Modal Analysis, Dynamic Properties and Horizontal Stabilisation of Timber Buildings

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Timber Structures
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Expression of gratitude

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Abstract

Engineers face new challenges as taller timber buildings are constructed. According to Eurocode 1-4, both horizontal deformations from static wind and acceleration levels shall be limited. Due to the low self-weight of wood, dynamic vibrations and acceleration levels can cause problems. The current knowledge in the field is limited and there is a need for increasing the understanding of dynamic properties in tall timber buildings. This research project has been a collaboration between Luleå Technical University and Sweco Structures AB, where the author has gained practical experience as a designer in parallel to the research studies.

The purpose of this research is to understand and describe the dynamic behaviour of tall timber buildings using FE-simulations, studying their dynamic properties, and comparing acceleration levels to comfort criteria. By varying different parameters, dynamic properties have been studied and compared with assumptions and recommendations in Eurocode 1-4.

In this study, buildings with cross-laminated timber panels (CLT) have been studied but also post-and-beam systems with trusses. Depending on the shape, layout and materials of the building, the dynamic properties of the building will vary: natural frequency, mode shape, modal mass, and modal stiffness. To assess the comfort of the building, the standard ISO 10137 has been used evaluating the natural frequency of the building and its peak acceleration. Simulations have been performed using finite element (FE) software where modal analyses have been performed. Over 250 simulations have been performed in this study.

Adding mass reduces the natural frequency and the acceleration level of the building, which is an appropriate measure if the building has a frequency below 1 Hz. Increased stiffness increases the natural frequency and reduces the acceleration level, which is suitable for buildings with a natural frequency over 2 Hz.

The empirical expression $f = \frac{46}{h}$ should be used with caution as it is based on measurements of concrete and steel buildings. The recommendation is to perform FE simulations until the empirical knowledge base is sufficient for timber buildings.

The placement of the stabilizing system is important for creating a balanced (symmetrical) system resulting in pure translation modes. Eurocode 1-4 presupposes 2D modes in the plane while asymmetry can create diagonal and even torsional modes, which Eurocode 1-4 cannot handle. Openings and asymmetry in the floor plan affect the dynamic properties of the building. The assumption that the building can be modelled as a homogeneous beam where the mass is evenly distributed can result in an over- or underestimation of the equivalent mass, which in turn can lead to an underestimation of the acceleration level, around 20% - 30%. It is
recommended that the equivalent mass is calculated from FE generated modal mass and mode shapes.

Acceleration levels vary over the building height depending on the mode shape. Timber buildings with a slenderness <3.9 have more or less a pure shear mode and with increasing height it shifts to a linear mode. For timber buildings, it is recommended to use the generated mode shape from FE simulations, and not those prescribed in Eurocode 1-4 as these can underestimate the acceleration levels, around 30%.
Sammanfattning


Syftet med forskningen är att förstå och beskriva det dynamiska beteendet hos flervåningshus i trä genom simulering, studera trähusens dynamiska egenskaper och jämföra accelerationsnivåer mot komfortkrav. Genom att variera olika parametrar har dynamiska egenskaper studerats och jämförts med antaganden och rekommendationer i Eurokod 1-4.

I denna studie har byggnader med korslimmade skivor (KL-trä) studerats men också pelarbalk system med fackverk. Beroende på byggnadens form, planlösning och material kommer byggnadens dynamiska egenskaper variera: egenfrekvens, modform, modalmasse och modalstylvhet. För att bedöma byggnadens komfort har standarden ISO 10137 använts som utgår från byggnadens egenfrekvens och maxacceleration. Simuleringar har utförts med hjälp av finita element (FE) program där modalanalys har använts. Över 250 simuleringar har utförts i denna studie.

Att addera massa reducerar byggnadens egenfrekvens och accelerationsnivå vilket är en lämplig åtgärd om byggnaden har en lägsta egenfrekvens under 1 Hz. Ökad styvhet ökar byggnadens egenfrekvens och reducerar accelerationsnivån vilket lämpar sig för byggnader med en egenfrekvens över 2 Hz.

Den empiriska formeln $f = \frac{46}{h}$ ska användas med försiktighet då den baseras på mätningar av betong- och stålbyggnader. Rekommendationen är att utföra FE-simuleringar tills den empiriska kunskapsbasen är tillräckligt stor för trähus.

Placeringen av det stabiliserande systemet är viktigt för att skapa ett balanserat (symmetriskt) system med rena translationsmoder. Eurokod 1-4 förutsätter 2D moder i planet medan asymmetri i byggnaden kan skapa diagonal och även vridmoder, vilket Eurokod 1-4 ej kan hantera. Öppningar och asymmetri i planlösningen påverkar byggnadens dynamiska egenskaper. Antagandet i Eurokod 1-4 om att byggnaden kan ses som en homogen balk där massan är jämst fördelad kan ge över- eller underskattning av den ekvivalenta massan, vilket i sin tur kan leda till en underskattning av accelerationsnivåerna, omkring 20%-30%.

Accelerationsnivåerna varierar över byggnadens höjd beroende på modformen. Trähus med en slankhet $< 3.9$ har mer eller mindre skjuvmod och med ökad höjd övergår den till en linjär modform. För trähus rekommenderas att använda en genererad modform från FE-simulering,
inte de modförmor som finns föreskrivna i Eurokod 1-4 då dessa kan leda till en underskattning av accelerationsnivåerna omkring 30%.
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Introduction

Timber construction has grown in popularity during recent years and the competition of building the tallest timber building in the world is on. In 2015 Treet was completed with a height of 49 m (14 storeys) and is together with Brock Commons, Canada with a reported height of 53 m the two tallest (2018) residential timber buildings in the world (Malo, et al., 2016, Fast, et al., 2016). Treet has timber truss systems to stabilise the structure and additional concrete slabs to increase the mass (Malo, et al., 2016). Brock Commons is a hybrid system where two concrete cores are used to stabilise the structure, timber columns are used for the vertical load-carrying system and CLT slabs with concrete topping are used as floor elements (Fast, et al., 2016). The contenders are Mjøstårnet in Brumunddal Norway (81 m, 18 storeys) and HoHo Tower in Vienna, Austria (84 m, 24 storeys), both under construction (Abrahamsen, 2017, Woschitz & Zotter, 2017).

There are different types of structural systems used for timber buildings; light-frame, massive timber, and post-and-beam systems. Light-frame systems consist of a timber frame with sheathing connected to the frame by nails or screws. Gypsum is often used as the sheathing material, but plywood and particle board are also popular. Massive timber systems are usually based on cross-laminated timber (CLT) panels. Glulam columns and beams are common in post-and-beam systems. To stabilise the buildings for horizontal loads, there are different types of stabilising systems. Both light-frame and massive timber systems are stabilised through wall and floor diaphragms. For post-and-beam systems, diagonal bracing or moment resting connections can be used.

In the design phase, both ultimate and serviceability limit states must be satisfied. For serviceability limit state (SLS), requirements on deformations and displacements, vibrations and cracks are set to ensure the function of the building elements (Lüchinger, 1996). According to Eurocode 1-4 (2005), two requirements in serviceability limit state need to be satisfied: maximum horizontal displacements from static along-wind load and accelerations from the along-wind load. Due to the low mass of timber, compared with concrete and steel, the structural system needs to be anchored to the foundation to resist potential uplift forces from horizontal wind forces. Wind load is by nature dynamic and varies over time. Acceleration from wind may be significant and cause discomfort for inhabitants.

Important dynamic characteristics of a tall building are the natural frequency, the mode shape, and the damping. Theoretical, empirical or numerical methods can be used to predict the natural frequency and mode shape (Chopra, 2012). Stiffness, mass and damping of a building are parameters that affect its dynamical behaviour. The stiffness and mass are represented by structural material properties, the geometrical placement of the structural system, and the floor layout. Other parameters that may affect the dynamic properties of a structural system are the stiffness in connections, the additional mass from non-structural elements, and live load placement. Damping reduces the magnitude of motion and dissipates energy from
structural vibrations. Passive damping systems have fixed properties while active damping relies on active mechanisms. Damping originating from structural materials is a passive damper and the damping value differs depending on the structural system (Ali & Moon, 2007).

In Eurocode 1-4 (2005), an expression for estimating the natural frequency for buildings with a height over 50 m can be found. Similar expressions can be found in other building codes (Johann, et al., 2015). The mode shape of the building can be pure bending, pure shear or a combination of both. Depending on the structural system and layout, equations for estimating the mode shape can be found in Eurocode 1-4 (2005). The natural frequency and mode shape are used when analysing the acceleration levels to evaluate the comfort.

There are several ISO standards and building codes which present requirements on acceleration levels (ISO 6897, 1984) (SS-ISO 10137, 2008) (Tamura, et al., 2004). The comfort criteria are based on users’ perception of motion. There are several mechanisms in the sensory system connected to human sensitivity i.e. the vestibular, visual and auditory sensory systems (Burton, et al., 2006). Acceleration levels should be limited due to discomfort that can even cause nausea.

Both in Eurocode 1-4 (2005) and the Swedish national annex EKS 10 (BFS 2015:6 EKS 10, 2015) equations for calculating the along-wind acceleration are presented. Alternative methods may be found in other national annexes and building codes, but the basic phenomena remain the same (Kwon & Kareem, 2013).

The current engineering knowledge concerning dynamic behaviour of tall timber buildings is limited and there is a need for increasing the understanding in the area. From an interview study made by the author, 9 respondents with experience in timber structures were interviewed regarding serviceability limit state issues (Näslund, 2015). Several of the respondents pointed out that dynamic properties will be a challenge when designing higher timber buildings. Many of the respondents asked questions on how the analysis should be performed, in what situations dynamic properties in a timber building need to be considered, and what comfort criteria should be used in the design. This thesis contributes with answers to and understanding of these questions.

**Purpose**

The main purpose of this research is to understand and describe the dynamic behaviour of buildings with a timber structure by performing dynamic simulations of multi-storey timber buildings, study their dynamic behaviour, and compare it to comfort criteria. Secondly, by varying different parameters dynamic properties are studied and analysed against assumptions and recommendations in Eurocode 1-4 (2005). Finally, to expand the knowledge and propose guidelines for the design process of tall timber buildings.
**Scope**

The assumptions in this study are Swedish conditions for wind load and material properties from products available on the Swedish market in concordance with European standards. A fictitious floor plan has been used in the study and it is based on a common building type (a tower block) and timber building technologies used in the Nordic countries. Simulations have been performed by using the finite element modelling software Autodesk ® Robot™ Structural Analysis. To numerically predict the natural frequencies, mode shapes and associated dynamic properties, modal analysis has been used. The modal analysis employs the stiffness and the mass of the structure to predict the basic dynamic properties, but it does not simulate an actual wind load case as in a time history analysis. Empirical formulas to predict the natural frequency and mode shape have been studied and compared with the numerical results. For the acceleration estimations, the Swedish annex EKS 10 (2015) has been used together with complementary equations in Eurocode 1-4 (2005). The acceleration levels have been evaluated against ISO 10137 (2008). Measurements of existing timber buildings and numerical predictions of timber buildings have been summarised and compared with the results in this research. Parallel to the research, the author has worked as a structural engineer at Sweco Structures AB. The work has included conceptual design of high-rise timber buildings, which has rendered the author knowledge and understanding of the topic and how the research problem should be formulated.

**Papers**

This thesis is based on the following papers:
Edskär, I previously Näslund, I.
Lidelöw, H. previously Johnsson, H.

I  *Wind-induced vibrations in timber buildings – parameter study of CLT residential structures*
   Edskär, I. & Lidelöw, H.

II  *Dynamic properties of cross-laminated timber and timber truss building systems*
    Edskär, I. & Lidelöw, H.
    Engineering Structures, submitted July 2018, minor revision December 2018

III  *Dynamic properties of timber buildings – the effects of openings*
     Edskär, I. & Lidelöw, H.
     European Journal of Wood and Wood Products, submitted December 2018

IV  *Dynamic properties of timber buildings – the effects of asymmetrical floor plans*
    Edskär, I. & Lidelöw, H.
    European Journal of Wood and Wood Products, submitted December 2018
V  *Horizontal displacements in medium-rise timber buildings: basic FE modeling in serviceability limit state*
  Näslund, I. & Johnsson, H.
  Conference - RILEM - Materials and Joints in Timber Structures - Recent Developments of Technology (2013)

VI  *Stiffness of sheathing-to-framing connections in timber shear walls: in serviceability limit state*
  Näslund, I. & Lidelöw, H.
  Conference - World Conference on Timber Engineering 2014 (2014)

The author’s contribution in all papers has been: collecting the theoretical background and data, performing numerical simulations, generating and analysing the result, and writing the manuscript including drawing the figures. The co-author contributed with comments and advice on the analysis, interpretation of results, and minor writing efforts. In *Paper I*, the co-author had a more extended role with the set-up of the studied parameters.

**Additional publications**

Two technical reports have been published within the research project:

*Engineers’ views on serviceability in timber buildings*
  Näslund, I.

*Finite element modelling of high-rise timber buildings: dynamic analysis*
  Edskär, I.
Theoretical background

Light timber-frame structures are often stabilised using diaphragm action in the sheathing fastened to the studs. However, the stiffness and strength of a light timber-frame does not permit buildings higher than approximately 8 storeys. When aiming for taller buildings massive timber building systems or post-and-beam structures are useful, sometimes combined with concrete into hybrid structures.

Massive timber building systems can use cross-laminated timber (CLT) panels to stabilise the building through wall and floor diaphragms. For post-and-beam building system, diagonal bracing systems can be used to stabilise the structure. Floor plans can have different geometries, but mid- and high-rise buildings are organised around the elevator shafts and point block buildings are often used. The shaft can be placed in the central or peripheral parts of the floor plan and can form part of the stabilising system, Figure 1a and b. Ali and Moon (2007) categorise the structural systems for tall buildings as interior or exterior structures depending on the lateral system. An interior structure would be to stabilise the building through making use of diaphragm action in elevator shafts, while an exterior system would utilise rigid frames, diaphragm action or trusses in walls on the perimeter of the floor plan. To avoid torsion, the stabilising system needs at least two constituents with a lever arm between them e.g. two shafts or one shaft and a strong wall, Figure 1c and d. Using a single shaft would imply that an exterior system is needed to obtain enough torsional stiffness (Lorentsen, 1985).

Figure 1 Stabilising system a) Tower block, shaft in the middle b) shaft in the peripheral parts c) two shafts d) shaft and a strong wall. Z-direction is out-of-plane
Serviceability limit state

The function of a building in serviceability limit state and thus the comfort for the user require limiting large deformations and uncomfortable vibrations. Two serviceability requirements need to be satisfied according to Eurocode 1-4 (2005): maximum along-wind horizontal displacement and the characteristic standard deviation, $\sigma_z(z)$, of acceleration along-wind at building height $z$.

The interest of wind as phenomena increased in the 1930s and 1960s due to the skyscraper boom. During that time, the development of building materials made progress. Due to increased material stiffness, slender building elements could be used which in turn resulted in decreased mass in the building. There was also a reduction in damping because heavy masonry elements became out of fashion and welding and pre-stressing became more widely used. Due to these developments, the dynamic behaviour of tall buildings become the limiting factor for constructing taller and taller buildings in the 1960s (Davenport, 2002).

Wind load is by nature dynamic and varies over time and a rule of thumb is that for structures with their lowest natural frequencies at 1 Hz or below, the resonance response from wind may be significant (Holmes, 2001). Buildings with natural frequencies around 1 Hz and lower may thus cause discomfort for the user of the building. This rule of thumb applies to tall steel and concrete buildings and may not be valid for timber buildings since the typical range or the measured natural frequency of contemporary timber and hybrid buildings is between 1.0 Hz to 4.0 Hz, see Table 1 in the section Dynamic properties of timber buildings.

Horizontal static displacement

An equation for the static wind load is presented in Eurocode 1-4 (2005) and in the Swedish national annex EKS 10 (2015). The external wind pressure is given by:

$$w_e = q_p(z)c_{pe}$$

(1)

Where $q_p(z)$ is the peak velocity pressure and $c_{pe}$ is the pressure coefficient for external pressure. The Swedish national annex (ibid) present method for calculate the peak velocity pressure. A fifty-year return period should be used. There is no limit established for static displacement in Eurocode 5 (2009) or Eurocode 1-4 (2005). In an earlier version of Eurocode 5 (2009) the maximum horizontal displacement was set to H/300 where H is the height. The recommendation in the German design code at the time was H/500 (Källsner & Girhammar, 2008). There are uncertainties around what criteria should be used and engineers often use “best practices”. Some engineers use H/500 as the global criteria and H/300 or H/250 as local criteria for horizontal displacement (Näslund, 2015). Limits for horizontal displacement are important to avoid cracking of building elements, but could also play a role in ensuring enough stiffness to decrease problems with discomfort.
Acceleration

In Eurocode 1-4 (2005) two methods are presented for calculating the standard deviation of the along-wind acceleration, \( \sigma_X(z) \), Annex B and Annex C. Alternative methods may be found in national annexes. The method applied in this research was the Swedish national annex, EKS 10 (2015). The method presented in EKS 10 (ibid) is based on the old Swedish snow and wind load code BSV 97 (BSV 97, 1997), which in turn is based on amongst others the old ISO 4354 (ISO 4354, 1990) and the old Eurocode 1 (ENV 1991-2-4, 1995). Equations for the standard deviation of the along-wind acceleration presented in EKS 10 (2015) and in Eurocode 1-4 (2005) are quite similar. Some differences can be found between the spectral densities used for describing wind turbulence. EKS 10 (2015) uses the von Karman model and Eurocode 1-4 (2005) uses Solari (BSV 97, 1997) (Lungu, et al., 1996). The basic wind velocity for the calculations in this research is measured by SMHI, the Swedish Meteorological and Hydrological Institute (SMHI, u.d.), and the equations in the Swedish national annex, EKS 10 (2015), are adapted to fit the measured wind velocity value. Any combination of equations and wind velocities that are calibrated to each other can be used and the results should be similar. The standard deviation of acceleration is given by Eq. (2) (BFS 2015:6 EKS 10, 2015):

\[
\sigma_X(z) = \frac{3 \cdot I_v(h) \cdot R \cdot q_m(h) \cdot b \cdot c_f \cdot \phi_{1,x}(z)}{m_e}, \quad \text{(2)}
\]

\( I_v(h) \) is the turbulence intensity at height \( h \), \( R \) is the square root of the resonant response, \( q_m(h) \) is the mean pressure at height \( h \), \( b \) is the width of the structure, \( c_f \) is the force coefficient, \( \phi_{1,x}(z) \) is the mode shape value at height \( z \), \( m_e \) is the equivalent mass per unit length, \( z \) is the height above the ground, and \( h \) is the height of the structure. The equation for acceleration, Eq.(2), assumes two-dimensional modes.

The limitation of acceleration levels in different standards (further described under Comfort section) are based on the r.m.s (root-mean square) acceleration value or the peak acceleration value. The return period of the wind in the standards varies between 1-10 years. The peak acceleration is obtained by multiplying the standard deviation of acceleration, \( \sigma_X(z) \), with a peak factor Eq. (3):

\[
a_{\text{peak}} = \sigma_X(z) k_p \quad \text{(3)}
\]

The peak factor is defined as the ratio between the maximum values of the fluctuating part of the response to the standard deviation of acceleration and is given by Eq.(4) (BFS 2015:6 EKS 10, 2015):

\[
k_p = \sqrt{2 \cdot \ln(v \cdot T)} + \frac{0.6}{\sqrt{2 \cdot \ln(v \cdot T)}} \quad \text{(4)}
\]
υ is the up-crossing frequency, T is the average time for the mean wind velocity, T = 600 seconds. The up-crossing frequency is based on the natural frequency of the evaluated structure Eq.(5):

\[ v = n_{1,X} \frac{R}{\sqrt{B^2 + R^2}} \]  \hspace{1cm} (5)

\( n_{1,X} \) is the natural frequency of the structure, which later on will be denoted with \( f \), \( R \) is the resonance response and \( B \) is the background response, further details refers to (BFS 2015:6 EKS 10, 2015) and (SS-EN 1991-1-4, 2005).

To consider different return periods, the following equation is presented in EKS 10 (2015):

\[ v_{Ta} = 0.75v_{50} \sqrt{1 - 0.2\ln\left(-\ln\left(1 - \frac{1}{T_a}\right)\right)} \]  \hspace{1cm} (6)

Where \( T_a \) is the return period and \( v_{50} \) is the characteristic basic wind velocity for a return period of 50 year. Eq. (6) cannot be used for a return period of one year since it is not valid for \( T_a = 1 \). For a return period of one year ISO 6897 (1984) suggests that the acceleration level should be taken as 0.72 times the acceleration level for a five year return period or a two year period can be used, which will give the same results. Malo, et al., (2016) used 0.73 times the basic wind velocity for a return period of 50 years. For a return period of two years a factor of 0.78 should be used for the basic wind velocity.

Dynamic properties

Buildings are often approximated as a uniform cantilever beam, fixed to the foundation when evaluating the natural frequency, Figure 2. To predict the natural frequency of a building theoretical models, empirical formulae, or numerical methods can be used. The natural frequency of the building can e.g. be calculated by approximated equations presented in Eurocode 1-4 (2005), textbooks on dynamics e.g. (Chopra, 2012) or by modal analysis in finite element (FE) software. Parameters that must be given are material properties such as stiffness and mass. The layout of the floor plan and the height of the building will also affect the natural frequency.

Figure 2 Cantilever beam a) distributed mass b) lumped mass
Theoretical and numerical models

For simple structures, a model based on the theory of an idealised cantilever beam with simple boundary conditions can be used. The dynamic system can be represented by either a lumped or a distributed mass system, Figure 2. Lumped masses are connected by elements (walls, columns) with a certain stiffness and damping. To solve the system and determine the natural frequencies, modal analysis can be used. The system can be scaled, and the dynamic behaviour is described by second order differential equations and results in a finite number of natural frequencies (Chopra, 2012).

The classic Euler-Bernoulli beam model estimates the natural frequency of beams. The theory only considers flexural deformations of a slender beam with linear elastic, homogeneous material. Equation (7) estimates the natural frequencies of a fixed uniform cantilever beam (Blevins, 1979):

\[ f_i = \frac{\lambda_i^2}{2\pi L^2} \sqrt{\frac{E}{m}} \quad i = 1,2,3, ..., n \]  

(7)

Where \( L \) is the length of the beam, \( E \) is the modulus of elasticity, \( I \) is the moment of inertia, \( m \) is the mass per unit length and subscript \( i \) represents the mode of vibration. For the first natural frequency \( \lambda_1 = 1.875 \) (ibid). For slender beams the flexural deformations will dominate, but for less slender beams shear deformation may become important and needs to be considered. Equation (8) estimates the natural frequencies of a fixed uniform cantilever beam, considering shear deformation only (ibid):

\[ f_i = \frac{\lambda_i}{2\pi L} \sqrt{\frac{\kappa G}{\rho}} \quad i = 1,2,3, ..., n \]  

(8)

Where

\[ \lambda_i = (2i - 1) \frac{\pi}{2} \]  

(9)

and \( \kappa \) is the shear coefficient, \( G \) is the shear modulus, and \( \rho \) is the mass density. The flexural beam model relates the frequency as proportional to \( 1/L^2 \) and in the shear beam model to \( 1/L \). The Timoshenko beam model considers, except from flexural and shear deformation also the effect of rotary inertia. The introduction of the shear deformation and rotary inertia reduces the natural frequencies compared with using only the flexural beam theory. However, the effect of rotary inertia is smaller than the effect of shear deformations. There is no general closed form solution for the frequency when combining flexural and shear deformations, but Dunkerley’s approximation can be used which estimates a lower bound of the natural frequency (ibid) expressed as:

\[ \frac{1}{f^2} = \frac{1}{f_f^2} + \frac{1}{f_s^2} + \cdots \]  

(10)
where $f_f$ is the fundamental frequencies from the flexural beam model and $f_s$ from the shear beam model. Tall buildings consist of several elements and can be very complex in structure therefore finite element (FE) software is useful to solve the multiple degree of freedom system. Modal analysis finds the mode shape with the lowest energy for natural vibration. Modal analysis is used to solve the free-vibration equation of motion expressed as (Chopra, 2012):

$$M\ddot{u} + K u = 0$$  \hfill (11)

Where $M$ and $K$ are the $N \times N$ matrices of mass and stiffness and $u$ denotes the generalised displacement:

$$u(t) = q_n(t)\phi_n$$  \hfill (12)

$\phi_n$ is the deflected shape and $q_n(t)$ can be described by a simple harmonic function, which gives the following algebraic equation also known as the eigenvalue problem:

$$[K - \omega_n^2 M] \phi_n = 0$$  \hfill (13)

Equation (13) has a nontrivial solution if

$$\det[K - \omega_n^2 M] = 0$$  \hfill (14)

The solution will give the eigenvalue of squared natural frequencies $\omega_n^2$ and the corresponding eigenvector $\phi_n$ to each eigenvalue:

$$\phi_n = \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_N \end{bmatrix} \quad n = 1, 2, \ldots, N$$  \hfill (15)

$n$ is the number of modes. The eigenvector is also known as the natural mode and represents the mode shape. Depending on the structural system the mode shape will be a bending mode, a shear mode or a combination of bending and shear. The bending mode for a cantilever beam is presented in Eq. (16):

$$\phi(z) = \cosh \left(\frac{\lambda_1 z}{L}\right) - \cos \left(\frac{\lambda_1 z}{L}\right) - \sigma_1 \left(\sinh \left(\frac{\lambda_1 z}{L}\right) - \sin \left(\frac{\lambda_1 z}{L}\right)\right)$$  \hfill (16)

Where $L$ is the length of the beam/height of the building, $z$ varies along the building height and for the first mode $\lambda_1 = 1.875$, $\sigma_1 = 0.734$. Equation (17) is for the first shear mode:

$$\phi(z) = \frac{\pi (2-1)z}{L}$$  \hfill (17)

These two equations represent pure bending mode and pure shear mode, Figure 3 (Blevins, 1979). The eigenvector is normalised to make the amplitude unique, $\phi_n$. The mode can be scaled so the eigenvector $(\phi_n)_n = 1$ at the specified coordinate $N$, the mode can be scaled at
the coordinate $N$ where the mode has it maximum displacement, or the mode is scaled so the
generalised mass or modal mass, $\bar{M}_n$, has a specified value (Chopra, 2012). The modal mass
is defined by:

$$\bar{M}_n = \phi_n^T M \phi_n$$

(18)

$\bar{M}_n = 1$ is often used and since the product of $\phi_n^T M \phi_n$ has the unit mass entails $\bar{M}_n = 1 \, kg$.
The generalised stiffness or modal stiffness for the $n$th mode is defined by:

$$\bar{K}_n = \phi_n^T K \phi_n$$

(19)

For the $n$th mode and multiplied by $\phi_n^T$ Eq. (13) can be written as:

$$\phi_n^T K \phi_n = \omega_n^2 (\phi_n^T M \phi_n)$$

(20)

The squared natural frequencies can then be related to the modal mass and modal stiffness by:

$$\omega_n^2 = \frac{\bar{K}_n}{\bar{M}_n}$$

(21)

Each resonance mode has an associated modal mass and modal stiffness (Jeary, 1997). For a
free vibration continuous systems the modal mass is defined by:

$$\bar{M}_n = \int_0^h m(z) \phi_n^2 dz$$

(22)

An assumption is made that the dimensions in X and Y directions, Figure 1, are constant and
do not vary along the building. If the building is irregular it needs to be considered in Eq.
(22). The equation of modal mass for 3D-system is by definition a volume integral, (Jeary,
1997).

The modal mass can be generated from the FE software directly or be calculated from the
eigenvector. Values of the eigenvector can be defined by taking points at certain levels of the
building where the maximum amplitude is generated, $\phi_N$. The modal mass can then be
calculated using:

$$\bar{M}_n = \frac{1}{\phi_N^2}$$

(23)

When dealing with ‘tall’ buildings, the building is considered slender and some assumptions
are often made. The building is considered to be ‘line-like’/linear and with a constant mass
over the height, but often the mass decreases with height since smaller dimensions are needed
to carry the weight. Another assumption is that the mass is lumped in certain points and a
normalised amplitude is used (Jeary, 1997). The mode shape can for tall buildings normally
be represented by a straight line, Eq. (24):

$$\phi(z) = \frac{z}{h}$$

(24)
The mode shape represented by a straight line is presented in Figure 3 with $\zeta = 1$.

Figure 3 Mode shape

If using the assumption for tall buildings and inserting (24) into Eq. (22) the modal mass will be:

$$M_n = m(z)\frac{h^3}{3h^2}$$

(25)

And for a pure shear mode Eq. (22) will be:

$$M_n = \frac{m(z)h}{2}$$

(26)

If $m(z)h$ is assumed to be the total mass of the building, $M_{tot}$ the modal mass will be:

$$\overline{M_n} = \frac{M_{tot}}{3}$$

(27)

and for shear:

$$\overline{M_n} = \frac{M_{tot}}{2}$$

(28)

Depending on the mode shape the modal mass will vary. Eq. (27) is often used as an approximation for tall buildings with a constant mass per unit height (Jeary, 1997). As seen in Eq. (22), (25) and (26) the modal mass is dependent on the mass distribution over the height, $z$. 
From a modal analysis which is performed on a 3D model, the mode shape is visually presented in 3D. The sum of modal masses in X- Y- and Z-directions represent the total mass for the associated vibration modes:

\[ \tilde{M}_n = \tilde{m}_X + \tilde{m}_Y + \tilde{m}_Z \]  

(29)

Where \( \tilde{m}_X, \tilde{m}_Y, \tilde{m}_Z \) are the modal mass in each direction X- Y- and Z-direction for the current mode (Autodesk Inc., 2015)

**Empirical formulas in Eurocode 1-4 and others**

Several empirical formulae to estimate the first natural frequency have been suggested by researchers (Ellis, 1980) (Kim & Kanda, 2008) (Lagomarsion, 1993) (Reynolds, et al., 2016b). In Eurocode 1-4 (2005) the fundamental frequency \( f \) can be estimated by Eq. (30) for multi-storey buildings with a height larger than 50 m.

\[ f = \frac{46}{h} \]  

(30)

Where \( h \) is the height of the structure in meters. The formula is based on a collection of 163 rectangular plan buildings with different types of materials; concrete, steel, and mixed (Ellis, 1980). To estimate the first natural frequency, different predictors were studied and compared with the measured frequency. The height was a better predictor than including the width of the building in the formula. Equation (30) has a correlation coefficient of \( R = 0.88 \). By looking at the frequency plotted against height in Ellis (1980), one finds that the largest scatter of data exists for lower buildings. However, there are many more data collection points for lower buildings than higher ones. From a collection of 185 different buildings (steel, reinforced concrete, mixed, pre-cast, masonry, unknown) following equation to predict the natural first natural frequency was arrived (Lagomarsion, 1993):

\[ f = \frac{50}{h} \]  

(31)

Where \( h \) is the height of the structure. In Lagomarsion (1993) data was also analysed with respect to structural material, Eq. (32)-(34):

\[ f = \frac{45}{h} \]  

steel buildings  

(32)

\[ f = \frac{55}{h} \]  

reinforced concrete buildings  

(33)

\[ f = \frac{57}{h} \]  

mixed buildings  

(34)

Two equations have been proposed by AIJ (2000) in Kim & Kanda (2008) for estimating the first natural frequency, one for reinforced concrete buildings and one for steel buildings:

\[ f = \frac{67}{h} \]  

reinforced concrete buildings  

(35)
\[ f = \frac{50}{h} \] steel buildings \hspace{1cm} (36)

Where \( h \) is the height of the structure. The equations are used for comfort assessment and have been derived from 68 reinforced concrete buildings and 137 steel buildings in Japan. In the National Building Code of Canada (2010) in Hu, et al., (2014), several empirical formulae are given to calculate the first natural frequency. Hu, et al., (2014) stated that the following equations could be used to predict the natural frequencies for a timber building:

\[ f = \frac{40}{h} \] braced frames \hspace{1cm} (37)

\[ f = \frac{20}{h^{3/4}} \] shear walls and other structures \hspace{1cm} (38)

Eq. (38) suggests a relationship between \( f \) and \( h \) that is not exactly inverse proportional. Measurements of timber buildings have been collected in Reynolds, et al., (2016b), and dynamic tests (ambient vibration tests) have been carried out on 11 timber buildings. Some of the buildings have the first floor in concrete and some are composite timber-concrete or timber-steel, but the main vertical and lateral load-carrying system is of timber, which includes cross-laminated timber, glulam and light timber frame. Based on the collected data a single relationship between height and natural frequency was found for timber buildings Eq. (39):

\[ f = \frac{55}{h} \] \hspace{1cm} (39)

It should be noted that the buildings may have different stabilising systems, which may behave differently under wind-induced vibration. In Figure 4 Eq. (30), (33), (35), (37), (38) and (39) are plotted. Eq. (35) and (37) constitute the lower and higher bounds. Studying Figure 4, the different empirical formulae display the same tendency, but with considerable spread.
The equivalent mass per unit length in Eurocode 1-4 (2005) is given by Eq. (40):

\[ m_e = \frac{\int_0^h m(z) \cdot \phi_1^2(z) \, dz}{\int_0^h \phi_1^2(z) \, dz} \]  

(40)

\( h \) is the height of the structure, \( m(z) \) is the mass per unit length, and \( \phi_1 \) is the first eigenmode. The equivalent mass is the modal mass presented per unit length and has the unit kg/m. The numerator is the modal mass and the denominator is the integral of the mode shape over the height of the building for a given mode. In this research, the term modal mass is used to described the modal mass in kg and equivalent mass is used to described the modal mass in kg/m. During calculations in FE software the eigenvector is normalised as in Eq. (18). Two alternative methods to arrive at the modal mass exist: either the modal mass in Eq. (40) is set to 1 and the eigenvector is normalised, or the software can produce the value of the modal mass The denominator is the integral of the mode shape over the height of the building where the amplitude has its maximum. The number of integration points can be set to the mesh size over the height. According to Eurocode 1-4 (2005) an approximate expression for the equivalent mass is the average value of the mass of the upper third of the structure or the total mass of the building divided by the total height:

\[ m_{e,\text{app}} = \frac{m_{\text{tot}}}{h} \]  

(41)

Inserting Eq. (24) and (27) into (40) will give this approximation.
Mode shape in Eurocode 1-4

In Eurocode 1-4 (2005), an expression for the mode shape of different types of buildings is stated:

$$\phi(z) = \left(\frac{z}{h}\right)^\zeta$$  \hspace{1cm} (42)

$\zeta = 0.6$ for slender frame structures with non load-distributing walls or cladding. $\zeta = 1.0$ for buildings with a central core plus peripheral columns or larger columns plus shear bracings. $\zeta = 1.5$ for slender cantilever buildings and buildings supported by central reinforced concrete cores. Mode functions with different $\zeta$-values are displayed in Figure 3. $\zeta = 1.5$ corresponds to a bending mode shape, while $\zeta = 1.0$ is a linear relationship. When solving the eigenvalue problem numerically in FE software, the eigenvector for each natural frequency will be generated and thereby the mode shape is captured.

Mode shapes in FE modal analysis

For a building with a square floor plan, a central core, and columns or walls along the perimeter, the three first modes are normally two orthogonal translational modes and a third torsional mode, Figure 5. For symmetric buildings the mode direction is controlled by the weak stiffness direction and aligns with the geometry of the building, in other words the direction with lowest energy. The mode direction can differ if the building is asymmetric and the weak stiffness direction of the structure may not be obvious (Jeary, 1997).

If the centre of mass is offset from the centre of rigidity the structure can undergo torsion. The structure may resist the torsion depending on the structural torsional stiffness. Eurocode 1-4 (2005) does not include torsional phenomena when checking accelerations caused by wind load. ISO 10137 (2008) allows for evaluation of torsional effects, as do other building codes (Kwon & Kareem, 2013). However, instead of pure translational modes the building may have a tendency to rotate (mixed mode), which Reynolds, et al., (2016a) discovered when evaluating two timber buildings with the same floor plan, one with a cross-laminated timber frame and one with a timber frame. Both buildings have a concrete core at the centre of one of the long sides of the building. The test results were compared with a finite element model. The mode shape showed rotation around the core for the first mode, the second mode was a translation mode and the third mode was in torsion.
Damping

Depending on structural systems, materials, connections etc. the damping in the building will vary. The magnitude of motion will be reduced more quickly in a building with high damping. Damping of a certain building can be measured once the building is completed. It is difficult to predict the damping ratio before the building is built. There are two types of damping systems, passive and active. Passive damping systems has a fixed property related to chosen layout and structural materials, while active dampers modify the system properties and realise on active mechanism. Damping of the structural system itself is a passive damper (Ali & Moon, 2007). The damping ratio can be non-linear depending on the amplitude of the motion (Reynolds, et al., 2014b). The damping properties are included in the resonance factor $R$ in Eq. (2). Both structural damping, aerodynamic damping and damping from special advices are included.
The total logarithmic decrement for damping is given by:

\[ \delta = \delta_s + \delta_a + \delta_d \]  

(43)

\( \delta_s \) is the logarithmic decrement of structural damping, \( \delta_a \) is the logarithmic decrement of aerodynamic damping and \( \delta_d \) is the logarithmic decrement due to special devices e.g. tuned mass dampers, sloshing tanks (SS-EN 1991-1-4, 2005). The logarithmic decrement of structural damping is defined by (Chopra, 2012):

\[ \delta_s = \frac{2\pi \xi}{\sqrt{1-\xi^2}} \]  

(44)

Where \( \xi \) is the damping ratio. In the old Swedish handbook “Snow and Wind Load” (BSV 97, 1997) values on the logarithmic decrement for timber structures with and without mechanical connections are presented. Structural damping is stated as \( \delta_s = 0.06 \) for a timber structure without mechanical connections and \( \delta_s = 0.09 \) for a timber structure with mechanical connections. The values correspond to about 1.0 % and 1.5 % damping ratio. In Eurocode 1-4 (2005) a value of \( \delta_s = 0.06 - 0.12 \) is tabulated for timber bridges, which corresponds to 1.0-1.9% damping ratio. In Table 1, dynamic properties are presented for measured buildings and values from numerical analyses. Damping ratio values are reported between 1.3-9.1% for buildings with cross laminated timber as the main load bearing structure, 1.4-2.4% for post-and-beam systems and 1.0-6.9% for hybrid structures. For steel and reinforced concrete structures, the damping ratio, tabulated in Eurocode 1-4 (2005) are 0.8% and 1.6% respectively. The presented values in Eurocode 1-4 (2005) are not related to the amplitude or height of the building but from measured timber buildings the damping ratio varies with the amplitude and may have a non-linear behaviour (Reynolds, et al., 2014b). The damping ratio tends to decrease with increased height (Reynolds, et al., 2016b).

**Dynamic properties of timber buildings**

The number of dynamically tested timber buildings is growing. In the following Table 1, a summary of measured, numerically calculated and/or estimated (theoretical or empirical models) timber buildings are listed. In Figure 6, the values are plotted.
<table>
<thead>
<tr>
<th>Number in Figure 6</th>
<th>Building type</th>
<th>Height [m]</th>
<th>Measured</th>
<th>Simulated/estimated</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$f_1$ [Hz]</td>
<td>$\xi_1$ [%]</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>CLT</td>
<td>9.1</td>
<td>3.9</td>
<td>5.4</td>
<td>(Hu, et al., 2014)</td>
</tr>
<tr>
<td>2</td>
<td>CLT, post-and-beam</td>
<td>12.6</td>
<td>3.2</td>
<td>1.3</td>
<td>(Hu, et al., 2014)</td>
</tr>
<tr>
<td>3</td>
<td>CLT</td>
<td>21</td>
<td>2.7</td>
<td>3.20-5.60</td>
<td>(Hu, et al., 2014b)</td>
</tr>
<tr>
<td>4</td>
<td>CLT, timber frame, first storey in concrete</td>
<td>21</td>
<td>2.45</td>
<td>5.20-9.10</td>
<td>(Hu, et al., 2014)</td>
</tr>
<tr>
<td>5</td>
<td>CLT, first storey concrete</td>
<td>27</td>
<td>2.26</td>
<td>1.9</td>
<td>(Hu, et al., 2014a)</td>
</tr>
<tr>
<td>6</td>
<td>CLT core, post-and-beam</td>
<td>29.5</td>
<td>1.1</td>
<td>3</td>
<td>(Hu, et al., 2016)</td>
</tr>
<tr>
<td>7</td>
<td>CLT core, post-and-beam</td>
<td>74.8</td>
<td>0.6</td>
<td>1.5</td>
<td>(Johansson, et al., 2016)</td>
</tr>
<tr>
<td>8</td>
<td>CLT</td>
<td>31-68.2</td>
<td>2.78-0.97</td>
<td>1.5</td>
<td>(TräCentrumNorr (TCN), 2016)</td>
</tr>
<tr>
<td>9</td>
<td>Post-and-beam</td>
<td>18.3</td>
<td>2.8</td>
<td>1.4</td>
<td>(Hu, et al., 2014)</td>
</tr>
<tr>
<td>10</td>
<td>Post-and-beam, concrete slab every fourth storey</td>
<td>45</td>
<td>0.98-1.03</td>
<td>1.6-2.4</td>
<td>(Hu, et al., 2016)</td>
</tr>
<tr>
<td>11</td>
<td>Post-and-beam, concrete slab in the upper part</td>
<td>81</td>
<td>0.33</td>
<td>1.9</td>
<td>[]</td>
</tr>
<tr>
<td>12</td>
<td>CLT, concrete core</td>
<td>15.6</td>
<td>4.03-4.08</td>
<td>4.9-6.9</td>
<td>(Hu, et al., 2016)</td>
</tr>
<tr>
<td>13</td>
<td>Post-and-beam with concrete core</td>
<td>21.8</td>
<td>2.1</td>
<td>1.5</td>
<td>(Hu, et al., 2014)</td>
</tr>
<tr>
<td>14</td>
<td>Post-and-beam with concrete core and shear walls</td>
<td>22.1</td>
<td>2.7</td>
<td>1</td>
<td>(Hu, et al., 2016)</td>
</tr>
<tr>
<td>15</td>
<td>Solid timber, concrete core</td>
<td>23.9</td>
<td>2.33-2.34</td>
<td>1.82-2.90</td>
<td>(Feldmann, et al., 2016)</td>
</tr>
<tr>
<td>16</td>
<td>Post-and-beam with concrete core, CLT</td>
<td>53</td>
<td>0.5</td>
<td>1.5</td>
<td>(Fast, et al., 2016)</td>
</tr>
</tbody>
</table>

1. Without non-structural elements (partition wall, finishing etc.)
2. Mass from non-structural material was not included
3. Magne Aanstad Bjertnæs, Sweco Norway AS, personal communication
The buildings have been divided in three types CLT, Post-and-beam, and Hybrid. For CLT the main stabilisation is through shear wall elements. For Post-and-beam, a bracing/truss system with diagonals in timber is used for stabilisation. In the Hybrid type, most of the buildings have concrete parts e.g. a core or shear walls in concrete for the main stabilising system and timber elements for vertical loads and load distribution. To measure dynamic properties ambient vibration tests (AVT) have been made in most cases, but also a few forced vibration tests (FVT) have been carried out (Hu, et al., 2014) (Reynolds, et al., 2014b) (Reynolds, et al., 2014a) (Hu, et al., 2016) (Fjeld Olsen & Hansen, 2016) (Feldmann, et al., 2016). For simulated cases, FE software and modal analysis have been used (Johansson, et al., 2016) (TräCentrumNorr (TCN), 2016) (Fast, et al., 2016). Flexural beam models have been used in theoretical estimations (Reynolds, et al., 2014b) (Reynolds, et al., 2014a). In Reynolds, et al., (2014b) and Hu, et al., (2016) measurements have been taken during construction so the effect of non-structural elements such as partition walls, finishing etc. has been evaluated. The natural frequencies decreased when cement screed was added in Reynolds, et al., (2014b) and for Hu, et al., (2016) the natural frequencies increased when partition walls, finishing etc. were added. The frequency of the measured buildings is 1.02 Hz to 4.08 Hz with a variation in height of 9.1 m up to 53 m. Simulated values are between 0.33 Hz to 2.78 Hz with a height variation between 31-81 m.

Figure 6 Empirical formulas together with measured, simulated and estimated natural frequencies
**Comfort criteria**

Human’s sensitivity to building motions is an interaction of several mechanisms in the sensory system. Some of the mechanisms are vestibular, visual, and auditory sensory systems (Burton, et al., 2006). To predict the human response to vibration (vestibular), the aerospace industry performed several experiments in the 1950s and 1960s. Since it was questionable if the result could be used on tall buildings, where frequencies are usually smaller than 1 Hz. Chang (1973) proposed threshold levels for acceleration based on theoretical extrapolation of the aerospace industry data. Chang (1973) recommended that a comfort limit under 0.5% g is not perceptible, 0.5% g to 1.5%g is perceptible, and 1.5% to 5.0%g is annoying.

Based on a large volume of data and sources Irwin (1978) has presented comfort curves regarding human response, for perceptible and acceptable levels of acceleration for different types of structures. For buildings used for general purposes, the comfort curves are given for a probability that not more than 2% of the occupants complains about a motion-caused by a ten minute wind storm with a return period of 5 years. The work presented by Irwin (1978) has later been adapted in the ISO 6897 standard (ISO 6897, 1984), where r.m.s (root mean square) values of the acceleration are used (Johann, et al., 2015). ISO 6897 (1984) gives guidelines to evaluate the response of buildings and off-shore structures to low-frequency horizontal motion, 0.063-1.00 Hz. As in Irwin (1978), ISO 6897 (1984) is based on a five-year return period for the wind velocity and r.m.s acceleration values. The probability of the occupants in the building complaining about a motion with a one year return period is estimated to be 12 %. To meet the criteria of 2 % probability using a one year return period, ISO 6897 (ibid) suggests that the acceleration level should be taken as 0.72 times the acceleration level for a five year return period.

In 2008 a new standard was published, ISO 10137 (2008), to evaluate the comfort criteria of wind-induced vibrations in buildings. The standard uses a one year return period for wind velocity to calculate the peak acceleration and covers a frequency range of 0.06 - 5 Hz. The standard presents acceptable levels both for buildings used as “offices” and “residences”, see Figure 7. The curve for “residences” is close to the 90% level of the probabilistic perception threshold presented in Tamura (2009), which is used in the Japanese standard, AIJ-GBV-2004 (Tamura, et al., 2004). The “offices” curve in ISO 10137 (2008) is 3.5 times the evaluation curve “general purpose” in ISO 6897 (1984) with a one year return period, see Figure 7. 3.5 is a typical peak-factor for wind-induced vibrations. The criteria in ISO 10137 (2008) are approximately based on mean values (Rainer, 2005). The ISO 10137 (2008) reflects a more severe comfort criterion than ISO 6897 (1984). The comfort criteria in both standards utilises deterministic evaluation of human comfort but probabilistic methods have also been proposed (Johann, et al., 2015).
ISO 6897 (1984) is based on r.m.s acceleration and ISO 10137 (2008) on peak acceleration. When it comes to evaluating the human response there are two different lines of thought regarding accelerations over a reference period of 20 – 60 min. The first line of thought is that only the peak values of the highest amplitude of motion will be remembered by humans, the smaller ones tends to be forgotten. The second line of reasoning explains that the number of cycles above some threshold and intensity will determine the perception of building motion (Boggs, 1995). The r.m.s and peak acceleration values are linked, by using a peak factor for the vibration waveform one can convert between them (Tamura, 2009). The peak factor can vary between $\sqrt{2}$ to 4.0 depending on waveform/signature of the vibration, Eq. (4). The AIJ-guideline 2004 (Tamura, et al., 2004) and ISO 10137 (2008) use the peak acceleration. The main reason according to Tamura (2009) is that the peak value is easier to understand for designers, owners and occupants. Tamura (2009) also stated that the peak acceleration with a one year return period relates more to daily comfort than extreme occurrences.
Method

Research process

The research project was initiated in 2012 as a collaboration between Luleå University of Technology and Sweco Structures AB (Sweco). Sweco is a consultant firm with a structural engineering division. The focus was set on multi-storey timber buildings and connections. The first studies, which are presented in Papers V and VI, were on horizontal displacements from static wind load in the serviceability limit state. A 2D model of stabilising shear walls was created, with studs, sheathing materials, connectors within the wall, connectors between storeys and sylomers (flanking transmission reduction material). A post-and-beam truss system was also modelled. Limits for horizontal displacement are lacking in the current Eurocode 5 (2009) version or Eurocode 1-4 (2005), which could lead to extensive cracking in structures unless limits are enforced.

During autumn 2014 the author performed interviews with structural engineers/designers, timber frame suppliers, and development managers. The results are presented in Technical Report I. The focus was on serviceability limit state in taller timber buildings. The experience and knowledge with the respondents varied, which resulted in a wide perspective of the area. The main outcomes were: dynamic properties will be a challenge when designing taller timber buildings and there is a need for increasing the knowledge of how and when dynamic analysis should be considered. Thereby, continued research into the dynamics of taller timber buildings became natural.

The author has in parallel to the research worked as a structural engineer at Sweco Structure AB. The work has mainly been related to the conceptual design of high-rise timber buildings. Some of the work during the years has been at Sweco Norway AS together with Rune B. Abrahamsen (CEO Moelven Limmre and former employee at Sweco Norway) and Magne Aanstad Bjertnaes (team leader Sweco Norway, Lillehammer). Rune and Magne are the structural engineers behind Treet and Mjøstårnet.

During 2015 the author worked with a feasibility study of Mjøstårnet. Different structural concepts were suggested in the feasibility study to solve the dynamic issues and decrease the acceleration levels. Examples that were studied: the structural layout of the bracing, varying the amount of additional mass and placement of the mass, and larger cross-sections of the columns in the corner. The work at Sweco has contributed to an increased knowledge and insight into what challenges structural engineers are facing, but also brought understanding to the author to formulate a purpose for the research and guide method choices.

A need for understanding how different parameters affect dynamic properties and acceleration levels formed the idea of Paper I where parameters were studied for a cross-laminated timber panel system by gradually increasing building height, changing the building footprint, vary
the damping ratio, increase the stiffness and adding concrete screed to floors. In Paper II the same floor layout as in Paper I was used to study the dynamic properties of a post-and-beam system with diagonal bracing and compare them to the results in Paper I. Empirical formula to estimate the first natural frequency and the expression for the mode shape presented in Eurocode 1-4 (2005) were analysed and compared with numerical results. Tall building assumptions (Jeary, 1997) were evaluated for high-rise timber buildings.

The cross-laminated timber system in Paper I showed a tendency to rotate in the first mode. Therefore, asymmetrical aspects such as size and placement of openings and floor layout were studied in Papers III and IV. The intention was also to provide knowledge about situations phasing engineers in their everyday work. A floor layout with a hybrid solution, a concrete shaft, was also studied in Paper IV. Solutions with a higher mass are often suggested when trying to solve dynamic issues, although Papers I and II show that timber buildings often fall in the range of natural frequencies where an increased mass does not improve the comfort.

**Analysis method**

To predict the acceleration levels the natural frequency of the building, the mode shape and the equivalent mass must be known (see theory chapter). These can be evaluated once the building is constructed, but the situation for an engineer is to determine these properties beforehand. By using finite element software, the natural frequency, mode shape, and modal mass can be generated by modal analysis, see Eq.(11) and further in the theory chapter. By performing an operational modal analysis (OMA) or an experimental modal analysis (EMA) modal properties of a structure can be estimated (Brincker & Ventura, 2015). These test techniques can be performed during or after the structure is built and the results will only reflect that specific structure. Another numerical alternative to predict the acceleration levels is a Time History Analysis (THA) (Autodesk Inc., 2015) (Chopra, 2012). THA uses modal analysis to predict the natural frequency and the mode shape to subsequently generate acceleration levels over time. Since THA is an analysis over time, a wind load function is needed in the model and the damping of structural elements must be included.

**Cases**

In this research, the following studies have been carried out:

In Paper I the dynamic properties of a fictitious CLT building was studied where the following parameters were included:

- number of storeys (10-22)
- footprint size (20x20m² – 28x28m²)
- damping ratio (1.5%-6.0%)
- wall stiffness (E = 13,600 MPa - E = 31,000 MPa)
- wall density (10 kN/m³ - 25 kN/m³)
parameters have been combined to explore possible interaction effects

In Paper II the dynamic properties of two building types with the same floor plan as in Paper I were studied, a CLT building and a post-and-beam building with diagonal bracing. The slenderness of the building was varied. Empirical formulae on natural frequencies and mode shape were studied. Tall building assumptions were also evaluated.

- Number of storeys expressed as slenderness h/d (0.9-3.3)
- Building systems (2)

In Paper III the effect of openings on dynamic properties was studied where size and placement of openings were evaluated. The floor plan was simplified to four perimeter walls and floors at every 3 meters.

- Number of storeys expressed as slenderness h/d (1.5-4.5 + 9.0)
- Size of openings (2 sizes)
- Placement of openings (symmetric/asymmetric)

In Paper IV the dynamic properties resulting from a floor plan from four different building types were studied with varying placement of the elevator shaft. The floor plan from Paper I was re-used and the placement of the shaft varied.

- Number of storeys expressed as slenderness h/d (1.5-4.5 + 9.0)
- Placement of elevator shaft (3 positions)
- Placement of elevator shaft and extra wall added opposite the shaft
- Building system types (3 systems: CLT, hybrid concrete/CLT, and concrete)

In Papers V and VI the horizontal displacement of a 2D model of a timber-frame shear wall and a truss system with diagonal bracing was studied and compared to horizontal displacement criteria.

In the sections below more details around the papers are described. In total in excess of 250 different structures have been simulated as the basis for the analysis and conclusions in this thesis.

Paper I

In Paper I a fictitious building with a floor plan based on existing multi-storey residential timber buildings in Sweden was studied, Figure 8 with a footprint of 20x20m². For the structural frame, CLT panels was assumed to stabilise through shear walls and diaphragm action in the floor elements. The footprint is quadratic, with a CLT core placed almost in the middle to accommodate for staircase and elevator. Each storey contains four apartments connected to the core. In Figure 8 a 3D-model, a floor layout, and a section are presented. The floor plan shows the load carrying structural elements. The building also has glulam columns and beams to support the structural floors. The height of each storey is 3 m. For the load-
bearing CLT walls and floor elements a thickness of 170 mm was used. Additional information regarding composition of external walls, load-bearing inner walls/apartment separation walls, core shaft, floor slab and roof refer to Paper I.

Figure 8 Fictitious building in Paper I, a) 3D-mode, b) floor layout (Z-direction is out-of-plane, c) section
Five parameters were varied in the parameter study: number of storeys, building footprint, damping ratio, wall stiffness, and wall density. The variations of the parameters are presented in Table 2 with a total of 27 cases.

Table 2 Cases and variation of parameters, Paper I

<table>
<thead>
<tr>
<th>Case</th>
<th>w [m]</th>
<th>h [m]</th>
<th>Storeys</th>
<th>Footprint [m²]</th>
<th>ξ [%]</th>
<th>Wall stiffness [MPa]</th>
<th>Wall density [kN/m³]</th>
<th>Extra mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>30</td>
<td>10</td>
<td>20 x 20</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>36</td>
<td>12</td>
<td>20 x 20</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>42</td>
<td>14</td>
<td>20 x 20</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>48</td>
<td>16</td>
<td>20 x 20</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>54</td>
<td>18</td>
<td>20 x 20</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>60</td>
<td>20</td>
<td>20 x 20</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>66</td>
<td>22</td>
<td>20 x 20</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>22</td>
<td>30</td>
<td>10</td>
<td>22 x 22</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>24</td>
<td>30</td>
<td>10</td>
<td>24 x 24</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>26</td>
<td>30</td>
<td>10</td>
<td>26 x 26</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>-</td>
</tr>
<tr>
<td>11</td>
<td>28</td>
<td>30</td>
<td>10</td>
<td>28 x 28</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>20</td>
<td>30</td>
<td>10</td>
<td>20 x 20</td>
<td>3.0%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>-</td>
</tr>
<tr>
<td>13</td>
<td>20</td>
<td>30</td>
<td>10</td>
<td>20 x 20</td>
<td>4.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>-</td>
</tr>
<tr>
<td>14</td>
<td>20</td>
<td>30</td>
<td>10</td>
<td>20 x 20</td>
<td>6.0%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>-</td>
</tr>
<tr>
<td>15</td>
<td>20</td>
<td>30</td>
<td>10</td>
<td>20 x 20</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>-</td>
</tr>
<tr>
<td>16</td>
<td>20</td>
<td>30</td>
<td>10</td>
<td>20 x 20</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>-</td>
</tr>
<tr>
<td>17</td>
<td>20</td>
<td>30</td>
<td>10</td>
<td>20 x 20</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>10</td>
</tr>
<tr>
<td>18</td>
<td>20</td>
<td>30</td>
<td>10</td>
<td>20 x 20</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>15</td>
</tr>
<tr>
<td>19</td>
<td>20</td>
<td>30</td>
<td>10</td>
<td>20 x 20</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>20</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>30</td>
<td>10</td>
<td>20 x 20</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>25</td>
</tr>
<tr>
<td>21</td>
<td>20</td>
<td>30</td>
<td>10</td>
<td>20 x 20</td>
<td>1.5%</td>
<td>31,000</td>
<td>25</td>
<td>-</td>
</tr>
<tr>
<td>22</td>
<td>20</td>
<td>66</td>
<td>22</td>
<td>20 x 20</td>
<td>1.5%</td>
<td>31,000</td>
<td>25</td>
<td>-</td>
</tr>
<tr>
<td>23</td>
<td>24</td>
<td>36</td>
<td>12</td>
<td>24 x 24</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>-</td>
</tr>
<tr>
<td>24</td>
<td>28</td>
<td>42</td>
<td>14</td>
<td>28 x 28</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>-</td>
</tr>
<tr>
<td>25</td>
<td>20</td>
<td>42</td>
<td>14</td>
<td>20 x 20</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>3 kN/m³ on all storeys</td>
</tr>
<tr>
<td>26</td>
<td>20</td>
<td>42</td>
<td>14</td>
<td>20 x 20</td>
<td>1.5%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>3 kN/m³ on storeys 10-14</td>
</tr>
<tr>
<td>27</td>
<td>20</td>
<td>42</td>
<td>14</td>
<td>20 x 20</td>
<td>3.0%</td>
<td>Table 3</td>
<td>Table 3</td>
<td>-</td>
</tr>
</tbody>
</table>

Case 1 was referred to be reference building. For cases 2-7, the number of storeys has been varied from 12 to 22 in steps of 2 storeys. The building footprint has been varied in cases 8-11 with a step increase of 2 meters, 22 x 22 m² to 28 x 28 m². In cases 12-14 the damping ratio, ξ, has been varied between 3.0% to 6.0% with a step of 1.5. The wall stiffness has been varied in cases 15 and 16. For the wall stiffness the elastic modulus was varied simulating a change in materials while the thickness of the panel remained fixed. E = 13,600 MPa corresponds to the stiffness of glulam GL30h and E = 31,000 MPa corresponds to C25/30 concrete (mean values) according to Table 3. The wall density was changed in cases 17-20. The stiffness and thickness of the wall remained the same. The density of the wall was increased from 10 kN/m³ to 25 kN/m³. A change from 5 kN/m³ (reference building) to 25 kN/m³ corresponds to exchanging timber to concrete. For cases 21-27 the parameters have been combined to explore possible interaction effects. The material for the reference building and the 22 storey high building was changed to C25/30 concrete both for walls and slabs (cases 21 and 22), 12 storeys high with a footprint of 24 x 24 m² and 14 storeys high with 28 x 28 m² (cases 23 and 24, same aspect ratio as the reference building). The 14-storey building was also studied with the following combinations (cases 25-27): damping ratio of 3.0 %, extra mass at all storeys.
and extra mass at the tenth to fourteenth storey. 3 kN/m² was added as extra mass, which corresponds to a 50 \% live load of 2.0 kN/m² and a concrete layer of 80 mm.

In total 27 simulations have been performed in this study.

**Paper II**

In *Paper II* two building types have been evaluated, one with CLT elements and one with a post-and-beam structure. The building types have firstly been evaluated in ultimate limit state on a 14-storey high (42 m) building to get a sense of the approximate dimensions of elements. The utilisation is between 65-90\%. Connections and openings have not been checked in the model. The building types have been studied for a height of 18-66 m, which corresponds to 6-22 storeys. A step of 4 storeys was used.

The CLT structure has the same composition as in *Paper I* except the thickness and layer setup of the CLT elements. The thickness of the wall is 200 mm and it consists of five layers. The floor element is made of seven layers where the outmost parts has two parallel layers to increase the stiffness in the stiff direction. The thickness of the floor is 280 mm.

The post-and-beam building type has columns in the perimeter of the façade and in the core. Beams are connected between the columns. The same floor elements were used as in the CLT building type. Columns and beams were placed to reduce the span of the floor element. A diagonal bracing system in the façade was used to stabilise the building type. The diagonals span over two storeys, Figure 9. For the columns a dimension of 450x450 mm was used. For the beams 200x400 mm and for the diagonals 300x450 mm were used. Grade GL30h was assumed for all columns and beams. The post-and-beam building type was assumed to have the same insulation, gypsum, surface finishing etc. as the CLT building type (*Paper I*). The difference between the systems is only the load bearing structure and the stabilising system.

In total 10 simulations have been performed in this study.
In Paper III the effect of openings has been studied by using the same footprint as in Papers I and II, but the structure has only the outer walls and floors left – the inner walls were neglected and also the non-structural mass to isolate the phenomenon of size and placement of openings.

Opening sizes of (height x width) 1.5x1.0m² and 1.5x2.5 m² have been studied with 4 openings in each wall that is assumed to have openings. 6 different configurations of opening placement have been studied, Figure 10:

- In case O no openings are added,
- Case 1X has openings on one wall in the X-direction
- Case 2X has openings on both walls in the X-direction
- Case 1X1Y has openings on one wall in X-direction and one wall in Y-direction
- Case 1X2Y has openings on one wall in X-direction and on both walls in Y-direction
- Case 2X2Y have openings on all walls.

For all cases, the height of the building has been varied which is represented by the building slenderness, $h/d$ where $h$ is the height and $d$ is the depth of the building. For all cases the depth remains the same, 20 m. The slenderness has been varied between 1.5 to 4.5 with a step
of 0.6. To evaluate a very high building $h/d = 9.0$ was added. The wall and floor elements are the same as in *Paper II*.

![Diagram of building configurations](image)

**Figure 10 Placement of openings, Z-direction is out-of-plane**

Additional simulations were performed on case 1 in *Paper I* (recalculated in *Paper II*, case B). The building in *Paper I* had openings for windows in all 4 outer walls, where 4 larger openings were placed along the left wall on the Y-axis, while the building was symmetric in X-plane, Figure 8. The following simulation was additional been performed:

- Case 2X1Ya without non-structural mass
- Case 2X1Yb without the 4 larger openings on the Y-axis
- Case 2X1Yc without non-structural mass and 4 larger openings on the Y-axis

In total around 80 simulations have been performed in this study.

**Paper IV**

In *Paper IV* the floor layout has been studied by varying the placement of the elevator shaft. The same floor layout as in *Paper I* has been used. The openings from windows have been removed, but the door openings are left. Four different cases have been studied with varying placement of the elevator shaft, Figure 11. The placements of the shaft are denoted I, II and III, with an additional case III – Hybrid with the same floor layout but an extra wall added opposite the shaft. For each case, four different building systems are analysed: 1) *Timber* where all structural elements are in CLT 2) *Concrete*, where all structural elements are in concrete 3) *Hybrid I* where all structural elements are in CLT except from a concrete shaft, and 4) *Hybrid II*, where the shaft is in concrete and a 7m long concrete wall on the opposite side is added while the rest of the structure is CLT. *Hybrid II* has only been studied for floor layout III. The wall and floor elements are the same as in *Paper II*. Non-structural mass was not included in the study.

In total 70 simulations have been performed in this study.
Figure 11 Floor plans used for modal analysis of timber, concrete and hybrid building systems

Papers V and VI
In Papers V and VI a 2D-model of a timber-frame wall with two different sheathing materials, plywood and particle board, and a post-and-beam structure with diagonal bracing was studied, Figure 12, and compared with the static horizontal displacement limits of $H/500$ and $H/300$. The studied walls form part of a complete building. The height of the wall and the number of wall segments along the width of the wall was varied.

In total 54 simulations were performed in this study.
The finite element software Autodesk ® Robot™ Structural Analysis (Robot), version 2015 and 2017, has been used in Papers I-IV.

**FE-elements and orthotropic properties**

The CLT elements were modelled as four-node quadrilateral shell elements with orthotropic material properties. For the reinforced concrete, isotropic material properties were used. The columns and beams supporting the floors and in the post-and beam structures were modelled as beam elements with six degrees of freedom in each end.

In the software, there are different way to handle orthotropic material. Either by input on material properties in the weak and stiffness direction. From the input the software produces the stiffness matrix for a shell element. In Paper I the material properties for the CLT elements was given by supplier, Table 3 (ETA-13/0684, 2013). The orthotropic material properties were considered by defining the shell element one property in the stiff direction and one in the weak direction.
Table 3 Material properties and densities, mean values. *Paper I*

<table>
<thead>
<tr>
<th>Material</th>
<th>Element type</th>
<th>Elastic modulus [MPa]</th>
<th>Shear modulus [MPa]</th>
<th>Poisson’s ratio [-]</th>
<th>Density [kg/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLT, 170 mm (bending)</td>
<td>Four-node quadrilateral shell elements, orthotropic material properties</td>
<td>5220</td>
<td>2767</td>
<td>0.02</td>
<td>~ 400</td>
</tr>
<tr>
<td>CLT, 170 mm (tension/compression)</td>
<td>Four-node quadrilateral shell elements, orthotropic material properties</td>
<td>3974</td>
<td>3641</td>
<td>0.02</td>
<td>~ 400</td>
</tr>
<tr>
<td>Glulam GL30h, 190 mm x 270 mm</td>
<td>Beam element</td>
<td>13,600</td>
<td>-</td>
<td>-</td>
<td>430</td>
</tr>
<tr>
<td>Reinforced concrete, C25/30</td>
<td>Four-node quadrilateral shell elements, isotropic material properties</td>
<td>31,000</td>
<td>31,000</td>
<td>0.2</td>
<td>2500</td>
</tr>
</tbody>
</table>

Later it was concluded that the software did not handle the shear modulus properly and the shear modulus was given too high a value in *Paper I*. Some of the results were recalculated in *Paper II* were the stiffness matrix was manually define in the software by Eq. (45) (Gustavsson, 2017) and later adopted in *Papers III and IV*.

$$ C = \begin{bmatrix} 
D_{11} & D_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\
D_{21} & D_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & D_{33} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & D_{44} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & D_{55} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & D_{66} & D_{67} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & D_{76} & D_{77} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & D_{88} 
\end{bmatrix} $$

(45)

Where:

- \([D_{11} - D_{33}]\) flexural stiffness (plate element)
- \([D_{44} - D_{55}]\) shear stiffness (plate element)
- \([D_{66} - D_{88}]\) axial stiffness in plane (shell/membrane)

In *Papers II-IV* graded C24 (SS-EN 338, 2009) was used for all layers in the CLT element with a E-modulus of 11,000 MPa, a G-modulus of 690 MPa, and a density taken as 400 kg/m³.

**Mass**

In *Papers I and II* all masses from non-structural elements were applied as uniform loads [kN/m² or kN/m] converted in the software to masses when running the modal analysis. The software included the dead load in the modal analysis. The Z-direction for the mass was ignored in the analysis to avoid local vibrations in floors; X and Y-directions (transversal vibrations) were set to active, Figure 8b.
Connections

Connections between CLT elements (Papers I and II) were modelled as fixed except the connection between wall and floor elements that was assumed to be pinned. The walls were assumed to be continuous through the building and fixed to the ground. In Paper I pinned connections between walls and foundations and between walls were tested to compare with fixed connections. No differences in global results such as mode shape or acceleration were observed in the modal analysis. In Malo, et al., (2016) the stiffness of connections in a truss system was varied. Rotational stiffness decrease did not have any effect on the mode shape, while an axial connection stiffness lower than 25% of the member stiffness affected the mode shape and increased the acceleration. (Malo, et al., 2016) assumptions of modelling connections with many fasteners as a single element was validated in Paper VI where simplified models were shown to give reliable results. Landel, et al., (2018 tested the dynamic behaviour of a single truss element (18.5m). The results were compare with a 2D FE model where the connections were modelled by spring and stiffness according to Eurocode 5 (2009). Spring connections obtained better dynamic results compare with rigid and pinned connections. In Papers III and IV the connections were modelled as fixed and the walls were assumed to be continuous through the building and fixed to the ground. By allowing rotation between wall and floor elements, the natural frequency and the modal mass does not change expect for case 1X1Y where the modal mass increases around 8%. However, the overall behaviour was the same.

In Papers I and II the columns are continuous through the building and fixed to the foundation. The diagonals and the beams are have pinned connections to the columns.

Paper V and VI

For the 2D FE model Matlab has been used with a finite element toolbox, CALFEM (Dahlblom, et al., 1986). The light frame model consists of three basic elements, a frame member, a sheathing member and fasteners. Frame members are modelled as beam elements with three degrees of freedom. Sheathing member is modelled as a plane stress isotropic element with two degrees of freedom in each node. The fasteners between the frame member and the sheet are modelled with two springs in the translation direction in the corner. The studs are pinned connected to the top and bottom rail so an additional degree of freedom where added in nodes that contains frame members. The studs and top/bottom rail will not have the same rotation. The damping material is modelled by single spring pair in the translation direction and only takes shear and compression load. Each floor is connected by hold-downs which are modelled with a spring in vertical direction that only take load in tension. The bottom rail is hindered to move in the translation direction in the model. Details of connected elements are presented in Figure 13.
The sheets are nailed along the perimeter but to simplify for the finite element model a numerical calibration to arrive at the stiffness of the corner nails simulating nails along the perimeter of the sheathing was performed, Figure 14, *Paper VI*.

![Figure 13 Detail of connected members, light frame system. *Paper V*](image)

![Figure 14 Simplify FE model for modelling nails. *Paper VI*](image)

The post and beam system consist of columns and beams which are modelled as beam elements. The diagonals are also modelled as a beam element (can take both compression and tension) and connected with bolts to the column. The bolted connection is represented by a rotational spring and the columns are pinned connected to the beams and the beams are modelled as continuous. The bottom columns are hindered to move in translational direction. Details of connected elements are presented in Figure 15. For both cases the wind load is distributed to each beam element node, the dead load is also applied to all beam element nodes.
Figure 15 Detail of connected elements in post-and-beam structure. Paper V

For details refer to Paper V and VI.

**Calculation process**

To evaluate the dynamic behaviour several steps are taken during the calculation process where both FE software and hand calculations are used.

- Input to the FE-software to perform modal analysis:
  - Geometry of the building and elements
  - Material properties such as density and the stiffness matrix for materials
  - Boundary condition definition
  - Definition of connections between elements
  - Definition of loads (if present)
- Output from the FE-software (for each mode)
  - Natural frequency
  - Eigenvectors (X,Y,Z)
  - Modal mass (X,Y,Z)
  - Total mass
  - Displaced coordinates for nodes
- Hand calculations from the FE output
  - Equivalent mass, based on modal mass and eigenvectors (mode shape) deducted from the software, Eq.(40)
  - Acceleration levels, Eq.(3)
  - Acceleration levels depending on mode shape
- Plot resulting acceleration level in diagram from ISO 10137 (2008), Figure 7
Results and analysis

Recalculation of results from Paper I

The shear stiffness of CLT was attributed too high a value in Paper I. In Paper II some of the results were recalculated and the author has also recalculated select cases in Paper I for verifications. The reduced shear stiffness and change in thickness of the wall and floor for a 10 storey (30 m) high building resulted in a reduced first natural frequency from $f_1 = 3.31 \, \text{Hz}$ to $f_1 = 2.40 \, \text{Hz}$ and the acceleration levels increased from $a_{\text{peak},1} = 0.033 \, \text{m/s}^2$ to $a_{\text{peak},1} = 0.042 \, \text{m/s}^2$ at the top floor. The mode shape changed from bending to bending/shear, which was expected since the material stiffness was reduced. The recalculated results did not affect the conclusions from the findings in Paper I. The recalculated results are presented in Figure 16, Paper II. Cases B, C, D, and E represent the cases 1, 3, 5, and 7 in Paper I. The natural frequency decreases with increased height and higher acceleration levels result, Figure 16. The results indicate that timber buildings might fall short of comfort criteria already at a slenderness of 1.5 and to meet comfort limits at higher slenderness a change in stiffness, mass or damping is required.

![Figure 16 Increasing number of storeys, paper II](image)

Effect of changing parameters

In Figure 17, a summary of the results from Paper I is presented, and it shows the effect of changing stiffness, damping, and mass on the natural frequency and acceleration levels. The natural frequency and peak acceleration values should not be seen as absolute, they are just an indication of the effect of changing parameters. Oosterhout, (1996) found a similar tendency, with the same behaviour as found here. From the parameter study presented in Johansson, et
al., (2015) where the bending stiffness, mass, and damping were increased 2 and 3 times, the same behaviour was found only shifted in natural frequency and acceleration ranges. An increased stiffness increase the natural frequency and decrease the acceleration level, increase mass decrease both the natural frequency and acceleration and the damping only affect the acceleration level by reduce the acceleration level with increased damping, Figure 17.

The prediction curve for acceleration levels can be divided into 3 zones, Figure 17. Zone I, $f = 0.05 - 1 \text{ Hz}$, zone II, $f = 1 - 2 \text{ Hz}$, and zone III, $f = 2 - 5 \text{ Hz}$. In Zone I, the acceptable acceleration levels increase with decreased natural frequency, in Zone II the acceleration levels are constant and strictest and in Zone III the acceptable acceleration levels increase with increase natural frequency. Dependent on the natural frequency of the structure different change in the structural system is appropriate to meet the comfort limit in ISO 10137 (2008).

For $f = 0.5 \text{ Hz}$ the peak acceleration limit for residence is $a_{\text{peak}} = 0.054 \text{ m/s}^2$. By assuming a sinusoidal vibration, the magnitude of amplitude becomes around 2 mm while assuming along-wind vibration the amplitude becomes 5 mm, which is far below the static criteria $H/500$.

![Figure 17 Effect of changing stiffness, damping, and mass on natural frequency and acceleration levels](image)

**Stiffness**

By increasing the stiffness, the natural frequency increases and the acceleration is reduced. Increased stiffness can be achieved by changing the material properties i.e. either E-modulus or G-modulus (*Paper 1, cases 15 and 16*) or the thickness of the elements. Non-structural elements have not been considered in the simulations but may have an effect on the structural performance. The non-structural elements are often not included in the analysis due to lack of
knowledge of the contribution. From the measurement of a six storey timber frame building it was seen that the natural frequency increased when stairway and internal plasterboard was added to the structure (Ellis & Bougard, 2001). The natural frequency increased 25%-67% depending on mode direction (brickworks of 2 storeys were included in that measurement). The brickwork cladding also had an effect on the natural frequency. Since the plasterboard is a part of the stabilising system it is difficult to judge the effect of non-structural elements. Reynolds, et al., (2014b) also reported effect from non-structural elements on the natural frequency.

From the measurements of Treet the natural frequency for the first mode direction is in the range of 0.92-1.03, Table 1. The measured values are 23-37 % higher than simulated ones. The estimated value for the first natural frequency $f_1 = 0.75$ Hz is placed in zone I, Figure 17, while most of the measured ones are in zone II. An over- or underestimated stiffness can lead to an unsafe evaluation of the human comfort. High stiffness increases the natural frequency and can give values in zone III where the accepted acceleration levels increase with increased natural frequency. In the case of Treet the stiffness may have been underestimated or the higher natural frequency is a result of the contribution of the non-structural elements.

**Stiffness of connections**

Pinned connections were considered between wall and floor elements in Papers I and II. Pinned connections between walls and foundations and walls to walls did not result in any difference compared with rigid connections when performing a modal analysis. (Landel, et al., 2018) concluded that connections, in a truss system, modelled by spring and stiffness according to Eurocode 5 (2009) obtained better dynamic results than rigid and pinned connections. The tests were based on a single truss element, (18.5m) and 2D models in FE software. However, the global effect was small and the mode shape did not change when the connection stiffness was tested in modal analysis of the truss system of Treet (Malo, et al., 2016).

In Papers V and VI all connections were modelled as springs with the purpose to evaluate the effect on static horizontal deformations. The reasoning in Paper VI showed that connections can reliably be modelled using simple axial springs to represent groups of fasteners, which contributes to faster calculation times for models. Malo, et al., (2016) confirmed that axial stiffness is more important to incorporate in dynamic analysis than rotational stiffness. In Papers V and VI the surrounding structure and 3D effects were not evaluated. The contribution of 3D effects might lead to a stiffer structure than analysing separate parts of the structure since there are several elements in different directions contributing to the overall stiffness. Estimated values on natural frequency tends to be lower than measured ones, Table 1 (building 3, 4 and 10), and by modelling connections with springs the structure become weaker, which decreases the natural frequency.
Mass

By adding additional mass or increasing the material density both the natural frequency and the acceleration level decreases (Paper I, case 17-22, 25 and 26). Additional mass can be a concrete casting on a timber floor element or concrete element. The natural frequency decrease with increased mass which also was showed in Reynolds, et al., (2014b) where the natural frequency decreased, 2.70 Hz to 2.45 Hz (9 %) when additional 50 mm of cement was applied on the floor.

By using quasi-permanent load value (ψ2, mean value), 30 % of the live load can be added to the structure. For a residential building 30 % of live load corresponds to around 25 mm concrete. In this research, the live load has not been considered and a decreased natural frequency can be excepted. For all currently existing tall timber buildings, concrete elements has been used in the structural system, either as concrete slabs or shafts (Malo, et al., 2016), (Abrahamsen, 2017) (Fast, et al., 2016). The mass of the concrete elements contributes to the overall mass and decreases the natural frequency and the acceleration level. The simulated natural frequencies for Treet, Mjøstårnet and Brock Commons are all below 1 Hz, Table 1, and are placed in zone I where a decrease in natural frequency allows higher acceleration levels, Figure 17.

Figure 18 represent the mode shape and acceleration levels for a 14 storey (case C, Paper II) building for the first two orthogonal modes, mode 1 and 2. The dotted lines are the comfort limits for residences and offices (a = 0.04 m/s² / 0.06 m/s²). Adding extra mass (3 kN/m²) to storeys 10-14 has almost the same effect on acceleration levels as extra mass on all storeys, (Paper I, cases 25 and 26, recalculated values are presented). For mode 1, Figure 18a, the acceleration levels are not satisfactory for residential buildings from storey 9 and up. With added mass to storeys 10-14, the threshold moves up to storey 12. The requirements for office buildings are satisfied. For mode 2, Figure 18b, the acceleration levels are not satisfactory from storey 12 and up, but with an added mass to storeys 10-14 a satisfactory acceleration level is reached on the top floor (storey 14). The method of adding mass to the building is used both in Treet and Mjøstårnet to reduce the acceleration levels (Malo, et al., 2016) (Abrahamsen, 2017). The natural frequency decreases for the first mode from 1.65 Hz to 1.16 Hz when mass on all storeys is considered and to 1.26 Hz with extra mass on storeys 10-14. For mode 2, the natural frequency is reduced from 1.69 Hz to 1.19 Hz with mass on all storeys and to 1.29 Hz with extra mass on storeys 10-14. The building remains in zone II, Figure 17. For buildings in zone III, adding mass may be counterproductive whence the natural frequency decreases, the comfort limit become stricter, Figure 17.
Figure 18 14 storeys, a) mode 1 b) mode 2

Damping ratio
The damping ratio, $\zeta$, affects the acceleration level, which decreases with increased damping ratio, Figure 17. Increasing the damping ratio from 1.5% to 3.0% for a 14 storey building has almost the same effect on acceleration levels as adding mass, Figure 18. Damping ratio is an uncertain parameter since the damping can only be determined after the building is completed. From earlier measurements, a large variation of damping ratios in timber buildings was found, Table 1. For a building with CLT elements the damping ratio varies between 1.3-9.1%, for post-and-beam structures it varies between 1.4-2.4% and for hybrid buildings the
damping varies between 1.0-6.9 %, Table 1. The average damping ratio from 6 measured CLT structures is 3.43%. The average for post-and-beam structures is 1.7 %, which is close to the suggested value in Eurocode 1-4 (2005) for timber bridges, 1.9 %, which in turn has been used both in Treet and Mjøstårnet, Table 1. For hybrid structures the average values is 3.0 % The measured value for timber buildings are higher than values of damping ratios for buildings presented in Eurocode 1-4 (2005), where a damping ratio of 1.6% is presented for reinforced concrete buildings, 0.8% for steel buildings, and 1.5% for mixed structures of concrete and steel. For the hybrid timber-concrete structure, the damping ratio is in the same range as for the CLT structure, which may indicate that the energy dissipation in a hybrid structure occurs in the timber elements and connections. Higher damping ratios were collected for lower building heights, Table 1, and the damping ratio decreases with increased building height, which was found in Reynolds, et al., (2016b).

In this research a damping ratio of 1.5% been used. Looking at the collected measured damping ratios, a higher value could have been used and as a consequence presented acceleration values should decrease as seen in Figure 18. Figure 19 shows peak acceleration, Eq.(3), values from a modal analysis of a building with varying damping ratios but the same geometry. The relationship between peak acceleration level and damping ratio is not linear. By changing the damping ratio from 1.5% to 3.0% the acceleration level decreases around 30% for the 14–storey building. (Reynolds, et al., 2014b) showed that the damping ratio varied with the acceleration level and that the damping ratio increased after installation of non-structural elements.

![Figure 19 Acceleration vs damping ratio, 14 storey mode 1](image)
Natural frequency

In Papers I and II measured, simulated, and estimated values for natural frequencies of timber frame buildings have been presented (Table 1). Together with the empirical equations \( f = 46/h \) and \( f = 55/h \) the values are presented in Figure 20. The letters a and b stand for measured and simulated/estimated values respectively. Buildings that have been measured in two steps are labelled with 1, for measurements taken before partition walls, finishing etc. were added to the building. The measured values follow the same trend as the empirical predictions but most of the measured, simulated, and estimated values have a higher natural frequency than \( f = 46/h \). The higher natural frequency might be due to higher stiffness in the materials, connections and added components such as partition walls in the actual building compared with the simulated one.

![Figure 20 Empirical formulations together with measured, simulated and estimated natural frequencies](image)

The choice of empirical formula affects the acceleration level: \( f = 46/h \) is more conservative than \( f = 55/h \) since decreased natural frequency results in increased acceleration levels. The expression \( f = 46/h \) is valid for buildings with \( H > 50 \)m, which will give natural frequencies under 1 Hz and higher acceleration levels are then acceptable with increased height, zone I Figure 17, which is in the range of the ISO 6897 (1984) standard, 0.063-1 Hz. Most of the measured timber building have a natural frequency above 1 Hz where \( f = 46/h \) is not valid.

Several of the empirical formulae are applicable to different structural materials (Eq.(32)-Eq.(36) Japan) and two of the formulae are categorised using stabilising system, Eq.(37) and
Eq. (38) (Canada). The formulae for concrete structures Eq. (33)) and (35) give a higher estimated natural frequency than the formulae for steel structure Eq. (32) and (36). For higher buildings 7, 10, 11 and 16 in Figure 20, the natural frequency is close or under the curve of $f = 46/h$. For building 10, 11 and 16 additional mass has been added to the structure, which thereby the natural frequency decreases. To building 7, mass from non-structural elements has not been added. It should be noted that only building 10 has been measured and the other values (buildings 7, 11 and 16) are simulated/estimated values. Since Lagomarsion (1993) suggested to categorise formulae depending on material, it might be appropriate to have different formulae for hybrid structures and pure timber structures for estimating the first natural frequency. Some of the empirical formulae are categorised based on stabilising system (National Research Council of Canada, 2010) (Hu, et al., 2014), but in Figure 20 there is no clear difference between stabilising systems. Since there are few tall timber and hybrid structures ($h > 50$ m) built and measured the data is limited to judge the appropriateness of the empirical formulae. At lower heights ($16 < h < 30$ m) the measured values range between $f = 46/h$ and $f = 55/h$ and for building with $h < 16$ m, the natural frequency is below the curve $f = 46/h$. At that height the boundary conditions affect the behaviour. However, for building 12, which is a hybrid structure, the natural frequency is above $f = 55/h$. The concrete core may contribute to a stiffer structure and thereby result in a higher first natural frequency.

The numerical estimations of natural frequency presented in Paper II and in TräCentrumNorr (2016) are higher than measured ones. The numerical values (Paper II) represent a curve $f = 65/h$, Figure 20. Since there are several parameters that affect and are connected to the results, it is difficult to specifically determine the cause. The fictitious building is a tower block and has a core with lever arms to the perimeter of the outer walls where both the inner and outer parts of the structure contribute to the stiffness. Non-structural elements have not been investigated but according to Hu, et al., (2014) and Ellis & Bougard, (2001), the natural frequency increases when non-structural elements are added to the structure. Mass from live load, non-structural elements such as staircase, elevator shaft, technical installation (ventilation, water and satin, electrical) and windows are not included in the study. The additional mass from windows is $1,260$ kg/storey, the weight of windows is around $35$ kg/m². The mass from windows corresponds to a concrete layer of $14$ mm on a storey and adding mass from 30% of the live load will give a total concrete layer of $38$ mm. For a 14-storey building (Case C), additional mass from windows and live load decreases the first natural frequency around 11% to $1.47$ Hz, which is closer to the curve $f = 55/h$, Figure 20.

**CLT vs P & B**

Table 4 presents results from FE simulations from Paper II. In Paper I the CLT structure shows a tendency to rotate while the post-and-beam structure (Paper II) has two pure orthogonal modes and the third in torsion. The results showed that the two building types had similar first natural frequencies and the CLT system weighs around 10% more than the post-
and-beam system. The total mass/m³ is 96 kg/m³ which is almost 1/3 compared with the concrete building in Yang, et al., (2004).

In Paper II, acceleration levels calculated with generated modal mass and mode shape were compared with acceleration levels calculated with an approximation of equivalent mass, Eq.(41). In Figure 21, the acceleration levels for cases A-E are presented for mode 1. The approximation, $m_{e,app}$ is 14-22 % higher than $m_e$ derived from the generated modal mass and mode shape for cases A-E, Table 4. Higher equivalent mass reduces the acceleration levels and by using the approximation, acceleration levels can be on the unsafe side, e.g. case C in Figure 21.

Since the stiffness and mass of the post-and-beam system (Paper II) are not reduced by openings and the structural elements is placed symmetrically, the structure behaves more like a cantilever beam with linearly distributed mass over the height expect from the concentrated mass at floor levels. The concentrated mass contributes to a higher equivalent mass since the mass becomes lumped at each storey, Table 4 cases F-J. The difference between approximated and generated equivalent mass are between 6-11%, which decreases with increased height, Figure 22. Figure 22 presents $m_e$ for mode 1 and $m_{e,app}$ for case A-E plus data from cases with a slenderness of 4.5 and 9.0. However, the effect of a non-linear distribution of the mass decreases with increased height and the building approaches a continuous system with linearly distributed mass, Figure 22. Even though the modes shift direction, the difference between $m_e$ and $m_{e,app}$ decreases with increased height/slenderness. The use of Eq.(41) can over- or underestimate the equivalent mass depending on the structural layout and give acceleration levels on unsafe side.

Table 4 Results from FE simulations – mode 1, Paper II

<table>
<thead>
<tr>
<th>Case</th>
<th>h [m]</th>
<th>$m_{tot}$ [kg]</th>
<th>$f_1$ [Hz]</th>
<th>Mode shape</th>
<th>Generated mode $m_e$ [kg/m]</th>
<th>$a_{peak}$ [m/s²]</th>
<th>Approximation $m_{e,app}$ [kg/m]</th>
<th>$a_{peak,app}$ [m/s²]</th>
<th>$m_{e,app}/m_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>18</td>
<td>696,826</td>
<td>4.11</td>
<td>Y*</td>
<td>33,610</td>
<td>0.018</td>
<td>38,713</td>
<td>0.016</td>
<td>1.15</td>
</tr>
<tr>
<td>B</td>
<td>30</td>
<td>1,152,788</td>
<td>2.40</td>
<td>Y*</td>
<td>31,625</td>
<td>0.042</td>
<td>38,426</td>
<td>0.034</td>
<td>1.22</td>
</tr>
<tr>
<td>C</td>
<td>42</td>
<td>1,608,750</td>
<td>1.65</td>
<td>Y*</td>
<td>31,658</td>
<td>0.066</td>
<td>38,304</td>
<td>0.055</td>
<td>1.21</td>
</tr>
<tr>
<td>D</td>
<td>54</td>
<td>2,064,711</td>
<td>1.23</td>
<td>Y*</td>
<td>32,766</td>
<td>0.090</td>
<td>38,235</td>
<td>0.078</td>
<td>1.17</td>
</tr>
<tr>
<td>E</td>
<td>66</td>
<td>2,520,673</td>
<td>0.94</td>
<td>Y*</td>
<td>33,486</td>
<td>0.120</td>
<td>38,192</td>
<td>0.106</td>
<td>1.14</td>
</tr>
<tr>
<td>F</td>
<td>18</td>
<td>635,594</td>
<td>4.14</td>
<td>Y</td>
<td>39,509</td>
<td>0.017</td>
<td>35,311</td>
<td>0.019</td>
<td>0.89</td>
</tr>
<tr>
<td>G</td>
<td>30</td>
<td>1,050,735</td>
<td>2.47</td>
<td>Y</td>
<td>38,458</td>
<td>0.034</td>
<td>35,025</td>
<td>0.037</td>
<td>0.91</td>
</tr>
<tr>
<td>H</td>
<td>42</td>
<td>1,465,876</td>
<td>1.67</td>
<td>Y</td>
<td>37,849</td>
<td>0.052</td>
<td>34,902</td>
<td>0.056</td>
<td>0.92</td>
</tr>
<tr>
<td>I</td>
<td>54</td>
<td>1,881,017</td>
<td>1.21</td>
<td>X</td>
<td>37,497</td>
<td>0.080</td>
<td>34,834</td>
<td>0.086</td>
<td>0.93</td>
</tr>
<tr>
<td>J</td>
<td>66</td>
<td>2,296,158</td>
<td>0.91</td>
<td>X</td>
<td>37,143</td>
<td>0.111</td>
<td>34,790</td>
<td>0.118</td>
<td>0.94</td>
</tr>
</tbody>
</table>

* the mode has a tendency to rotate
Figure 21 Acceleration levels for generated and approximated equivalent masses for the CLT system - mode 1

Figure 22 Equivalent mass vs slenderness, (case A-E and h/d = 4.5 and 9.0)

**Bending and shear modes**

From *Paper II*, the generated mode shape for cases A and F with slenderness of 0.9 had at lower heights a bending tendency but with increasing building height the mode shifts to shear i.e. the EI/GA-ratio for the “beam” representing the building changes with the height Figure
23. With increased slenderness (and/or height), the mode shape shows more bending at lower heights, Figure 24. A pure bending mode shape is not present in Figure 24. The mode shape is a combination of both bending and shear. The post-and-beam system (cases F and J) is stiffer than the CLT system (cases A and E) and the bending mode shape will appear at lower heights. The constant $\zeta = 1.5$ in Eurocode 1-4 (2005) is set to represent a slender building that will deform in bending which timber buildings, at these heights ($h < 66m$), do not. A mode shape constant, $\zeta$, between 0.6 and 1.0 seems to be more suitable for timber buildings, Figure 23 and Figure 24.

The normalised mode shape for floor plan III are presented in Figure 25 and Figure 26 for slenderness ratios 1.5, 3.9 and 9.0 represented by normalised height (Paper IV). For the Timber structure, a shear mode shape appears at slenderness of 1.5 (both modes 1 and 2) and with increased slenderness the mode shape shows more bending behaviour, Figure 25 a and b which is in line with results in Paper II. At a slenderness of 3.9 the Timber structure has an almost linear mode shape. A slenderness of 2.0-2.5 corresponds to a 13-16 storey building (if the width is 20 m) and a shear mode can be expected for high-rise timber buildings, which can be problematic given the low shear modulus of timber. The Hybrid I and Hybrid II structures, Figure 25 c-d and Figure 26 a-b, shows a linear behaviour at lower slenderness ratios (1.5). For the second mode of Hybrid I and at a slenderness of 1.5, the shear mode shape appears. The Hybrid I structure is symmetric in the Y-direction and the concrete walls in the shaft contribute to increasing the stiffness with less shear behaviour as a result. The concrete structure, Figure 26 c and d, has more or less a linear mode shape at a slenderness of 1.5 and with increased slenderness the mode shape changes to bending.
Figure 24 Mode shape, cases E and J, Paper II
Figure 25 Mode shape for floor plan III for Timber, Hybrid I
Acceleration levels at different heights in the building are dependent on the mode shape, Eq. (2). A shear mode shape will give larger acceleration levels at lower heights of the building compared with a bending mode. In Figure 27 a and b the variation of acceleration levels for a 14 storey building (case C in Paper II) with different mode shapes are presented, mode 1 and 2. Values of acceleration are also presented by using generated equivalent mass $m_e$ and approximated $m_{e,app}$. The thicker lines represent calculations made with approximation of equivalent mass and the thin lines represent calculations with generated equivalent mass. The dotted lines are acceleration limits for residence and offices. The applied mode shape does not affect the acceleration on the top of the structure but at lower heights, the mode shape will give a variation in acceleration levels, Figure 27 a and b.
In Table 5, acceleration levels at storey 13 are presented for modes 1 and 2. The first column presents acceleration levels calculated with generated equivalent mass, \( m_e \), and generated mode shape. The second column presents acceleration levels calculated with generated equivalent mass, \( m_e \), and assumed mode shape with \( \zeta = 1.5 \). The third column presents acceleration levels calculated with approximated equivalent mass, \( m_{e,app} \), and generated mode shape. The last column presents acceleration levels calculated with approximated equivalent mass, \( m_{e,app} \), and assumed mode shape with \( \zeta = 1.5 \).
Table 5 Acceleration levels at storey 13, generated vs approximation

<table>
<thead>
<tr>
<th>Storey 13</th>
<th>$m_e$ $\phi =$ generated</th>
<th>$m_e$ $\phi =$ assumed $\zeta=1.5$</th>
<th>$m_{e,app}$ $\phi =$ generated</th>
<th>$m_{e,app}$ $\phi =$ assumed $\zeta=1.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>0.063</td>
<td>0.054</td>
<td>0.052</td>
<td>0.045</td>
</tr>
<tr>
<td>Mode 2</td>
<td>0.047</td>
<td>0.041</td>
<td>0.050</td>
<td>0.044</td>
</tr>
</tbody>
</table>

The acceleration levels vary and in this case, mode 1, the approximation, $m_{e,app}$ and mode shape, $\zeta = 1.5$, give an acceleration level that is 29% less than the acceleration level from the generated equivalent mass, $m_e$, and the generated mode shape. For mode 2, the approximated equivalent mass, $m_{e,app}$, and the generated mode shape will give the highest acceleration level while generated equivalent mass, $m_e$, and assume mode shape with $\zeta = 1.5$ the lowest, a difference of 19%. The acceleration levels can be on the unsafe side if the equivalent mass and mode shape are not properly handled. Choosing $\zeta = 1.5$ can give acceleration levels on the unsafe side when evaluating acceleration levels at lower heights of the building. The mode shape with $\zeta = 1.5$ corresponds to a bending mode and according to Eurocode 1-4 (2005), the type of buildings it should be used for are slender cantilever buildings and buildings supported by a central reinforced concrete core.

Effect of openings

In Paper III and IV the effect of openings and asymmetric floor plans was studied. In cases where the centres of mass and rigidity do not coincide, the structure tends to rotate and/or have a diagonal translation. Openings of 1.5x2.5 m² reduce the stiffness and the mass of the structure and results in increased natural frequency. However, smaller openings (1.5x1.0 m²) do not affect the dynamic properties to any larger extent. For all cases (O – 2X2Y), the structure had mainly two orthogonally translation modes and one torsional mode. Case 1X1Y mode 2, Table 6, stands out from the other cases since the mode direction was diagonal. The results from the FE software have amplitudes in two directions. The large modal mass in case 1X1Y may be due to the diagonal mode direction where the amplitude has components in both the X- and Y-directions with the modal mass being a summation of the mass times the squared amplitude in each node. In the other cases the modal mass is more or less associated with one direction, Table 6. The appropriateness of using Eurocode 1-4 (2005) is questionable for this type of mode, since the equation in Eurocode 1-4 (2005) and the Swedish National Annex EKS 10 (2015) assume two-dimensional vibration modes which case 1X1Y is not.
| Case | m_{\text{base}} [kg/storey] | X_{\text{CM}} [m] | Y_{\text{CM}} [m] | X_{\text{CR}} [m] | Y_{\text{CR}} [m] | \varepsilon_x [m] | \varepsilon_y [m] | h/d | Mode 1 \( M_1 \) [kg] | \( \tilde{m}_{1x} \) / \( \tilde{M}_1 \) | \( \tilde{m}_{1y} \) / \( \tilde{M}_1 \) | Mode 2 \( M_2 \) [kg] | \( \tilde{m}_{2x} \) / \( \tilde{M}_2 \) | \( \tilde{m}_{2y} \) / \( \tilde{M}_2 \) |
|------|----------------|-----------------|-----------------|-----------------|-----------------|----------------|----------------|-----|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| O    | 63,630         | 0               | 0               | 0               | 0               | 0              | 0              | 1.5 | 291,111        | 1.00            | 0.00            | 289,896        | 0.00            | 1.00            |
|      |                 |                 |                 |                 |                 |                |                | 3.9 | 610,446        | 1.00            | 0.00            | 608,785        | 0.00            | 1.00            |
|      |                 |                 |                 |                 |                 |                |                | 9   | 1,100,494      | 1.00            | 0.00            | 1,098,987      | 0.00            | 1.00            |
| 1X   | 62,437         | 0               | 0.191           | 2.333           | 0               | 2.142          | 1.5            | 205,687        | 0.98            | 0.02            | 280,329        | 0.00            | 1.00            |
|      |                 |                 |                 |                 |                 |                |                | 3.9 | 497,029        | 0.99            | 0.01            | 576,162        | 0.00            | 1.00            |
|      |                 |                 |                 |                 |                 |                |                | 9   | 1,057,506      | 0.00            | 1.00            | 1,025,699      | 1.00            | 0.00            |
| 2X   | 61,244         | 0               | 0               | 0               | 0               | 0              | 1.5            | 286,332        | 1.00            | 0.00            | 270,990        | 0.00            | 1.00            |
|      |                 |                 |                 |                 |                 |                |                | 3.9 | 657,066        | 1.00            | 0.00            | 546,443        | 0.00            | 1.00            |
|      |                 |                 |                 |                 |                 |                |                | 9   | 1,020,637      | 0.00            | 1.00            | 1,160,846      | 1.00            | 0.00            |
| 1X1Y | 61,244         | 0.195           | 0.195           | 2.333           | 2.333           | 2.13           | 1.5            | 259,501        | 0.55            | 0.45            | 497,206        | 0.45            | 0.55            |
|      |                 |                 |                 |                 |                 |                |                | 3.9 | 652,356        | 0.62            | 0.38            | 907,654        | 0.38            | 0.62            |
|      |                 |                 |                 |                 |                 |                |                | 9   | 1,735,922      | 0.60            | 0.40            | 1,538,637      | 0.40            | 0.60            |
| 1X2Y | 60,051         | 0               | 0.199           | 2.333           | 0               | 2.135          | 1.5            | 278,437        | 0.00            | 1.00            | 167,992        | 0.95            | 0.05            |
|      |                 |                 |                 |                 |                 |                |                | 3.9 | 622,052        | 0.00            | 1.00            | 435,032        | 0.99            | 0.01            |
|      |                 |                 |                 |                 |                 |                |                | 9   | 953,745        | 1.00            | 0.00            | 1,102,000      | 0.00            | 1.00            |
| 2X2Y | 58,858         | 0               | 0               | 0               | 0               | 0              | 1.5            | 269,983        | 1.00            | 0.00            | 270,197        | 0.00            | 1.00            |
|      |                 |                 |                 |                 |                 |                |                | 3.9 | 588,208        | 1.00            | 0.00            | 585,276        | 0.00            | 1.00            |
|      |                 |                 |                 |                 |                 |                |                | 9   | 1,047,724      | 1.00            | 0.00            | 1,043,546      | 0.00            | 1.00            |

From the generated modal mass and mode shape, the equivalent mass for all cases has been calculated according to Eq.(40) and the approximate equivalent mass has been calculated according to Eq.(41). The results are presented Figure 28 for all slenderness ratios. For symmetric cases (O, 2X, 2X2Y), Figure 28 a, c and f, the equivalent mass, \( m_e \), is around 5% higher than \( m_{e,\text{app}} \) for both modes 1 and 2. The only difference is the reduced mass due to openings. The higher value of \( m_e \) is due to the concentrated mass on each third metre from the floor structure. For the asymmetric cases 1X and 1X2Y, Figure 28 b and e, \( m_e \) is around 30% less than \( m_{e,\text{app}} \) for the second mode and decreases with increased slenderness. In the case 1X1Y, Figure 28 d, a large difference between \( m_e \) and \( m_{e,\text{app}} \) is shown and it is attributed to the calculation method of modal mass, where a simplification was made in Eq.(22). Due to the asymmetry depending on the placement of openings, the structure becomes non-homogenous and the equivalent mass may be underestimated or overestimated. A reduction of 20% of the equivalent is almost linearly proportional to the acceleration. The standard deviation of acceleration in Eq.(2) is also dependent on the natural frequency and the resonance factor, \( R \), so the reduction is not exactly a one-to-one relation. But overall, it can be stated that an under- or overestimation of the equivalent mass changes the acceleration levels with almost the same percentage.
Figure 28 Equivalent mass vs slenderness, a) case O b) case 1X c) case 2X d) case 1X1Y e) case 1X2Y f) case 2X2Y
**Timber, hybrid, and concrete structures**

A comparison between timber, hybrid, and concrete structures was conducted in *Paper IV* where asymmetrical floor layouts were studied, Figure 11. The asymmetrical placement of the shaft generated an eccentricity between the centre of mass and rigidity along the Y-axis and which causes a tendency to rotate.

**Timber**

For the timber structure, independent of placement of the shaft, the second mode is a pure translation mode in Y-direction, Figure 29b. The first mode is a pure translation when the shaft is placed in the middle of the structure (placement I) while for placements II and III the structure shows more tendency to rotate around the shaft, Figure 29a and c. This is due to the shaft being the stiffest part of the structure. Due to the rotation, all mass of the structure does not contribute to the vibration mode and the equivalent mass, $m_e$, is reduced compared with the approximation $m_{e,\text{app}}$. The difference is greatest for a slenderness of 1.5 where $m_{e,\text{app}}$ is 25% higher than $m_e$, Figure 30 a. The difference decreases with increased slenderness as also seen in *Paper III*. The modal mass for placement III, mode 1, in the Y-direction is only 1.1% of the total modal mass for a slenderness of 1.5 (*Paper IV*) and the mode is seen as pure translation. Acceptable acceleration levels for residences are achieved at a slenderness of 1.5, but not more. This corresponds to a timber building that is 30 m high i.e. 9-10 storeys. The natural frequency for slenderness 1.5 is $f_1=3.60$ Hz and $f_2=3.64$ Hz. If using the limits for offices, the slenderness ratio 2.1 can be allowed i.e. a building height of 42 m or 13-14 storeys. The natural frequency is $f_1=2.46$ Hz, at slenderness of 2.1, and by increasing the stiffness, maybe a few more storeys can be achievable. By using the approximation, $m_{e,\text{app}}$, an acceleration level that is 20% smaller (slenderness of 1.5) is generated, which is on the unsafe side.

**Hybrid I**

The concrete shaft in *Hybrid I* will both increase the stiffness and the mass, but the natural frequencies are almost the same as in the timber case. For *Hybrid I* the natural frequencies are $f_1=3.68$ Hz and $f_2=3.94$ Hz, for a slenderness of 1.5 and placement I of the shaft. The added concrete will increase the equivalent mass, but the structure starts to rotate around the shaft when the shaft is placed in the outer part of the structure (placement II and III), Figure 29c. The *Hybrid I* solution with floor plan III is quite similar to the buildings presented in Reynolds, et al., (2016a). Both the simulations presented here and the work in Reynolds, et al., (2016a) showed that the buildings have a tendency to rotate around the core. A pure torsional mode does not occur, most of the displacements are still in the translation direction and the equivalent mass, $m_e$, is reduced compared with the approximation, $m_{e,\text{app}}$ Figure 30 b. Additional mass from the shaft will not help as much to reduce the acceleration in mixed mode as if it was a pure translation mode. The modal mass in Y-direction is 5% of the total modal mass (*Paper IV*). Torsional acceleration may need handling and that part is missing in
Eurocode 1-1-4 (SS-EN 1991-1-4, 2005). Equations for torsional accelerations can be found in (Kwon & Kareem, 2013). At a slenderness of 2.1 acceptable levels are reached both for residences and offices even though the structure has a tendency to rotate that reduces the equivalent mass, Figure 30 b.

**Hybrid II**

The concrete wall together with the concrete shaft, Figure 11 creates enough torsional stiffness so that pure translation in the first two orthogonal modes is obtained except at a slenderness of 1.5 where a tendency to rotate occurs even though the eccentricity is 6.56 m between centre of mass and rigidity (Paper IV). The concrete wall and concrete shaft create a balanced system and for a slenderness above 1.5 the calculated equivalent mass is close to the approximation, Figure 30 c. Acceptable acceleration levels are obtained at a slenderness of 2.1 for residences and 2.7 for offices, which is four more storeys than Timber and Hybrid I.

**Concrete**

Concrete is around 3 times stiffer (E-modulus) and weighs around 5-6 times more than timber. The natural frequencies for Concrete cases are 18-63% higher than the Timber cases and decreases with increased height. For a slenderness of 1.5 the natural frequency is $f_1 = 5.85$ Hz which decreases to $f_1 = 2.41$ Hz for a slenderness of 2.7. The high stiffness reduces the tendency to rotate even when the shaft is placed at the outer wall (floor plan III). For all floor plans (I-III) the calculated equivalent mass is within +/- 10% of the approximation, Figure 30 d. At a slenderness of 9.0 acceptable acceleration levels for residences is satisfied due to the high mass. The approximation of equivalent mass, $m_{e,app}$, at any slenderness, seems to have good agreement with the generated $m_e$. 
Figure 29 Mode directions a) translation in X-direction b) translation in Y-direction c) Translation in X-direction with a tendency to rotate d) Translation in Y-direction
With symmetrical floor layouts the torsional stiffness of the building become crucial and non-pure translation mode may occur. Adding a concrete shaft to the structure is thus a good measure if wanting to build higher, but care must be taken to place the shaft as to avoid torsion. The challenge with constructing a building with a hybrid system is to transfer the forces to the elevator shaft and any stabilising walls to ensure that the effect is consistent.

**Interaction effect between openings and floor plan**

The engineer should beware of the effect openings and floor layout has on the dynamic performance of timber buildings. In Table 7 acceleration levels for the first two orthogonal modes are presented for cases O-2X2Y (Paper III), Timber structure with floor layout I (Paper IV) and cases 2X1Y and 2X1Ya-c which represent case B in Paper II where non-structural mass and openings along one wall are varied.

The non-structural mass decreases the natural frequency of case 2X1Y from 3.07 to 2.4 Hz for mode 1, which is 22 % smaller. Case 0-2X2Y could be expected to be in the range of 1-2 Hz
if the non-structural mass was added and stricter acceleration levels are set, Figure 17. For cases 2X1Y and 2X1Yb the natural frequency increases moderately, 3.8 % for mode 1 and 6.5 % for mode 2 but the reduction of the acceleration is around 20 % due to a more symmetric building. It should be observed that case 2X1Yb changes mode direction when the openings are removed and the X-direction of the building becomes the weak direction, which was the second mode for case 2X1Y and 2X1Ya.

Timber structure with floor layout I has the natural frequencies \( f_1 = 3.60 \text{ Hz} \) and \( f_2 = 3.64 \text{ Hz} \) which is similar to case O and 2X1Yc. The core and the lever arm do not contribute to any larger extent compared with cases O and Timber. The non-structural mass has a larger impact on the natural frequency than the openings on one wall (cases 2X1Ya- 2X1Yc), but the openings create asymmetry in the layout and affect the equivalent mass and thereby the acceleration level.

**Table 7 Acceleration levels for selected cases**

<table>
<thead>
<tr>
<th>Case</th>
<th>( h/d )</th>
<th>( f_1 ) [Hz]</th>
<th>( f_2 ) [Hz]</th>
<th>( a_1 ) [m/s(^2)]</th>
<th>( a_2 ) [m/s(^2)]</th>
<th>( a_{1,\text{limit residence}} ) [m/s(^2)]</th>
<th>( a_{2,\text{limit residence}} ) [m/s(^2)]</th>
<th>( a_{1,\text{limit office}} ) [m/s(^2)]</th>
<th>( a_{2,\text{limit office}} ) [m/s(^2)]</th>
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<tr>
<td>O</td>
<td>1.5</td>
<td>3.14 3.16</td>
<td>0.044 0.044</td>
<td>0.063 0.063</td>
<td>0.094 0.095</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1X</td>
<td>1.5</td>
<td>2.54 3.15</td>
<td>0.057 0.044</td>
<td>0.051 0.063</td>
<td>0.076 0.094</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2X</td>
<td>1.5</td>
<td>2.08 3.13</td>
<td>0.073 0.044</td>
<td>0.042 0.063</td>
<td>0.062 0.094</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1X1Y</td>
<td>1.5</td>
<td>2.39 2.66</td>
<td>0.061 0.055</td>
<td>0.048 0.053</td>
<td>0.072 0.080</td>
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</tr>
<tr>
<td>1X2Y</td>
<td>1.5</td>
<td>2.12 2.46</td>
<td>0.071 0.058</td>
<td>0.042 0.049</td>
<td>0.064 0.074</td>
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<td></td>
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<tr>
<td>2X2Y</td>
<td>1.5</td>
<td>2.08 2.11</td>
<td>0.073 0.071</td>
<td>0.042 0.042</td>
<td>0.062 0.063</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Timber, I</td>
<td>1.5</td>
<td>3.60 3.64</td>
<td>0.033 0.031</td>
<td>0.072 0.073</td>
<td>0.108 0.109</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2X1Y</td>
<td>1.5</td>
<td>2.40 2.47</td>
<td>0.044 0.031</td>
<td>0.048 0.049</td>
<td>0.072 0.074</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>2X1Ya</td>
<td>1.5</td>
<td>3.07 3.16</td>
<td>0.052 0.038</td>
<td>0.061 0.063</td>
<td>0.092 0.095</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>2X1Yb</td>
<td>1.5</td>
<td>2.49 2.63</td>
<td>0.031 0.036</td>
<td>0.050 0.053</td>
<td>0.075 0.079</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2X1Yc</td>
<td>1.5</td>
<td>3.19 3.37</td>
<td>0.037 0.042</td>
<td>0.064 0.067</td>
<td>0.096 0.101</td>
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</tbody>
</table>

Safe

Unsafe for residence

Unsafe for residence and office
Discussions and conclusions

In this research over 250 simulations of multi-storey timber buildings have been performed to study their dynamic behaviour and compare it to comfort criteria. Different parameters, such as stiffness, mass, and damping have been varied and analysed against recommendations in Eurocode 1-4 (2005). The effects of asymmetrical floor plans and placement of openings have been studied.

*Increasing mass is not the only solution*

Depending on a building’s natural frequency, different changes in the structural system are appropriate to reduce the acceleration levels with respect to ISO 10137 (2008). Increased mass, especially on the top floor of the building decreases the natural frequency and acceleration levels which is suitable in Zone I. In Zone III increased stiffness is preferred since it increases the natural frequency and decreases the acceleration levels.

![Figure 31 Peak acceleration vs natural frequency, discussion on zones](image)

Timber buildings can fall into zones I, II, and III. As shown in this study, many timber structures present themselves in zone III. This makes e.g. ISO 6897 (1984) difficult to apply for estimating acceleration levels, since the frequency range covered is 0-1 Hz. When structures present themselves in zone III, it is important to remember that adding mass to the structure can be contra productive and lead to a structure that comes closer to the limiting curve. When comparing, ISO 10137 (2008) reflects a more severe comfort criterion than ISO 6897 (1984), Figure 7. The static criterion H/500 or other selected criterion is met if the dynamic criteria is fulfilled.
**Empirical equations are not valid**

Eq.(30) should be used with care since it relies on measurements of concrete and steel buildings and is only valid for buildings with a height \( h > 50 \text{ m} \) i.e. buildings with a natural frequency below 0.92 Hz. There are few existing timber buildings with these properties. If opting to include empirical formulas such as Eq.(30) in Eurocode 1-4 (2005), equations valid for timber and timber-concrete structures would need to be established since these are missing. However, since timber buildings experience problems with comfort criteria due to dynamic motions at lower heights compared with concrete and steel buildings the assumption that the building is a tower block characterised solely by its height is non-valid. Consequently, the recommendation is to perform FE simulations instead of relying on empirical estimations until the empirical knowledge base is large enough.

**Asymmetrical floor plans should be avoided**

The placement of the stabilising system in the floor plan is important to create a balanced system (symmetric) to achieve pure translation modes. In the current standard, Eurocode 1-4 (2005) or EKS 10 (2015) 2D, diagonal modes in the XY plane, cannot be handled since the standards assume 2D modes in the YZ or XZ plane. Two orthogonal modes are preferable since torsional stiffness of timber structure is low and also since equations for evaluating torsional modes is missing in Eurocode 1-4 (2005) or EKS 10 (2015). By placing the stabilising system far from the rotational centre, organise symmetry in the floor plan and create long lever arms a high torsional stiffness can be achieved.

**Proper handling of equivalent mass is needed for timber buildings**

Already at heights of 10 storeys, timber buildings have problems with meeting the criteria for acceleration levels. Thus, the assumption of having a homogeneous, vibrating beam element is not valid and using it might lead to the design of structures that have undesired acceleration levels. Furthermore, the assumption of a homogeneous beam is also applied to the calculation of equivalent mass, Eq. (41). For mid-rise timber buildings this assumption is not valid and the equivalent mass can be overestimated by 20-30%. Instead, engineers are recommended to use the generated equivalent mass from FE simulations along with the generated mode shape to accurately represent timber and hybrid structures.

**Openings affect the stiffness and mass of CLT structures simultaneously**

In a post-and-beam structure with diagonal bracing the stiffness and mass is not reduced by openings for windows and doors as they are in a CLT structure. Thereby, the post-and-beam structure behaves like a homogeneous cantilever beam and the approximation, Eq.(41), is valid given that the truss system is symmetric. The concentrated mass from the floor elements on each third meter will yield a higher generated equivalent mass, \( m_e \), compared with the approximation \( m_{e,app} \), which is based on a linear mass distribution while the mass from the floor is lumped.
The mode shape is seldom bending for timber structures
The acceleration levels will vary along the height of the structure depending on the mode shape, Eq.(2). Eurocode 1-4 (2005) suggests different mode shapes depending on the structural system and material Eq.(42). Timber buildings with a slenderness $< 3.9$ have more or less a shear mode shape. Then, the mode shape becomes linear with increased slenderness. A shear mode shape will give larger acceleration levels at lower heights of the building compared with a bending mode. Therefore, the use of $\zeta = 1.5$ in Eq. (42) for timber structures is not recommended (corresponds to a bending mode). If in need to use the mode shape equation with associated factors in Eurocode 1-4 (2005) a value of $\zeta = 0.6 – 1.0$ is recommended for timber buildings. However, modal analysis to determine the mode shape is the preferred method.
Further Research

The non-structural elements are not included in the FE model except as masses. As seen in Ellis & Bougard (2001) and Reynolds, et al., (2014b) the natural frequency changes during construction when non-structural elements are added. In what manner they contribute to the overall behaviour of the structure is still to be explored and especially how they should be included in an FE model.

In higher timber buildings \( f < 1 \text{ Hz} \) adding mass decreases the acceleration level. Appropriate solutions can be adding mass too top floors or using a hybrid solution with a concrete shaft. Timber hybrid solutions with a concrete shaft can undergo torsion, *Paper I* and (Reynolds, et al., 2016a). The current Eurocode 1-4 (2005) does not propose a method to handle torsional behaviour. The standard ISO 10137 (2008) can be used to evaluate torsional behaviour and further research is to address the torsional behaviour in timber structures and propose a method that can be incorporated in Eurocode 1-4 (2005).
Guidelines when designing timber buildings with respect to dynamic comfort criteria

A schematic figure of the design process when evaluating acceleration levels is presented in Figure 32.

1 Conceptual design
The interplay between architects and structural engineers becomes more important when aiming for higher timber buildings. The size of the footprint and the slenderness of taller timber buildings play a significant role in achieving acceptable acceleration levels. Already at a slenderness of 1.5 (10 storeys) timber buildings can have problems with the comfort criteria depending on the structural layout. Therefore, engineers are encouraged to perform a modal analysis using FE software to determine the lowest natural frequency and mode shape. The choice of stabilising system and placement of stabilising elements affect the dynamic properties along with the size and placement of openings. The engineer should strive for the first two modes being pure orthogonal translation modes and iterate design until these conditions are achieved. Take note of the position of the design proposal in Figure 31 according to ISO 10137 (2008) remembering that many timber buildings have their lowest natural frequency at 1-3 Hz.

2 Stiffness and mass
Timber weighs around 1/5 of concrete and has much lower stiffness. This affects both the natural frequency and acceleration levels. A dynamic analysis is performed in serviceability limit state where mean values on stiffness and mass should be used (quasi-permanent load value, $\psi_2$). A CLT structure is more sensitive to reduced torsional stiffness due to openings compare to a post-and-beam structure. Added mass will reduce the natural frequency and acceleration levels and depending on the position in Figure 31 the action of adding mass will be positive or negative. 30% of the live load and masses from non-structural elements (insulation, boards, windows, inner walls etc.) should be added to the structure before simulations. The effect of connections is still not explored to any large extent and may be assumed rigid due to 3D complexity. Modelling connections as semi-rigid is correct when looking at the behaviour of a single joint. However, measurements of natural frequency in timber buildings show an increase in natural frequency of 23-37% compared with simulations. This points to the fact that the connections should be modelled either as semi-rigid or fully rigid.

3 Natural frequency and mode shape
Empirical formulae for natural frequencies should be used with care since they rely on measurements of concrete and steel buildings. The empirical formula in Eurocode 1-4 (2005), $f = 46/h$, may underestimate the natural frequency of a timber building. The acceleration levels vary along the height of the building depending on the mode shape.
A shear mode shape will give larger acceleration levels at lower heights of the building compared with a bending mode. A shear mode can be expected for high-rise timber buildings. Therefore, the use of $\zeta = 1.5$ in Eq.(42) for timber structures is not recommended (corresponds to a bending mode). If in need to use the mode shape equation with associated factors in Eurocode 1-4 (2005) a value of $\zeta = 0.6 – 1.0$ is recommended for timber buildings. However, modal analysis to determine the mode shape is the preferred method.

4 Equivalent mass

The equivalent mass is calculated by using Eq.(40) where the numerator is the modal mass which can be generated by FE software. The denominator is the integral of the mode shape over the height of the building where the amplitude has its maximum. The number of integration points can be set to the mesh size over the height.

$$m_e = \frac{\int_0^h m(z) \phi_1^2(z) dz}{\int_0^h \phi_1^2(z) dz} \quad (46)$$

According to Eurocode 1-4 (2005) an approximate expression for the equivalent mass is the average value of the mass of the upper third of the structure or the total mass of the building divided by the total height:

$$m_{e, app} = \frac{m_{tot}}{h} \quad (47)$$

The approximation assumes a homogeneous structure and linear distribution of the mass. Asymmetrical placement of the stabilising system and larger openings may affect the equivalent mass and the approximation is not valid. The approximation may under- or overestimate the equivalent mass and can give acceleration levels on the unsafe side. The engineer is recommended to use the generated modal mass and the associated mode shape from an FE software for a timber and hybrid structure.

5 Damping

For a post-and-beam structure a damping ratio of 1.9 % can be used and for a CLT structure a damping ratio of 2-2.5% may be used. Damping ratio of hybrid structures may be around 2-3%. There are still too few measurements to give an exact value.

6 Acceleration

The acceleration is calculated according to EKS 10 (2015) (Swedish regulation) where some parts refers to Eurocode 1-4 (2005). Since higher timber buildings are more sensitive to wind-induced vibrations a wind tunnel test may be appropriate to determine the terrain type and the basic wind velocity for a specific case - often a lower basic wind velocity can be adopted. The Swedish Meteorological and Hydrological Institute can also give more specific basic wind velocities from their meteorological observations.

7 Evaluation of acceleration

According to EKS 10 (2015), ISO 6897 (1984) is recommended. The standard is valid
for natural frequencies between 0.063-1 Hz. Timber buildings may fall into a range where ISO 6897 (1984) is not valid. The standard is a general advice in the Swedish code which allows engineers to use other standards where ISO 10137 (2008) which has a larger span of natural frequencies 0.06-5 Hz is recommended.

In the end, it is the engineer’s responsibility to create and validate FE models and judge the appropriateness of approximations.
Figure 32 Schematic figure of the design process
References


Autodesk Inc., 2015. Technical documentation for calculating the modal mass: Autodesk Inc..

Autodesk Inc., 2015. Theoretical basis for time history analysis: Autodesk Inc..


ETA-13/0684, 2013. *Solid wood slab element to be used as a structural element in buildings*


ISO 6897, 1984. *Guidelines for the evaluation of the response of occupants of fixed structures, especially buildings and off-shore structures, to low-frequency horizontal motion (0.063 to 1 Hz)*. u.o.: International Organization for Standardization.


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Reynolds, T. et al., 2014b. Ambient vibration tests of a cross-laminated timber building. *Institution of Civil Engineers*.

SMHI, [Online] Available at: [http://smhi.se/](http://smhi.se/)


