the region of the “knee” in real current-voltage curves. The precision of the first derivative method for a single Langmuir probe is in some cases decreased by the round-off or the total absence of the “knee” which leads to a less distinctive peak of the derivative. Furthermore, since the derivative has to be smoothed, errors can add up during the procedure. The emissive probes also known as “hot” Langmuir probes are simple devices that can provide reliable measurements of the plasma potential. As explained in Chapter 1, the thin filament of the probe is heated by a DC or AC current till the point when thermionic electron emission occurs, described by Equation (1.7). In the same time the voltage of the probe is swept as in the case of a regular single Langmuir probe, known also as a “cold” probe. Thus, an emissive probe without a heating current behaves as a classical single Langmuir probe. The emitting behaviour occurs only for probe potentials under the plasma potential, since the electrons can only travel up potential gradients.

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\end{cases} $$

(2.85)

The emitted current, $I_{em}$ is considered to be constant for bias voltages under the potential of plasma. For more positive bias potentials compared to the plasma potential, the emitted current decreases exponentially. The term $\psi$ accounts for the orbital motion of electrons and can be approximated by using the thick sheath theory [3]:

$$ T_e = \frac{2}{3} \frac{1}{en_e} \int_0^\infty F(e) de \quad [\text{eV}]. $$

(2.83)
\[
\psi = \frac{2}{\sqrt{\pi}} \sqrt{1 + \frac{e(V_b - V_p)}{k_BT_w}}.
\] (2.86)

At low filament temperatures, \(T_w\) the probe collects a current defined as follows [3, 12]:

\[
I_{cw} = \begin{cases} 
I_{es}\psi' - I_{is}\exp \left[-\frac{e(V_b - V_p)}{k_BT_i}\right] & V_b \geq V_p \\
I_{es}\exp \left[\frac{e(V_b - V_p)}{k_BT_e}\right] - I_{is} & V_b < V_p 
\end{cases}
\] (2.87)

Again, the term \(\psi'\) accounts for OML effects and is expressed as follows [3]:

\[
\psi' = \frac{2}{\sqrt{\pi}} \sqrt{1 + \frac{e(V_b - V_p)}{k_BT_e}}.
\] (2.88)

Since the heated emissive probe combines both emission and collection, the total probe current is the sum of the currents previously presented. For bias potentials under the plasma potential, the total current is as follows:

\[
I(V_b)_{V_b<V_p} = I_{es}\exp \left[\frac{e(V_b - V_p)}{k_BT_e}\right] - (I_{em} + I_{is}).
\] (2.89)

The first term of the right-hand side of Equation (2.89) predicts the increase in electron current when the bias voltage approaches the plasma potential. This term is reduced by the constant negative electron emission current, hence the prominent "knee" that the trace of such a probe exhibits. At the floating potential the total net current is null, thus:

\[
I_{es}\exp \left[\frac{e(V_f - V_p)}{k_BT_e}\right] = (I_{em} + I_{is}).
\] (2.90)

After applying a natural logarithm to both sides of the Equation (2.90), the following is obtained [12]:

\[
V_p = V_f - \frac{k_BT_e}{e} \ln \left(\frac{I_{em} + I_{is}}{I_{es}}\right).
\] (2.91)

In a low emission regime the sum of the emitted current and the ion saturation current is smaller than the electron saturation current, hence the plasma potential is larger than the probe’s floating potential. Based on this theoretical result, the floating emissive probe method implies the gradually increase in the emitted current, via an increase in the filament heating current, \(I_h\), until the emitted current approaches the electron saturation current. At this point, the probe’s floating potential approaches the plasma potential and ideally reaches it. The accuracy of this method is first of all influenced by the voltage drop along the probe’s filament. This issue can be overcome by using a symmetric circuit based on a voltage divider bridge, as it is explained in the following chapter. However, both the floating emissive probe and the inflection point techniques can provide just a lower bound approximation for the plasma potential since an intrinsic limitation force the probe to float at large emissions at 1-2\(T_e\) under the actual plasma potential [13, 54].

Figure 2.17 depicts the floating potential of an emissive probe as a function of the heating current. The probe was placed at 30 mm away from the tip of a hollow cathode operating at 340 W and 1.90 mg/s xenon mass flow rate with a background pressure of 7.2E-2 Pa. As the heating
current increases, the emitted current increases as well and the natural logarithm term in Equation (2.91) tend to cancel out. Thus, the floating potential approaches the plasma potential. For this particular situation the plasma potential of the plateau is around 16.25 V. In Figure 2.18 are depicted the emissive probe traces for different heating current. Again, the point where the floating potentials of the probe traces tend to converge is approximated as the plasma potential.

**Figure 2.17:** Heating characteristic of an emissive probe used for the floating emissive probe method.

**Figure 2.18:** Emissive probe traces for different heating currents. The position of the plasma potential is inferred by the convergence of the traces floating potentials.
Previous studies showed that the uncertainty of this method in identifying the start of the plateau is typically around 0.1\(T_e\) [60, 61].

The floating potential method explained so far implies significant electron emission which in turn perturbs the local surrounding plasma. This inconvenient can be circumvented by a more precise but in the same time more sophisticated technique. The inflection point method was extensively discussed in numerous studies and at this point a comprehensive description exists [60, 61, 62]. Two sub-methods, based on the same principles, can be outlined: the inflection point in the limit of zero emission and the inflection point of a warm, but not emissive probe. Both methods rely on the first derivative of the total current of the emissive probe.

The main idea of the inflection point in the limit of zero emission is to follow the inflection point of the current-voltage characteristic as the emission of the probe decreases to zero. As the space charge effects are reduced during the process, the inflection point reaches the plasma potential as the emission goes to zero [61, 63]. The inflection point is determined by the peak of the first derivative of the probe's current. When the probe’s bias voltage is smaller than the plasma potential, the probe’s current is expressed by Equation (2.89). On the other hand, when the potential becomes more positive than the plasma potential, the probe’s current is as follows [12]:

\[
I(V_b)|_{V_b>V_p} = I_{es} \frac{2}{\sqrt{\pi}} \left[ 1 + e(V_b - V_p) k_B T_e - I_{em} \exp \left\{ - \frac{e(V_b - V_p)}{k_B T_w} \right\} \right]^{1/2} \frac{1 + e(V_b - V_p)}{k_B T_w}. \tag{2.92}
\]

The first derivatives of Equations (2.89) and (2.92) are as follows:

\[
\left. \frac{dl}{dV_b} \right|_{V_b>V_p} = \frac{eI_{es}}{k_B T_e} \exp \left\{ - \frac{e(V_b - V_p)}{k_B T_e} \right\} \left. \frac{dl}{dV_b} \right|_{V_b<V_p} = \frac{eI_{es}}{k_B T_e} \sqrt{1 + e(V_b - V_p) k_B T_e} - \frac{2eI_{em}}{k_B T_w} \exp \left\{ - \frac{e(V_b - V_p)}{k_B T_w} \right\}. \tag{2.93}
\]

When the previous equations as plotted against the probe bias potential, the sharp peak corresponds to the plasma potential.

The inflection point in the limit of zero emission techniques implies finding the inflection points (peaks of the first derivative of the probe current) at some emission levels and then extrapolating to zero emission gives the plasma potential [28, 63]. The linear extrapolation technique was used in several previous studies leading to a measured potential within \(T_e/10\) of the plasma potential [60, 61]. Figure 2.19 depicts the first derivative of the total probe current when the probe worked at different heating currents. The data corresponds to the same conditions as for the floating emissive probe method. The first derivative was computed using a second order central difference approximation, as presented in Equation (2.80). For each derivative plot the peak corresponds to the inflection point.

By measuring the collected current with the “cold” probe, it was possible to find the emitted currents corresponding to the different heating currents by subtracting the “cold” probe data from the “hot” probe ones. At this point a plot of the emitted current against the inflection points found using the derivative method is constructed, as presented in Figure 2.20. By linear
extrapolation of the data towards zero emission, the plasma potential is found. Using this technique the plasma potential was approximated to 19.3 V.

Figure 2.19: First derivatives for the emissive probe total current for different hearing currents. The peak of the derivative tends to drift towards smaller potentials as the emitted current increases.

A second inflexion point technique implies a “warm” probe heated with a current under the point of emission. In this way the probe is kept clean during the collecting process. The peak of the first derivative of the probe’s current signalises the position of the inflection point, thus the plasma potential. In the previous example a “warm” probe at 0.5 A gives a plasma potential approximation of 19.1 V. Since less measurements are taken the uncertainties increase, previous studies showing an uncertainty in the range of 0.5\( T_e \) [60, 61].

Figure 2.20: Linear extrapolation of the inflection points for an emissive probe.

In order to appreciate the difference between the methods Table 2.1 depicts a summation of the results given by different methods for emissive probes data analysis and for the case presented throughout this section. Using the inflexion point method for the “cold” probe depicted in Figure 2.9, the electron temperature for this particular case was found to be around...
Thus, the lower and upper bounds of the plasma potential can be approximated. Moreover, the same method depicted in Figure 2.9 approximates the plasma potential at 19.9 V. The most reliable methods remain the inflection point in the limit of zero emission followed by the inflection point of a “warm” probe. Since those techniques imply numerical derivation the processing time is increased but when properly applied the results agree with theoretical predictions and give an accurate measure of the electric potential.

Table 2.1: Plasma potential obtained with different emissive probe methods for a point situated at 30 mm away from the tip of a hollow cathode operating at 340 W and 1.90 mg/s xenon mass flow rate.

<table>
<thead>
<tr>
<th>Method</th>
<th>$V_{p-}$ (V)</th>
<th>$V_p$ (V)</th>
<th>$V_{p+}$ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floating emissive probe</td>
<td>16.05</td>
<td>16.25</td>
<td>16.45</td>
</tr>
<tr>
<td>Floating emissive probe + $1.5T_e$</td>
<td>19.05</td>
<td>19.25</td>
<td>19.45</td>
</tr>
<tr>
<td>Inflection point in the limit of zero emission</td>
<td>19.10</td>
<td>19.30</td>
<td>19.50</td>
</tr>
<tr>
<td>Inflection point of a “warm” probe</td>
<td>18.10</td>
<td>19.10</td>
<td>20.10</td>
</tr>
</tbody>
</table>

2.4 Physics of Retarding Potential Analysers

Besides the information about the electron temperature, electron and ion number densities and the plasma potential, information about the ion velocity distribution function can emphasise the average voltage utilization efficiency and bring valuable knowledge on loss and accelerating mechanisms within the plume plasma of hollow cathodes and Hall thrusters. Moreover, knowing the distribution of the energetic back-flowing ions that are responsible for cathode keeper erosion is of great importance in assessing the cathode lifetime. This type of information can be delivered by a retarding potential analyser (RPA). Having a simple design, as presented in Section 1.4.3 of Chapter 1, a RPA is an useful plasma diagnostic device that can provide, after a simple data analysis, the ion energy per charge distribution function.

Once the electron repeller grid have screened out the contribution of the electrons from the input plasma current, the ion repeller, biased at a specific retarding voltage, selects just the ions with a high enough energy to overcome the electric potential. The ion current reaching the collector grid characterised by the area $A_c$ is as follows [42]:

$$I(V_b - V_p) = eA_c \int_{v_{\text{min}}}^{\infty} vF(v)dv. \tag{2.94}$$

Assuming Maxwellian distribution, the minimum velocity $v_{\text{min}} = \sqrt{2e(V_b - V_p)/M}$ and the ion energy as $\epsilon = Mv^2/2$, Equation (2.94) becomes [42]:

$$I(V_b - V_p) = \frac{eA_c}{M} \int_{v_b-v_p}^{\infty} F(v)dv. \tag{2.95}$$

Based on Equation (2.95) a simple analysis technique for the RPA data ensues. In virtue of Equation (2.95) the first derivative of the measured RPA current-voltage trace is proportional to the ion energy per charge distribution, as the energy is also $\epsilon = e(V_b - V_p)$. On this basis, for plasmas with one ion species, the probe current equals the plasma ion current and the energy
distribution function of the collected particles, \( f(V_b) \) equals the ion energy per charge distribution function, \( f(V_a) \), where \( V_a \) is the average ion acceleration potential \([6]\):

\[
\frac{dI}{dV_b} = \frac{dI}{dV_a} = -\left( \frac{A_g F}{N_A M} \right) Z^2 n_i f(V_a). \tag{2.96}
\]

where \( A_g \) is the open area fraction of the ion repeller grid, \( F \) is Faraday constant, \( N_A \) is Avogadro’s number, \( M \) is the molecular mass of the propellant and \( Z \) is the ion charge state of the ion species. Thus for singly charged ions plasmas the first derivative of the RPA I-V trace is proportional to the ion energy per charge distribution function.

\[\text{Figure 2.21: Typical RPA current-voltage characteristic (blue) and its first derivative (green). The data corresponds to a point situated at 50 mm off the centreline and 10 mm downstream from a hollow cathode operating at 520W with a discharge voltage of 26V and a mass flow rate of 1.90 mg/s of xenon.}\]

The current-voltage characteristic of a RPA and the normalised first derivative of the data are presented in Figure 2.21. The magnitude of the distribution function is difficult to resolve accurately since the repeller open area fraction is unknown \([6]\). However, the location of the distribution peak is a high fidelity measurements delivering information about the most probable ion potential \(V_{mp}\) used to compute the local average ion acceleration potential for all the ion species \([6]\).

The local average ion acceleration potential is found when the local electric potential is subtracted from the most probable ion potential \([6]\):

\[V_a = V_{mp} - V_p. \tag{2.97}\]

The only accelerating mechanism, baring the losses, is the discharge voltage \(V_d\). Since in real conditions the loss mechanism should be taken into account, the average ion acceleration potential is as follows \([6]\):

\[V_a = V_d - V_{\text{loss}}. \tag{2.98}\]

This last equation leads to the definition of the voltage utilization efficiency expressed as follows \([6]\):
\[ \eta_v = (1 - \beta) = \frac{V_{mp} - V_p}{V_d} = 1 - \frac{V_{loss}}{V_d} \]  \hspace{1cm} (2.99)

where \( \beta \) is the ion acceleration potential loss fraction.

Other information that can be extracted from the ion energy per charge distribution function is the spread in the distribution. This particular feature is characterised by the half width at half-maximum (HWHM). As the discharge voltage increases, the HWHM increases as well. In general the ions with energies per charge greater than the discharge potential are created when multiply charged ion decreases its charge during a collision. In this situation the ion charge state is decreased, whereas the ion kinetic energy remains constant [6].

2.5 Thruster propulsion principles

Electric propulsion devices are spacecraft propulsion systems with internal reaction mass which produce thrust by expelling the reaction mass with a specific exhaust velocity. While the chemical propulsion systems, i.e. chemical rockets, eject gas jets with exhaust velocities in the range of 3-4 km/s [31], a Hall Effect thruster ejects a mix of energetic charged particles at velocities up to 10-20 km/s [31]. Some basic definitions of thrust and efficiency of Hall thrusters are given hereinafter to support the experimental data for an external discharge plasma thruster presented later in the current research.

The simplest definition for the thrust generated is the product between the propellant mass flow rate and the exhaust velocity of the ionized propellant:

\[ Thrust = \dot{m} \cdot u. \]  \hspace{1cm} (2.100)

The exhaust velocity of the ionized propellant can be found using Equation (1.11) if the acceleration potential is known (for example from RPA measurements). Moreover, if the total kinetic power of the exhaust propellant is defined as:

\[ P_k = \frac{1}{2} \dot{m} u^2, \]  \hspace{1cm} (2.101)

And assuming that the level of kinetic power that is imparting to the beam ions depends on how efficiently the total input discharge power is used, the thrust efficiency may be defined as follows:

\[ \eta_T = \frac{P_k}{P_d}. \]  \hspace{1cm} (2.102)

Combining the last three equations, thrust efficiency can be computed using thrust measurements, power consumption and propellant mass flow rate measurements:

\[ \eta_T = \frac{Thrust^2}{2 \dot{m} P_d}. \]  \hspace{1cm} (2.103)

Thrust efficiency is a very powerful state parameter for an electric propulsion drive encompassing the thrust-to-power ratio defined ultimately as:

\[ \frac{Thrust}{P_d} = \frac{2 \eta_T}{u}. \]  \hspace{1cm} (2.104)

A rapid interpretation of Equation (2.104) would lead to the fact that a decrease in the input power may lead to a decrease in \( u \) and hence an increase in the thrust-to-power ratio. However, it was shown in many studies that a decrease in the input power leads to an abrupt decrease in the thrust efficiency, leading in some extreme cases to an opposite effect in the case of thrust-to-
power ratio. Hence, understanding the different mechanisms that contribute to the definition of the thrust efficiency is of great importance in order to provide solid basis for future electric drives design especially in the low discharge range (lower than 300V).

A simple breakdown of the thrust efficiency various constituents implies the definition of two main efficiencies: energy efficiency, $\eta_E$, and mass utilisation efficiency, $\eta_M$:

$$\eta_T = \eta_E \eta_M. \tag{2.105}$$

In turn, the energy efficiency can be defined as follows [25, 57]:

$$\eta_E = \eta_C \eta_V. \tag{2.106}$$

The first term in the right-hand side of Equation (2.106) is the current utilization efficiency defined as the ratio between the beam current, $I_{beam}$, and the discharge current, $I_d$, considered as the sum of the free electron current and the ionization electron current [57]:

$$\eta_C = \frac{e \bar{m}}{M I_d} (f_i Q) = \frac{I_{beam}}{I_d}. \tag{2.106}$$

The term $(f_i Q)$ defines the average charge of the exhausted gas/plasma mix, accounting for both neutral and all multiple ionization species [41]. The voltage utilization efficiency, $\eta_V$, was previously defined in Equation (1.10).

The mass utilization efficiency equals the propellant efficiency corrected by a factor that takes into account the thruster beam divergence. As any other reaction drives, during operation an electric propulsion drive produces a plume, or beam. The plume is characterized by an envelope defined by a divergence angle. Figure 2.22 presents the characteristics of a general plume for an electric propulsion thruster. Hence, a correction factor to the thrust is applied due to the divergence of the beam-in the assumption that the beam diverges uniformly [25]. The correction is defined as follows:

$$\eta_B = (\cos \theta)^2. \tag{2.107}$$

The propellant efficiency describes the fraction of the propellant neutrals that is converted into ions and thus, accelerated, can be defined as follows [25]:

$$\eta_P = \frac{\alpha^2 I_{beam} M}{e \bar{m}}. \tag{2.108}$$

Figure 2.22: Schematic of the thruster beam plume showing the half angle of the beam and the direction of the thrust.

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$$\eta_P = \frac{\alpha^2 I_{beam} M}{e \bar{m}}. \tag{2.108}$$
The beam current may contain doubly charged particles, therefore the correction factor \( \alpha \) accounts for this situation [25]:

\[
\alpha = \frac{I^+ + \frac{1}{\sqrt{2}} I^{++}}{I^+ + I^{++}},
\]

\( (2.109) \)

where \( I^+ \) is the singly charged ions current and \( I^{++} \) is the doubly charged ions current. The ratio \( I^{++}/I^+ \) has small values in Hall thrusters, around 10% being the most usual value [25]. At this point the thrust efficiency can be fully described as follows:

\[
\eta_T = \eta_c \eta_v \eta_p \eta_B.
\]

\( (2.110) \)

Various plasma diagnostic techniques may help in measuring the efficiencies presented so far instead of relying just on \( \eta_T \) given by thrust measurements. In this way the underlying physics that dictates the functioning of electric drives can be better understood. The ion beam current as well as the beam divergence can be measured using Faraday probes, while an RPA together with an emissive probe, for precise plasma potential measurements, can reveal information about the ion acceleration potential. Efficiency and thrust data correlated with plasma characteristics data, as number density and electron temperature, provide a deep insight on the operation of electric drives, giving solid foundations for the numerical analyses and an overall better grasp on the mechanisms involved in plasma formation and plume dynamics, and this is what the present research aims for.

### 2.6 Previous research

Intensive research has been conducted in order to characterise the plume dynamics of hollow cathodes and low voltage Hall thrusters. However, there is still lack of comprehension especially for the erosionless full external discharge thrusters’ plume. The current research bolsters for the first time the experimental data for plume plasma characterisation of low-voltage full external discharge plasma thruster (XPT). The extensive plasma plume probing of the XPT offers new insights in the peculiarities of plasma parameters spatial distribution patterns of the external discharge low voltage thrusters. As it is commonly accepted, a low voltage Hall thruster is considered to operate below 300 V. At low discharge voltages the efficiency of those devices drops and sometimes the thrust-to-power ration decreases. The highest level of thrust-to-power ratio of a low voltage thruster was recorded to be in the range of 85 mN/kW [41, 57]. XPT achieves an efficiency of \(~25\%\) at \(~250\) W, which is comparable to SPTs, in spite the fact that the propellant is directly injected into open space [36]. Although previous studies revealed important characteristics of different configuration of wall-less [47] or external discharge [19, 67] Hall thrusters, as thrust and efficiency measurements together with numerical analysis of the dynamics of the plumes, at present date there is no research centred on experimental plasma plume diagnostics of this innovative type of electric drives. Nevertheless, there is important research on plasma plume diagnostics for low voltage classical Hall thrusters.

Brown [6] conducted an intensive research on low voltage Hall thrusters operating on xenon in the range of 105 to 300 V, reaching a thrust-to-power ratio of 86.4 mN/kW at 120V. The efficiency of the thruster dropped from 41% at 120 V to 34% at 100 V. Langmuir probes, Faraday probes and RPA measurements were conducted in order to characterise the dynamics of the plume and efficiently identify the loss mechanisms. The electron temperature was calculated using Langmuir probe traces and it was in the range of 1.2-1.3 eV at a radial distance of 1 m from thruster exit plane, while the thruster was operating at 100 V and 20 mg/s xenon.
mass flow rate. The research provides a solid understanding on Langmuir probe and RPA design and data analysis. In the case of RPA measurements a broadening of the ion energy per charge distribution function towards higher energies was recorded when the discharge voltage increases.

Another ample study on the relation between the plume plasma parameters and the efficiency of a low voltage Hall thruster was conducted by Ross [57]. Faraday probes and RPA were used to identify the beam current and beam divergence of a Hall thruster operating at voltages between 100 and 300 V with xenon mass flow rates between 3 and 5 mg/s. Efficiencies between 15% and 55% were accounted, while the discharge current was increasing with the decrease in voltage and increase in mass flow rates. The maximum thrust-to-power ratio was found to be in the range of 60 mN/kW at 250 V and dropped to just 30 mN/kW at 100 V with a reduction in efficiency of more than 65%. The loss mechanism was analysed as a function of magnet current as well. Double Langmuir probes were used for plasma probing, while the floating potential was recorded using the technique described in Section 2.2.4. The plasma parameters were extracted from the coefficients of a custom made fitting curve for the raw probe traces. Low current perturbations were recorded during the procedure and the uncertainties in electron temperature measurements were assessed to be around 30%. This uncertainty propagated in plasma potential results leading sometimes to an anomalous rise in plasma potential as the axial distance from the thruster exit plane increased. Data provided by Ross for the plasma number density, electron temperature and electric potential were useful for comparison with the data collected for the XPT since the discharge voltages were the same. However, Ross thruster operated at much higher xenon mass flow rates, thereby just a qualitative comparison was possible. Plasma density varied between $7 \cdot 10^{18} / \text{m}^3$ and $2 \cdot 10^{17} / \text{m}^3$ between 5 mm and 20 mm away from thruster exit plane and on thruster centreline. Between the same locations the temperature varied between 15 and 3 eV and the plasma potential between 90 and 30 V, for discharge voltages of 150 V, 200 V and 300 V.

Lobbia et al. [45, 46] used the time resolved electron energy distribution functions to characterise the plume of a Hall thruster operating at 200 V and 24 mg/s of xenon mass flow, leading to plume plasma number densities between $4 \cdot 10^{16} / \text{m}^3$ and $2 \cdot 10^{17} / \text{m}^3$.

Herman and Gallimore [27] performed Langmuir probe experiments of the near plume of the NASA’s Evolutionary Xenon Thruster. The electron energy distribution functions showed single-peak structures near the orifice and then gave way to double-peak EEDFs downstream in the double layer. Those secondary peaks in the EEDF were considered to be formed by the accelerated particles travelling across the thruster double layer in radial direction.

Linnell and Gallimore [44] used high speed Langmuir probes to map the internal and external plasma of a Hall thruster operating with xenon and krypton. As 300V and when operating with a 10 mg/s xenon mass flow rate the plasma density varied between $2 \cdot 10^{18} / \text{m}^3$ at 5 mm downstream the channel exit plane and $5 \cdot 10^{17} / \text{m}^3$ at around 20 mm axial distance, while the temperature dropped from 15 eV to 5 eV. Similar results were outlined by Nakles et al. [51] for the BHT-200 Hall thruster.

Azziz [3] work envisaged the probing of the plume plasma of a Hall thruster operating at discharge voltages between 225 V and 300 V using Faraday probes and emissive probes. The thermal model of the emissive probe is a good tool for designing a suitable emissive probe for different plume approximations. The emissive probe was used in “hot” mode to identity the plasma potential via the floating probe method and in “cold” mode for plasma density and
electron temperature measurements. When operating at 250V with a 0.85 mg/s xenon mass flow rate, the thruster discharge current reached 0.8A. In this operational configuration the plasma potential measured at a radius of 25 cm downstream the thruster exit plane varied between 5 V and 11.5 V. At the same distance the number density was between $5 \cdot 10^{15}$ /m^3 and $2 \cdot 10^{17}$ /m^3, while the electron temperature reached a peak of 2.8 eV.

Hollow cathode plume diagnostics is of great importance due to the problem of rapid erosion of the orifice and any other component in contact with the plume. This process reduces the lifetime of the cathode and since this device is a key component in the functioning of Hall and ion thrusters, the lifetime of the entire assembly is influenced. Moreover, hollow cathodes can become good stand-alone ion thrusters if the emitted ion current can be somehow increased to a level comparable to existing ion drives [13]. Thus, a deep understanding of the hollow cathode behaviour can improve the development of such devices.

High energy ion bombardment is thought to be as one of the main mechanisms that lead to the rapid erosion of the cathode keeper and orifice [4]. Goebble et al. [23, 24] and Goebble and Katz [25] predicted that the plasma potential undergoes a steep increase in the near-orifice region and this could be a source of high energy ions that appear to be focused and travelling mostly in radial direction [23, 24, 25].

Behlam [4] conducted Langmuir probe measurements for a low current hollow cathode and presented a comparison between several data analysis methods. The most robust turned to be the Druyvesteyn EEDF, based on the fact that according to Godyak [20, 21] in low pressure plasmas the electron distribution seems to be non-Maxwellian. This statement is supported by Mikellides and Katz [49] arguing that the low collision level in the plume plasma forces the EEDF to deviate from being Maxwellian. Hence, Druyvesteyn method has a validity independent on sheath size and its practical derivation was defined by Lieberman and Lichtenberg [43].

On low current hollow cathodes ensues Asselin [2] work as well. The near plume of such a device was probed using Langmuir and emissive probes. When the cathode was operated on xenon the electron temperatures computed using the thin sheath approximation slope method and Druyvesteyn EEDF were found to be in the same range. On the other hand, the electron number density extracted from the EEDF was one order of magnitude smaller than the one computed using OML theory. The author presents a brief empirical definition of uncertainties brought by the Langmuir probe data analysis. The accuracy of the OML and thin sheath approximation theories is about 20% for electron temperature and 50% for the plasma number density, based on the results formulated by Linnell and Gallimore [44]. In the case of the EEDF computed using Druyvesteyn method, the numerical differentiation can amplify any noise especially in the high energy tail. Averaging the EEDF obtained from several Langmuir traces for the same point leads to better results, as it was previously showed in Herman and Gallimore [27] work. Moreover, the EEDF contains also negative areas carrying no physical meaning. It was assumed that those areas build up 10% out of the total area of the distribution [2]. Last, since the method is based on the accurate knowledge of the plasma potential, uncertainties in computing this quantity propagate in the EEDF computation.

While the previously mentioned studies employed external anode plates during the hollow cathode experiments, Crofton and Boyd [13] proposed a study with a standalone cathode working just with the discharge between the cathode tube and the keeper. Previous studies suggested that the emitted ion energy distributions have dramatically differences in the two configurations [13]. In their study Crofton and Boyd operated the cathode with keeper
discharges between 4 V and 12 V and mass flow rates of 0.125 mg/s and 0.5 mg/s. A RPA was used to provide the ion energy per charge distribution functions (IDF). The experimental results revealed a good agreement with the Maxwellian distributions but the numerical codes failed to reproduce the broad high energy tails of the recorded distributions in radial directions. The study also concluded that the peaks of the IDF were recorded at relative low energies and the highest IDF was found when the probe was directly viewing the cathode orifice (centreline). The distribution has a broad characteristic towards lower energy, implying the high propensity of the cathode to produce low energy ions.

A comprehensive presentation of plasma diagnostics is provided by Rai [55], while the Langmuir probe, emissive probe and RPA physics is well described by Leal-Quirós [42]. The physics of particle collection by electrostatic probes as well as the problematic of Debye shielding are extensively treated by Conde [12] and Furno et al. [18].

Merlino [48] gives an easy to follow description of the physics and working principles of Langmuir probes, while Williams [69] provides a straight-forward simple description of data analysis for thin sheath assumption. More in depth comprehensive display of all existing data processing methods for single and double Langmuir probes is given by Korolov [37]. An important reference for double Langmuir probe data analysis using thin sheath approximation was found in Dote [14]. The algorithm allows for a rapid computation of the electron temperature followed by ion number density but may induce high uncertainties. Useful were also the Langmuir probe techniques for Hall thrusters described by Reid and Gallimore [56] and Kai [32].

In the case of emissive probe experimental data analysis, Sheehan and Hershkowitz [60, 61] exhaustively describe the different manners for obtaining the plasma potential from emissive probe traces. A comparison between the different methods, accounting for uncertainties usually related to the electron temperature, reveals that the most reliable methods are the inflection point in the limit of zero emission and the warm probe method.

2.7 Summary on fundamental principles of plasma diagnostics and thruster propulsion

This Chapter presented an exhaustive description of the plasma physics underlying the mechanisms of charge particle collection by electrostatic probes, departing from the kinetic theory for plasma, through energy distribution functions with an emphasis on the Druyvesteyn electron energy distribution function and reaching the aspects that define the various data analysis techniques. Subsequently, data analysis techniques for single and Double Langmuir probes are highlighted as the thin sheath and orbital motion theories together with Druyvesteyn electron energy distribution function methods. Emissive probe data processing methods are depicted as well as a straight-forward method to extract information from RPA curves. In the end, the main principles of thruster propulsion, as definition of efficiency and thrust-to-power ratio are outlined together with a review on the previous research on hollow cathodes and Hall thrusters.
CHAPTER 3: Experimental setup for plasma plume diagnostics

Hereinafter the various experimental setups used for plasma plume diagnostics for the hollow cathodes and the external discharge plasma thruster are presented. The experiments took place in two vacuum facilities of ISAS/JAXA and, in order to increase the manoeuvrability of the probes and the accessibility to different locations in the probed plasma, a three-axis positioning system was utilised. A detailed description of the different probes design and manufacturing is presented together with a test matrix for each conducted experiment.

3.1 Vacuum facilities

The commercial cathode and the laboratory model cathode were tested in the Cathode Vacuum Chamber of ISAS/JAXA, presented in Figure 3.1-left. This chamber has a diameter of 0.6 m and a length of 1 m, being capable to maintain a base pressure of $3 \cdot 10^{-4}$ Pa. This level of vacuum is established by two EDWARDS 0.4 kW (STP-Ixa4506C) turbomolecular pumps helped by two ULVAC 3.7 kW (VD901) oil sealed rotary mechanical pumps. At a maximum speed of 24200 rpm the two turbomolecular pumps are capable of evacuating 4300 L/s of xenon [38]. Two ICF70 vacuum flanges were used as feedthroughs for the probe and the positioning system. Pressure was monitored via a Vista CC-10 crystal/cold cathode combination wide range gauge corrected for xenon and located on the upper part of the chamber, close to its half length.

The second vacuum chamber used for the XPT experiments was the Space Science Chamber of ISAS/JAXA, depicted in Figure 3.1-right. The chamber has a diameter of 2.5 m and a total length of 5 m. The lowest level of vacuum was recorded when the XPT was run with 1.47 mg/s anode mass flow rate and 0.29 mg/s cathode mass flow rate and it was around $9.14 \cdot 10^{-3}$ Pa. Usual operation during XPT experiments took place at vacuum levels around $5 \cdot 10^{-3}$ Pa. The vacuum level of the chamber was monitored with an Anelva M-336MX crystal ion gauge corrected for xenon.

3.2 Auxiliary systems and equipment

The gas supply was xenon in all the experiments regulated at approximately 0.125 MPa from the xenon main tank at 2.5 MPa. The gas passed then though a Moriba STEC-E440 mass flow rate control unit calibrated for argon and used for xenon with a correction factor of 0.971 (1.3/1.4).

Several power supplies were used to provide the bias voltage of the probes. For the Langmuir probes, emissive probes and RPA ion repeller grid a Matsusada high speed bi-polar power
supply was employed capable to generate a ±120 V voltage sweep. The sweep was controlled by an Iwatsu FG-350 function generator producing a 100 Hz sinusoidal wave at 100 ms sweeping time and with a duty cycle of 30%. No attenuation was applied to the output of the function generator. For the emissive probes an extended range Ex-1125U2 DC power supply capable to output up to 6 A, and 500 V was providing the heating current for the filament. Two more DC regulated power supplies, a Kikusui PMC250-05A and TP070-1 were employed in order to apply the constant voltage for the electron repeller grid and the collector of the RPA. The laboratory model hollow cathode was supplied by three main power supplies capable to deliver high currents (cathode-anode and heater power supplies) and high voltages (keeper power supply).

During the experiments probe's I-V traces were recorded by a Yokogawa DL750 ScopeCorder at regular sample rate of 5 kS/s on a trigger mode. The trigger signal was provided by the function generator. The data was displayed in X-Y mode in order to visualise in real time the shape of the recorded trace. This allowed for a rapid interpretation of the quality of the trace, leading to the decision of repeating the measurement at that specific location or not. The raw data was exported as .CVS files and subsequently analysed using semi-automated programs based on the analysis theories presented in Chapter 2.

The temperature of the different regions of the hollow cathodes and XPT as well as the temperature of the stepper motor of the positioning system assembly closest to the plasma were monitored using thermocouples connected to a Graphtec midi Logger GL840 capable to provide a sampling rate up to 1 kS/s. The floating voltage of the double Langmuir probe in the XPT experiments was measured using an Iwatsu SS-0130R 200MHz voltage probe connected between the bipolar power supply negative output and the common ground.

### 3.3 Hollow cathodes and anodes

Two different hollow cathodes were tested in order to find the far-field plume plasma characteristics as the number density, electron temperature, plasma potential and the electron energy distribution functions or the ion energy distribution functions. Hereinafter some specifications of the tested hollow cathodes are presented together with the test matrices.

#### 3.3.1 Commercial hollow cathode

The first cathode tested was a high current commercial hollow cathode with a thermionic emitter made from a material with a low work function. Although the exact material remained unknown, most probably tungsten, tantalum impregnated with barium oxide or lanthanum hexaboride were used for the emitter. Upon heating those materials can easily emit electrons and by applying an electric field the electrons can be pulled out the cathode and establish a discharge with an external anode.

The plasma plume of the cathode was probed using a double Langmuir probe. By grounding one of the probe's electrodes to the common plasma ground the system behaved as a single Langmuir probe. Thus, both single and double Langmuir probe traces were recorded. Table 3.1 outlines the operating conditions during which the probe measurements were conducted. The cathode tests were run in the small ISAS/JAXA cathode vacuum chamber.

After heating the emitter the cathode was ignited and the primary discharge between the cathode and the keeper was established. An external annular graphite anode \( (\Phi_{\text{in}} = 50 \, \text{mm}, \Phi_{\text{out}} = 200 \, \text{mm}) \) was placed at 103 mm downstream the cathode tip in order to establish the secondary discharge. During high current operation the keeper was turned off.
Several locations along the cathode centreline were probed, starting with 13 mm downstream the cathode tip. The axial position of the probed locations is presented in Figure 3.2-bottom. Since the positioning system was placed behind the anode, a long metallic arm was used to attach the probe to the system. The experimental setup is depicted in Figure 3.2-top. Since the discharge powers were high, especially in the last two operating modes, the residence time of the probe in the plume was limited to 0.5 s. Hence, the positioning system allowed an axial motion in y-direction as well as a radial motion in x-direction. Probe’s collected current was calculated via the voltage drop across a 149.15 Ω cement resistor.

<table>
<thead>
<tr>
<th></th>
<th>Mode 1</th>
<th>Mode 2</th>
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</thead>
<tbody>
<tr>
<td>$I_d$ (A)</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td>$V_d$ (V)</td>
<td>17</td>
<td>16</td>
</tr>
<tr>
<td>$P_d$ (W)</td>
<td>340</td>
<td>640</td>
</tr>
<tr>
<td>$\dot{m}$ (mg/s)</td>
<td>1.43</td>
<td>1.43</td>
</tr>
<tr>
<td>$P_c$ (Pa)</td>
<td>$3.5 \cdot 10^{-2}$</td>
<td>$3.9 \cdot 10^{-2}$</td>
</tr>
</tbody>
</table>

The probe’s bias voltage was swept between ±40 V with a triangular 1 kHz function. The voltage was limited to this range in order to avoid the self-emission regimes that would complicate the data analysis as well as the discharges between the probe’s electrodes especially in high density plasma regions. The data logger sample rate was set at 50 kS/s. In this specific
configuration the voltage step was 0.1 V. This very first set of experiments was intended to be a try-out for the Langmuir probe diagnostic system, providing experience for the experiments that followed. Still, the experimental session outputted pertinent and valuable results that may be used in adjusting the numerical codes on which the design of the cathode is based.

3.3.2 High current laboratory model hollow cathode

The high current laboratory model hollow cathode was developed and built at ISAS/JAXA and then tested with xenon for discharge currents of 10-50 A in order to establish the voltage-current characteristics and to identify the mode transition occurrence point. For the first time the cathode had its plasma plume diagnosed using Langmuir probes, emissive probes and retarding potential analyser in order to confirm the results obtained with the Hybrid-PIC model simulations [38].

The cathode has a classical design. A 25 mm long LaB$_6$ emitter is kept in place inside the Φ14 mm graphite cathode tube by a graphite sleeve and an Inconel spring. A graphite heater is wrapped around the cathode tube and then a heat shield formed of a multi-layer tantalum foil covers the heater. The cathode tube has a Φ3 mm orifice through which the electrons are extracted by the Φ40 mm graphite keeper electrode. The keeper electrode has a Φ6.3 mm orifice. Figure 3.3 shows the cathode placed inside the vacuum chamber before the probe experiments.

In order to start up the cathode the emitter should be heated up to the point of thermionic emission. This happens when the heater power is around 300 W. As the keeper voltage is set to 250 V, once the temperature of the cathode tube reaches around 850°C the primary discharge occurs between the keeper electrode and the internal plasma. This temperature seems high for a conventional sheathed hollow cathode because of insufficient thermal optimization [38]. The temperature of the cathode tube was recorded using a K-type thermocouple inserted thorough the orifice made on the keeper electrode at 47 mm from the end of the cathode tube, as it can be seen in Figure 3.3. At this point the secondary discharge can be established between the cathode and the external anode and then the keeper and the heater can be turned off, the cathode being in self-sustained operation.

![Figure 3.3: Laboratory model hollow cathode inside the vacuum chamber with the K-type thermocouple attached. The plume plasma diagnostic system can be seen on the left side of the picture, while in the background there is the external annular graphite anode and the positioning system.](image)
The voltage-current characteristic of the cathode, depicted in Figure 3.4, was established for two different mass flow rates in the ISAS/JAXA small vacuum chamber. The same external annular graphite anode used for the first commercial cathode experiments was utilised for this cathode and placed at a distance of 37 mm downstream. During the experiments the keeper electrode was let floating. The discharge voltage tends to decrease when the discharge current is increased for a constant mass flow rate and becomes lower when the mass flow rate is increased. For a 1.90 mg/s mass flow rate the voltage remains over 20 V up to 50 A. On the other hand, for 2.86 mg/s mass flow rate there is an abrupt change in the voltage between 30 and 40 A, this point corresponding to the mode transition between the plume and spot modes.

![Figure 3.5: Top: Schematic of the experimental setup for the high current laboratory model hollow cathode probe experiments (not to scale). Bottom: Position of the probed points, in mm with different probes (Langmuir, cold and hot emissive, RPA) (not to scale).](image-url)
Plume diagnostic measurements were conducted for the cathode in the small ISAS/JAXA cathode vacuum chamber using the graphite annular anode placed at 60 mm downstream the cathode tip, as depicted in Figure 3.5. Several diagnostic types were used in order to characterise the plume and to offer a first glance on the dynamics of the plasma generated by this laboratory model. Langmuir, emissive probes and RPA were used in the experiments.

Emissive probe measurements were conducted for several positions in the plume, as depicted in Figure 3.5-bottom. The probe was used in both “cold” and “hot” modes. When in “cold” mode the probe behaved a classical single Langmuir probe, offering information about the electron temperature, plasma number density and the plasma potential. When in “hot” mode the plasma potential was found in order to provide results with higher accuracy to be used together with the RPA data. During the experiments with the emissive probe the voltage was swept from -20 V to +70 V within 50 ms with a sinusoidal 10 Hz function allowing for a step in voltage of 0.1 V. The collected current was measured across a 100 Ω cement resistor at the end of a voltage divider bridge composed of two 220 Ω cement resistors. Data was recorded by the data logger with a sample rate of 5 kS/s. In the case of the “hot” emissive probe the current was provided by a DC power supply from 0.5 A to 1.75A. Table 3.2 outlines the operating conditions of the cathode during the emissive probe measurements.

| Table 3.2: Operating conditions of the laboratory model hollow cathode for which “cold” and “hot” emissive probe measurements were conducted. |
|---|---|---|---|---|
| Plume 1 | Plume 2 | Spot 1 | Spot 2 |
| $I_d$ (A) | 19.8 | 19.8 | 20.5 | 39.6 |
| $V_d$ (V) | 27.2 | 26 | 19 | 17.6 |
| $P_d$ (W) | 538.36 | 514.8 | 389.5 | 696.96 |
| $m$ (mg/s) | 1.90 | 2.86 | 1.90 | 2.86 |
| $P_c$ (Pa) | $2.5 \cdot 10^{-2}$ | $6.5 \cdot 10^{-2}$ | $3.9 \cdot 10^{-2}$ | $6.7 \cdot 10^{-2}$ |

The results for the electron temperature and the plasma number density obtained using the “cold” emissive probe were double-checked by a new series of data taken using a double Langmuir probe. The same positions were probed as in the previous case. The probe bias voltage was swept between ±70 V with a step of 0.1V by a 10 Hz sinusoidal function in 50 ms. The collected current was calculated via the voltage drop across a 146 Ω cement resistor and then recorded by the data logger with a sample rate of 5 kS/s. The operational conditions of the cathode during the double Langmuir probe measurements are presented in Table 3.3.

| Table 3.3: Operating conditions of the laboratory model hollow cathode for which double Langmuir probe measurements were conducted. |
|---|---|---|---|---|---|
| Plume 2 | Plume 3 | Spot 1 | Spot 2 | Spot 3 |
| $I_d$ (A) | 20 | 30.2 | 20 | 40.3 | 30 |
| $V_d$ (V) | 24 | 23.4 | 19 | 15.3 | 15.4 |
| $P_d$ (W) | 480 | 706.68 | 380 | 604.5 | 462 |
| $m$ (mg/s) | 2.86 | 2.70 | 1.90 | 2.86 | 2.86 |
| $P_c$ (Pa) | $1.3 \cdot 10^{-1}$ | $1.5 \cdot 10^{-1}$ | $4.6 \cdot 10^{-2}$ | $5 \cdot 10^{-1}$ | $2.6 \cdot 10^{-1}$ |

The results obtained using those plasma diagnostic methods allowed for a consistency analysis with the numerical results obtained using a Hybrid-PIC model described in a previous study [38]. Another goal was to probe a position into the plume plasma domain using a retarding potential analyser. The probe was deployed off-centreline at 50 mm and 10 mm away from the
cathode tip as depicted in Figure 3.5-bottom. In this way the ion energy per charge distribution functions in radial direction were computed for the two operating modes highlighted in Table 3.4. For this experiments the first grid of the RPA was let floating, the electron repeller grid and the collector grid were biased to -50V, while the ion repeller grid voltage was swept manually from 0 to 120V.

Table 3.4: Operating conditions of the laboratory model hollow cathode for which RPA measurements were conduced.

<table>
<thead>
<tr>
<th></th>
<th>Plume 1</th>
<th>Spot 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_d$ (A)</td>
<td>20</td>
<td>39.6</td>
</tr>
<tr>
<td>$V_d$ (V)</td>
<td>26.84</td>
<td>17.45</td>
</tr>
<tr>
<td>$P_d$ (W)</td>
<td>536.8</td>
<td>691</td>
</tr>
<tr>
<td>$\dot{m}$ (mg/s)</td>
<td>1.90</td>
<td>2.86</td>
</tr>
<tr>
<td>$P_e$ (Pa)</td>
<td>$5 \cdot 10^{-2}$</td>
<td>$6 \cdot 10^{-1}$</td>
</tr>
</tbody>
</table>

The voltage step was 2 V up to 15V and 5 V up to 120V. The collected current was measured across a 93 kΩ metal film resistor. Section 3.5.4 presents a detailed analysis on the design particularities of the RPA.

All the probes were mounted on the positioning system by the means of a long metallic arm. Since the system allowed for a high speed motion in x-y plane, the residence time for the emissive and Langmuir probes in the plume was limited to 0.5 s. The design particularities of the probing instruments are presented in Section 3.5.

3.4 External Discharge Plasma Thruster (XPT)

The external discharge plasma thruster (XPT) is a prototype of an erosion free low power Hall thruster capable of producing and sustaining a fully external plasma discharge [33, 35, 36]. The thruster has a simple annular Hall thruster geometry based on three main parts, as depicted in Figure 3.6: a metallic annular anode plate that serves as propellant distributor, SmCo permanent cylindrical hollow magnets accommodated by a metallic magnetic bed and the external cathode. The propellant neutrals are injected through the four slots performed in the anode plate, as seen in Figure 3.8. Since this way of feeding the propellant leads to an azimuthally non-uniform flow rate, the electron conductivity is affected [36]. The most important geometrical parameters of XPT are the thickness of the dielectric wall, the magnetic system geometry, the anode plate thickness and the number and width of the propellant injection slots [35, 36].

In order to confine the electrons and produce the external discharge directly in the outer space, the XPT is characterised by a plasma-lens magnetic field with a maximum strength of about 0.13 T, when the dielectric wall thickness of 1 mm, at the anode centreline and at 0 mm from anode plate [36]. The magnetic field strength drops rapidly and as for 5 mm away from the anode plate and on the thruster centreline 0.055 T was recorded [36]. At 20 mm away from the anode the strength reaches 0.007 T.

The relatively high magnetic field throughout the plasma leads to a highly magnetised plasma, especially in the near plume. As the magnetic field strength inside the plume increases, the electron and ion Larmor radii, defined by Equation (1.4) and (1.6), decrease. In the case of single Langmuir probes the thin sheath approximation data analysis is based on the total electron saturation current collected by the probe. Previous studies [30, 69] showed that as the
electron Larmor radius deceases approaching the probe’s electrode radius size, a reduction in the collected electron saturation current is noticed. This leads to an underestimation of the electron number density. However, the electron temperature remains independent of the magnetic field strength [69]. Thus, for the XPT double Langmuir probes were used in order to avoid these issues. Based on ion collection, the probe can accurately provide the local ion number density, since the ion Larmor radius remains larger than the electrode’s size even for high magnetic field strengths.

Previous studies [35, 36] presented some performance results of the XPT operated on xenon with the external HCN-252 cathode. Thrust and efficiency measurements as well as voltage-current characteristics measurements were conducted for both 1 mm and 2 mm wall thickness, as suggested in Figure 3.7. The voltage-current characteristics (Figure 3.7-(a)) showed a general behaviour as in the case of conventional Hall thrusters. As the discharge voltage was increased, the discharge current increased as well due to the increase in the mean electron ionization energy. Once the ionization fraction reaches its maximum, a further increase in the discharge voltage outputs a constant current or slightly increasing discharge current, the changes in electron mobility being responsible for this last feature [36]. By comparing the two wall configurations it is possible to state that as the wall thickness increases the discharge current increases as well. This is due to a reduction in the magnetic field strength at the edge of the anode plate which implies increasing electron conductivity [36].

Thrust measurements are presented in Figure 3.7-(b). The general behaviour is of a linearly increasing thrust when the discharge voltage is increased. Thrust level spans between 0.5 mN and 17.4 mN, proving the versatility of the thruster [36]. For this range of thrust the specific impulse of XPT-2 mm was between 108 s and 1240 s, comparable to the conventional Hall thrusters operated in the same power range [36].
Anode efficiency data are showed in Figure 3.7 (c). XPT-2 mm can offer up to 25% efficiency when operated at 250 V and 1.43 mg/s of anode mass flow rate. It is believed that for higher discharge voltages the efficiency can be improved since the mass utilization efficiency can be increased [36].

Figure 3.7: Left: XPT performance results for 1 mm wall thickness. Right: XPT performance results for 2 mm wall thickness; (a) voltage-current characteristics, (b) thrust, (c) anode efficiency [35, 36].
Discharge current oscillations were measured for the XPT-1 mm. The results [36] showed that for small mass flow rates the oscillations remain low but have a sharp increase when the discharge voltage is increased from 200 V to 250 V [36]. As the mass flow rate is increased, the peak-to-peak value of the oscillations increases but show a decrease trend with the increase in discharge voltage. The high frequency oscillations in the discharge current had low amplitudes, while in the range of low frequencies the highest amplitude of 0.5 A (30% of the nominal discharge current) was recorded for 150 V when the anode mass flow rate was set to 1.43 mg/s [36]. This is believed to be caused by an instable breathing mode due to the overlapping of the acceleration and ionization regions in the near field plume of the XPT where the peak in neutral number density is expected as well.

Double Langmuir probe measurements were conducted for the XPT plasma plume. The closest axial position to the anode plate was 5 mm. The XPT had a 1 mm thick dielectric wall. During the experiments the thruster was run together with the HCN-252 hollow cathode. The cathode keeper current was always set to 0.3 A, and the heater current to 5 A. A constant xenon mass flow rate of 0.29 mg/s was run through the cathode. Both the cathode and the thruster were kept floating.
Several points along the thruster centreline were probed starting with the axial distance of 13 mm away from the anode plate and till 63 mm downstream. The position of the points is depicted in Figure 3.9, while the operating conditions of the thruster during the probing are outlined in Table 3.5.

Table 3.5: Operating conditions of the XPT for which double Langmuir probe measurements were conducted for the points along the centreline depicted in Figure 3.9. The heater voltage is $I_h$, the anode mass flow rate is $m_a$, and the cathode mass flow rate is $m_c$.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$V_d$ (V)</th>
<th>$I_d$ (A)</th>
<th>$I_k$ (A)</th>
<th>$I_h$ (A)</th>
<th>$P_d$ (W)</th>
<th>$P_c$ (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>150</td>
<td>0.25</td>
<td>0.3</td>
<td>5</td>
<td>30</td>
<td>3.94 $\times 10^{-3}$</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>0.3</td>
<td>0.33</td>
<td></td>
<td>60</td>
<td>4.03 $\times 10^{-3}$</td>
</tr>
<tr>
<td>3</td>
<td>250</td>
<td>8.25</td>
<td></td>
<td></td>
<td>82.5</td>
<td>6.36 $\times 10^{-3}$</td>
</tr>
</tbody>
</table>

The Langmuir probe experiments aimed also to provide mapping of the XPT plume plasma. For that 25 position within the thruster x-z plane were probed, as suggested in Figure 3.10. The goal was to provide a map for the plasma number density, electron temperature and electric potential encompassing the anode surface at four axial positions away from the anode plate, 5, 8, 13 and 18 mm. For the central point at (0, 0) the data was subtracted by averaging four individual traces. Data was collected at 250 V discharge voltage and for three different mass flow rates, as presented in Table 3.6.

Table 3.6: Operating conditions of the XPT for which double Langmuir probe measurements were conducted at the locations depicted in Figure 3.10.

<table>
<thead>
<tr>
<th>Mode 3</th>
<th>$V_d$ (V)</th>
<th>$I_k$ (A)</th>
<th>$I_h$ (A)</th>
<th>$P_d$ (W)</th>
<th>$m_a$ (mg/s)</th>
<th>$m_c$ (mg/s)</th>
<th>$P_c$ (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>250</td>
<td>0.3</td>
<td>5</td>
<td>82.5</td>
<td>0.48</td>
<td>0.29</td>
<td>3.94 $\times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>212.5</td>
<td>0.85</td>
<td>0.29</td>
<td>6.46 $\times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>362.5</td>
<td>1.45</td>
<td>0.29</td>
<td>8.5 $\times 10^{-3}$</td>
</tr>
</tbody>
</table>

The probe was attached to the positioning system via a metallic interface. The residence time of the probe’s tip inside the plasma was limited to 0.5 s, the positioning system providing fast movements in three directions covering the x-y and x-z planes of the thruster. The probe bias voltage was swept between $\pm 60$ V with a step of 0.1 V by a 10 Hz sinusoidal function in 50 ms. The collected current was calculated via the voltage drop across a 15 $\Omega$ cement resistor and
recorded by the data logger at a sample rate of 5 kS/s. The voltage of the bi-polar power supply feeding the probe was measured half way between the electrodes using a 200 MHz voltage probe in order to provide the probe's floating potential.

![Figure 3.10: Position of the probed points on the XPT x-z plane, in mm (not to scale). The measurements were conducted for four axial positions along the centreline at 5, 8, 13 and 18 mm away from the anode plate.](image)

3.5 Plume diagnostic systems

Beside the probes, Langmuir, emissive or RPA, the plume diagnostic system was composed of a high speed positioning system and several interfaces in order to attach the probes to this system. Hereinafter a brief description of each different part is presented.

3.5.1 Three axis stage and probes interfaces

In order to provide small residence time for the probes within the plume just one motion axis was not enough. Moreover, for the XPT plume mapping movements in three directions were needed. Thus, an Original Mind 3-axis high speed positioning system was employed, as depicted in Figure 3.11.

The system is composed of three brush stepper motors, allowing for linear translations across a system of shafts in a pulley-like way. The maximum strokes possible for each axis as well as the deviation when a full stroke motion is done are presented in Table 3.7. The positioning accuracy of the device was given by the manufacturer to be ±0.1 mm. Since during all the experiments the strokes were limited to half of the maximum stroke, the total uncertainty in position is approximated to be of ±0.5 mm.
At motors feeding rate of 300% the positioning system was delivering a speed of 16 mm/s with an acceleration of 19.6 mm/s². The system was controlled via an Original mind CNC controller using the Artsoft Mach 3 Professional 2.0 software with custom made G-codes, containing the potions to be probed and allowed times for each instruction. The temperature of the motor closest to the plume was monitored with a thermocouple and kept under 100°C. Regular temperature range during the experiments was 60-70°C. The highest temperatures were recorded during operation in the small ISAS/JAXA cathode vacuum chamber. In order to avoid demagnetization of the motors’ magnetic material and thus loosing mobility the experiments were stop whenever the temperature was passing over 100°C.

Table 3.7: Positioning system data.

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum stroke</td>
<td>150</td>
<td>150</td>
<td>80</td>
</tr>
<tr>
<td>(mm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deviation at full stroke (mm)</td>
<td>0.6</td>
<td>0.2</td>
<td>0.9</td>
</tr>
<tr>
<td>(mm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Accuracy (mm)</td>
<td>±0.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The probes were mounted to the positioning system using aluminium interfaces, as presented in Figure 3.12. For the experiments conducted in the small vacuum chamber, the geometry of the chamber allowed only for placing the positioning system behind the anode. Therefore, a U-shape 32 cm long aluminium arm was built. The arm was reinforced with a second metallic piece in order to confer stability and damp the oscillatory behaviour of the long structure. However, some oscillations still persisted and in order to overcome their effect on the measurements precision some extra times were included during the movement sequence. At its one end the arm was attached to the positioning system, while at the other end the probes were mounted. For the Langmuir and emissive probes a small rectangular 20 mm x 50 mm aluminium interface was built with a central Φ0.3 mm groove to accommodate the probe (Figure 3.12-a). The RPA interface was a simple L-shape aluminium piece with a Φ22 mm hole and four additional screw slots in order to fix the front cap of the probe (Figure 3.12-b). In the case of the XPT measurements longer Langmuir probes were used. This time the probe was attached.
directly to the positioning system using a rectangular 80 mm x 50 mm aluminium interface with a central Φ0.3 mm groove to accommodate the probe (Figure 3.12-c).

![Figure 3.12: Different metallic interfaces for probes’ mounting on the positioning system: (a)-small rectangular interface for Langmuir and emissive probes attached at the end of the metallic arm; (b)-RPA interface attached at the end of the metallic arm; (c)-large rectangular interface for long Langmuir probes attachment to the positioning system.](image)

### 3.5.2 Langmuir probes design and construction

Several cylindrical double Langmuir probes were designed and built according to the limitations imposed by the experimental setup, i.e. size of the vacuum chamber and location of the positioning system, and by the characteristics of the plume plasmas that were about to be tested.

Based on data from numerical analysis, the Debye length (Equation (2.22)) and the electron-neutral and electron-ion collision mean free paths were computed. Then, by applying Equations (2.23) and (2.24) for specific electrode radii, the variations of the parameter ξ and the Knudsen number were computed. For the hollow cathodes and the XPT the electron number density was expected to be in the range of $10^{17} - 5 \cdot 10^{18}/m^3$, while the electron temperature should not overpass 10 eV in the case of the cathodes and may reach up to 15 eV for the XPT. The neutral number density was assumed to be around $1.6 \cdot 10^{19}/m^3$ in the case of XPT. Figure 3.13 presents the variation of parameter ξ when the radii of the probe were 0.125 mm and 0.2mm. The limits for the thin sheath and orbital motion limited theories are marked by the dotted black lines. Since the plasma characteristics range is large, probes having those dimensions can work in both regimes and also there are regions where none of the theories can be applied in order to correctly analyse the data. In all the case the mean free paths were much larger compared to the probe’s radii, hence the assumption of a collisionless plasma remain valid through the entire domain considered.
Figure 3.13: Parameter $\xi$ computed for different electron number density and electron temperature for two probes with $r_p 0.125$ mm (left) and 0.2 mm (right). The dotted lines represent the limits for the thin sheath theory ($\xi = 3$) and orbital motion limited theory ($\xi = 10$).

Moreover, the end effect was not of concern. To assess for this effect Equation (2.29) was used for electrode lengths of 2 mm and 5 mm. The ion drift velocity, defined by Equation (2.30), was computing assuming that the ion temperature is one tenth of the electron temperature [4, 25], assumed to be in the order of 1.5 eV inside the cathode’s insert. In the case of XPT this velocity can be approximated to the exhaust velocity of the thruster. The parameter $\tau_I$ was found to be much larger than one for all the case.

Three types of double Langmuir probes were built with the dimensions specified in Table 3.8. Tungsten wire was used for the electrodes, protruding from boron nitride tubes. All the ceramic tubes used were $\Phi2$ mm double bore $2x\Phi0.4$ mm. For longer probes two tubes were fixed together using ceramic paste. A schematic of the double Langmuir probe is presented in Figure 3.14. The electrodes have equal areas and the tungsten wires are coupled to BNC cables. In the case of XPT the length of the probe electrodes as well as the radii are the highest since the probe had to withstand large, high energy electron currents.

Figure 3.14: Schematic of a double Langmuir probe (dimensions in mm, not to scale).

A detailed diagram of the setup used to power and collect data from the Langmuir probes is presented in Figure 3.15. The circuit of a double probe is easy to setup, no plasma grounding is needed, lower biasing voltages can be applied, the acquisition system can be very basic and the results can be more precise although the spatial resolution is decreased. The function generator
controls the high speed bi-polar supply though a trigger line. Another trigger line controls the data logger so that when the voltage is swept the data logger records it. It was avoided to record the bias voltage coming directly from the power supply though a BNC T-connector, since the noise was amplified due to high voltage levels. Signal from the function generator was used instead with a conversion factor of 12. The resistor box contained the termination resistor (150 Ω for cathodes and 15 Ω for XPT) across which a voltage drop was measured due to current collection by the probe. The box provided all the connections between the different parts of the circuit, replicating the double Langmuir probe circuit depicted in Figure 1.5-left. A BNC interface was used to provide transition between the BNC cables outside the vacuum chamber and the feedthrough cable connected to the vacuum chamber flange.

<table>
<thead>
<tr>
<th></th>
<th>Commercial cathode</th>
<th>Laboratory cathode</th>
<th>XPT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_{\text{tube}} ) (mm)</td>
<td>100</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>( l_p ) (mm)</td>
<td>2</td>
<td>2.5</td>
<td>5</td>
</tr>
<tr>
<td>( r_p ) (mm)</td>
<td>0.125</td>
<td>0.15</td>
<td>0.2</td>
</tr>
</tbody>
</table>

**Table 3.8:** Sizes of the double Langmuir probes using during the experiments.

![Schematic of a double Langmuir probe circuitry.](image)

The double Langmuir probes behaved as single Langmuir probes if one of the electrodes is grounded. This was possible thanks to the compact design of the probe's circuit inside the resistor box that allowed for a “plug and play” configuration.

### 3.5.3 Emissive probes design and construction

An emissive probe was designed and built in order to provide more accurate data about the plasma potential on the same location the RPA was deployed in the plume of the laboratory model hollow cathode. When the heater current was null the probe behaved as a classical single
Langmuir probe. An optimal design for the probe can be attained using a thermal model [68] based on the heat transfer analysis of the probe's filament. The probe is formed of a dielectric material tube and the emitting portion of the probe is a small filament made of conducting wire. The thermal model considers the equilibrium between the different heat fluxes that affect the filament as follows [68]:

\[ Q_e + Q_i + Q_{oh} = Q_{em} + Q_r + Q_c. \]  

Therefore, the sum of the heat due to electron collection current, \( Q_e \), ion collection current, \( Q_i \), and external current applied to the probe, \( Q_{oh} \) (ohmic heating) should be balanced by the emitted heat due to the emitted current, \( Q_{em} \), and heat lost due to radiative cooling, \( Q_r \), and conduction through the surrounding ceramic walls, \( Q_c \). Balance equilibrium can be obtained for a specific filament temperature and is influenced by the amount of heating current applied \( I_h \), the bias voltage \( V_b \) and the size of the filament. Hence, the filament should have a small diameter in order to reach emission at low current but in the same time should be big enough to avoid the melting due to the plasma temperatures. The heating current should also be small to avoid the melting of the filament but high enough to start and sustain electron emission.

![Schematic of an emissive probe](image-url)

Figure 3.16: Schematic of an emissive probe (dimensions in mm, not to scale).

Tungsten was chosen for the filament for its high melting point. The probe was built using a \( \Phi 0.1 \) mm wire forming a loop with a total length of 3.57 mm out of a 100 mm long, \( \Phi 2 \) mm double bore \( 2 \times \Phi 0.4 \) mm boron nitride tube. In order to keep the filament in position, two more \( \Phi 0.3 \) mm tungsten wires were introduced in the ceramic tube bores. A schematic of the emissive probe is depicted in Figure 3.16. The small size of the filament diameter would decrease even more the parameter \( \xi \) allowing for thin sheath assumption.

The emissive probe circuit is presented in Figure 3.18-left, while the experiment setup diagram is depicted in Figure 3.18-right. A voltage divider bridge was used to measure the collected current, improving the accuracy by decreasing the impact of the voltage drop across the filament on the results. However, the resistance of the circuit when no plasma is present was measured around 2Ω. As the heating current was increased, the voltage drop increased as well, as Figure 3.17 shows. This explains the shift of the plasma potential position towards lower voltages when the peak of the first derivative of the emitting probe was calculated.

The heating current was varied between 0.5 and 1.75 A, with a “warm” non-emitting probe at 0.5 A and strongly emitting probe at 1.75 A. The resistor box depicted in Figure 3.20-right accommodates the voltage divider bridge and a 100 Ω termination cement resistor across which
the voltage drop is recorded using the data logger. The probe's voltage is swept using the high speed bi-polar supply controlled by the function generator as in the case of Langmuir probes.

![Variation of the voltage drop through the emissive probe circuit as a function of probe's heating current when no plasma is present.](Figure 3.17)

**Figure 3.17:** Variation of the voltage drop through the emissive probe circuit as a function of probe's heating current when no plasma is present.

![Schematic of the emissive probe circuit setup.](Figure 3.18)

**Figure 3.18:** Left: Emissive probe circuit using a voltage divider bridge formed of two identical cement resistors of 220 Ω. Right: Schematic of the emissive probe circuit setup.

### 3.5.4 Retarding potential analyser design

The retarding potential analyser used during the laboratory model cathode experiments was designed and built at Tsukizaki Laboratory from ISAS/JAXA [29] and used for ion thrusters.
plume characterisation. The probe has a relatively simple geometry. Two brass cups, frontal and posterior, hold together three insulator washers via four long screws. The insulators accommodate three metallic grids build out of chemical etched tungsten mesh. The fourth grid is placed on the frontal cap and kept floating during the experiments. A description of each grid purpose was presented in Section 1.4.3 of Chapter 1. Moreover, a detailed diagram of the probe is depicted in Figure 3.19.

Two important design parameters should be taken into account when sizing a retarding potential analyser: the grid opening size and the distances between the grids. Both parameters are dictated by the characteristics of the plasma that flows through the probe. Figure 3.20 highlights two possible cases. If the grid opening size is smaller than the thickness of the plasma sheath formed across the grid, the plasma sheath is uniformly distributed across the grid. If a negative potential is applied to the grid only the ions are able to cross the sheath and enter through the opening, while the electrons are effectively repelled. On the other hand, if the grid opening size is bigger than the plasma sheath thickness, the plasma can enter through the opening, forming a region that allows a continuous flow through the opening. Thus, upon a negative bias the grid is inefficient, since the plasma sheath shields the flow and both the electrons and the ions can pass.

As presented in the diagram from Figure 3.5-bottom, the RPA was deployed in one location into the plasma plume of the laboratory model cathode. Numerical simulations [38] revealed that at this location the plasma parameters lead to a plasma Debye length of about $3 \times 10^{-5}$ m. As for an effective shielding the grid opening size should be smaller than the Debye length, the RPA grids were changed with 500 mesh tungsten grids having openings of 0.025 mm.

![Figure 3.19: Schematic of the retarding potential analyser (dimensions in mm, not to scale).](image)

The second design parameter is the distance between the grids. In the original design the distance between the floating grid and the electron repeller was about 17 mm, while between the electron repeller and the ion repeller and ion repeller and collector was of 4 mm. For the RPA measurements for the laboratory model cathode just the grid meshes were replaced, while the insulator washers that are dictating the distances between the grids remained the same. Thus, the numerous problems encountered during those particular measurements. Faulty grid spacing can induce space charge limitations, occurring after the electron repeller. Influenced by the potential difference between the grids, the charge density can alter in turn this potential
difference. For instance, the ion repeller potential can be greater than the one applied. In this case a shift in the ion energy per charge distribution functions towards lower energies can be seen, since ions that should be collected are repelled. In order to avoid this situation, proper grid spacing especially between the electron repeller and the ion retarding grid should be computed by equating the Bohm flux with the Child-Langmuir flux [6]:

\[
Grid \ spacing \ < \frac{2}{3} \lambda_D \left[ \frac{2}{\exp(-1)} \right]^{1/4} \left[ \frac{V}{T_e} \right]^{3/4}.
\] (3.2)

The potential difference between the grids, \(V\) is a limiting factor for the grid spacing. If the Debye length computed previously is used and a potential difference between the electron repeller and the ion retarding grid of 50 V is considered, the distance between those two grids should be at maximum 0.34 mm, thus much smaller than the one of the RPA original design. The floating grid may slow down the incoming plasma flux and decrease the local plasma density, leading to an increase in the Debye length at the entrance of the electron repeller grid. Still, the distance between the grids of the original design is inappropriate for the cathode plasma plume measurements.

The electrical diagram of the RPA is presented in Figure 3.21. The electron repeller was biased to -50 V with respect to the common plasma ground and the collector, a grid itself, was biased to -50 V as well. The potential of the ion retarding grid was swept from 0 to 120 V with respect to the common ground. The collected ion current was computed using the potential drop measured across a 93 kΩ metal film resistor. The schematic of the entire RPA experiment circuitry setup is shown in Figure 3.22. The resistor box provides all the connections necessary to establish the electrical circuit presented in Figure 3.21.

Figure 3.20: Comparison between the plasma sheath thickness, influenced by the grid bias voltage, and the opening size of the grid. Left: Ideal case-plasma sheath covers the openings of the grid and, in this case, the electron repelling mechanism is efficient. Right: Faulty case-plasma can enter through the opening and the potential barrier imposed by the grid bias is inefficient.

Figure 3.21: RPA electrical diagram.
3.5.5 Related issues to the invasive plasma diagnostic techniques

During the experiments with Langmuir probes, emissive probes and RPA several issues were noticed, providing insights on the peculiarities of those plasma diagnostic techniques.

**Grounding**

First and foremost, since all those techniques are invasive procedures the contact with the plasma implies a well defined common ground. In the case of emissive, single Langmuir probes and RPA the definition of a single common ground for both the plasma and the diagnostics system is of high importance leading to flawless results. A faulty grounding sequence induces erroneous results and in some situation the impossibility of recording intelligible I-V traces. The double Langmuir probes do not need a ground since the electrodes are floating. However, the floating potential can be easily found by measuring the potential drop half way between electrode, thus between the supply and the common plasma ground.

**Electrodes contamination and probe cleaning**

Another important issue that ensues from the fact that the probes are in direct contact with the surrounding plasma is the contamination of the electrodes. Contamination can be easily observed when analysing double or single Langmuir probes traces. For a double Langmuir probe the ideal I-V trace has a symmetrical profile that passes though the origin. In reality, plasma non-uniformities induce a drift in the trace position on both vertical and horizontal directions, as suggested in Figure 3.24. Contamination of the electrodes can be assumed when the slope of the ion saturation regions increase and the ion saturation current is reduced, leading to an underestimation in ion number density. The phenomenon is depicted in Figure 3.26. The round-
off between the retarding field and ion saturation regions can be accentuated by the increase in the plasma sheath thickness as the voltage is increased. In the case of equal areas electrodes if one of the electrodes is contaminated the I-V trace undergoes a shift towards the clean electrode, since the saturation current collected by the contaminated electrode is reduced.

Due to contamination, metallic or carbon particles can build up and form a bridge between the electrodes of a double Langmuir probe. In this case a current is drawn by the probe similar to the capacitive current that is to be presented later. A routine check the resistance level between the two lines of the double Langmuir probe should indicate a very high level. It was appreciated that a level in the orders of MΩ would allow for reliable results.

In these situations the solution is to change the probe or to clean the electrodes using very fine sandpaper (CAMI 200 to 400). In the case of Hall thrusters the probe can be cleaned in-situ by approaching the electrodes to the more dense regions inside the plume, while the sweeping voltage encompasses high potentials. Ion bombardment help cleaning the surface of the probe to certain extend. The mechanism proved to be effective in the case of XPT. In the case of hollow cathodes, the low ion concentration inside the plume hinders the efficiency of the aforesaid mechanism.

The sweeping voltage range should be chosen in order to provide, when possible, clear saturation regions. However, high potentials can induce, especially for the double Langmuir probes, electric discharges between the electrodes. This phenomenon occurs especially in high density plasma regions where the path between the electrodes is easily established. The discharges between the electrodes or between the electrode of a single probe and the

![Figure 3.23: Qualitative comparison between an ideal and real double Langmuir probe traces depicting the issues induced by electrodes contamination and discharges between electrodes.](image)
surrounding plasma can be noticed in the I-V traces as sharp spikes, defining a localised abrupt rising in the collected current. In most of the experiments the probe bias voltage limits were adjusted according to the appearance of the I-V trace at different locations within the plume plasmas.

**Discharge current perturbations**

As the metallic tip of the probes and the dielectric tube enter the plasma, perturbations in the discharge current can be noticed. The amplitude of these perturbations can vary, depending on the location of the probe, the operating conditions of the cathode/thruster and the dimensions of the probe. In order to produce reliable probe measurements, the perturbations should be as small as possible. Figure 3.25 depicts the discharge current perturbation with respect to the nominal discharge current of the XPT operating at 250 V and 1.43 mg/s anode mass flow rate.

![Graph showing discharge current perturbation amplitude](image)

**Figure 3.24:** Recorded discharge current perturbations for the XPT when a double Langmuir probe was intruding the plasma at various locations across the anode plate plane and at several axial distances away from the anode.

![Graph showing oscillation amplitude](image)

**Figure 3.25:** Discharge current oscillation amplitude for the XPT [36].

It can be clearly seen that as the distance from the anode plate decreases the current perturbations are increasing reaching a maximum of 0.3 A, thus about 20% of the nominal discharge current of 1.51 A, when the probe was at 5 mm away from the anode plate. The perturbations in the discharge current showed in Figure 3.25 contain also the intrinsic discharge current oscillations of the thruster. Figure 3.26 shows the oscillation amplitude for different
anode mass flow rates and discharge voltages for XPT. At 0.48 mg/s anode mass flow rate the oscillations undergo a step increase when the discharge voltage approaches 250 V. A cumulative effect of both phenomena-probe induced perturbations and intrinsic current oscillations-may explain the impossibility of recording reliable I-V traces for some operational conditions and locations within the plume.

**Capacitive current**

During operation of double Langmuir probes and RPA the line capacitance of the system may induce relatively high capacitive current. This appears due to the cable and electrodes line capacitance $C$ and may become important for high frequency probe voltage sweeping. This current is monitored in the absence of plasma and is defined as follows [57]:

$$I_{cap} = C \frac{dV_p}{dt}. \quad (3.3)$$

![Capacitive current variation for a 40 Hz bias voltage sweep for a 2.5 mm long double Langmuir probe](image)

**Figure 3.26:** Capacitive current variation for a 40 Hz bias voltage sweep for a 2.5 mm long double Langmuir probe in the absence of plasma.

As outlined in Figure 3.27, for a ±80 V 40 Hz sinusoidal sweep of a double Langmuir probe bias voltage in the absence of plasma the capacitive current reaches about $8 \cdot 10^{-6}$ A. The capacitive current is influenced by the sweeping frequency, the length of the probe’s feeding lines, the length of the electrodes and their diameter. However, the amount of capacitive current was still with at least one order of magnitude under the saturation current levels during the experiments, therefore its effects on the reliability of the results were neglected.

### 3.6 Summary on experimental setup for plasma plume diagnostics

In this Chapter the vacuum facilities in which the experiments took place were presented together with the various auxiliary systems used during the plume diagnostics for hollow cathodes and external discharge plasma thruster. Subsequently, the experimental logic and the probed domains are suggested by the means of several diagrams. The probes’ schematics and their circuitry are also exhaustively described. Each probe is mounted on the three-axis positioning system via custom-made mechanical interfaces. The former provides fast positioning of the probes in the plasma plume, assuring small residence time for the probes in order to reduce any damage upon the electrodes. In the end several issues related to the invasive plasma
diagnostic techniques are discussed as "learned lessons" from the experiments. The most influential issue related to these types of plasma diagnostic techniques remains the contamination of the electrodes, in the case of Langmuir probes and emissive probes, and the contamination of the grids of the retarding potential analyser.
CHAPTER 4: Hollow cathodes plasma plume characterisation

By now exhaustive reviews of the different electrostatic and energy analysis plasma diagnostic methods were presented together with a description of the experimental setups for the hollow cathodes and XPT plumes probing. Hereinafter the results for the two hollow cathodes are underlined and discussed. Different analysis techniques are employed and the results are compared in a comprehensive way. Moreover, where the case, consistency analyses with numerical data are presented.

4.1 Commercial hollow cathode plasma plume characterisation

The first cathode tested was the commercial hollow cathode. As previously described, the cathode was tested together with an external graphite annular anode placed at 130 mm from the cathode orifice. The position of the anode influences the plasma parameters distributions and makes the comparison with other experimental results difficult. Double Langmuir probes were used to characterise three cathode operating conditions, while data from single Langmuir probes were collected for two high current operating conditions. The thin sheath limited theory, the orbital motion limited theory and the Druyvesteyn electron energy distribution functions method were employed and check of the plasma sheath thickness enforced the validity of one or the other theory. The error bars in the following graphs are derived according to the assumptions presented in Section 4.1.5.

4.1.1 Double Langmuir probe results and discussion

Double Langmuir probe data were available for three operating conditions defined by the same mass flow rate of xenon, 1.43 mg/s and two different current levels 20 A and 40 A. Five positions along the cathode centreline were probes, starting with 13 mm. Raw double probe traces were first smoothed and then processed using the method described in Figure 2.13. This method assumes collisionless, unmagnetised plasma and the thickness of the plasma sheath around the electrodes is much smaller compared with the probe’s radius. Hence, the Knudsen number should be much greater than 1, the parameter $\xi$, defined in Equation (2.23), should be much greater than 1 or, according to Chen [8, 9] it should be above 10. If $\xi$ is smaller than 3, according to Chen, the probe works in orbital motion limited regime. In between the limit margins of this parameter neither thin sheath nor the orbital motion limited theories are fully valid. Plasma number density, approximated assuming quasi-neutrality from the ion number density, was computed using Equation (2.60), while the electron temperature was derived using Equation (2.70). A smoothed I-V double Langmuir trace is depicted in Figure 4.1

Plasma number density approximated using the ion number density and assuming plasma quasi-neutrality showed decreasing trends as the distance from the cathode orifice increased, ranging from $4.82 \cdot 10^{18}$ /m$^3$ at 13 mm and 40 A to $5.68 \cdot 10^{16}$ /m$^3$ at 43 mm and 10 A. The results are presented in Figure 4.2. For both 20 A and 40 A modes the reduction in plasma number density is about one order of magnitude between the closest point to the orifice and the 43 mm point. An increase in plasma number density can be observed as the power lever increases due to higher ionization levels.

Numerical analysis for hollow cathodes tested with external annular anodes [38] predicted for the laboratory model high current hollow cathode electron temperatures in the range of 2-5
eV, at a discharge current of 30 A. Double Langmuir probe measurements for the commercial cathode revealed similar experimental data for 20 A and 40 A operating conditions, as depicted in Figure 4.3. A probable drop in neutral density, as the distance from the cathode orifice increases, induces a decrease in electron-neutral collisions which may lead, in turn, to an increase in the electron temperature. This feature can be due to Joule heating or anomalous plasma resistivity conditions. In general, in this particular experimental setup the electron temperature should undergo a steep increase in the near-plume region (5-10 mm away from the cathode orifice) followed by an almost constant region and ending with a constant or slightly increase trend towards the anode position. The profile can be influenced by the distance between the cathode orifice and the external anode, by the geometry of the latter and it proved sensible to different cathode mass flow rates.

![Figure 4.1: Smoothed data of a double Langmuir I-V trace corresponding to 13 mm from the cathode orifice, on the centreline, at 40 A and 1.43 mg/s of xenon mass flow rate.](image1)

![Figure 4.2: Double Langmuir probe: Plasma (ion) number density variation along the cathode centreline at 1.43 mg/s xenon mass flow rate. Uncertainties of ±19.5% are mainly associated to the assumption of stationary plasma.](image2)


**Figure 4.3:** Double Langmuir probe: Electron temperature variation along the cathode centerline at 1.43 mg/s xenon mass flow rate. Uncertainties of ±20% are mainly associated to the time variation of the ion saturation current.

Having both the plasma number density and the electron temperature approximated using the double probe data, the parameter $\xi$ can be computed in order to provide information about the thickness of the plasma sheath. As it can be seen in Figure 4.4, based the Debye length computed using the plasma parameters, $\xi$ parameter is always bigger than 10 for the 40 A case, while for 20 A lies in between 3 and 10. This implies that the thin sheath assumption can be employed for the 40 A case, while for the 20 A operating condition neither the thin sheath nor the orbital motion limited theories are correctly describing the working regime of the probe. However, since single Langmuir probe data was available, a second approximation of the sheath thickness was possible.

![Double Langmuir probe](image)

**Figure 4.4:** Double Langmuir probe: $\xi = r_p/\lambda_D$ parameter. The black dotted lines mark the application margins of the orbital motion limited theory ($\xi < 3$) and the thin sheath approximation theory ($\xi > 10$).

### 4.1.2 Single Langmuir probe results and discussion

*Single Langmuir probe in thin sheath assumption*

First, the thin sheath limited (TSL) theory was applied to the single probe traces in order to get a first approximation for the plasma number density and electron temperature. The electron temperature was computed using the slope method presented in Figure 2.9. The inverse of the
slope of the linear fit to the retarding field region provides the electron temperature as defined in Equation (2.58). A second linear fit of the electron saturation region provides the electron saturation current used in Equation (2.59) to compute the plasma number density. An example of a smoothed single Langmuir probe I-V curve is presented in Figure 4.5.

The plasma number density results, presented in Figure 4.6, displayed similar trends as in the case of double probe results and the magnitude ranged from $6.5 \cdot 10^{18}$ m$^{-3}$ at 13 mm and 40 A to $4 \cdot 10^{17}$ m$^{-3}$ at 43 mm and 20 A.

The electron temperature showed a monotonically decreasing trend for 20 A ranging between 2.6 and 3.1 eV, as depicted in Figure 4.5. On the other hand, at 40 A, a slightly increasing profile can be seen in Figure 4.7. For this operating condition a peak of about 4 eV is reached at 23 mm away from the cathode orifice, the electron temperature remaining always above 3.5 eV for all the positions probed along the cathode centreline.

**Figure 4.5:** Smoothed data of a single Langmuir I-V trace corresponding to 23 mm from the cathode orifice, on the centreline, at 20 A and 1.43 mg/s of xenon mass flow rate.

**Figure 4.6:** Single Langmuir probe: Plasma (electron) number density variation along the cathode centreline at 1.43 mg/s xenon mass flow rate using thin sheath limited theory (slope method). Uncertainties of ±50% are suggested for the TSL method by Herman and Gallimore [27].
For 20 A operating condition the plasma number density approximated using the thin sheath limited theory for the single probe (slope method) was almost 80% in average higher than the one computed using the double probe thin sheath limited theory. In the case of 40 A the difference is reduced to a 25% in average. In the case of the electron temperature, at 40 A the difference in the electron temperatures estimated by the two methods is less than 0.5% in average. In the 20 A case the temperatures computed using the single probe data were with 1 eV, in average, lower than in the ones computed with the double probe data.

Using the single probe data for the plasma parameters, a quick computation of the electron-ion and electron-neutral mean free path allowed for the Knudsen number approximation. For all the positions probed along the cathode centreline the plasma was collisionless and the probe’s end effects were not of concern. Figure 4.8 highlights the values of parameter $\xi$ computed using the Debye length resulted from the single probe thin sheath limited plasma parameters. For the 40 A case the probe worked in thin sheath regime in all the probed positions. This time the results shows that also in the case of 20 A the single probe worked in thin sheath limited regime,
except for the last two points at 28 and 43 mm away from cathode orifice. This proves that the size of the probe is suited for the thin sheath approximation for both single and double probe data, therefore the plasma number density and electron temperature computed using the double probe data are valid as well. The plasma number density results are equally valid for single and double probe data, considering that the discrepancy between the two data sets regarding this parameter is under the total uncertainty level for the thin sheath limited assumption theory. In the case of electron temperature, the results obtained using double probe revealed trends that are in agreement with the numerical simulations [38], whereas the uncertainties in the magnitude are in the same range for both single and double probe analysis methods, being about 25-30%.

**Single Langmuir probe in orbital motion assumption**

Orbital motion limited (OML) theory was applied for the single probe data taken for 20 A operating condition. Although for the points situated at 28 and 43 mm away from cathode orifice neither the thin sheath nor the orbital motion limited theories are fully valid, it was interesting to find the magnitude of the plasma density and compare it to the results obtained using the thin sheath theory.

The single probe curves revealed no electron saturation region, thus it was impossible to apply the orbital motion limited theory for electron collection. This leads to the impossibility of subtracting the orbital temperature using this method. However, an estimation of the ion number density can be found by the means of the method presented in Figure 2.11. Equation (2.63) states that in the OML regime the ion number density is directly proportional to the square root of the slope of the linear fit to the squared ion saturation current.

![OML ion number density, n_i (m^-3)](https://www.example.com/figure4.9.png)

**Figure 4.9:** Single Langmuir probe: Plasma (ion) number density variation along the cathode centreline at 1.43 mg/s xenon mass flow rate using orbital motion theory. Uncertainties of ±50% are suggested for the TSL method by Herman and Gallimore [27].

Figure 4.9 depicts the ion number density variation along the thruster centreline obtained with the OML method. The trend displayed remains consistent with the ones obtained using thin sheath limited theory for single probe and double probe. However, the magnitude of the plasma density is up to one order of magnitude smaller than in the TSL approximation for the single probe but just 30% in average smaller than the data obtained using the double probe traces. The plasma density obtained using this method can be used just for the positions for which none of the theories is valid. In this case the plasma density can be interpreted as a weighted value between the OML prediction and the TSL prediction using the parameter $\xi$. If the point is closer
to the TSL region the plasma number density found with TSL method weights more and if the point is closer to the OML region, the plasma number density found with OML method weights more. This method was previously used in several studies [57].

4.1.3 Druyvesteyn electron energy distribution function method

Druyvesteyn electron energy distribution function (EEDF) is a powerful method that has the big advantage of not being sensitive to the plasma sheath thickness. It implies data from single Langmuir probe and a comprehensive presentation of the method was already included in Chapter 2 Section 2.2.5. As previously explained, the probe’s bias voltage is measured during the experiments with respect to the common ground, while the plasma was at plasma potential. Thus, a conversion of the bias voltage should be done in order to properly describe the electron energies with respect to plasma potential. This implied the subtraction of the bias voltage from plasma potential, the latter becoming a reference level for the EEDF.

For the following results the plasma potential was found as the average between the plasma potential computed using the peak of the first derivative of the probe’s collected current and the approximation given by the slope method of the TSL theory. While the first derivative was found, the current second derivative was computed and the EEDF was calculated using Equation (2.79). The accuracy of the plasma potential level has a significant impact in the approximation of the electron temperature as well as plasma number density. Both the first and second current derivatives present high level of noise, therefore smoothing is necessary in order to isolate the important parameters. The smoothing process can alter the position of the plasma potential, and in the case of single curves with no distinct or even absent "knee" the procedure is hard to be weighted with other results. Independent emissive probe measurements for the plasma potential would have produced more reliable data. The EEDFs profiles along the cathode centreline for 20 A and 40 A cases are presented in Figure 4.10 and Figure 4.11, respectively. The electron plasma density and electron temperature were calculated departing from the EEDF by using Equation (2.81) and (2.82). The results along the centreline are depicted in Figure 4.12 and Figure 4.13, respectively for both operating conditions.

![Druyvesteyn EEDFs for 20A](image)

**Figure 4.10:** Druyvesteyn electron distribution functions (EEDF) for a discharge current of 20 A and 1.43 mg/s xenon mass flow rate.
Figure 4.11: Druyvesteyn electron distribution functions (EEDF) for a discharge current of 40 A and 1.43 mg/s xenon mass flow rate.

Figure 4.12: Single Langmuir probe: Plasma (electron) number density variation along the cathode centreline at 1.43 mg/s xenon mass flow rate using Druyvesteyn EEDF method.

As the plume plasma at the axial positions probed is considered to be collisionless, the EEDF profile is determined mainly by the local electric field and by the characteristics of the near-field plasma. In general, the Druyvesteyn method predicts higher electron temperatures than the other methods, since it places more weight on the higher electron energies than the Maxwellian distribution does [2, 4]. High electron temperatures are in the same time associated to low collision rate and thus, lower electron number densities are computed. The electron number densities computed by taking the first moment of the EEDFs are with one order of magnitude smaller than the ones predicted by the double probe method, ranging between $5.85 \cdot 10^{16}/m^3$ and $1.53 \cdot 10^{17}/m^3$ at 20 A and between $(1.25 \div 2.24) \cdot 10^{17}/m^3$ at 40 A. Chen [9] states as a general rule that single Langmuir probe I-V traces with no inflection point characterise low densities plasmas.

Electron temperature trends subtracted from the Druyvesteyn EEDFs display an almost constant variation for the 20 A condition with a drop of about 1eV around 23 mm location. The
magnitude varies between 4.85 and 5.82 eV, being almost two times higher than the temperatures predicted by the double probe method or the single probe TSL slope method. For the 40 A operating condition the EEDFs predicts a decreasing temperature profile along the centreline from a maximum of 9.84 eV to 5.96 eV. The magnitudes in this case are almost three times higher than the ones predicted by the other analysis methods.

![Graph showing EEDF electron temperature variation along the cathode centreline for 20 A and 40 A operating conditions.](image)

**Figure 4.13:** Single Langmuir probe: Electron temperature variation along the cathode centreline at 1.43 mg/s xenon mass flow rate using Druyvesteyn EEDF method.

The geometry of the anode has a great influence on the electron distribution along the centreline of the cathode. During the experiments an annular anode was employed. The EEDFs profiles, presented in Figure 4.10 and Figure 4.11, display in both operating conditions shifts of the distribution peak towards lower energy as the distance from the cathode orifice increased. Although the density decreases in the same time and the collision frequency reduces as well, low electron energies can be explained by the fact that the electrons are accelerated off-centreline towards the anode, depleting the centreline of high energetic electrons. In the case of 40 A, a secondary peak started to form at axial positions closer to the cathode orifice revealing an increasing fraction of higher energy electrons in this region. Moreover, the EEDFs can be altered significantly if the electrons are accelerated towards the anode to speeds comparable to the thermal speed. All those effects are hard to quantify and may induce high uncertainties to the measured plasma parameters. Despite the contradictions in the measured plasma density and electron temperature with the other methods, the Druyvesteyn EEDFs provide more insights on electron behavioural trends along the cathode centreline. When used together with independent plasma potential measurements, as from emissive probes, this method is a powerful and robust tool in plasma diagnostics.

A last method for electron temperature computation implies an indirect approximation of this parameter based on its relation with the plasma potential and probe’s floating potential. If both plasma potential and floating potential are known, the electron temperature can be approximated as follows [57]:

$$T_e = \frac{2(V_p + V_f)}{\ln(2M/m)}$$  \hspace{1cm} (4.1)

As previously stated the plasma potential was computed as the average between the approximations offered by the TSL slope method and the peak of the first derivative of the probe
current. The results are presented in Figure 4.14. For the 20 A case the plasma potential monotonically decreases with about 3 V across the measured domain, while in the 40 A operating condition a steeper decrease of about 4.5 V is recorded. The electron temperature follows the trends imposed by the plasma potential, as depicted in Figure 4.15. The temperature of the electrons remains almost constant across the domain with a magnitude of 2.5 eV in average when the cathode was operating at 20 A and 3.5 eV in average for 40 A case. The trends are in good agreement with the approximations offered by the TSL slope method, while the magnitudes are in average 30% lower.

Figure 4.14: Single Langmuir probe: Plasma potential variation along the cathode centreline at 1.43 mg/s xenon mass flow rate as the average between the potential approximated using the current 1st derivative method and the one approximated using the TSL slope method.

Figure 4.15: Single Langmuir probe: Electron temperature variation along the cathode centreline at 1.43 mg/s xenon mass flow rate derived from the plasma potential $V_p$ and the probe’s floating potential $V_f$.

4.1.4 Uncertainties in measured plasma parameters

Foremost, the position accuracy provided by the positioning system is about ±0.5 mm, since the maximum strokes of the x- and y-axis are around 70 mm, thus half the maximum stroke of the device. The uncertainties in positioning the probe’s tips are rather low compared to the main
source of positioning error: the oscillations inherited from the movement of the long mechanical arm that hold the probes in position. Although the arm was designed with a high rigidity constant in order to minimize de oscillations and damping times were included during the experiments in order to reduce even more the uncertainties in position, it turned out to be difficult to achieve a motion without oscillatory behaviour. However, axial positioning was not influenced by the oscillation and the fact that the raw I-V traces did not present significant additional noise that could have been induced by strong vibrations of the arm, it was assumed that the oscillations remained within low limits.

In the case of single Langmuir probe measurements previous studies estimate errors of ±30% for the electron temperature and ±50% for the plasma number density for both TSL and OML methods [2, 4, 27]. Although high, the errors can be considered to be consistent across all the measurements [2, 4]. Moreover, of great importance are the shapes of the parameters trends rather than their absolute values, since the electrostatic probes come with several limitations that hinder the knowledge of the absolute value of the plasma parameters to a high degree of accuracy.

In the case of Druyvesteyn EEDF method the account for errors becomes complex and prevents a quantification of their magnitudes. When all the constitutive parameters that enter into the EEDF computation are known to a high degree of accuracy, Herman and Gallimore predict an error or ±8% in the determination of the EEDF using the harmonic method presented in Equation (2.79) [27]. This error propagates without amplification in the electron number density and electron temperature estimates computed based on the EEDF moments. Moreover, the error in the plasma parameters estimate can be decreased by averaging several EEDFs for the same location in the plume. In this way the noise present within the high energy tails can be reduced. This was not the case of the present research because of the lack of data. Another factor that has a great influence in the computation of EEDFs is the magnitude of the plasma potential. The uncertainties in finding the plasma potential lie on the smoothing effects and the differentiation as well as in the accuracy of identifying the peak of the first derivative. Since the plasma potential was computed as an average between the peak of first derivative method and the TSL slope method, the error of the later method are also inherited. The second differentiation of the probe’s current introduces an amplification of the noise, hence data smoothing has to be applied which in terms affects the accuracy of the EEDF.

The double probe data analysis technique used for this research came with its own error analysis method [53]. For the error in the electron temperature estimation, the following equation should be considered:

\[
\frac{k_B T_e}{e} = -\frac{\sum I_i}{A_p \left( \frac{dI}{dv_b} \right)_{v_b=0} - \left( \frac{dI}{dv_b} \right)_{I_{sat}}}.
\] (4.2)

As it can be seen the absolute value of the saturation current does not influence the electron temperature. Moreover, the fitting error for the retarding field region \((dI/dv_b)_{v_b=0}\) is usually under ±1% for all the points, using this method [53]. In the same time, for the saturation regions fitting curves slopes, as presented in a previous study, the error induced to the electron temperature is less than ±5% even if the slope encompasses ±200% error [53]. The main influential factor remains the fluctuation of the ion saturation current. This error was measured as a coefficient of variation defined as the ratio between the standard deviation and the mean value of the ion saturation current for several curves taken at the same location in the plume.
Several traces were available for the points situated at 13 mm and 18 mm away from the cathode orifice, on the centreline. The previously explained logic revealed two coefficients of variation of 0.0285 and 0.108. The higher value is considered as the basis of the measurements error and is assumed to be representative for all the locations on the centreline at which measurements were conducted. Thus, an error in the electron temperature was considered to be about ±20% and consistent throughout the probed domain.

In the case of the plasma number density computed using the double probe data the factor with the greatest impact in the accuracy of the ion number density derivation is the plasma drift velocity. Other errors associated to the difference between the two electrodes surfaces, the finite ion temperature effects or the direction of plasma flow with respect to the probe encompasses up to ±10% from the total error. The fact that the plasma is flowing, and still it is considered as being stationary in the analysis, brings a level of uncertainty in the magnitude of the plasma number density. The influence of the plasma flow upon the measurements is difficult to estimate due to the complex mechanism involved in the process. However, it may be stated that in the case of flowing plasma with a finite drift velocity, \( U \), the ions that are approaching the electrodes and are collected lead to an ion saturation current density \( j_{\text{is, flowing}} \) defined as follow [53]:

\[
j_{\text{is, flowing}} = 0.6n_i e \sqrt{\frac{U^2 + k_BT_e}{M}}. \tag{4.3}
\]

In the case of stationary plasma, Equation (4.3) is simply [53]:

\[
j_{\text{is, stationary}} = 0.6n_i e \sqrt{\frac{k_BT_e}{M}}. \tag{4.4}
\]

It is easy to see that the ratio between the two ion saturation current densities reveals the error factor that assumes non-stationary plasma conditions and can be used to account the error in plasma number density for the double probe method:

\[
\frac{j_{\text{is, flowing}}}{j_{\text{is, stationary}}} = \sqrt{1 + \frac{U^2M}{k_BT_e}}. \tag{4.5}
\]

Plasma drift velocity can be assumed as the velocity of the heavier species within the plume, i.e. ions and neutrals. The velocity is approximated to the speed of sound, assuming a neutral continuum flow [2, 4]:

\[
U = \sqrt{\gamma X_e R_X e T_i}. \tag{4.6}
\]

The ion temperature is approximated to be one tenth of the local electron temperature [25]. Applying Equation (4.5) for the electron temperatures presented in Figure 4.3 with the temperature error of 20% considered, the ratio was between 1.064 and 1.095 throughout all the measurements for electron temperatures ranging between 3.18 eV to 4.42 eV. Thus it may be considered that a maximum ±19.5% error can be assumed for the plasma number density due to the plasma drift velocity and the aforementioned effects.
4.2 High current laboratory model hollow cathode plasma plume characterisation

The high current laboratory model hollow cathode was tested in the ISAS/JAXA Cathode Chamber where single and double Langmuir probe, emissive probe and RPA measurements were taken for different operating conditions. The cathode was run on two different xenon mass flow rates, 1.90 and 2.86 mg/s, while the current was varied between 20 and 40 A. During the experiments it turned out to be difficult to attain the modes suggested by the voltage-current profiles presented in Figure 3.4. For instance, at a discharge current of 20 A and a xenon mass flow rate of 1.9 mg/s the cathode was supposed to run in a plume mode with a discharge voltage stabilised around 26 V. However, at this particular operating point the cathode remained in spot mode with the discharge voltage reaching 19 V. Plume mode was attained just when the mass flow rate was increased to 2.86 mg/s, when the voltage reached 25 V. This latter behaviour is considered anomalous, since in general when the mass flow rate is increased the mode transition from “noisy” plume mode to “quiescent” spot mode should occur. The voltage-current profiles presented in Figure 3.4 were obtained before the probe measurements began, thus the cathode outgassing may have an effect on the voltage-current characteristics.

Single Langmuir probe traces were recorded using the emissive probe in “cold” mode. Plasma parameters were computed using the thin sheath limited assumption by applying the slope method. The plasma number density was also approximated using the orbital motion limited assumption. Double Langmuir probe results were also derived, assuming thin sheath conditions by utilising both the slope and the fitting methods presented in Section 2.2.4. For one operating condition, 30 A and 2.70 mg/s mass flow rate, depicting a plume mode, numerical simulation data were available and a consistency analysis between those data and the experimental results is outlined hereinafter.

A retarding potential analyser was deployed in radial direction in order to characterise the ion energy per charge distribution. In order to compute the net acceleration voltage of the ions, the plasma potential had to be known to a high degree of accuracy. This was possible by using an emissive probe and several analysis techniques as the floating probe method, the “warm” emissive probe method or the inflection point to the limit of zero emission.

Hereinafter an extensive presentation of the results is given. The location of the probed points was previously depicted in Figure 3.5. In order to assure an easy reading of the following graphics, the error bars are not added. Nevertheless, at the end of this section an exhaustive presentation of the uncertainties associated with the results is underlined.

4.2.1 Langmuir probe results and discussion

Single Langmuir probe results

As previously explained, an emissive probe with zero heating current behaves as a regular single Langmuir probe. Data for three operating conditions were collected and analysed using both thin sheath and orbital motion limited methods. At a mass flow rate of 1.90 mg/s and 20 A the cathode should exhibit a “noisy” plume mode. However, the mode was “quiescent” spot mode with a discharge voltage stabilised at 19 V. The plume mode was achieved by increasing the mass flow rate to 2.86 mg/s. Data during both operations was recorded. At 39.6 A and 2.86 mg/s a spot mode was tested as well.
The variation of the plasma number density computed using the slope method of the thin sheath limited theory is shown in Figure 4.16. As previously explained, the plasma density does not vary much with the mass flow rate for the same power level. The variation can be sensed if the input power is modified, leading to higher densities for higher powers. As expected, the highest trend is recorded when the cathode was operated at 39.6 A, ranging from \(2.5 \cdot 10^{18} \text{ / m}^3\) to \(7.2 \cdot 10^{17} \text{ / m}^3\). For 20 A operating conditions, although the 1.9 mg/s case records a lower input power of about 390 W as against the 514 W for the 2.86 mg/s case, the plasma number density displayed higher values in the former case. This can be explained by the fact that the first case is a "quiescent" spot mode with a plume less divergent and collimated along the centreline, whereas the second case is a plume mode with a more divergent beam, depleting the centreline plasma electron population to some extent.

![Figure 4.16: Single Langmuir probe: Electron number density variation along the cathode centreline using thin sheath limited theory (slope method).](image)

The electron temperature variation profiles are presented in Figure 4.17. The magnitude of this parameter is influenced by the amount of input power and the trends along the centreline can be influenced by the position and geometry of the anode. The cathode was tested with an external graphite annular anode located at 60 mm away from the cathode orifice. The electron temperature varies between 1.41 and 4.51 eV. All the trends suggest a rise in temperature as the distance increases from the cathode orifice. This may be explained by a relative acceleration of the electrons towards the anode combined with a decrease in density, thus a relaxation of the collision regimes. Those mechanisms allow electrons to increase their energy. The oscillatory trends for the 20 A cases during the first 25 mm probed may be explained due to the plumes instabilities during experiments.

![Figure 4.17: Single Langmuir probe: Electron temperature variation along the cathode centreline using thin sheath limited theory (slope method).](image)
By using the thin sheath limited slope method the plasma potential can be approximated as well, as the intersection point between the two linear fits of the electron saturation and retarding field regions. Figure 4.18 outlines the results obtained using this method. The electric potential displayed almost constant trends with slightly increasing behaviour towards the anode and spanned between 16 and 31 V. The spatial distribution of the electric potential has a great influence on the electrons energy distribution, influencing the local electric fields.

Based on the electron temperature and plasma number density estimations, the Knudsen number was computed revealing collisionless plasma conditions along the entire domain probed. The Debye length was also derived and the variation of the parameter $\xi$ along the cathode centreline during the different operating conditions is depicted in Figure 4.19. As it can be seen for all the operating conditions the probe worked outside the margins of the TSL and OML theories, therefore neither TSL nor OML are fully valid for analysis. In this situations the results for the plasma density and electron temperature can be weighted with a function based on the value of the parameter $\xi$, as explained in several studies [27, 57]. The available single probe traces revealed no regions with linear dependency between the bias voltage and the squared electron current, as needed for the OML analysis. Therefore it was impossible to compute the electron temperature using this method. However, for the 20.5 A and 1.90 mg/s mass flow rate the squared ion saturation current revealed linear dependency with the bias voltage and the ion number density was approximated using the OML theory. Figure 4.20 presents the variation of the plasma number density computed using the TSL slope method and
the OML method. The green profile is the result of a direct weighting between the other two with coefficients proportional to parameter $\xi$. If $\xi$ lies between 3 and 10, the plasma number density can be assumed as follows:

$$n = \left(\frac{\xi - 3}{7}\right)n_{e,TSL} + \left(\frac{10 - \xi}{7}\right)n_{i,OML}.$$  \hspace{1cm} (4.7)

As the uncertainties associated with both TSL and OML methods reach as much as 50% in the case of the plasma number density estimations, such an approach does not influence much the accuracy of the parameter. As previously stated, the trends and their dependency on the input power and mass flow rate are more of importance.

![Figure 4.20: Single Langmuir probe: Plasma number density variation along the cathode centreline at 20.5 A and 1.90 mg/s xenon mass flow rate computed using the thin sheath limited (TSL) and orbital motion limited (OML) methods. The green profile depicts a weighted variation between the two aforementioned methods.](image)

**Double Langmuir probe results**

Electron number density profiles extracted from double Langmuir traces using the slope method are presented in Figure 4.21. The results correspond to the same operating conditions as in the case of single Langmuir probe. Unfortunately, for the 20 A cases data just for several positions close to the cathode orifice was available. However, as expected, the trends depict the same variation as displayed in the case of single probe measurements. The magnitude remained in the same order of magnitude, as in the previous results dataset, confirming the validity of the data for plasma number density obtained using the TSL method for the single probe. Once again, the difference between the spot and plume mode at 20 A is revealed, as the plasma number density at 1.90 mg/s case is higher than in the case of 2.86 mg/s, although the input power is lower.

As in the case of the commercial cathode, the electron temperatures estimated using the double probe slope method are higher than the ones obtained using the single probe TSL slope method. The results for the double probe electron temperature variations are shown in Figure 4.22. The electron temperature magnitude spans between 4.3 and 9.63 eV with higher temperatures for higher input powers. The temperatures are relatively high for these operating conditions and may be induced by unclear retarding field regions in the double probe I-V traces. This can be the reason for the high scattering trend along the probed domain in the case of 40 A operating condition. Moreover, the lack of data for the 20 A cases and the spreading of the values in the 40 A case barred a clear electron temperature trend interpretation for the double probe results. At this point, the electron temperature results provided by the single probe analysis
seem to agree with the numerical simulations [38], presented later in the section, and the results obtained for the commercial cathode, at similar input power levels.

![Figure 4.21: Double Langmuir probe: Electron number density variation along the cathode centreline computed using the slope method in the thin sheath assumption.](image)

Numerical simulations data were available for 30 A and 2.86 mg/s case, describing a plume mode operation with a discharge voltage of 24.2 V. Double probe traces were collected for 30 A operation and a mass flow rate of 2.70 mg/s. Due to the instability of the cathode during the experiments it was difficult to attain a plume mode for 2.86 mg/s. Each time the mass flow rate was increased over 2.70 mg/s the cathode operation changed to spot mode and the discharge voltage dropped below 20 V. Therefore, the closest operating condition to the one for which numerical data was available was the one of 30.2 A, 23.4 V and 2.70 mg/s mass flow rate. The results for the electron temperature and electron number density derived using the double probe slope method are presented in Figures 4.23 and 4.24, respectively. The graphics include the results for the spot mode attained at the same discharge current and 2.86 mg/s when the discharge voltage stabilized around 15.4 V. Once again, it may be seen that the spot mode is characterized by higher electron temperatures and plasma number densities although the input power is lower compared to the one of the plume mode. The electron temperature estimates range between 6.05 and 8.78 eV across the measured domain. Later in the chapter a consistency analysis between those experimental data and the numerical analysis data is provided in order to bolster the validity of the various probe analysis techniques.
**4.2.3 Emissive probe results and discussion**

“Hot” emissive probe traces were collected for the point where the RPA was deployed, in radial direction (x), 50 mm off the cathode centreline and at 10 mm away from the cathode orifice (y), as depicted in Figure 3.5. Data was collected during two operating regimes, the same for which RPA data existed. Two methods were used in order to analyse the emissive probe traces: floating emissive probe method and the inflection point of a “warm” non-emitting emissive probe method. Both techniques were described in Section 2.3. One additional data set was obtained from the “cold” probe traces by applying the TSL slope method. The results are summarised in Table 4.1. Moreover, for each method the lower and upper limits for the plasma potential magnitude are computed based on the uncertainties that each method brings in the analysis. The uncertainties are defined as fractions of the local value of the electron temperature, thus it was necessary that this parameter is known. For that, “cold” emissive probe traces were analysed using the TSL slope method in order to provide the electron temperature.

The I-V traces collected for the x-50/y-10 position did not allow for the method of the inflection point in the limit of zero emission to be applied. However, the “warm” emissive probe method results are in good agreement with the TSL slope method results, confirming that the latter can provide reliable plasma potential estimations. Moreover, the floating potential method
provided results in the same range as the other two methods when the correction of $1.5T_e$ was added. The plasma potential needed for the RPA measurements interpretation was calculated as an average between the different methods. For the 20 A case, the "warm" probe and TSL methods results were used, while for the 40 A the floating method result was used as well together with the two aforementioned.

Table 4.1: Plasma potential estimation for the point situated at 50 mm in radial direction from the cathode centreline at 10 mm away from cathode orifice in axial direction. The values are given with the lower ($V_{p-}$) and upper ($V_{p+}$) limits imposed by the various methods uncertainties. The electron temperature was computed using the TSL slope method for "cold" emissive probe traces.

<table>
<thead>
<tr>
<th>Operating condition</th>
<th>20 A, 26 V, 1.90 mg/s</th>
<th>40 A, 18V, 2.86 mg/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_e$ (eV)</td>
<td>1.88</td>
<td>2.2</td>
</tr>
<tr>
<td>$V_p$ (V)</td>
<td>$V_{p-}$</td>
<td>$V_p$</td>
</tr>
<tr>
<td>Floating emissive probe</td>
<td>17.49</td>
<td>17.68</td>
</tr>
<tr>
<td>Floating emissive probe + 1.5$T_e$</td>
<td>20.31</td>
<td>20.5</td>
</tr>
<tr>
<td>Inflection point of a &quot;warm&quot; probe</td>
<td>23.46</td>
<td>24.4</td>
</tr>
<tr>
<td>TSL slope method</td>
<td>-</td>
<td>24.5</td>
</tr>
</tbody>
</table>

Table 4.2: Plasma potential estimation for the point situated at 30 mm from the cathode orifice on the cathode centreline. The values are given with the lower ($V_{p-}$) and upper ($V_{p+}$) limits imposed by the various methods uncertainties. The electron temperature was computed using the TSL slope method for "cold" emissive probe traces.

<table>
<thead>
<tr>
<th>Operating condition</th>
<th>20 A, 26 V, 2.86 mg/s</th>
<th>20A, 19V, 1.90 mg/s</th>
<th>40A, 17V, 2.86 mg/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_e$ (eV)</td>
<td>1.94</td>
<td>2.24</td>
<td>3.21</td>
</tr>
<tr>
<td>$V_p$ (V)</td>
<td>$V_{p-}$</td>
<td>$V_p$</td>
<td>$V_{p+}$</td>
</tr>
<tr>
<td>Floating emissive probe</td>
<td>19.6</td>
<td>19.8</td>
<td>20</td>
</tr>
<tr>
<td>Floating emissive probe + 1.5$T_e$</td>
<td>22.51</td>
<td>22.71</td>
<td>22.9</td>
</tr>
<tr>
<td>Inflection point of a &quot;warm&quot; probe</td>
<td>17.23</td>
<td>18.2</td>
<td>19.17</td>
</tr>
<tr>
<td>Inflection point in the limit of zero emission</td>
<td>19</td>
<td>19.2</td>
<td>19.39</td>
</tr>
<tr>
<td>TSL slope method</td>
<td>-</td>
<td>17.4</td>
<td>-</td>
</tr>
</tbody>
</table>

Emissive probe traces were collected for a second point situated on the cathode centreline at 30 mm away from the cathode orifice. Data during three operating conditions was recorded and processed using the hereinbefore three methods and the inflection point in the limit of zero emission method. The results are presented in Table 4.2. As expected, the inflection point in the limit of zero emission delivered results in good agreement with the other methods results. Once again, the floating emissive probe method provides a plasma potential estimate close to the ones given by the other methods when the correction factor depending on the electron temperature is added.
The results revealed that for the location in the radial direction the plasma potential increased as the discharge voltage was increased, as depicted by the results in Table 4.1. In the axial direction location, results in Table 4.2, the trend seems to be reversed. In this case the electric potential displays and increasing trend as the discharge voltage decreases. This can be caused by the high current densities that can appear in the case of lower discharge voltages, since the plasma number density is lower and thus the plasma resistivity is higher. Whereas at high discharge voltages, the high plasma number density induces a reduction in the current density, leading to a lower electric potential.

4.2.4 Retarding potential analyser results and discussion

The retarding potential analyser was used in order to characterise the ion population in the radial direction of the cathode. The probe, described in Section 3.5.4, was placed in the radial direction, at 50 mm off the cathode centreline and 10 mm away from the cathode orifice, in axial direction. Despite the design issues of the RPA, i.e. the spacing between the grids, some measurements were possible for two operating conditions: a spot mode at 40 A and 2.86 mg/s mass flow rate with a discharge voltage of 18 V and a plume mode at 20 A and 1.90 mg/s with a discharge voltage stabilised at 26 V.

Several measurements were conducted during each operating conditions and ion energy per charge distribution functions were computed based on the method presented in Section 2.4. For the plume mode seven individual distribution functions were averaged and in the case of the spot mode two distribution functions were averaged. The process eliminated the noise in the low and high energies tails of the distributions and provided a more accurate peak location. The results are depicted in Figure 4.25.

For the plume mode, the HWHM of the ion energy per charge distribution function was about 11 V. In the case of the spot mode, the HWHM was about 4.5 V. As expected, higher HWHM is recorded for higher discharge voltage. This means that in the case of the plume mode the population of high energy ions is higher in radial direction than in the case of the spot mode. Those ions are considered to be the main cause for the keeper erosion [23, 24], drastically influencing the cathode lifetime [23, 24]. The spot mode revealed narrow ion energy per charge distribution function with the peak shifted towards lower energies and a very sharp high energy tail. In the case of the spot mode the collimated appearance of the cathode plume indicates the fact that most of the high energy ions are found in axial direction, since the plume divergence is low.

The source of high energetic ions remains under discussion, since intensive research is undertaken on this important issue that influences the lifetime of the cathode. Three phenomena can be responsible for the generation of high energetic ions during plume mode operation: formation of a potential hill in the near-field plume region, ion acoustic instabilities and the formation of a double layer in the orifice of the cathode due to charge separation [24, 25]. Previous studies [23, 24] showed that the "nosy" plume mode is characterised by RF plasma potential oscillations in the near-cathode plume that are induced by plasma instabilities: turbulent ion acoustic wave of high frequency (damped by Landau damping at low Mach numbers) and low frequency ion acoustic waves or ionization instabilities [24]. Once the plume mode is installed, the discharge voltage and the keeper voltage are characterised by relatively high oscillations, since the mode couples the oscillating voltages to the power supply connections [24]. In this modes high cathode erosion rates are possible [24].
The increase of the mass flow rate has as effect an extraionization which may collisionally inhibit, or damp the ionization instabilities in the near-cathode plasma [24], pushing the operation of the cathode towards the “quiescent” spot mode. The transition produces a collimated flow with an increase in density along the cathode centreline. However, high mass flow rates are considered to be harmful for the cathode insert [24].

Figure 4.25: Ion energy per charge normalised distribution functions for a point situated at 50 mm off the cathode centreline (radial direction-x) and 10 mm away from cathode orifice (axial direction-y).

The energetic ions fraction can be characterised by RPA distributions which can provide in the same time the local average ion acceleration potential, $V_a$.

Important information that can be subtracted from the distribution functions is the position of the most probable ion potential $V_{mp}$. This potential is known with respect to the plasma potential, $V_p$, and can deliver the local average ion acceleration potential, $V_a$, for all the ion species, defined by Equation (2.97). Once known, the average ion acceleration potential can provide an estimation of the average ion velocity, $U$, by the means of Equation (1.11). For the plume mode, the most probable potential was found to be 24.6 V, whilst for the spot mode was 20.3 V. The average ion acceleration potentials for the two cases are found by subtracting the local electric potential, previously found using the emissive probe, from the most probable potential. Table 4.3 emphasises the results for the ion velocity. In the following section a comparison with simulated data is presented.

Table 4.3: Retarding potential analyser results.

<table>
<thead>
<tr>
<th>Operating condition</th>
<th>$V_d$ (V)</th>
<th>HWHM (V)</th>
<th>$V_{mp}$ (V)</th>
<th>$V_p$ (V)</th>
<th>$V_a$ (V)</th>
<th>$U$ (m/s)</th>
<th>$\beta$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot mode</td>
<td>18</td>
<td>4.5</td>
<td>20.3</td>
<td>20.24</td>
<td>0.06</td>
<td>210</td>
<td>0.33</td>
</tr>
<tr>
<td>Plume mode</td>
<td>26</td>
<td>11</td>
<td>24.6</td>
<td>24.45</td>
<td>0.15</td>
<td>332</td>
<td>0.57</td>
</tr>
</tbody>
</table>

4.2.5 Consistency analysis

*Langmuir probe results and numerical simulation data*

Numerical simulation results from a previous study [38] were available for one operating condition: plume mode at 30 A, 24.2 V and 2.86 mg/s mass flow rate of xenon. The results were generated using a hybrid-PIC model based on an axisymmetrical flow and assuming quasi-neutrality in the bulk region of the plume plasma [38]. The computation domain assumed the
presence of an annular anode at the distance of 34 mm away from the cathode orifice. This fact
influences the spatial distribution of the electron temperature and electric potential along the
centreline. Taking into account that during the plume probing with electrostatic probes and RPA
the anode was placed at 60 mm away from the cathode orifice, it is difficult to establish a direct
comparison between the numerical simulation results and the experimental results. However, a
discussion can ensue.

Figure 4.26 shows the numerical simulation results for the electron temperature and electron
number density profiles along the centreline of the cathode for the same axial distances for
which experimental results were derived. As it can be seen, the plasma number density ranges
between $2 \cdot 10^{18}/m^3$ and $1.64 \cdot 10^{17}/m^3$, displaying a sharp drop in the first 5 mm of the
domain. The electron temperature profile presents a continuous increase up to the position of
the anode (34 mm) followed by an almost constant variation and in the end it is slightly
decreasing. The magnitude of the electron temperature spans between 2.86 and 3.38 eV.

![Graph showing electron number density and temperature variations along the cathode centreline](image)

**Figure 4.26:** Numerical simulation: Electron number density and electron temperature
variations along the cathode centreline for 30 A, 24.2 V and 2.86 mg/s of xenon mass flow rate.

![Graph comparing experimental and simulation electron temperatures](image)

**Figure 4.27:** Comparison between the electron temperature profiles along the cathode
centreline obtained using double Langmuir probe traces analysed with the
slope and fitting techniques and the numerical simulation data for 30 A, 24.2
V and 2.86 mg/s of xenon mass flow rate. Uncertainties of ±33% are mainly
associated to the time variation of the ion saturation current.
Experimental data for the same cathode operating condition were collected using double Langmuir probe. Two analysis techniques were used: the slope method and the fitting method, both presented in Section 2.2.4. Both methods delivered relatively high electron temperature estimations ranging between 4 and 7.82 eV, as depicted in Figure 4.27. These figures are in average 2.5 times higher than the simulation results for the same operating condition. For the commercial cathode it was shown that in general the double Langmuir probe techniques deliver higher electron temperature and plasma number density estimates than the TSL method for the single probes, for instance. In this particular case the experimental estimates are probably too high for portraying correctly the plasma reality. This can be due to the lack of distinct retarding field regions within the raw data and the inability of recording several clean traces for the same point due to cathode plume instabilities.

The situation for the plasma number density is however different. The numerical simulation results for the plasma number density variation along the thruster centreline lies between the estimates obtained using the fitting and the slope method for the double probe traces. The slope method estimates are in average 55% higher than the simulated ones, whereas the fitting method values are in average 64% smaller than the ones delivered by the numerical code. Taking into account the uncertainties that both the slope and the fitting methods have, the experimental results are in good agreement with the numerical simulation data in the case of the plasma number density.

![Figure 4.28](image)

**Figure 4.28:** Comparison between the electron number density profiles along the cathode centreline obtained using double Langmuir probe traces analysed with the slope and fitting techniques and the numerical simulation data for 30 A, 24.2 V and 2.86 mg/s of xenon mass flow rate. Uncertainties of ±17% are mainly associated to the assumption of stationary plasma.

Since the electron temperature results obtained using the single Langmuir probe data had shown lower magnitudes, it was interesting to compare those results with the numerical simulation data. Unfortunately, there were no single Langmuir probe experimental data for the operating condition of 30 A and 2.86 mg/s. However, data for 19.8 A and 2.86 mg/s (plume mode) and 39.6 A and 2.86 mg/s (spot mode) were available. As expected the simulated results lie in between the experimental data, as depicted in Figure 4.29. Since the anode position for the numerical simulation was placed at 34 mm from the cathode orifice, the slight increase in the electron temperature is observed before this axial position. In the case of experimental data, the same trend is observed before the axial position of 60 mm, where the annular anode was placed during the experimental tests. Although the input power in the case of numerical simulation is
greater than the input power of the 39.6 A case, the electron temperature profile for the latter case displays higher magnitudes, since the cathode operated in spot mode with a narrow beam, increasing the energetic electrons population along the centreline.

![Figure 4.29](image1.png)

**Figure 4.29:** Comparison between the electron temperature profiles along the cathode centreline obtained using single Langmuir probe traces analysed with the TSL slope method (19.8 A, 26 V and 39.6 A, 17.6 V at 2.86 mg/s) and the numerical simulation data for 30 A, 24.2 V and 2.86 mg/s of xenon mass flow rate. Uncertainties of ±30% are suggested for the TSL method by Herman and Gallimore [27].

![Figure 4.30](image2.png)

**Figure 4.30:** Comparison between the electron number density profiles along the cathode centreline obtained using single Langmuir probe traces analysed with the TSL slope method (19.8 A, 26 V and 39.6 A, 17.6 V at 2.86 mg/s) and the numerical simulation data for 30 A, 24.2 V and 2.86 mg/s of xenon mass flow rate. Uncertainties of ±50% are suggested for the TSL method by Herman and Gallimore [27].

Figure 4.30 presents a comparison for the electron number density between the numerical simulation data and the single probe experimental results. The highest densities were recorded for the 39.6 A spot mode, although it does not have the highest input power. The exhibited plume with a small divergence has as affect the creation of a denser region along the centreline. The numerical simulation data seems to be in perfect agreement with the experimental data for the 19.8 A plume mode. Despite the difference in the input power, both operating conditions describe plume modes, thus the density profiles are lower than for a similar input power spot mode. For the closest point to the cathode orifice, the plasma number density computed using
the numerical code is still higher than the experimental data for the lower power plume mode. Then the simulation data remains just slightly higher than the experimental data. This confirms that the data sets are in good agreement and the single Langmuir probe experimental results are representative for this cathode plasma plume characterisation.

**Langmuir probe and emissive probe results**

The electric potential was computed for all the points using single Langmuir probe data, while at the location of the RPA and for one point situated at 30 mm away from the cathode orifice on its centreline emissive probe data was collected. The Langmuir probe electric potential was derived from the TSL slope method. From the data presented in Tables 4.1 and 4.2 it can be stated that the difference between the plasma potential computed using single Langmuir probe data and the one computed using the different emissive probe techniques was between 0.15\(T_e\) and 1.1\(T_e\), proving the validity of the TSL slope method for electric potential computation.

The main influential factor on the electric potential magnitude is the discharge voltage. Therefore, for the same discharge voltage but different mass flow rates and discharge currents, the magnitudes of the plasma potential remain almost unchanged. Numerical simulation data available for 30 A case and 24.2 V discharge voltage can provide a good comparison term for the experimental data collected at 20 A and 26 V. For the point situated at 50 mm in radial direction and 10 mm in axial direction from the cathode orifice, numerical simulation provided an electric potential of 24.74 V [38]. Experimental data for the same point at 20 A and 26 V operating condition revealed a plasma potential of 24.45 V, 1.2% lower. On the other hand, the results for the point located at 30 mm away from the cathode orifice, on the centreline, show a disagreement. The simulated result was 26.74 V. The closest experimental result was computed using the corrected floating emissive probe method, the experimental value being with 2\(T_e\) smaller than the numerical data. For all the other experimental results derived using the emissive probe methods of the single probe TSL method lie between 3.8T_e and 4.9T_e under the simulated result. The discrepancy can be due to the complex effects induced by the plasma flow in the axial direction, effects that are less severe at the location of the point situated at 50 mm in radial direction and by the geometry of the domain. Once again, in the numerical simulated domain the anode is placed at 34 mm from the cathode orifice whereas during the experiments the anode was at 60 mm away.

**Retarding potential analyser results and numerical simulation data**

Notwithstanding the fact that no numerical simulation data was available for the same operating conditions for which RPA experimental data existed, an extrapolation between the existing numerical simulation results and the experimental figures was possible. Based on the RPA ion energy per charge distribution functions the ion velocity at the probe location was derived. At a discharge voltage of 18 V the velocity was estimated at 210 m/s, while for a discharge voltage of 26 V the velocity was estimated at 332 m/s. The numerical simulation data for a discharge voltage of 24.2 V provides an ion velocity at the same location of 288.2 m/s. The experimental results seem to be in good agreement with the numerical estimation, although the former are prone to high uncertainties due to the faulty design of the probe. Further research should consider an optimization of the RPA design for the particular case of the cathode plume plasma and several positions should be probed in order to prove the viability of the results.
4.2.6 Uncertainties in measured plasma parameters

Once again, as in the case of the commercial cathode the position accuracy provided by the positioning system is considered to be about ±0.5 mm since the maximum strokes on the x- and y-axis are around 70 mm, thus half the maximum stroke of the device. The main source for the positioning uncertainty remained the oscillations of the metallic arm holding the probes. In the case of RPA measurements those errors are excluded since the probe was stationary. Since the noise in the collected I-V traces remained in acceptable limits, the oscillations of the arm are considered to remain within low limits.

The uncertainties in plasma parameters estimations for the single Langmuir probe case, for both TSL and OML techniques were considered to be ±30% for the electron temperature and ±50% for the plasma number density, as previous studies suggested [2, 4, 27]. The same uncertainties may be applied for the double probe fitting method, as previously suggested by Hutchinson [30] and Linnell and Gallimore [44].

The uncertainties associated with the double probe slope method can be computed departing from the description presented in Section 4.1.5 and Equation (4.5). For the plasma number density, ±10% is assumed for the effects induced by the area difference between the electrodes, the finite ion temperature and direction of plasma flow [53]. The main uncertainty remains due to the non-stationary effects of the plasma. If an ion axial velocity of 700 m/s is assumed, based on the numerical simulation data, and since the electron temperature measured with the double probe ranged between 4.3 and 9.63 eV, Equation (4.5) reveals a factor between 1.03 and 1.07. Therefore, the maximum error for the plasma number density in the case of double probe measurements is of ±17%.

For the electron temperature measurements uncertainties brought by the double probe slope method, ±1% is considered for the fitting error for the retarding field region and ±5% for the fitting error for the ion saturation regions [53]. The main factor that induces high uncertainties is the time fluctuation of the ion saturation current. This error was measured, as in the case of the commercial cathode, as a coefficient of variation defined as the ratio between the standard deviation and the mean value of the ion saturation current recorded for several curves taken at the same location in the plume. Several traces were available for the points situated at 10 mm, 25 mm and 50 mm away from the cathode orifice, on the centreline. The coefficients of variation were 0.264, 0.213 and 0.174. The higher value is considered as the basis of the measurements error and is assumed to be representative for all the locations on the centreline at which measurements were conducted. Thus, an error in the electron temperature was considered to be about ±33% and consistent throughout the probed domain.

For the emissive probe results each analysis technique came with its own uncertainty level. For instance, the floating emissive probe method is believed to underestimate the plasma potential in most of the case with 1-2$T_e$ [13, 54], while the uncertainty in finding the beginning of the plateau can be of ±0.1$T_e$ [61]. The method of the inflection point of a “warm” but not emissive probe has an uncertainty of ±0.5$T_e$ while the method of the inflection point in the limit of zero emission comes with an uncertainty of ±0.1$T_e$, as suggested by Sheehan and Hershkowitz [61].

In the case of the retarding potential analyser measurements the uncertainty in providing the most probable potential was considered to be ±50% of the HWHM of the ion energy per charge distribution functions, as suggested by Brown [6]. In this case the uncertainty in the most
probable potential for the 40 A, 18 V spot mode is ±2.25 V, while for the 20 A, 26 V plume mode is ± 5.5 V.

4.3 Summary of hollow cathodes plasma plume characterization

The plasma plumes of two high-current hollow cathodes were probed using single and double Langmuir probes, emissive probes and retarding potential analyser. Various analysis techniques were used in the case of the single probe and emissive probe data in the attempt to reach an agreement with the main assumptions dictated by the plasma physics principles and, in the case of the laboratory model hollow cathode, with the numerical simulation data.

For the commercial cathode single and double Langmuir probe traces were recorded and analysed using the thin sheath assumption-slope method for single probe and double probe-and the orbital motion limited assumption and Druyvesteyn energy distribution function for single probe data. Both single probe TSL slope method and double probe slope method gave results in agreement for the plasma number density and electron temperature when parameter $\xi$ was larger than 10, limit for the thin sheath assumption, as suggested by Chen [8, 9]. When the parameter $\xi$ was neither larger than 10 nor smaller than 3, limit of the orbital motion theory, none of the two theories could provide fully trustful estimates for the plasma parameters. In this case a weighting of the results should be conducted according to the value of $\xi$. In general, the single Langmuir probe traces with no distinct "knee" are believed to characterise low density plasmas [8].

Another method would imply the derivation of electron energy distribution functions. Previous research [20, 21, 22] argued that for low pressure discharge plasmas the electron energy distribution is rather non-Maxwellian, being better described by the Druyvesteyn energy distribution function [2, 4, 22]. The method estimated for the commercial hollow cathode lower densities than the other methods and higher temperatures. This is because the Druyvesteyn method places more weight on the higher electron energies than the Maxwellian distribution does [4], and since high electron temperatures are in the same time associated to low collision rates, lower electron number densities are computed. However, when properly applied, the method can provide the most comprehensive view on the plasma plume dynamics.

The laboratory model hollow cathode showed during the tests a large plateau between 20 A and 30 A characterised by mode transition instabilities. Anomalous “quiescent” spot mode was attained for 20 A and 1.9 mg/s mass flow rate, while when increasing the mass flow rate “nosy” plume mode was reached. Single Langmuir probe traces were analysed using both TSL and OML theories, since the parameter $\xi$ computed based on the TSL results showed that most of the time the probe was in between the limits of the two theories. A weighted plasma number density distribution was calculated departing from the two theories. Since in the single probe traces the electron current, upon processing, did not show a clear trend necessary to compute the electron temperature using OML theory, this parameter was estimated only using the TSL assumption. Double probe results derived using the slope method predicted higher electron temperatures and the profiles displayed no clear trends. The consistency analysis with the numerical simulation data showed a good agreement with the experimental data obtained using the single probe traces, rather than double probe.

Emissive probe data was used in order to provide accurate plasma potential data for the retarding potential analyser measurements. From all the analysis methods for the emissive probe data, the most accurate are the inflection point in the limit of zero emission and the
inflection point of a “warm” but non-emissive probe. However, the single probe TSL slope method estimated plasma potentials close to the ones found using the aforementioned techniques which require more processing time and numerical differentiation. If corrected with the factor of \(1.57\) [13, 54], the plasma potential delivered by the floating emissive probe method agrees with the other results.

The retarding potential analyser was deployed in a radial position with respect to the cathode orifice and was used to provide the ion energy per charge distribution functions for a plume and a spot mode. Broadening of the distribution was observed for the plume mode, proving the fact that high energetic ions are produced in this mode, whereas the spot mode displayed a narrow distribution with the peak shifted towards lower energies. The fact that for this particular cathode the discharge mode can easily change even under the same conditions can be due to complex mechanisms and just the probe data prevent, for the moment, a comprehensive understanding of the situation. More RPA measurements in both radial and axial directions correlated with beam current scans can enlarge the view on the dynamics of the cathode plume and bolster the further design improvements.

Some conclusions can be highlighted regarding the probe measurements for the hollow cathodes. First, the probes should be better designed in order to assure the validity of the thin sheath theory over the entire probed domain. This may imply the construction of several probes with different design configurations that suits well different regions that are to be probed. More emissive probe data can provide a better overview on the different analysis techniques and allow the identification of the most reliable in terms of uncertainties and consistency with numerical simulation data. For the RPA design high importance should be given to the proper grid spacing in order to avoid the space charge situations. This can mean several designs for different plume regions and operating conditions, thus a versatile, simple and compact design that allows interchangeable configurations would be the best solution.

Regarding the Langmuir probe traces post-processing technique, the analysis flow should follow the subsequent logic. Single Langmuir probe traces should be analysed with the TSL slope method in order to have a first estimation of the plasma number density and electron temperature. The parameter \(\xi\) should be then computed and the results can be declared valid or not for the TSL if \(\xi > 10\). Double probe results computed using the slope theory are valid only for the points where single probe results in TSL assumption are valid. If \(\xi < 3\), OML theory is applied. Plasma number density can be approximated from the ion saturation current, while the electron temperature is derived from the electron saturation trend. If the latter cannot be identified, the electron temperature estimation remains the one given by the TSL slope method. If \(\xi\) is in between the two values, then a weighted plasma number density can be calculated as well as the electron temperature, if the OML theory delivered such information. If the plasma potential is known to a high accuracy level, using emissive probes for example, Druyvesteyn electron energy distribution functions can be calculated instead and the plasma parameters can be subsequently subtracted by taking moments of the distributions. When applied correctly the method proves to be robust and provides reliable results [4]. For the emissive probe the most accurate method remains the inflection point in the limit of zero emission, although it requires at least five different low-emission level traces for each probed point [61], long analysis times and numerical differentiation.
CHAPTER 5: External Discharge Plasma Thruster (XPT) plume characterisation

Beside the hollow cathodes plasma plume characterisation, a second important objective of this research was to bolster, for the first time, the numerical simulation and thrust and efficiency measurements for the external discharge plasma thruster (XPT) with insights on the plasma plume parameters. The main particularities of the innovative proof-of-concept low-power fully external discharge plasma thruster (XPT) were depicted in Section 3.4. Hereinafter the results of the plasma plume probing utilising double Langmuir probes are presented.

The experimental session for XPT plume plasma characterisation utilising double Langmuir probes envisaged two main objectives: plasma plume characterization along the thruster axis of symmetry and mapping of the far-field plume plasma. As it can be seen in Figure 5.1 (a), the XPT near-field plume is characterised by four distinct regions, corresponding to the propellant injection slots through the anode plate. This peculiarity has affects on the far-field plasma plume, influencing the spatial distribution of the plasma parameters. Thus, a first set of results depicts the plasma characteristics along the thruster centreline (axis of symmetry) for distances between 13 and 63 mm. The probed locations were presented in Figure 3.9 together with the experimental logic and the operating conditions for the XPT (Table 3.5). Measurements were conducted for three discharge voltages, 150, 200 and 250 V and three anode xenon mass flow rates, 0.48, 0.95 and 1.43 mg/s. During all the cases the cathode mass flow rate was kept constant at 0.29 mg/s. The background pressure was always under $8.95 \times 10^{-3}$ Pa.

Considering the particularity of the near-field plasma, it was of high concern to map the plasma across a wide region on the plume. In order to secure the measurements success the closest locations probed were on a plane situated at 5 mm away the anode exit plane. Numerical simulations results showed that at this distance the plume should be already axisymmetrical and considered to be a far-field plume. Figure 3.10 depicted the locations of the 25 points probed across four planes situated at 5, 8, 13 and 18 mm away from the thruster exit plane. The experimental logic was presented in Section 3.4 together with a review on the thruster operating conditions during this set of measurements. The thruster was operated at 250 V and three anode xenon mass flow rates, as in the previous case. Once again the cathode mass flow rate was kept constant at 0.29 mg/s.

![Figure 5.1: XPT during probing session: (a) front view, (b) side view. The cathode is positioned at 45° with respect to thruster axis and at 150 mm away from thruster centreline.](image-url)
The plasma diagnostic system was set as previously shown in Figure 3.8. The double Langmuir probes were formed of two cylindrical tungsten electrodes with a diameter of Φ0.4 mm and a length of 5 mm, protruding from a double bore 20 mm long ceramic tube. The thruster was operated during all the experiments with the external cathode situated at 45° with respect to thruster axis and at a distance of 150 mm away from thruster centreline. Moreover, the dielectric wall of the thruster had a thickness of 1 mm.

In the following sections the results obtained for the two experimental sets are outlined together with discussions that make parallels with the thrust and efficiency measurements and highlight issues as the influence of the cathode position and magnetic field configuration. In order to prove, to some extent, the validity of the experimentally measured plasma parameters, a consistency analysis with the numerical simulations results is presented. In the end the uncertainties in measured plasma parameters are assessed.

5.1 Plasma parameters variation along the thruster centreline

Double Langmuir probe raw traces were analysed using the slope technique in thin sheath assumption presented in Section 2.2.4, in order to estimate the plasma number density and the electron temperature. Plasma potential was derived using Equation (2.74) and base on the electron temperature estimation and the floating potential. The latter was measured using a voltage probe half-way between the two electrodes of the probe. Hereinafter there are presented the results for the plasma number density, electron temperature and plasma potential variations along the thruster centreline as functions of the discharge voltage and for constant anode mass flow rates. At 0.48 mg/s anode mass flow rate it was impossible to record proper double Langmuir probe traces, thus for this particular operating condition no plasma parameters estimations exist for distances higher than 33 mm away the thruster exit plane. A discussion on the plasma parameters trends can be formulated albeit this shortcoming.

5.1.1 Results

Plasma number density trends are depicted in Figure 5.2 (a-c) for each of the three anode mass flow rates. In order to assure an easy reading of all the following graphics, the error bars are omitted, nevertheless the uncertainty analysis is presented in Section 5.5.

At an anode mass flow rate of 0.48 mg/s the plasma number density undergoes a drop from $6 \cdot 10^{17}/m^3$ at 13 mm, at 250 V to $1.7 \cdot 10^{17}/m^3$ at 33 mm, at 150 V. The profiles seems to conserve both trend and magnitude as the discharge voltage is increased from 150 V to 200 V and then finally to 250 V. As the anode mass flow rate is increased to 0.95 mg/s, the plasma number density increases as well, reaching $1.6 \cdot 10^{18}/m^3$ at 13 mm for 250 V. As the distance from the anode exit plane increases, the density drops at $1.7 \cdot 10^{17}/m^3$ at 63 mm, at 150 V. The plasma density becomes sensitive to the discharge voltage, increasing with an increase of the latter. However, after 43 mm away from the anode plate the plasma number density level remains unchanged with the discharge voltage and for constant anode mass flow rates. When the anode mass flow rate is increased again the plasma number density reaches up to $2.8 \cdot 10^{18}/m^3$ at 13 mm and 250 V, displaying a rapid drop to $2 \cdot 10^{17}/m^3$ at 150 V at 63 mm. Once again, the plasma number density depicts a sensitive behaviour to the change in discharge voltage, keeping an almost value after 43 mm independent of the discharge voltage. The peaks in plasma number density profiles for 200 and 250 V at 1.43 mg/s between 18 and 33 mm are most probably data artefacts induced by plasma perturbations, since in those regions the plasma
number densities and the electron temperatures are high. It is considered that the profiles follow regular decreasing trends as in the other two anode mass flow rates cases.

![Graph showing plasma number density variation along the XPT centreline for different anode mass flow rates.](image)

**Figure 5.2:** Plasma number density variation along the XPT centreline for the anode mass flow rates of (a) 0.48 mg/s, (b) 0.95 mg/s and (c) 1.43 mg/s.

The electron temperature was computed using Equation (2.70) and the profiles along the thruster centreline are presented in Figure 5.3 (a-c) for each anode mass flow rate. The lowest electron temperature range is recorded for a 0.48 mg/s anode mass flow rate. The maximum temperature of 7 eV is reached for 250 V at 13 mm, whilst the minimum corresponds to 150 V at 33 mm of just 2.11 eV. The trends show an increase in the electron temperature as the discharge voltage is increasing. However, the trends preserve this feature up to a distance of 18 mm from the thruster exit plane, together with an overall decrease in the electron temperature. This region is followed by a relatively constant plateau and ends with a second decreasing region. In the latter regions there is a clear trend of the electron temperatures at 200 V to be higher than the ones recorded for 250 V. As the anode mass flow rate is increased at 0.95 mg/s, the electron
temperature reaches a maximum of 8 eV at 13 mm for 250 V. The trends show the same peculiarities as in the previous case, the total average drop in the electron temperature along the 50 mm probed is around 5.5 eV.

Figure 5.3: Electron temperature variation along the XPT centreline for the anode mass flow rates of (a) 0.48 mg/s, (b) 0.95 mg/s and (c) 1.43 mg/s.

For the highest anode mass flow rate, 1.43 mg/s, the electron temperature at 13 mm and 250 V reaches about 11 eV. The electron temperature profiles follow the same trends as for the previous cases, recording a smaller drop within the measured domain of about 4 eV.

The electric potential follows the electron temperature profiles trends and displays an increase with the increase of the anode mass flow rate and/or the increase of the discharge voltage. For 0.48 mg/s and 0.95 mg/s anode mass flow rates the plasma potential peaks a bit over 80 V and drops up to 60 V. It is believed that in the case of 0.48 mg/s the drop is even larger, but the lack of data prevents the knowledge of the level with certainty. When the thruster operated on the highest anode mass flow rate the plasma potential seemed to remain relatively constant along the probed domain around the level of 78 V.
Figure 5.4: Plasma potential variation along the XPT centreline for the anode mass flow rates of (a) 0.48 mg/s, (b) 0.95 mg/s and (c) 1.43 mg/s.

5.1.2 Discussion

A first look at the voltage-current characteristics for the XPT, presented in Figure 3.7 (a), provides useful insights on the dynamic range of the discharge current for different anode mass flow rates. Theoretically, for a certain mass flow rate, the maximum attainable beam current in a Hall thruster is reached when the propellant utilization efficiency approaches unity [25]:

\[ \eta_P = \frac{I_{beam}M}{em} \]  

(5.1)

For the anode mass flow rates utilised during the experiments the maximum beam currents would have been 0.353 A for 0.48 mg/s, 0.698 A for 0.95 mg/s and 1.05 A for 1.43 mg/s. The voltage-current characteristics shows that for the XPT in the range of 150 – 250 V, the discharge current attains the saturation, thus the ionization fraction reaches its maximum value, or is increasing slightly due to changes in electron mobility. During the experiments the discharge current increased with increasing discharge voltage, and in the case of 0.95 and 1.43 mg/s anode
mass flow rates it was higher than the maximum theoretical beam current for the same mass flow rates. This feature can be due to an increase in plasma non-uniformities that can induce high changes in electron mobility, especially in azimuthal direction, which may, in turn, lead to an increase in the discharge current. This may be the effect of a non-uniform propellant injection scheme [36]. If the discharge current increases, the current efficiency can be affected. Moreover, the voltage utilization efficiency and the beam divergence efficiency are very important terms that influence the net thrust, overall efficiency and specific impulse.

At 0.48 mg/s anode mass flow rate it was possible to observe a plasma number density almost insensitive to the discharge voltage increase. At this low mass flow rate the plasma number density has the smallest magnitude inducing a high sheath thickness, since $\lambda_p \propto 1/\sqrt{n_e}$. This affects mainly the plasma potential distribution and the voltage efficiency. As the voltage utilization of an ion is mainly dependent on the location where it is ionised, the portion of the anode potential that the ion sees influences directly its acceleration, thus exhaust velocity. In a thick sheath case the plasma potential undergoes a relatively slow decrease, thus the acceleration of the ions is less efficient and the voltage utilization efficiency is drastically decreased. As expected the thrust variation, depicted in Figure 3.7 (b) and anode efficiency variation in Figure 3.7 (c) display a slow increase as the discharge voltage increase, since the acceleration of the ions is increased, but with the lowest magnitudes due to the poor acceleration efficiency. The thrust remains in a narrow range of 2-3 mN, while the efficiency reaches 12-16% throughout the plateau.

As the anode mass flow rate is increased, the plasma number density increases as well. The anode mass flow rate was doubled from 0.48 to 0.95 mg/s. The plasma number density at 250 V, for example, increased 2.5 times for the axial position of 13 mm. Moreover, the plasma number density becomes sensitive to the change in the discharge voltage, revealing an increasing trend as the discharge voltage is increased. This proves an increase in the propellant utilisation efficiency with increasing discharge voltage. However, the discharge current increases as well with the increasing discharge voltage, over the maximum theoretical beam current for this mass flow rate. This can be translated into a decrease in the current utilization efficiency. Still, the high plasma number densities produce shrinkage of the plasma sheath and a better acceleration of the ions, leading to high voltage utilization efficiencies. In this situation the thrust, anode efficiency and specific impulse undergo steeper increase with the discharge voltage. At 150 V the thrust is almost double the thrust recorded for the same voltage at 0.48 mg/s mass flow rate, while for higher discharge voltage the thrust becomes up to 3 times higher. This is due to the correlation of higher ionization levels and a better acceleration process.

The increase in the mass flow rate of 50% to 1.43 mg/s produces an increase in the plasma number density of about 180% at 250 V and for 13 mm location. Higher plasma number densities are recorded, leading to even thinner plasma sheaths during the different discharge voltage operation. The enhanced Debye shielding leads to better ion acceleration, increasing the voltage utilization efficiency. As the plasma number density tends to drop faster at higher mass flow rates, the current density and the plasma resistivity increase, maintaining relatively constant high plasma potential all along the probed domain, as depicted in Figure 5.3 (c). The anode efficiency and the thrust increase as well as the specific impulse. If the discharge voltage was to be increased over 250 V it is believed that for both 0.95 and 1.43 mg/s anode mass flow rates the thrust, thrust efficiency and specific impulse would have increased even more due to better propellant utilization efficiencies, increase in plasma number density correlated with
enhanced Debye shielding and better ion acceleration. At this mass flow rate the discharge current reaches values up to 50% higher than the maximum theoretical beam current. Once again, plasma instabilities may induce high changes in the electron mobility, increasing in turn the electron fraction of the discharge current. For better overall performances for the XPT the discharge current should be decreased in order to enhance the current utilization efficiency.

The electron temperature profiles showed higher temperatures for higher anode mass flow rates and higher discharge voltages. The trends depict a decrease in the electron temperatures profiles along the probed domain, with higher drops for lower anode mass flow rates. This fact influences the distribution of the plasma potential along the thruster centreline. The plasma potential should remain almost constant along the probed domain and should notice small or no changes with the mass flow rates, being influenced mainly by the discharge voltage.

Taking into account the design particularities of the XPT with the four individual neutral injection slots leading to the formation of four distinct ionization regions, it is probably that the electron temperature profiles along the centreline are influenced by the non-uniformities in the near-field plume plasma and the transition between the near-field and far-field plume. Moreover, the discharge voltage has a major influence on the aspect of the plasma plume. As the voltage is increased the plume seems to be more collimated, leading to a higher beam divergence efficiency that influence in turn the increasing trends for thrust and efficiency.

The innovative design of the XPT and its complex plasma plume make any analysis of the plasma parameters difficult especially if the probed domain is restricted, for instance, just to the thruster centreline. Moreover, the lack of any other previous work on plasma plume diagnostics for this type of fully external discharge Hall thrusters makes the process of data interpretation arduous without comparison terms. To circumvent some of the demerits that this research faces, some more data sets were collected and analysed in order to provide a thoroughly view on the XPT far-field plasma plume dynamics.

5.2  Far-field plasma plume mapping

Several locations across the anode plane were probed at four axial distances from thruster exit plane, 5, 8, 13 and 18 mm, as it was depicted in Figure 3.10. The far-field plume plasma was diagnosed for a discharge voltage of 250 V and three anode mass flow rates of 0.48, 0.95 and 1.43 mg/s with a constant cathode mass flow rate of 0.29 mg/s throughout all the experiments. The probe’s tip had a length of 5 mm, thus the resolution of the measurements is dictated by this parameter.

The following maps for the plasma parameters were obtained by applying a linear interpolation between the data points and only data within 4 mm around the actual probed position is displayed. Maps for the plasma number density, electron temperature and plasma potential are computed for each of the four axial distances and for each of the three anode mass flow rates. In order to make the following discourse more cursive, the following maps are read as the geographical maps, keeping the cardinal directions logic. Thus, the upper part of the graphics is denoted as North (N), the lower part ad South (S), the left-hand side as West (W) and the right-hand side as East (E) together with the intermediate directions N-E, S-W, so on and so forth. The anode plate contour is plotted against each map in order to provide an intuitive reading of the maps. The blue contours define the four regions though which the propellant neutrals are injected, each region having six injection slots distributed across it. Moreover, each graphic has its individual colour bar legend, thus the colour schemes are the same but depicting
different magnitudes from one map to another. The external cathode orifice was placed 45° with respect to thruster axis at \( z = 150 \text{ mm} \) and \( x = 0 \text{ mm} \).

5.2.1 Results

**Plasma number density**

The far-field plasma plume maps representing the plasma number density distribution across the probed domain are depicted in Figure 5.5 and Figure 5.6. At the lowest anode mass flow rate of 0.48 mg/s, Figure 5.5 (a), the cathode mass flow rate of 0.29 mg/s represented a relatively large fraction of the anode mass flow rate, 65%. At 5 mm away from the anode plate the plasma number density displays a peak of \( 1.9 \cdot 10^{18}/\text{m}^3 \) at the point situated at S, decreasing towards the centre of the thruster where it reaches \( 1 \cdot 10^{18}/\text{m}^3 \). Higher plasma densities are observed in the lower part of the map, ranging between \( 3 - 7 \cdot 10^{17}/\text{m}^3 \), whereas the upper region is characterised by lower plasma number densities. At 8 mm away from the thruster exit plane the trend is maintained and high density regions are observed in the lower part of the map, peaking \( 8.5 \cdot 10^{17}/\text{m}^3 \) in the centre of the thruster. While in the lower part the density remains over \( 7 \cdot 10^{17}/\text{m}^3 \), in the upper part it drops under \( 5 \cdot 10^{17}/\text{m}^3 \). The trend is preserved also for the 13 mm and 18 mm axial distances, with a slight shift from S to S-W in the case of 18 mm. At those distances the density peak diminishes to \( 6.5 \cdot 10^{17}/\text{m}^3 \) at 13 mm and \( 4.5 \cdot 10^{17}/\text{m}^3 \) at 18 mm, both recorded in the S part, on the vertical axis of symmetry of the thruster. The low density regions characterise the N part of the maps, with the minimum values recorded around the positions of the upper propellant injection slots.

As the anode mass flow rate is increased to 0.95 mg/s, Figure 5.5 (a) and Figure 5.6 (a), with the cathode mass flow rate as one third the anode mass flow rate, the plasma number density increases as well. The peak of \( 3 \cdot 10^{18}/\text{m}^3 \) is recorded at 5 mm in the S part of the map. At this distance higher plasma number densities are present in the S-W part. At 8 mm the plasma number density peaks \( 2.3 \cdot 10^{18}/\text{m}^3 \) and it can be noticed a smoothing in the density that spans the S part of the map. The situation is preserved at 13 mm and 18 mm, although the peak in plasma number density of \( 1.2 \cdot 10^{18}/\text{m}^3 \) is shifted towards S-E at 18 mm.

The anode mass flow rate was then increased to 1.43 mg/s with the cathode mass flow rate as one fifth of the former. The results, presented in Figure 5.6 (b), show an overall increase in the plasma number density and a slower decrease of the maxima as the axial distance increases. At 5 mm away from the thruster exit plane the plasma number density map reveals two high density regions positioned at E – N-E and W – S-W with the maxima of about \( 3.1 \cdot 10^{18}/\text{m}^3 \) recorded at E and W positions. As the distance increases, a more uniform plasma number density structure is found at 8 mm. At 13 mm and 18 mm away from anode plate the localization of plasma number density depicts the trends found for the other two anode mass flow rate cases, with high density concentrations in the S part of the maps. The maximum of the plasma number density remains high, reaching \( 2.5 \cdot 10^{18}/\text{m}^3 \) at 13 mm in the centre of the map and \( 2 \cdot 10^{18}/\text{m}^3 \) at 18 mm in the S region. The low density regions remain the propellant injection slots.

On the centreline the plasma number density peaks at the axial distance of 8 mm away from anode plate, displaying decreasing trends towards lower and higher axial distances. In the central region, at axial distances smaller than 5 mm the plasma number density drops, since no ionization regions is present next to the anode plate. In the same time, around the positions of the propellant injection slots, the plasma number density should increase as the axial distance is
diminished, since here the ionization and acceleration regions form. Any non-uniformity in the formation of these regions influence the plasma number density distribution downstream.

**Figure 5.5:** Plasma number density maps for XPT operating at 250 V. Maps for the axial positions 5, 8, 13 and 18 mm away from thruster exit plane for 0.48 mg/s xenon anode mass flow rate (a) and at 5 and 8 mm away from thruster exit plane for 0.95 mg/s xenon anode mass flow rate (b). The thruster contour is superimposed to the map. The external cathode orifice is placed 45° with respect to thruster axis at z = 150 mm and x = 0 mm.
Figure 5.6: Plasma number density maps for XPT operating at 250 V. Maps for the axial positions 13 and 18 mm away from thruster exit plane for 0.95 mg/s xenon anode mass flow rate (a) and at 5, 8, 13 and 18 mm away from thruster exit plane for 1.43 mg/s xenon anode mass flow rate (b). The thruster contour is superimposed to the map. The external cathode orifice is placed 45° with respect to thruster axis at z = 150 mm and x = 0 mm.

Electron temperature

Figure 5.7 (a) depicts the electron temperature maps for the case of 0.48 mg/s anode mass flow rate. At 5 mm the electron temperature distribution across the probed domain seem to be uniform with temperatures between 10 and 13 eV, the maximum being attained close to the N-W propellant injection slot. As the distance from the anode plate increases, the electron
temperature displays a decreasing trend, dropping to around 4–5 eV at 18 mm. The relatively uniform distribution at 5 mm becomes more localised at 8 mm, with an increase in the electron temperature in the lower part of the map, while the peak in temperature is reached towards S–W injection slot. As the temperature decreases, the distribution preserves the trends of higher electron temperature in the S – S-W part.

At a higher anode mass flow rate, 0.95 mg/s, the electron temperature showed an increase in magnitude, as depicted in Figure 5.7 (b) and Figure 5.8 (a). Higher electron temperature are recorded in the lower part of the probed domain, with the peak moving from S towards the centre as the distance from the anode plate increases. In the same time a tendency of levelling in temperatures is observed when the distance increase from the thruster exit plane. At 5 mm the electron temperature peak of 13.4 eV is observed at S, while at 18 mm the temperature peak drops to 6 eV, with a “hotter” distribution in S-W region. The “colder” regions remained the ones around the propellant injection slots, as for the previous anode mass flow rate case.

At the highest anode mass flow rate of 1.43 mg/s, Figure 5.8 (b), the electron temperature reaches its maximum of 13.64 eV at 5 mm. At the same distance the electron temperature profile displays a high temperature region along the N-W – S-E, in correlation with the high plasma number density recorded for the same region, at the same conditions. At 8 mm, although the peak in electron temperature of 13 eV is isolated at the N-E propellant injection slot, the distribution shows a relatively uniform trend across the probed domain with temperatures between 7-9 eV. At 13 mm and 18 mm away from the anode plate the electron temperature distributions reveal a relative uniform trend across the domain of interest, especially in the central regions. However, the peaks in electron temperature of 9 eV and 6 eV are still recorded in the lower part with a slight shift from S, at 13 mm to S-W at 18 mm.

**Plasma potential**

An overall view on the spatial distribution of the plasma potential across the measured domain indicates a relatively uniform distribution of this parameter, compared to the distributions for the plasma number density and electron temperature. Moreover, as a general tendency, the peak in plasma potential is recorded at positions close to the propellant injection points, but the position changes with the anode mass flow rate and the distance from the thruster exit plane.

At the lowest anode mass flow rate the level of uniformity is the highest, as presented in Figure 5.9 (a). The electric potential peaks 120 V at 5 mm, with an average of 115 V in the central region. As the distance from the anode plate increases the plasma potential undergoes a drop, reaching an average value of 65 V at 18 mm, for the central regions.

As the anode mass flow rate is increased to 0.95 mg/s, a higher level of non-uniformity is observed in the plasma potential distribution especially at 5 mm and 13 mm positions, shown in Figure 5.9 (b) and Figure 5.10 (a). The electric potential drops from a maximum of 115 V recorded at 5 mm to an average of about 70 V at 18 mm, for the central regions. At this anode mass flow rate it appears the tendency of higher magnitudes for the lower part of the measured domain.

For 1.43 mg/s anode mass flow rate, Figure 5.10 (b), the electric potential displays the same trend at 5 mm as the electron temperature. A region with higher plasma potentials is revealed along a stripe from N-W to S-E propellant injection slots. The potential reaches around 130 V in the central region. The drop in potential towards the N-E region is of about 30 V and towards the S-W region of 60 V. At 8 mm, the central region depicts a uniform electric potential around 85 V,
while the peak of 110 V is attained at the N-E propellant injection slot. The electric potential distribution becomes relatively uniform for the right-hand side of the domain at 13 mm, peaking 90 V at S. At 18 mm the levelling is more accentuated, but still the peaks are recorded in the lower part of the probed domain, while the central region records an average electric potential of 70 V.

Figure 5.7: Electron temperature maps for XPT operating at 250 V. Maps for the axial positions 5, 8, 13 and 18 mm away from thruster exit plane for 0.48 mg/s xenon anode mass flow rate (a) and at 5 and 8 mm away from thruster exit plane for 0.95 mg/s xenon anode mass flow rate (b). The thruster contour is superimposed to the map. The external cathode orifice is placed 45° with respect to thruster axis at z = 150 mm and x = 0 mm.
Figure 5.8: Electron temperature maps for XPT operating at 250 V. Maps for the axial positions 13 and 18 mm away from thruster exit plane for 0.95 mg/s xenon anode mass flow rate (a) and at 5, 8, 13 and 18 mm away from thruster exit plane for 1.43 mg/s xenon anode mass flow rate (b). The thruster contour is superimposed to the map. The external cathode orifice is placed 45° with respect to thruster axis at $z = 150$ mm and $x = 0$ mm.

5.2.2 Discussion

The mapping of the far-field plasma plume of the XPT brings more insights into the dynamics of the plume structure of this particular type of fully external discharge Hal thruster. In order to place the following discussion on a comprehensive basis, one should bear in mind some important aspects regarding the design peculiarities of the XPT and the characteristics of the
operating conditions though which the probe measurements took place. First, the thruster is characterised by a relatively strong magnetic field compared with other conventional Hall thrusters operating at the same power levels [36]. Although the magnetic field strength rapidly drops to only 0.008 T at 18 mm away from anode plate, it reaches up to 0.06 T at 5 mm and 0.025 T at 8 mm. Those levels of the magnetic field strength define a relatively highly magnetised plasma, at least on the first 10 mm away from the anode plate.

**Figure 5.9**: Plasma potential maps for XPT operating at 250 V. Maps for the axial positions 5, 8, 13 and 18 mm away from thruster exit plane for 0.48 mg/s xenon anode mass flow rate and at 5 and 8 mm away from thruster exit plane for 0.95 mg/s xenon anode mass flow rate. The thruster contour is superimposed to the map. The external cathode orifice is placed 45° with respect to thruster axis at z = 150 mm and x = 0 mm.
Figure 5.10: Plasma potential maps for XPT operating at 250 V. Maps for the axial positions 13 and 18 mm away from thruster exit plane for 0.95 mg/s xenon anode mass flow rate (a) and at 5, 8, 13 and 18 mm away from thruster exit plane for 1.43 mg/s xenon anode mass flow rate (b). The thruster contour is superimposed to the map. The external cathode orifice is placed 45° with respect to thruster axis at z = 150 mm and x = 0 mm.

A second influential factor can be the position of the external hollow cathode. Moreover, the cathode mass flow rate can induce some effects on the plume especially when it becomes a high fraction of the anode mass flow rate. Another important aspect is the propellant injection system. The fact that the propellant neutrals are injected through four distinct propellant slots
can play a role in the formation of the near-field plasma plume with consequences on the
dynamics of the far-field plasma plume. In the same time, all the measurements were taken for
250 V discharge voltage which is known for giving the highest thrust and efficiency levels [36].

The plasma number density showed an increasing trend with the anode mass flow rate,
defining higher ionization levels and thus higher thrusts and efficiencies for higher mass flow
rates. For all three levels of anode mass flow rate the number density distributions showed a
tendency of localization in the lower part of the probed domain. This increase in plasma number
density in a specific region of the plume can be translated to shifts of the centre of thrust
towards the denser regions. Since those regions contain higher numbers of ions that are
accelerated though the electrical potential structure, the thrust axis can be influenced. This
phenomenon has to be avoided especially for in-space situations when any modification of the
thrust axis can have major repercussions on the spacecrafts attitude.

Localization of plasma number density can affect the electron conductivity, leading to higher
current densities in a particular region. For closer locations to the anode plate this can produce
harmful effect in the plasma confinement in the high magnetic field regions, leading to a
decrease in the beam divergence efficiency.

The first supposed reason for this induced localization in the plasma number density can be
the location of the external cathode and its mass flow rate. Previous studies showed that the
position of the cathode in a thruster-cathode configuration can influence the plasma parameters
distribution in the far-field plume [64, 65, 70]. Firstly, the plume could be deflected due to the
high cathode mass flow rate, compared to the anode mass flow rate. However, this possibility is
less likely to be the main cause of the issue since the localization of the plasma number density
trend is maintained also for cathode mass flow rates lowered to only one fifth of the anode mass
flow rate. Moreover, the cathode ion and neutral drift velocities are much lower than the
thermal velocities of the electrons and ions inside the plume plasma and the electron drift
velocity. The cathode can induce a plume asymmetry by the creation of an increased ion flux
together with a non-uniform ingestion of the cathode neutrals. This can induce an increase in the
azimuthal ion velocity. An increase in the slow ion flux at the azimuthal position of the cathode
can be observed at high cathode mass flow rates due to the increase in the charge-exchange
collisions that quench the cathode’s emitted ions [70]. The electrons emitted by the cathode can
also create differences in the thruster plasma number density, influencing the ionization rates at
different positions [70]. Nevertheless, all those phenomena should be localised to azimuthal
positions close to the injection point, thus close to the azimuthal position of the cathode (x=0
mm, z=150 mm), but the maps display the contrary.

In this situation it seems that other mechanism can influence the localisation of the plasma
number density within the far-field plume of the XPT. Another explanation for the plasma
number density localization in the far-field plume can be driven from the magnetic field
structure and the propellant injection system. Any azimuthal non-uniformity in the magnetic
field structure correlated with propellant injection non-uniformity can lead to higher ionization
rates localized at specific azimuthal locations across the anode plate. Since on the centreline the
plasma number density peaks around the axial position of 8-10 mm, at this point the plasma
structure formed at each of the four injection propellant slots merge and give place to a
symmetrical plume with a density and electron temperature concentration around the central
region of the thruster. If the injection of the propellant is non-uniform and associated with any
significant variations in the magnetic field structure around a specific azimuthal position across

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the thruster exit plane, the ionization rate can be increased in a certain region. For sure it is very
difficult and not advisable to find just one ground that can endorse the experimental results and
it is the case of a concurrence of various complex mechanisms that can be accounted for the non-
uniform distribution of the plasma number density in the XPT far-field plume.

As the plasma number density, the electron temperature distribution across the probed
domain showed non-uniformities. The tendency was towards more uniform distributions as the
distance from the thruster exit plane increased, but closer to the anode plate the localization of
the electron temperature cannot be considered as a minor effect. In most of the cases, the peaks
in electron temperature were recorded in regions with smaller plasma number density since in
those regions the plasma collisionality regime is decreased. Nevertheless, for all the anode mass
flow rates the localization of the electron temperature was noticed and could have been induced
by a beam electron current concentration, leading to higher Joule heating, or by anomalous
plasma resistivity [25]. Higher electron currents within some regions of the beam, surpassing
the ion currents can have affect on the current efficiency, and therefore on the overall efficiency
and thrust.

The high levels of plasma number density prove once again the fact that high mass utilisation
efficiencies can be attained at this level of discharge voltage and as the anode mass flow rate is
increased. Moreover, the Debye shielding is enhanced for high plasma number densities, and as
the anode mass flow rate is increased the acceleration process is improved, leading to high
voltage utilisation efficiencies and better thrust and efficiency levels.

The electric potential structure within the plume has great influence on the acceleration of
the ions produced in the ionization regions. The plasma potential structure is influenced by the
plasma number density and electron temperature due to the Debye shielding, and preserved
along the far field plume by the same parameters. The configuration of the plasma equipotential
lines can be related to the beam divergence. Although for the 0.48 mg/s anode mass flow rate
the plasma potential distribution if almost uniform across the probed domain, the low level of
ionization hinder an efficient acceleration process due to low plasma number densities and thick
plasma sheath. As the anode mass flow rate is increased, the ionization rate is enhances, the
Debye shielding is improved as well and the acceleration process becomes efficient, although the
plasma potential structure along the far-field plume does not preserve a uniform distribution.

The azimuthal non-uniformities in the plasma number density and electron temperature can
induce variations in the electron conductivity and thus, producing variations in the electron and
ion currents. This can be correlated to breathing mode transitions and lead to the discharge
current oscillations and affecting the current efficiency. Since the ionization and acceleration
regions for the XPT are very close, or even overlapping, and the neutral injection takes place at
very small distance away from those regions, the breathing mode stabilization criterion can be
violated in some particular operating conditions [36].

As stated previously, considering the complexity of the phenomena involved in the creation
of near-field plasma for the XPT, additional research should be conducted in order to account for
the weight of each influential factor. Research on the influence of the cathode position and its
mass flow rate to the plume dynamics can bring more light on the issue. Beam divergence and
beam currents should be also measured and in this way the current efficiency can be computed
departing from experimental data. An optimization of the propellant injection system can lead to
different results and a more comprehensive understanding of the impact of such strong
magnetic fields in the far-field plasma can help to position the discussion on the right track.
5.4 Consistency analysis with experimental results

The experimental results obtained for the discharge voltage of 250 V and the anode xenon mass flow rate of 1.43 mg/s were compared with the numerical simulations results computed for the same XPT operating conditions. The simulation model used is called 2D3V fully kinetic Particle-in-Cell (PIC)/ Direct Simulation Monte Carlo (DSMC), previously presented in several studies [10, 34]. Similar numerical results for the XPT at a discharge voltage of 300 V and anode xenon mass flow rate of 1.36 mg/s were presented in another study [33]. The consistency analysis aims to compare the experimental results obtained for the points situated on the centreline and on the radial direction, from N-E to the centre of the thruster, passing across the propellant injection slot, at 13 mm and 18 mm away from the anode plate with the numerical simulation data obtained at the same positions.

Figure 5.11 depicts the comparison between the experimental results and the numerical simulation data for the plasma number density for the points situated across the centreline. Numerical data was available for axial distances up to 43 mm away from the anode plate. The experimental data was in average 41.7% higher than the numerical simulation results. The difference can be within the uncertainty of the plasma number density, proving that the experimental results are in relative agreement with the numerical simulation data, as it concerns the magnitude, and the displayed trend is similar.

![Figure 5.11: Comparison between measured and simulated plasma number density profiles along the centreline for 250 V and 1.43 mg/s anode xenon mass flow rate. The density axis is in logarithmic scale. The uncertainties are mainly associated to the assumption of stationary plasma and span between ±367% and ±580%.](image1)

![Figure 5.12: Comparison between the measured and simulated electron temperature profiles along the centreline for 250 V and 1.43 mg/s anode xenon mass flow rate. The uncertainties are mainly associated to the time variation of the ion saturation current and span between ±14.9% and ±45.3%.](image2)
For the electron temperature, the comparison is depicted in Figure 5.12. Numerical data was available only up to the axial distance of 30 mm. In this case the difference between the two data sets was in average 22.5%, with the experimental data higher than the numerical simulation results. Again, the difference can be considered as within the uncertainty level for the experimental results, upholding the relative agreement between the two data sets.

Figure 5.13 shows the comparison between the experimental and simulation results for the plasma number density in radial direction at 13 mm and 18 mm away from the anode plate. Once again, the difference between the two data sets is about 42.6% for both 13 mm and 18 mm cases. Moreover, the trends are similar in this case too. On the other hand the situation changes for the electron temperature in radial directions, as presented in Figure 5.14. The simulated data revealed higher temperatures with 93% in average over the one obtained during the experiments, for both 13 mm and 18 mm cases. This difference cannot be considered within the uncertainty limits for this experimental method, although the trends seem to be similar.

**Figure 5.13:** Comparison between measured and simulated plasma number density radial profiles for 250 V and 1.43 mg/s anode xenon mass flow rate at 13 and 18 mm away from anode exit plane. The density axis is in logarithmic scale. The uncertainties are mainly associated to the assumption of stationary plasma and are considered to be ±570% for 13 mm and ±689% for 18 mm.

**Figure 5.14:** Comparison between measured and simulated electron temperature radial profiles for 250 V and 1.43 mg/s anode xenon mass flow rate at 13 and 18 mm away from anode exit plane. Uncertainties of ±47.8% are mainly associated to the time variation of the ion saturation current.

However, the results do not fully validate or invalidate the experimental data since it was shown that the electron temperature and the plasma number density spatial distributions...
displayed high non-uniformities across the probed domain. The code assumes a symmetrical far-field plume, thus the results can be different than the experimental one to a high extent, while any match in magnitude or trends can be due to pure hazard in the case of the radial direction locations. In the case of the centreline, the trends show the agreement between the experiment and numerical simulation, while the magnitudes should not be fully trusted.

5.5 Uncertainties in measured plasma parameters

The position accuracy provided by the positioning system is considered to be about ±0.5 mm since the maximum strokes on the x- and y-axis are around 70 mm and on z-axis around 30 mm, thus less than half the maximum stroke of the device. Small uncertainties in position of the probe’s tip can be induced by a limited amount of oscillations caused by the 20 mm long probe tube. Moreover, the spatial resolution in the case of far-field plume mapping offered by the probe is 5 mm, equal to the electrodes length.

The uncertainties associated with the double probe slope method can be computed departing from the description presented in Section 4.1.5 and Equation (4.5). For the plasma number density, ±10% is assumed for the effects induced by the area difference between the electrodes, the finite ion temperature and direction of plasma flow [53]. The main uncertainty remains due to the non-stationary effects of the plasma. In the case of the XPT those effects can have a large influence on the measured results since the plume is composed of heavy ions with relatively large drift velocities, compared to the ones obtained for hollow cathodes. The ion axial velocities were computed departing from the specific impulse at different anode mass flow rates and discharge voltages [36] and approximating this velocity to the exhaust velocity derived from Equation (1.3). Equation (4.5) was then used to approximate the ratio between the plasma number density in flowing plasma and in stationary plasma, using the electron temperatures along the centreline, presented in Figure 5.3 (a-c) and for the plume mapping, depicted in Figure 5.7 and Figure 5.8. In the case of the centreline probed locations, Equation (4.5) revealed a factor varying from 2.74 to 7.5, increasing with the increase in the discharge voltage and the increase in the anode mass flow rate. Thus, the measurement error for the plasma number density for the centreline results, presented in Figure 5.2 (a-c), spans between 164% and 650%, as summarised in Table 5.1. Moreover, the error are increasing with the distance away from the anode plate, the minimum is estimated for the closest point, 13 mm, whereas the maximum error is estimated for the 63 mm location. In the case of the plume mapping results of plasma number density the measurement error seemed to increase with the distance away from the anode plate and with the increase in the anode mass flow rate. The measurement error spans between 84% and 700%, as presented in Table 5.2.

Table 5.1: Measurement errors for the plasma number density results along the thruster centreline for discharge voltage of 150, 200 and 250 V and 0.48, 0.95 and 1.43 mg/s anode mass flow rates.

<table>
<thead>
<tr>
<th>$V_d$ (V)</th>
<th>150</th>
<th>200</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_a$ (mg/s)</td>
<td>Error (%)</td>
<td>Error (%)</td>
<td>Error (%)</td>
</tr>
<tr>
<td>0.48</td>
<td>5000</td>
<td>164</td>
<td>313</td>
</tr>
<tr>
<td>0.95</td>
<td>6500</td>
<td>218</td>
<td>472</td>
</tr>
<tr>
<td>1.43</td>
<td>7100</td>
<td>235</td>
<td>421</td>
</tr>
</tbody>
</table>

The uncertainty in the electron temperature measurements brought by the double probe slope method is composed by several errors defined hereinafter. The fitting error for the
retarding field region is considered to be maximum ±1%, while ±5% is accounted for the fitting error for the ion saturation regions [53]. The main factor that induces high uncertainties in the electron temperature remains the time fluctuation of the ion saturation current. This error is measured as a coefficient of variation defined as the ratio between the standard deviation and the mean value of the ion saturation current observed for several curves taken at the same location in the plume. For the locations along the thruster centreline, the coefficients of variation are depicted in Figure 5.15 (a-c). This coefficient varies between 0.0152 and 0.455.

Table 5.2: Measurement errors for the plasma number density far-field plume mapping results for 250 V and 0.48, 0.95 and 1.43 mg/s anode mass flow rates.

<table>
<thead>
<tr>
<th>$V_d$ (V)</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_a$ (mg/s)</td>
<td>0.48</td>
</tr>
<tr>
<td>U (m/s)</td>
<td>8000</td>
</tr>
<tr>
<td>Error (%)</td>
<td>Error (%)</td>
</tr>
<tr>
<td>y (mm)</td>
<td>min</td>
</tr>
<tr>
<td>5</td>
<td>74</td>
</tr>
<tr>
<td>8</td>
<td>139</td>
</tr>
<tr>
<td>13</td>
<td>171</td>
</tr>
<tr>
<td>18</td>
<td>218</td>
</tr>
</tbody>
</table>

Figure 5.15: Coefficient of variation for the electron temperature measurement error along the thruster centreline for discharge voltage of 150, 200 and 250 V and (a) 0.48 mg/s, (b) 0.95 mg/s and (c) 1.43 mg/s anode mass flow rates.

Thus, the measurement error in electron temperature presented in Figure 5.3 (a-c) spans from 7.5% to 51.5%. For the far-field plume mapping electron temperature measurements, the
coefficient of variation across the probed domain remained between 0.0367 and 0.418 for all the anode mass flow rates and for distances between 5 mm and 18 mm away from the thruster exit plane. This implies that the error in electron temperature measurements depicted in the maps from Figure 5.7 and 5.8 is between 9.67% and 47.8%.

The high uncertainty in the plasma number density do not hinder the discussion about the plasma parameters trends along the thruster centreline or about the spatial distributions presented in the far-field plasma plume mapping section. Moreover, a discussion about how the measurement results influence the XPT overall performances is still assured despite the uncertainty levels.

### 5.6 Summary of XPT plasma plume characterization

Double Langmuir probe measurements were conducted for the low-voltage external discharge plasma thruster (XPT). Various positions along the thruster centreline were probed together with several other locations across horizontal planes located at different axial distances away from thruster exit plane. The latter measurements were used to derive far-field plasma plume maps for the main plasma parameters: electron temperature, plasma number density and plasma potential. Results for several anode mass flow rates and discharge voltages were discussed and related to the overall performances of the XPT.

The high plasma densities observed for high discharge voltages and anode mass flow rates proved the increase in ionization rate, and thus mass utilization efficiency, as the former parameters are increased. The results were then extrapolated to the XPT thrust, efficiency and specific impulse levels and an agreement was found between the probe measurements and the thruster performances. High levels of electron temperature and relatively uniform plasma potential structures revealed high voltage utilization efficiencies at high discharge voltages and high anode mass flow rates. The increase in the discharge current with higher discharge voltages can be due to plasma non-uniformities caused by a non-uniform propellant injection scheme, affecting in turn the current efficiency at high discharge voltages and anode mass flow rates.

Far-field plume plasma maps revealed non-uniform plasma number density and electron temperature distributions across the entire probed domain. The increase in electron temperature in some regions within the plume can be influenced by Joule heating or anomalous resistivity situations. The localization of the plasma number density in the lower part of the thruster exit plane can induce a shift in the thrust axis as well as a localized increase in the electron current density that can affect the overall current efficiency. Plasma confinement in the regions close to the anode plate can be also influenced by the localization in plasma number density. The relatively uniform plasma potential structures observed along the plume at 250 V indicates good acceleration efficiency and thus, voltage utilization efficiencies.

Plasma non-uniformities observed in the far-field plume maps are the output of complex mechanisms that influence mainly the formation of the near-field ionization and acceleration regions. The propellant neutrals injection scheme can be an influential factor in this equation. The position of the cathode may affect as well the dynamics of the plume plasma and further research should be undertaken on this issue. Any non-uniformity in the azimuthal magnetic field can also contribute to the localization tendency of the plasma parameters in the far-field plume. Further research on plume currents and plume divergence can provide a more comprehensive vision upon the effect of the plume dynamics on the XPT performances.
Although the plasma number density computed using the slope method for the double Langmuir probe working in thin sheath assumption bears high uncertainties associated with the stationary plasma assumption, the trends observed in the previous analysis can be considered independent of this issue, allowing the ensued discussions. The analysis technique for the uncertainty in the case of plasma number density should be revised in order to assure appropriate estimations of this specific plasma parameter.

The spatial resolution can be also increased by reducing the electrodes length without the risk of the end effects, since the plasma drift velocities are high enough in the case of XPT. The reliability of the results can be increased by reducing the probed domain and increasing the number of traces collected for the same location.
CHAPTER 6: General conclusions and suggested future work

6.1 General conclusions

The present thesis is the result of a research period at the Institute of Space and Astronautical Science of the Japanese Aerospace Exploration Agency, ISAS/JAXA, at Funaki Laboratory of the Department of Space Flight Systems that followed the path of plume plasma diagnostics for space electric propulsion drives. During the experimental sessions two high-current hollow cathodes and an innovative prototype of a low-voltage fully external discharge plasma thruster (XPT) had their plasma plumes diagnosed using electrostatic probes and retarding potential analyser (RPA).

The main objectives of the research were firstly to develop and test plume plasma diagnostic instruments based on specific design requirements dictated by the physics of plasma-probe interactions, to conduct preliminary testing for the Langmuir probes for a commercial cathode plume, followed by a more extensive plume diagnostics for a laboratory model hollow cathode and in the end to map the far-field plasma plume of the erosion free low-voltage external discharge plasma thruster (XPT). All the objectives were fully fulfilled and the results and ensued discussions were displayed in the current thesis.

The first step was to understand the physical processes underlying the electric propulsion, in particular the hollow cathodes and Hall Effect thrusters operation. Plasma diagnostic techniques were briefly presented with an emphasis on the main techniques utilised during the research: Langmuir probes, emissive probes and retarding potential analyser. Short descriptions of the design characteristics and operational logic were presented in Chapter 1.

This introduction was followed by an exhaustive presentation of the fundamental principles of plasma diagnostics and thruster propulsion in Chapter 2. The discourse departed from the plasma kinetic theory with a presentation of energy distribution functions as the non-Maxwellian, Druyvesteyn distribution function. Studies \[2, 4, 21\] showed that in low pressure discharge plasmas the electron energy distribution tend to be non-Maxwellian, therefore Druyvesteyn electron energy distribution function proved to better describe the reality. Next, the theory of charge collection by cylindrical probes was described step by step in order to uphold the physical principles of the emissive and Langmuir probes.

Chapter 2 contains a comprehensive description of the main analysis techniques for Langmuir probes, emissive probes and RPA current-voltage (I-V) traces. In the case of single Langmuir probe traces several analysis techniques can be employed in order to obtain plasma parameters as the electron temperature, plasma number density and plasma potential. An important parameter that describes the interaction between the probe electrode and the surrounding plasma is the ratio between the probe’s electrode radius and the plasma Debye length, defined as parameter \(\xi\) in Equation (2.23). Chen \[8, 9\] suggested that if \(\xi > 10\), the probe works in the thin sheath limited regime (TSL), whilst for \(\xi < 3\), orbital motion limited regime (OML) is established. Therefore several analysis techniques ensue based on this main consideration. Both TSL and OML theories assume collisionless plasmas, therefore the Knudsen number, defined by Equation (2.24) should be larger than 10 \[4\]. A Double Langmuir probe does not disturb the plasma as much as a single probe since the system “floats” and follows any change in plasma potential, having no plasma ground. Both the slope and fitting methods presented in the thesis assume TSL case, thus the probe has to be properly designed to assure
this working regime. Druyvesteyn method for electron energy distribution functions provides an analysis tool independent of the sheath thickness. Although more robust and offering an extensive view on the dynamics of the plume, the method is more complex involving numerical differentiation and accurate knowledge of the plasma potential, thus is prone to errors if applied incorrectly. The method implies that the second derivative of the probe current with respect to the bias voltage is proportional to the electron energy distribution function.

Emissive probes are used for accurate estimation of the plasma potential. Several analysis techniques were described as the floating emissive probe method, the inflection point in the limit of zero emission or the inflection point of a “warm” but non-emissive probe method.

RPA operation is based on several biased grids that repel the electrons (electron repeller) and provide a low pass filter (ion repeller) for ions with certain energies, while a collector provides the current due to the ion collection. First derivative of the probe current with respect to the ion repeller grid bias voltage is proportional to the ion energy per charge distribution function. The peak of the distribution provides information about the most probable potential for the ions and if the local plasma potential is known the local average ion acceleration potential can be found and subsequently the ion drift velocity can be derived.

Chapter 3 presented a comprehensive exhibit of the experimental facilities and different auxiliary systems used during the plume plasma diagnostic experiments. The design of the Langmuir probes, emissive probes and RPA was outlined together with the experiments logic and schematics and probes' electrical diagrams. Langmuir probes and emissive probes were built using double bore ceramic tubes and tungsten wires and filaments of different sizes. The RPA was designed at Tsukizaki Laboratory of ISAS/JAXA, while the grids made of tungsten metallic meshes were changed. Probe design was driven by the assumptions on plasma properties within the plumes of the hollow cathodes and XPT. A positioning system was used in order to provide three-axis motion for the probes, while the latter were mounted on the former using several mechanical interfaces. The most delicate issues related to the electrostatic probes and retarding potential analyser remains the contamination of the electrodes and grids. This situation induces uncertainties in the plasma parameters estimation. In the case of XPT it was possible to clean the probe in the high ion density regions, while for the hollow cathodes this process is not effective. Regular change of the probes or cleaning using very fine sandpaper can improve the results accuracy.

A commercial and laboratory model high-current hollow cathodes were tested in the Cathode Vacuum Chamber at ISAS/JAXA and their far-field plasma plumes were diagnosed utilising single and double Langmuir probes, emissive probe and RPA. Several positions along the cathodes centreline were probed, while for the laboratory model cathode the RPA was deployed in radial direction. Single probe traces were analysed by both TSL and OML assumptions, according to the value of the parameter $\xi$. Double probe traces were processed using the slope and the fitting method, both assuming TSL case. In general, TSL gave better approximation for the plasma parameters and in agreement with the numerical simulation data, for example in the case of the laboratory model cathode. Druyvesteyn method applied for single Langmuir traces in the case of the commercial cathode predicted lower plasma number densities and higher electron temperatures than the other methods. This is because the method places more weight on the high energy electrons in the distribution tails, assuming collisionless plasmas associated with low plasma number densities [4]. For the laboratory model cathode a plateau between 20 A and 30 A was found to be characterised by mode transition instabilities, leading to changes in the
discharge mode for the same operation conditions. Emissive probe data provided plasma potential estimation for the RPA. Broad ion energy per charge distribution function (IEDF) was derived for the “noisy” plume mode, revealing the presence of high energetic ions that may induce high keeper erosion rates, as suggested in previous studies [23, 24, 49]. At higher mass flow rate the “quiescent” spot mode was attained. The IEDF was narrow with a peak shifted towards lower energies.

The third objective was to characterise for the first time the plasma plume of the innovative proof-of-concept low-power fully external discharge plasma thruster (XPT). The propellant injection scheme is composed of four individual neutral injection slots, leading to the formation of four ionization and acceleration regions close to the anode plate. This peculiarity has great importance in the formation of near-field plasma, and therefore on the far-field plasma. Several positions along the thruster centreline were probed using double Langmuir probes in order to avoid the effects of the high magnetic field. The thruster was operated at constant discharge voltage of 150, 200 and 250 V and anode xenon mass flow rate was varied between 0.48 and 1.43 mg/s with a constant cathode mass flow rate of 0.29 mg/s. The results for the plasma number density, electron temperature and plasma potential underlain the XPT performances results obtained prior the diagnostics experiments. The discharge current was observed to increase for higher voltages mainly due to plasma non-uniformities in azimuthal direction induced by the propellant neutrals injection scheme [36]. Such behaviour can affect the current efficiency of the XPT.

Far-field plume plasma parameters maps were derived from the data obtained for several positions across horizontal planes situated at four axial distances from the anode plate. The discharge voltage was kept at 250 V, while the anode mass flow rate was varied between 0.48 and 1.43 mg/s with a constant cathode mass flow rate of 0.29 mg/s. Localization in the plasma number density and electron temperature was observed, while the plasma potential structure remained relatively uniform especially at high axial distances. Localization of plasma number density can induce a shift in the thrust axis and may affect the plasma confinement close to the anode plate, leading to a decrease in beam divergence efficiency. Electron temperature localization, due to Joule heating or anomalous plasma resistivity, can induce variations in the beam current, affecting in turn the current efficiency of the thruster, and therefore the overall performance. Relatively uniform plasma potential structures prove the enhanced acceleration mechanism at high discharge voltages and the increase in thrust and efficiency of the thruster. However, the localization trends for both plasma number density and electron temperature can be the outcome of complex mechanisms. The non-uniform propellant injection scheme is the first factor supposed to lead to the results obtained. Any non-uniformity in the azimuthal magnetic field can induce localized, higher ionization rates, while the position of the cathode and its mass flow rate especially when it becomes an important fraction of the anode mass flow rate can contribute to the overall observed trends. A consistency analysis was conducted based on numerical simulation data for both centreline and plume results. At 250 V and 1.43 mg/s anode mass flow rate, the centreline experimental results were in agreement with the numerical simulation data for both trends and magnitude. The experimental results in radial direction showed the same trends as the simulated results, while the magnitudes were in agreement only for the plasma number density. However, taking into account the high non-uniformity of the plasma number density and electron temperature observed in the plume maps for those parameters, the trends should be trusted just in the case of centreline results, while the
magnitudes and the trends for the radial direction cannot be used for a reliable comparison, since the numerical simulation data was obtained assuming plume symmetry.

Uncertainty analysis for the plumes measured parameters were conducted for the cathodes and the XPT results. The uncertainty in measured plasma number density and electron temperature for the cathodes experiments were between 15 and 50% for TSL and OML theories for single probe traces and slope and fitting methods for double probe traces. Druyvesteyn EEDF method comes with an uncertainty of 8% in the estimation of the distribution function, while the exact uncertainty in plasma number density and electron temperature it was rather difficult to estimate. Emissive probe methods come with different uncertainties related to the local electron temperature. The floating probe method has the highest uncertainty of 1-2\(T_e\) [13, 54], while the inflection point in the limit of zero emission provides results with an accuracy within \(\pm 0.1T_e\) [61]. For the RPA, the uncertainty in providing the most probable potential for a specific IEDF was assumed to be \(\pm 50\%\) of the half width at half maximum (HWHM) of the IEDF, as previously suggested by Brown [6]. In the case of XPT the uncertainty in measuring the plasma number density, assuming flowing plasma conditions and the method described by Oshio [53], spanned between 74\% and 700\%. In the case of the uncertainty in measured electron temperature, the time variation of the ion saturation current for the double probe traces revealed error levels under 50\% throughout the probed domain.

### 6.2 Suggested future work

Based on the discourse on the results of the present research, some suggestions were identified in order to improve the quality of plasma plume diagnostics for hollow cathodes and XPT as well as some considerations on performance improvement for XPT.

Both Langmuir and emissive probes design should be improved in order to assure the validity of thin sheath theory for the data processing. Therefore, several probes should be designed according to the plasma parameters estimations for the plume locations of interest. Since the slope method for double Langmuir probe traces assumes a thin sheath limited regime, the probe’s electrodes should be designed according to the results obtained with a single probe having the same electrode area as the double probe. This first estimation of plasma parameters leads to the calculation of the parameter \(\xi\), which dictates the size of the double probe electrodes so that it works in TSL regime, assuring the validity of the results.

RPA design should be optimised according to the plasma parameters at the locations the probe is deployed. Therefore previous Langmuir probe data should estimate the plasma number density and the electron temperature. Suitable grid spacing is crucial for the RPA proper functioning, since it assures the avoidance of space charge situations.

TSL slope method provides reliable estimations for the plasma parameters when properly applied and if the probe actually worked in this regime. Druyvesteyn EEDF is the most robust method and gives the most detailed description of plasma dynamics, if correctly applied. In order to reduce the uncertainties, the plasma potential should be known accurately beforehand, thus emissive probe measurements should be conducted as well. The uncertainties can be further reduced if the differentiation scheme is applied to a curve fit to the raw Langmuir traces, eliminating the propagation and enhancement of noise in the second derivative results.

In the case of hollow cathodes it is of great importance to understand the mode transition and how to quench the high energetic ions produced during plume mode. This latter mechanism
should not, if possible, imply the increase in the cathode mass flow rate or the increase in the input power. More RPA measurements in both radial and axial directions for different operating conditions can provide a more comprehensive understanding on plasma plume dynamics. The results should be associated with accurate estimations of the plasma potential structure across the plume, thus more emissive probe measurements should be conducted. Plasma number density and electron temperature measurements with Langmuir probes can be used to improve numerical simulation codes and to estimate the plasma parameters in the near-field plume.

The measurements conducted for the external discharge plasma thruster (XPT) can be considered as ground basis for future measurements, a comparison term and a database to support ongoing computational codes. The probe's electrodes length can be decreased in order to increase the spatial resolution. To avoid the effects induced by electrodes contamination, an external cleaning system can be attached to the probe. Neutral injection towards the tip of the probe when the probe is in high plasma density regions can provide a highly effective cleaning. First and foremost, the propellant neutrals injection scheme should be revised and improved in order to reduce the plasma non-uniformities and obtain better current efficiencies by reducing the discharge current for the same discharge voltage level. Moreover, further research on how the position of the cathode and its mass flow rate influences the plume plasma parameters can be conducted together with a better understanding on how the strong magnetic field influences the spatial distribution of the plasma parameters in the plume at small axial distances away from the anode plate. For a comprehensive understanding of the plume dynamics, beam current should be characterised and information about plume divergence are needed to account for their effects on the overall performances. Faraday probe measurements of the beam current and RPA measurements can enlarge the knowledge on current utilization efficiency, voltage utilization efficiency and beam divergence efficiency, while mass utilization efficiency can be derived from the thrust measurements, knowing all the other efficiencies.
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