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No-Go of Quantized General Relativity

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Abstract

In this article we show: i) The impossibility of actively "quantizing" general relativity. ii) That the key to quantum gravity - a theory for "deducing" the macroscopic theory of general relativity - is to explain, from a fundamental microscopic theory, why the inertial mass is proportional to the gravitational mass, $m_i/m_g = const$, in the classical limit.

The equivalence principle, the universality of free-fall, is the foundational stone of general relativity.

Consider

$$m_i a = G\frac{m_g M}{r^2}, \tag{1}$$

where m_i is the inertial mass ("resistance" to acceleration), m_g passive gravitational mass (gravitational "test-charge") and M active gravitational mass (gravitational "source-charge"), giving for the acceleration

$$a = G\frac{m_g}{m_i}\frac{M}{r^2}. \tag{2}$$

So *if*

$$\frac{m_g}{m_i} = const, \tag{3}$$

then

$$a = const\frac{M}{r^2}, \tag{4}$$

resulting in that *all* masses free-fall equally if only gravity is at work (as already Galileo observed).

Usually one assumes that the proportionality constant is unity, $m_i = m_g$, but one could also have *e.g.* $m_i \gg m_g$, with a corresponding up-scaling of the numerical value of Newton's gravitational constant G. This would, in the classical case, "explain" why gravity seems to be such a weak interaction.

In any case, this "equivalence principle" is what makes it possible to "transform away" gravity at each point by using small (infinitesimal) local reference frames free-falling throughout space. It is when all these infinitely many, infinitely small, inertial frames (inside which special relativity, *i.e.* flat Minkowski spacetime, is valid) are smoothly glued together into a *global* reference frame that the curved metric of general relativity arises.

Free-falling "test-particles" (of presumably negligible mass/energy) then are assumed to move along geodesic lines in the curved spacetime required by the equivalence principle, giving all predictions of classical general relativity.

As the (static) gravitational potential around a spherical mass (*e.g.* a microscopic "earth-satellite" system) is *mathematically* identical to the (static) Coulomb potential around a charge (*e.g.* proton-electron system), by using the substitution

$$\frac{e^2}{4\pi\epsilon_0} \to G m_g M, \tag{5}$$

we get the quantum gravitational case from the familiar (and analytically solvable) Hydrogen case, complete with wavefunctions ψ_{nlm}, just by changing from the Bohr-radius (a_0) to the gravitational Bohr-radius (b_0) [1],

$$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{e^2 m_i} \to b_0 = \frac{\hbar^2}{G m_i m_g M}. \tag{6}$$

Fortunately, dynamical effects are negligible to a very high degree of accuracy, both for Hydrogen and, especially, for the gravitational case.

We are now at a position to calculate the average acceleration of specific quantum states from Ehrenfest's theorem [2].

$$\frac{d}{dt}\langle \vec{p} \rangle = \langle \vec{F} \rangle = -\langle \nabla V \rangle. \tag{7}$$

In our case we have

$$\vec{p} = m_i \vec{a}, \tag{8}$$

and

$$V = -\frac{G m_g M}{r}, \tag{9}$$

giving the magnitude of the expectation (= mean) value of the acceleration, using the known wave functions

$$\langle a \rangle = \langle \frac{m_g}{m_i} \frac{GM}{r^2} \rangle = \frac{2}{(2l+1)n^3} \frac{G^3 m_i m_g^3 M^3}{\hbar^4}. \tag{10}$$

Even though $\langle a \rangle$ has the right dimensions (ms^{-2}) we see immediately that:

- The inertial and gravitational masses do *not* cancel in the quantum case (as they do in the classical case), *even* if they are proportional (or the same).
- Different quantum states (n, l) have *different* average accelerations, even for the n^2 number of degenerate levels (equal n , different l).

Thus, quantum mechanical averages do *not* adhere to the equivalence principle.

For individual quantum states it is even worse, as (the standard form of) quantum mechanics does not even attempt to describe individual quantum behavior. It is simply taken to be random and unknowable in principle (*e.g.* decay of *single* neutron, hit of *single* photon on detector screen in double-slit experiment, etc). Also, as individual quantum particles have no trajectories (due to $[\vec{x}, \vec{p}] \neq \vec{0}$) acceleration cannot even be defined for them. (Neither can geodesics.) It is somewhat different in deterministic, non-local hidden-variable theories, like de Broglie-Bohm [3], [4], but they are on the other hand manifestly non-covariant due to their non-local features, thus violating general relativity in other ways. Then again, we *know* from Bell's theorem [5] and its tests [6], [7], [8], and many more recent ones, that nature does have a non-local aspect to it, so covariance might only apply to (some of our) observables. It may turn out that the unknown, and searched for, fundamental theory is explicitly non-local (perhaps in some unobservable background) and relativistic covariance/invariance only applies at our level[1] - perhaps even due to some trait of human consciousness itself.

The equivalence principle is thus senseless in the microscopic regime, removing the fundamental backbone of general relativity and invalidating it at the quantum level. The classical principle of using arbitrarily small freely falling reference frames is simply rendered inapplicable by the more fundamental quantum principle. There is therefore no chance of "quantizing" general relativity into some kind of quantized metric/spacetime. If a general theory of quantum gravity exists it must be of a fundamentally different kind, for instance some scenario where gravity emerges at the macroscopic level but is non-existent microscopically. The key to quantum gravity would then *not* be to "quantize" the classical theory, *but* to understand and explain why $m_i \propto m_g$ at the macroscopic level. If that could be accomplished all of classical general relativity would follow, even if it would turn out that gravity is simply absent on the microscopic level. The only result we need is that $m_i/m_g = const$, as $F \propto r^{-2}$ is a consequence of isotropy in 3D-space.

[1] "...that is certainly the cheapest solution. Behind the apparent Lorentz invariance of the phenomena, there is a deeper level which is not Lorentz invariant." [9]

If gravity does not "see" the smallest scales (because it does not *exist* there, being a purely macroscopic "coarse-graining" effect needed to produce the equivalence principle) all its UV-divergencies, singularities, etc, would simply evaporate/disappear as mathematical artifacts unrelated/unapplicable to the physical world, as they arise only as distances go to zero. Also, the cosmological constant problem, where the quantum vacuum fluctuation contribution to the cosmological constant deviates from the cosmologically inferred one with a factor $\sim 10^{120}$, would be removed. Moreover, even if there may by dynamical ripples in spacetime, *i.e.* gravitational waves, on the classical level, they need not be composed of gravitational quanta ("gravitons").

For very large n and $l = l_{max} = n - 1$ (closely connected to Kepler's law, see [1]), we get that the average expectation value $\langle r \rangle$, and the individual most probable distance $\tilde{r}$ (where the probability density peaks) approach each other[2], and

$$\langle a \rangle \to \frac{m_g}{m_i} \frac{GM}{\langle r \rangle^2} = \frac{m_g}{m_i} \frac{GM}{\tilde{r}^2}, \tag{11}$$

giving the correct correspondence limit and the retrieval of the equivalence principle at the (semi-)classical level. This is also the regime of conducted experimental tests, such as the gravitational interferometric COW-experiment (Colella-Overhauser-Werner) [10], and the experiment claiming to having "measured" a quantum gravitational state for the first time [11], [12], [13], [14]. See also the discussion of the latter experiment in [1]. These use neutrons (m_i, m_g) near the surface of the earth, in the gravitational field of the whole earth (M), resulting in enormous $n \simeq 8 \times 10^{17}$ ($b_0 \simeq 10^{-29}$ m) [1], giving no insight into, nor any test of, "true" quantum gravity, as this would require small to moderate n, far from the correspondence limit. For neutrons near the earth surface, the relative uncertainty is

$$\frac{\Delta r}{\langle r \rangle} = \frac{\Delta r}{\tilde{r}} = \frac{\sqrt{\langle r^2 \rangle - \langle r \rangle^2}}{\langle r \rangle} = (2n)^{-1/2} \simeq 7.9 \times 10^{-10}, \tag{12}$$

and as in this case $\langle r \rangle = \tilde{r} \simeq R_\oplus \simeq 6.4 \times 10^6$ m, the positional uncertainty is only $\Delta r \simeq 5 \times 10^{-3}$ m, making trajectories/geodesics valid in a coarse-grained sense. Also, eq. (10) gives $\langle a \rangle \simeq 9.8\, ms^{-2}$. So [10] does not provide "...the first verification of the principle of equivalence in the quantum limit" as erroneously stated in that article, but rather in the correspondence limit.

To conclude, as shown above it is futile to try to actively "quantize" the classical theory of general relativity, as quantum mechanics violates its very foundation.

[2] That $\langle r \rangle \neq \tilde{r}$ generally is due to the probability density being asymmetric around $\tilde{r}$. Also, the individual radial probability densities have $n - l$ local maxima. Only for $l = l_{max}$ has it got a unique maximum.

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