



## Damage Detection in Structures – Examples

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### Abstract

Damage assessment of structures includes estimation of location and severity of damage. Quite often it is done by using changes of dynamic properties, such as natural frequencies, mode shapes and damping ratios, determined on undamaged and damaged structures. The basic principle is to use dynamic properties of a structure as indicators of any change of its stiffness and/or mass. In this paper, two new methods for damage detection are presented and compared. The first method is based on comparison of normalised modal shape vectors determined before and after damage. The second method uses so-called  $l_1$ -norm regularized finite element model updating. Some important properties of these methods are demonstrated using simulations on a Kirchhoff plate. The pros and cons of the two methods are discussed. Unique aspects of the methods are highlighted.

**Keywords:** mode shape damage detection, finite element model updating,  $l_1$ -norm regularization.

### 1. Introduction

Structural damage can cause loss of load bearing capacity with far-reaching consequences. To prevent that risk, early identification of damage, preferably by non-destructive testing, is of great importance. Non-destructive techniques such as ultrasonic methods, radiography, magnetic particles, eddy currents and acoustics emissions are capable to detect and locate damages even if they are not visible on the surface of the structure. The main disadvantages of these methods are that they require inspection of very small areas and that they therefore need a prediction of the damage location in advance as well as that they need to have access to that part of the structure [1, 2].

Nowadays, structural health monitoring (SHM) systems, especially for large structures, e.g. bridges, high rise buildings, cultural heritage structures, can be based on another non-destructive testing technique, namely vibration-based monitoring. Here the global behaviour of a

structure is described by its dynamic properties, such as natural frequencies, mode shapes and damping ratios and measured by accelerometers.

A basic assumption of the vibration-based damage detection is that the structural damage causes changes in the physical properties of the structure such as stiffness and/or mass properties which in turn cause changes in its dynamic properties.

There is a variety of vibration-based damage detection methods that are being widely investigated by many researchers because of their global sensitivity to damage. In general, all these methods can be divided into two groups: model-based and model-free. Model-based methods require an accurate finite element model (FEM) of the structure in question that helps to physically justify the location and the level of damage. Usually it is done via a FEM updating procedure driven by changes in natural frequencies and/or mode shapes. On the other hand, model-free damage detection methods are most often based on signal processing techniques and utilize, for example, changes in natural frequencies [3]–[5],

modal flexibility [6], modal curvature [7] and modal strain energy [8].

In this paper, two methods for detection of very localized damage are presented and compared. The first method is a new model-free damage detection method named Mode Shape Damage Index (*MSDI*) [9]. It is based on comparison of normalised modal shape vectors determined in undamaged and damaged state of the structure

The second method is a sensitivity-based finite element model updating method [10]. It uses the difference between weighted natural frequencies and mode shapes before and after damage. The method is targeted for detection of very local damage by using  $l_1$ -norm regularization technique. This, also so-called sparse solution technique, just quite recently come to the literature on structural damage detection from compressed sensing [11, 12, 13].

The validity and comparison of the presented methods was demonstrated using numerical models of a Kirchhoff plate. Total four damage scenarios were simulated, where a damage was modelled by reducing the modulus of elasticity of selected finite elements in the numerical plate model by 13 % and 20 %.

## 2. Mode shape damage index method

This method is only based on variation of mode shapes in between of undamaged and damaged state. It consists of four stages:

### Stage 1: Normalization of mode shapes

The mode shapes (eigenvectors) are usually subjected to a scaling procedure, in order to compare two sets of mode shapes. There are two common type of normalization: normalization to unity and mass normalization. In structural damage identification it is assumed that the mass of the structure is not changing significantly, and it can be taken as a unity matrix, furthermore taking in account the orthogonality of mode shapes we have:

$$\phi_i^T [M] \phi_i = 1 \quad (1)$$

where  $\phi_i$  is mass normalized modal vector, calculated as linear combination of scaling factor and un-scaled mode shape amplitude.

### Stage 2: trace of MAC matrix

The Modal Assurance Criterion (MAC) is used to determine the level of correlation between mode shapes determined for undamaged and damaged state of the structure. MAC value varies between 0 and 1. MAC criteria that composes mode  $i$  and  $j$  has the following form:

$$MAC_{k,l} = \frac{|\{\phi^u\}_k^T \{\phi^d\}_l|^2}{(\{\phi^u\}_k^T \{\phi^u\}_k)(\{\phi^d\}_l^T \{\phi^d\}_l)} \quad (2)$$

where  $\{\phi^u\}_k$  and  $\{\phi^d\}_l$  represent mass normalized sets of vectors for undamaged ( $k^{\text{th}}$ ) and damaged conditions ( $l^{\text{th}}$ ). Values near unity means that two mode shapes of two states are identical, which means modal vectors are consistent. While values close to zero show that compared mode shapes are dissimilar.

The diagonal elements of the MAC matrix contain only pair mode shapes and indicate which modes are most affected by the damage. The basic idea of the MSDI method is to use only diagonal elements of MAC matrix in form:

$$\gamma_{trMAC} = \left( \sum_{k=l=1}^n MAC_{tr} \right)^2 \quad (3)$$

where  $\gamma_{trMAC}$  represents squared value of the summed MAC matrix trace. Value  $\gamma_{trMAC}$  varies between 0 and  $n^2$ , where value  $n$  denotes number of compared mode shapes vectors. Values equal  $n^2$  means that compared mode shapes for two states are identical. If values are different from  $n^2$  and streaming towards zero it means that compared modal shapes are dissimilar and structure is affected by damage. Previous statement can be valid only if there are no measurement errors or uncertainties.

### Stage 3: modification of MAC matrix

The original MAC criterion is a statistical indicator that is sensitive to large differences and relatively insensitive to small differences in mode shapes. The aim of modified MAC matrix ( $\Delta MAC$ ) is to exclude or decrease influence of those modal shapes which are dissimilar when comparing

undamaged and damaged state of structure. Therefore, diagonal elements of  $\Delta MAC$  matrix are defined as follow:

$$\Delta\alpha_{kl} = \Delta\alpha_{kl}^{Y_{trMAC}} \quad (4)$$

where  $\alpha_{kl}$  and  $\Delta\alpha_{kl}$  represent diagonal elements of original MAC and modified  $\Delta MAC$  matrix. If there is no damage in between two stages of measurement, the stiffness of the structure will not be changed. Therefore, if there is no change in stiffness, the modified MAC matrix is equal to the original MAC.

In this way, the modified MAC matrix ensures that only those modal shapes which are almost identical can be useful for damage detection. Modified MAC matrix can be useful to overcome difficulties caused by measurement uncertainties and errors.

#### Stage 4: Mode shape damage index

A new method for structural damage detection and localization based only on modal shapes is given in this form:

$$|MSDI_i| = \left( \frac{\bar{\phi}_i^u - \bar{\phi}_i^d}{\bar{\phi}_i^u} \right) \leq 0 \quad (5)$$

The coefficients  $\bar{\phi}_i^u$  and  $\bar{\phi}_i^d$  represent modification of squared normal mode shape vectors and can be computed using the following equation:

$$\bar{\phi}_i^u = \sum_{k=1}^n ((\phi_i^u)_k^2 \Delta\alpha_{kl}) \quad (6)$$

$$\bar{\phi}_i^d = \sum_{l=1}^n ((\phi_i^d)_l^2 \Delta\alpha_{kl}) \quad (7)$$

The proposed MSDI method utilized coefficients related to the global behaviour of the structure. When a structural stiffness matrix is changed (damage induced), the original modal shape vectors are changed as well. To emphasize difference between two sets of vectors their value is squared and multiplied with corresponding element from the modified MAC matrix. Finally, all sets of modified mode shape vectors are summarized thereby representing coefficients for damage detection.

The presented method only considers modal shapes that are almost identical and excluding

dissimilar modal shapes. Theoretically, if there are no damages in between two stages, the stiffness of the structure will not be changed, provided there are no measurement uncertainties and errors. Therefore, if there is no change in stiffness, the modified MSDI index is equal to zero.

### 3. $l_1$ -norm regularized finite element model updating method

Compared to the MSDI method described in the previous section, this method requires considerable computational efforts. The gain is that it not only can precisely localize but also rather exactly quantify the level of point damage(s) using quite few number of measurement locations. At large, this method needs an accurate finite element model of the structure in question and an optimization algorithm that finds the damage parameter vector which minimizes the difference between the measured and numerically predicted structural dynamic properties which is also called residual.

Let us consider the structural dynamic properties as functions of the damage parameter vector  $\alpha$ , i.e. the eigenvalue  $\lambda(\alpha)$  ( $\lambda_k = (2\pi f_k)^2$ , where  $f_k$  is the natural frequency) and the eigenvector (mode shape)  $\phi(\alpha)$ . We define the residual by

$$r_{\lambda_j}(\alpha) = w_{\lambda_j} \left( 1 - \frac{\lambda_j(\alpha)}{\lambda_j^{mea}} \right) \quad (8)$$

$$r_{\phi_j}(\alpha) = w_{\phi_j} \left( \frac{(\phi_j^{mea})^T \phi_j(\alpha)}{\|\phi_j(\alpha)\|_2^2} \phi_j^{mea} - \phi_j(\alpha) \right) \quad (9)$$

where  $r_{\lambda_j}$  is the  $j^{th}$  component of the eigenvalue residual and  $r_{\phi_j}$  is the residual vector for the  $\phi_j$  mode shape,  $w_{\lambda_j}$  and  $w_{\phi_j}$  are corresponding weights and the upper index *mea* is referring to the measured quantity.

Now using Taylor series expansion, small changes in the dynamic properties or residual  $r$ , can be linearly related to small changes  $\Delta\alpha$  in the damage parameters around  $\alpha$

$$r = S \Delta\alpha$$

where the sensitivity matrix  $S \in R^{m \times n}$  is defined by

$$S_{ij} = \frac{\partial r_i}{\partial \alpha_j}$$

Typically, the number of residuals are much less than the number of damage parameters, i. e.  $m \ll n$ .

The derivatives of the residual corresponding to non-repeated eigenvalues with respect to the damage parameters are found from the following widely used parameterized undamped eigenvalue problem [10, 14, 15, 16, 17, 18].

$$\begin{aligned} K(\alpha)\phi_k(\alpha) &= \lambda_k(\alpha)M\phi_k(\alpha) \\ K(\alpha) &= K^0 - \sum_{i=1}^n \alpha_i K_i \\ \alpha_i &= \frac{E_i^0 - E_i}{E_i^0} \end{aligned} \quad (10)$$

and Equation (1). Here  $M$ ,  $K$  are square system mass and stiffness matrices respectively,  $\lambda_k$  is the  $k^{th}$  eigenvalue and  $\phi_k$  is the corresponding  $k^{th}$  eigenvector. Usually one assumes that the mass of the structure does not change after the damage is introduced. The matrix  $K(\alpha)$  is the improved stiffness matrix of the parameterized model.  $K^0$  is the initial global stiffness matrix corresponding to the undamaged structure,  $K_i$  is the constant stiffness matrix for the  $i^{th}$  element or substructure (group of elements) representing the unknown model property and location. The dimensionless damage parameters  $\alpha$  are chosen according to the simple isotropic damage theory [19]. In this theory, the damage is described by a reduction in bending stiffness.  $E_i^0$  and  $E_i$  are the undamaged and damaged elasticity modulus for the  $i^{th}$  element or group of elements.

Then instead of solving the minimization problem

$$\min_{0 \leq \alpha_i \leq 1} \frac{1}{2} \|r(\alpha)\|_2^2 \quad (11)$$

we solve the corresponding linear problem

$$\min_{0 \leq \alpha_i \leq 1} \frac{1}{2} \|S\Delta\alpha - r(\alpha)\|_2^2 \quad (12)$$

where  $r(\alpha)$  is the non-linearized residual.

Assuming that the damage is very local, i. e. is associated only with few locations on a structure, we look for a damage parameter vector which is sparse, i. e. contains just a few non-zero elements. The most simple and intuitive measure of sparsity of vector  $x$  as a solution of the underdetermined system of linear equations  $Ax = b$ , where  $A \in R^{m \times n}$  for  $m < n$ , is by counting the number of nonzero entries in it or using, so-called,  $l_0$  –"norm"

$$\|x\|_0 = \#\{i: x_i \neq 0\} \quad (13)$$

The norm is quoted because it is not really a norm, since it does not satisfy the homogeneity property. Unfortunately, the  $l_0$  –"norm" regularization problem is computationally difficult. That is why, for simplicity, the closest convex relaxation  $l_1$  –norm is used [20, 21]. So we formulate the following convex optimization problem

$$\min_{0 \leq \alpha_i \leq 1} \frac{1}{2} \|S\Delta\alpha - r(\alpha)\|_2 + \|\Delta\alpha\|_1 \quad (14)$$

For problem there exist a number of effective solvers, for example, interior-point methods. Here, we solve this problem with cvx open source Matlab toolbox [22] by relaxing this to the quadratically constrained problem

$$\begin{aligned} &\min_{0 \leq \alpha_i \leq 1} \|\Delta\alpha\|_1 \\ &\text{s.t.} \\ &\frac{1}{2} \|S\Delta\alpha - r(\alpha)\|_2^2 < \varepsilon, \\ &0 \leq \alpha_i \leq 1 \end{aligned} \quad (15)$$

where  $\varepsilon$  is the accepted threshold for the difference between the residual and its linear approximation.

#### 4. Simulated damage scenarios

We test our methods on a Kirchhoff plate with size  $0.55 \text{ m} \times 0.5 \text{ m} \times 0.01 \text{ m}$ . The model is built using four-node-shell elements with each node having six degrees of freedom: three translational and three rotational. The size of each finite element is  $0.05 \text{ m} \times 0.05 \text{ m}$ . Thus the model contains 110 elements. The plate is fixed on all sides.

Simple point damage is simulated by reducing the elasticity modulus for the chosen element. We considered four damage scenarios (Figures 1-4 a):

- Scenario 1 – damaged element 51 with 13 % reduction of modulus elasticity,
- Scenario 2 – damaged element 16 with 20 % reduction of modulus elasticity,
- Scenario 3 – damaged element 58 with 13 % reduction of modulus elasticity,
- Scenario 4 – damaged element 58 and 16 with 13 % reduction of modulus elasticity,

In all simulations the noise is modelled as follows

$$\hat{\psi} = \psi(1 + \varepsilon\rho) \quad (16)$$

where  $\hat{\psi}$  is the estimated vibration parameter,  $\psi$  is the corresponding exact parameter,  $\rho$  is a normally distributed random variable with zero mean and a variance of 1.0,  $\varepsilon$  is the variation equal to 0.001 [c.f. 11].

## 5. Damage detection results based on presented methods

In Figures 1-4 the true damages (a) and the results (b-c-d) of the damage identification based on the two methods described in this paper are presented. Both two methods are tested with the mode shape only data. For the used noise model, see Equation 16, they produced surprisingly good and very good damage localization without any natural frequency information which is usually used in the identification [2, 11, 12].

According to presented results, it can be noted that for the chosen noise model simple MSDI method found the damage localization pattern “comparable” with the mathematically overwhelmed  $l_1$ -norm minimization, but at the cost that much more measurement points were required, see Figures 1-4 b-c.

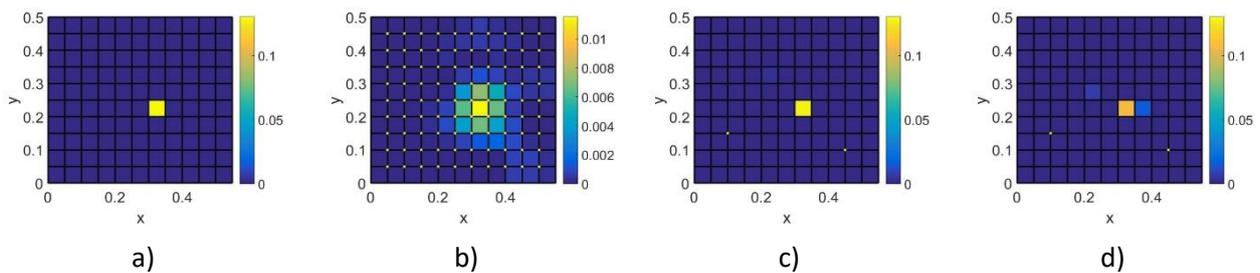


Figure 1. Damage scenario 1 and damage detection results. a) True damage, b) Damage pattern found by MSDI method based on first 20 mode shapes, c) Damage  $\approx 12.68\%$  found by  $l_1$ -norm regularized FEM updating based on first 6 mode shapes, d) Damage  $\approx 10.74\%$  found by  $l_1$ -norm regularized FEM updating based on first 4 natural frequencies and mode shapes. Measurement locations in all simulations are denoted by yellow stars.

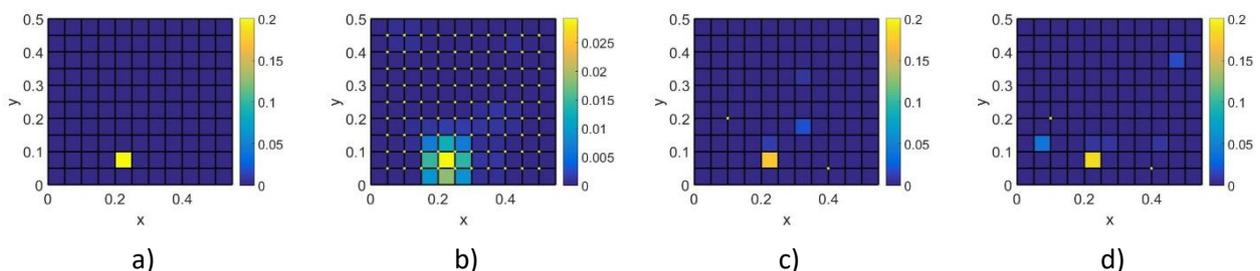


Figure 2. Damage scenario 2 and damage detection results. a) True damage, b) Damage pattern found by MSDI method based on first 20 mode shapes, c) Damage  $\approx 17.25\%$  found by  $l_1$ -norm regularized FEM updating based on first 12 mode shapes, d) Damage  $\approx 18.71\%$  found by  $l_1$ -norm regularized FEM updating based on first 11 natural frequencies and mode shapes. Measurement locations in all simulations are denoted by yellow stars.

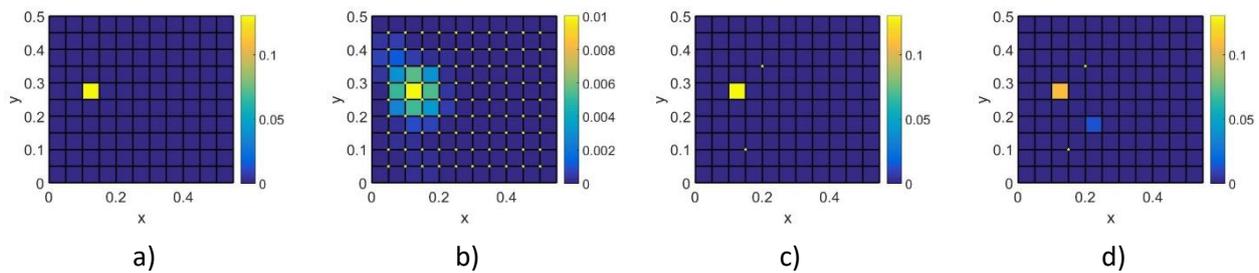


Figure 3. Damage scenario 3 and damage detection results. a) True damage, b) Damage pattern found by MSDI method based on first 20 mode shapes, c) Damage  $\approx 12.96\%$  found by  $l_1$ -norm regularized FEM updating based on first 6 mode shapes, d) Damage  $\approx 10.79\%$  found by  $l_1$ -norm regularized FEM updating based on first 5 natural frequencies and mode shapes. Measurement locations in all simulations are denoted by yellow stars.

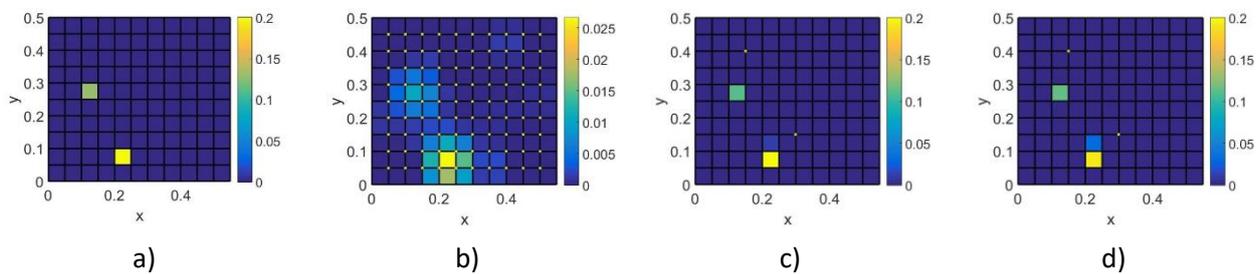


Figure 4. Damage scenario 4 and damage detection results. a) True damage, b) Damage pattern found by MSDI method based on first 20 mode shapes, c) Damage  $\approx 11.06\%$  and  $\approx 19.94\%$  found by  $l_1$ -norm regularized FEM updating based on first 12 mode shapes, d) Damage  $\approx 11.27\%$  and  $\approx 19.06\%$  found by  $l_1$ -norm regularized FEM updating based on first 11 natural frequencies and mode shapes. Measurement locations in all simulations are denoted by yellow stars.

Disadvantage of MSDI method, that it is not capable to quantify the level of damage, but it can only compare damage in terms that one location is more or less damaged than another one.

On the other hand, the  $l_1$ -norm regularized finite element model updating method also produced promisingly very good results for the damage level identification for what it requires just very few measurement locations, see Figures 1-4 c. Moreover, adding also natural frequencies to the identification process, see Figures 1-4 d, the total number of modes used in the analysis can be minimized with this method.

Some more results are given in an accompanying paper [23].

## 6. Conclusions and future work

In this paper we compared two mathematical approaches for damage detection: a simplified requiring less calculations (MSDI method) and a

more complicated demanding more computations ( $l_1$ -norm regularized finite element model updating method). For the reasonably low considered noise level they produced comparable results in damage localization. Because the first approach is model-free it can be attractive to use this in cases when no finite element model is available and only an approximation of the damage location is required. However, for doing this a MSDI method requires a dense net of measurement locations. On the other hand when a finite element model of the structure is presented the  $l_1$ -norm regularized finite element method for detection of point damage(s) gives a very accurate solution using just very few measurement locations.

It would be interesting to test how good these methods are for higher noise levels. What is the result of the damage localization with MSDI method when the number of measurement points decreases? It would also be interesting to find another way to apply  $l_1$ -norm regularization so

that no finite element model is required, for example, for the frequency response function.

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