Pearls From a Lost City: The Lvov School of Mathematics

by Roman Duda

translated from the 2007 Polish original

by Daniel Davies

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HISTORY OF MATHEMATICS, VOL. 40, PROVIDENCE, RI: AMERICAN MATHEMATICAL SOCIETY, 2014, XII + 231 PP., ISBN: 978-1-4704-1076-6

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In 1795, Poland was partitioned among Austria, Russia, and Prussia, and only at the end of 1918 did it reemerge as an independent country. There followed a period of remarkable development in science. Great mathematical centers were created both in Warsaw (with Sierpiński, Mazurkiewicz, and Kuratowski) and in Lvov (with Banach, Steinhaus, and Mazur). In Cracow, there were Zaremba and Żorawski, but unfortunately they worked separately so that no mathematical school was formed there before World War II.

The Lvov School of Mathematics, the topic of the book reviewed here, was a group of mathematicians in the Polish city of Lvov (then known as Lwów, today Lviv in Ukrainian) who worked together in the period from 1920 to 1945 under the leadership of Stefan Banach and Hugo Steinhaus, and often convened in the *Scottish Café* (Kawiarnia Szkocka) to discuss mathematical problems. Though without a notable previous mathematical tradition preceding it, the Lvov school made major contributions to what is now called functional analysis (analysis of functionals, or infinite-dimensional analysis). The results and names of Banach, Kac, Kuratowski, Mazur, Orlicz, Saks, Schauder, Sierpiński, Steinhaus, and Ulam, among others, are well known in the field.

The book aims to present the Lvov school, identifying and describing the time and place of its main historical events, the key individuals and their followers, the school's greatest achievements, and their significance. An attempt is also made to assess the influence of the school on mathematics in Poland as well as around the world. The book is divided into seven parts: I. Background, II. The Golden Age: Individuals and Community, III. The Golden Age: Achievements, IV. Oblivion, V. Historical Significance, VI. List of Lvov Mathematicians, VII. Bibliographies.

In the first part we find a short historical discussion of the University and the Polytechnic in Lvov, Polish mathematics at the turn of the twentieth century, the role of Sierpiński who was at the Lvov University in the period 1908–1914, the importance of Janiszewski's program at the Warsaw University (1915–1920), and the overall state of mathematics around 1920, focusing on those areas where Polish mathematics would later distinguish itself so highly during in the interwar years.

In the second part, Duda presents mathematicians in Lvov after World War I. He shows that the social life of the Lvov mathematicians was intimately connected with their mathematics. The “Golden Age” was characterized by humor, friendship, and playful competitions. Mathematical discussions often took place in cafés near the university, first in “Roma” and a year later in the “Scottish Café.” The atmosphere in Lvov was one of enthusiastic collaboration. People were interested in each other's mathematical problems. The overall enthusiasm was perhaps an expression of Polish joy of the country's newly recovered independence. The *Stammtisch* of Banach, Mazur, and Ulam was the most important one in the Scottish Café. Mathematical conversations led to exciting new theorems that were often written only on the tabletops. In 1935 Banach's wife, Łucja, bought a thick notebook to be kept in the Scottish Café for the mathematicians to record their problems, conjectures, comments, and solutions. The notebook was available to any mathematician who asked for it at the café. This legendary book became known as *The Scottish Book*. Stefan Banach wrote the first problem on July 17, 1935, and Hugo Steinhaus the last one on May 31, 1941. Prizes were announced for the solution of some of the problems: a cup of coffee, one small beer, five small beers, a bottle of wine, a bottle of wine of measure > 0 , lunch at the “Dorothy,” one kilo of bacon, or a live goose! The goose was offered for the solution of Problem 153, an approximation property posed by Mazur in 1936. In 1972 Per Enflo constructed a separable Banach space without the approximation property, that is, a space without a Schauder basis. He presented a lecture on the solution in Warsaw and Mazur duly handed him a “live goose” as he had promised in 1936. A Polish newspaper published a photo of the moment but unfortunately we don't find it in the book.

Today the original Scottish Book is in the custody of the Banach family; a copy is in the Library of the Mathematical Institute of the Polish Academy of Sciences in Warsaw. In May 1979, the history of the Scottish Book was the subject of a conference at the North Texas University in Denton; Ulam, Kac, Erdős, and Zygmund were among the participants. *The Scottish Book: Mathematics from the Scottish Café*, published in 1981, is an outgrowth of the conference. I have contributed comments to a positive solution of Problem 87 posed by Banach (pp. 161–170). Moreover, jointly with A. Plichko and V. Mykhaylyuk we solved Problem 188 posed by Eidelheit, and published in [MMP11]. Duda fails to mention that the last of the anecdotes at the end of Chapter 10 is about Juliusz Schauder.

Visitors to Lvov included well-known figures from abroad such as Zermelo, von Neumann, Lebesgue, Emil Borel, Montel, and Leray. The last chapter in Part II is a kaleidoscope of mathematical types in Lvov. Every mathematician is presented as an individual with specific gifts, interests, and viewpoints. There are photographs of Banach, Steinhaus, Mazur, Kuratowski, Ulam, and Schauder; Łomnicki, Stożek, and Auerbach are presented without photos. Kac, Kaczmarz, Orlicz, Ruziewicz, Schreier, and Żyliński are described briefly but in my opinion more could have been said of them.

Part III, “The Golden Age,” describes the most important achievements of the Lvov School of Mathematics in functional analysis, probability theory, measure theory, and partial-differential equations.

Banach’s doctoral thesis contains the first axiomatic definition of a “Banach space.” We note that Łomnicki was not Banach’s advisor (p. 85). Banach’s famous fixed-point theorem (also known as Banach’s contraction principle) was also part of his thesis.¹ The next few chapters present achievements of the 1920s in probability theory, measure theory, and operator theory. Highlights include the interesting work of Steinhaus and Kac in developing probability without adopting Kolmogorov’s axiomatics, and Banach’s work around the well-known *measure problem* that led to the Banach-Tarski paradox in 1924 and continued with Banach’s interest in measure-theoretic questions for the rest of his life. The chapter on “Operator Theory in the 1920s” presents the history and formulation of the Hahn-Banach theorem on extension of functionals in the real and the complex case, which is one of the fundamental theorems in functional analysis (completeness is not needed in the formulation). The original Hahn-Banach theorem was proved only for the real case (Hahn 1927, Banach 1929). The complex case was proved in 1938 by Bohnenblust-Sobczyk and Soukhomlinov. More on the history of the Hahn-Banach extension theorem can be found in Pietsch’s book [Pi07, pp. 39–40].

The chapter on Banach’s monograph: S. Banach, *Théorie des opérations linéaires*, Monografie Matematyczne, Tom 1, Warszawa 1932, is titled “Polishing the Pearls,” whereas the Polish original title was “Framing the Pearl.” This is Banach’s most influential work with fundamental theorems in functional analysis, including the Hahn-Banach extension theorem (real case), the Banach-Steinhaus theorem on uniform boundedness, the open mapping theorem, and the Banach fixed-point theorem. His notation and terminology were widely adopted. This is followed by a chapter on “Operator Theory in the 1930s: The Dazzle of Pearls,” which outlines and describes the main achievements of the Lvov School in operator theory: spaces (Banach spaces, Orlicz spaces, Schreier spaces, B_0 -spaces, and L^∞ -space as a dual to L^1), three fundamental theorems (Hahn-Banach theorem on extending functionals, Banach-Steinhaus theorem on uniform boundedness, and the open mapping theorem), and other interesting results. The theory of Banach spaces initiated and substantially developed in Lvov is now referred to as classical Banach space theory. The next golden age was the period 1950–1980 when Polish mathematicians would also make significant contributions. References for this material include A. Pełczyński’s plenary address at the Warsaw ICM in 1983, and the books by Lindenstrauss and Tzafriri (1977, 1979) and Pietsch [Pi07].

The third part ends with a discussion of a number of topics, for which the Lvov School did not have enough time or energy, nor information, on the more peripheral subjects its members studied. Examples of the former include nonlinear functional analysis (with Juliusz Schauder’s results such as the fixed-point theorem in Banach spaces, open invariance theorem, Leray-Schauder topological degree, and Banach and Mazur-Orlicz polynomial operators between Banach spaces), the theory of partial differential equations (Schauder’s results from 1932–1937, including Schauder’s a priori estimates concerning the regularity of solutions: 1934, 1937), and the theory of linear topological spaces (Mazur’s locally convex spaces from 1935, B_0 -spaces of Mazur and Orlicz).

The fourth part, “Oblivion,” concerns the destruction of the Lvov School. The chapter titles outline these tragic years: “Ukrainization the Soviet Way (1939–1941),” “The German Occupation (1941–1944),” and “The Expulsion of Poles (1945–1946).” The time when it was possible to concentrate on mathematics ended on September 1, 1939, with the German onslaught. On September 22, 1939, Germany ceded Lvov to the Soviet Union as agreed in the Molotov-Ribbentrop Pact of August 23, 1939. The Soviets closed down both of the mathematical departments in Lvov (at the Jan Kazimierz University and at Lvov Polytechnic), and in November that same year replaced them with the Ukrainian University, renaming them on January 8, 1940, as the Ivan Franko University (for a major Ukrainian intellectual) and the Ukrainian Polytechnical Institute. Mathematics didn’t suffer much at first, with Banach remaining a dean of the Department of Physics and Mathematics (the only Polish dean at the Ukrainian University) with M. Zarickij as the vice dean. Academic visits of Soviet scientists in Lvov were relatively frequent, and several mathematicians from Lvov visited Moscow and Kiev. Even so, life was very far from normal and Soviet propaganda with socialist ideology was the political priority. There were endless meetings and reschedulings.

The German occupation of Lvov started one week after the launch of Operation Barbarossa and it lasted three years (from June 30, 1941, until July 27, 1944). In July 1941 several professors were arrested and executed, including the mathematicians K. Bartel, A. Łomnicki, S. Ruziewicz, W. Stożek, and K. Vetulani. The Lvov colleges, including the university and the polytechnic, were closed by the German authorities. Only elementary schools continued, and from spring 1942 a two-year State Commercial School in Polish were opened instead of universities; a few Lvov mathematicians found employment there. Several professors had to feed the lice in Professor Weigl’s Bacteriological Institute, including Banach, Knaster, and Orlicz.² The following Lvov mathematicians did not survive the German occupation (1941–1944): H. Auerbach, K. Bartel, M. Eidelheit,

¹Renato Caccioppoli observed in 1930 that Banach’s proof works for complete metric spaces. Therefore, at least in Italy and Russia, the result is known as the Banach-Caccioppoli fixed-point theorem.

²Rudolf Stefan Weigl (1883–1957) was a Polish biologist and inventor of the first effective vaccine against epidemic typhus. He is remembered for his employment and protection of Polish intellectuals, Jews, and members of the Polish underground by employing them as lice-feeders, among other things, as was the case of Stefan Banach. His vaccines were smuggled into ghettos in Lvov and Warsaw, saving countless lives, until the Institute was shut down by the Soviet Union following their 1944 anti-German offensive. More information can be found in [A14].

J. Hercberg, M. Jacob, A. Łomnicki, S. Ruziewicz, J. Schauder, J. Schreier, L. Sternbach, W. Stożek, K. Vetulani, and M. Wojdysławski. In the sixth chapter of this part, “List of Lvov mathematicians,” there is a collection of 55 “Mathematicians Associated with Lvov” together with their short CVs, in alphabetical order. Phrases such as “shot by the Germans” recur throughout this tragic list.

On July 27, 1944 the Soviet Army, with the help of the Polish Home Army, liberated Lvov. The colleges started functioning again—the university opened in September and the Polytechnic in October 1944. In October 1944, S. Mazur was summoned to Moscow, where he was informed about the new Polish borders and about the plan to resettle Poles. In the aftermath of the war, Lvov together with large areas of Eastern Poland were annexed by the Soviet empire. From April 1945 to May 1946, all surviving Poles in the area, including the rest of the living mathematicians, were expelled from their home city. Banach died in Lvov on August 31, 1945, and was buried at the Łyczakowski Cemetery in Lvov at the Riedel family tomb. And so the Lvov School of Mathematics ceased to exist.

Finally, in the last three parts of the book, Duda considers the historical significance of the Lvov School, presents a list of mathematicians associated with Lvov, and furnishes a three-part bibliography. He points out, giving several examples, that many important results obtained in Lvov were either published too late or not at all. He also provides examples of weak points of the Lvov school following the Pełczyński-Semadeni article of 1969. I mention two of them:

- (1) discussion of real Banach spaces at the expense of spaces defined over complex numbers;
- (2) concentration on bounded linear operators at the expense of unbounded ones.

Despite the weaknesses, the Lvov School of Mathematics nonetheless was a first-rate concentration of talent, and its legacy occupies a permanent place in the history of mathematics. Banach’s name above all will remain permanently in mathematics.

Like any book, this one is also not free of misprints or errors. I compiled a comprehensive list but have not published it yet. For further reading about mathematicians and mathematics in Lvov, see [JM11] and [Ma81]. A more complete list (including Polish literature) appears in my MR3222779 review.

Many journal articles have been devoted to various aspects of mathematics in Lvov or to biographies of Lvov mathematicians, but Duda’s book is the first comprehensive exposition. The first original Polish edition of the book [*Lwowska Szkoła Matematyczna*, Wydawnictwo Uniwersytetu Wrocławskiego, Wrocław, 2007] generated much interest in Poland, and went on to become Poland’s Best Academic Book of 2008. I believe that the English version will also be successful. The second edition of Duda’s Polish version of the book [Du14] was published in 2014. Unfortunately, the second edition is out of print and I cannot report the type of modifications that were made. In summary, I conclude that Duda’s book is a must for everyone interested in the history of functional analysis or in the history of mathematics in Poland.

I thank Tushar Das, University of Wisconsin, and Margaret Stawiska-Friedland, *Mathematical Reviews*, for helpful suggestions.

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