

Mathematical Method Re-examined for Assessment of Ground Contaminated by Radioactive-Contaminated Groundwater

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Abstract

In this paper we re-examined the transfer equation of radioactive substances in the ground and offer a more realistic transfer equation and other equations available for assessment of the ground contamination from radioactive-contaminated groundwater. The transfer equation takes into account kinematic and hydrodynamic considerations on mass conservation of mobile radioactive substances in a porous medium that typifies the ground. The other equations available for contamination assessment are concerned with deposition in contaminated areas and discharge flow of contaminants to the areas contiguous to contaminated area. The equations are derived on the understanding that disintegration of the radioactive substances adhering onto solids in the ground porous medium occurs as a sink term during the transfer of mobile radioactive substances. Finally, it is noted that the discharge of groundwater due to advective flow will be predominant in comparison to the discharge by diffusion.

Keywords: Radioactive contamination, Transfer equation, Deposition, Assessment, Discharge, Groundwater.

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1. Introduction

To assess ground contamination due to groundwater polluted by radioactive substances, it is essential to analyze the transfer of radioactive substances. The analyses required need to consider both the subsurface concentration of radioactive substances within the contaminated areas, and the discharge rates of the radioactive substances toward contiguous areas. The question we must consider here is whether different equations should be developed for the analysis in concentration – taking note of the fact that the radioactive substances will be transferred in solute phase form in the porous media and be adhered to the solids in the porous medium during transfer. Note also that radioactive substances will disintegrate in both the solute and adhesion phases. These phenomena need serious considerations in developing the governing equation (Spitz & Moreno 1996). The purpose of this paper is to (a) a physically reasonable governing equation that takes into account the transfer of radioactive substances -- kinematically and hydrodynamically -- based on the mass conservation law of mobile radioactive substances in a porous medium (Yong et al. 2010, Pusch et al. 2011, 2017) and (b) opine on the useful equations that can be used to assess the potential contamination of contiguous areas.

2. Governing Equation

2.1 Mass Conservation law of mobile

The mass conservation law of mobile radioactive substances in a minute volume of porous medium can be described as follows,

$$\frac{\partial m}{\partial t} + \text{div} \mathbf{q}_m + S_{ink} = 0 \quad (1)$$

where m is the mass of radioactive substances in the solute phase, which is described as

$$m = \theta C ,$$

θ is the volumetric water content,

C is the concentration of solute,

\mathbf{q}_m is the flux of radioactive substances
and S_{ink} refers to the sink.

2.2 Transfer of Radioactive substances

The adhesion of radioactive substances to solid particles of the porous medium occurs during the transfer of the radioactive substances. Disintegration of radioactive substances occurs in both the solute and adhesion phases. It then is reasonable to assume that the sink term S_{ink} is obtained as the sum of disintegration in both the solute and adhesion phases.

$$S_{ink} = \lambda\theta C + \frac{\partial S}{\partial t} \tag{2}$$

where λ is the disintegration constant and S is the mass of radioactive substances adhered per unit time on the solids.

$$\frac{\partial(\theta C)}{\partial t} + div\mathbf{q}_m + \lambda\theta C + \frac{\partial S}{\partial t} = 0 \tag{3}$$

On the assumption of a linear relationship between C and S at a low solute concentration, we will obtain the relationship $S = kC$, where k is the partition coefficient (Yong et al. 1992).

Equation (1) then is described as follows:

$$R \frac{\partial(\theta C)}{\partial t} = -div\mathbf{q}_m - \lambda\theta C \tag{4}$$

where R is the retardation coefficient:

$$R = 1 + \frac{k}{\theta} \tag{5}$$

2.3 Deposition of radioactive substances

Radioactive substances are deposited in both the solute and adhesion phases during their transfer in a porous medium. Deposition in the solute phase results in the increment of concentration of the radioactive substances, while in the adhesion phase it is a difference between an amount of adhesion on the solids and their disintegration. The amount of deposition then is described as follows:

$$\frac{\partial D_p}{\partial t} = \frac{\partial(\theta C)}{\partial t} + \left(\frac{\partial S}{\partial t} - \lambda S \right) \quad (6)$$

where D_p is the deposition of radioactive substances in the contaminated areas.

Using the retardation coefficient, Equation (6) is:

$$\frac{\partial D_p}{\partial t} = R \frac{\partial(\theta C)}{\partial t} - \lambda S \quad (7)$$

The integral from 0 to t of Equation (7) gives:

$$D_p = \theta C + (1 - \lambda t) S \quad (8)$$

3. Flux of radioactive substances

The flux of radioactive substances in a porous medium is described by the sum of solute flow due to both mechanical dispersion and convection (Bresler et al.1982, Yong et al. 1992).

Writing in one-dimensional form where x denotes horizontal direction,

$$q_m = -\theta D_e \frac{\partial C}{\partial x} + v_w \theta C \tag{9}$$

where D_e is the mechanical dispersion coefficient, which is given by

$$D_e = D_H + D_M \tag{10}$$

D_H is the hydrodynamic dispersion coefficient, and D_M is the effective molecular diffusion coefficient, $D_H \cong Lv_w$, where L is the dispersivity parameter and v_w is the advective velocity of water which is q_w / θ , where q_w is the flux of water in the porous medium.

$$D_M = \tau D_0 \tag{11}$$

D_0 is the molecular diffusion coefficient in water and τ is the tortuosity factor, which is

$$\tau = (l / l_e)^n \quad (n=1 \sim 2) \tag{12}$$

where l is the macroscopic average path of diffusion, and l_e is the actual tortuosity path along which a particle moves.

4. Velocity of water

To analyze the flow rates of the radioactive substances by polluted groundwater flow toward contiguous areas, the velocity of groundwater flow is assumed to be governed by Darcy's law (Pusch, et al. 2018).

In non-equilibrium state of the groundwater where the surface level changes with time (Pusch et al. 2018), the water velocity is described as:

$$-div\mathbf{V}_w = C_s \frac{\partial h}{\partial t} + Q \quad (13)$$

where \mathbf{v}_w is the velocity of groundwater,
 h is the hydraulic head,
 C_s is the specific storativity ($\partial\theta / \partial h$),
 and Q is the sink or source term.

The components of velocity of groundwater are:

$$V_{w_{x,y}} = -K_{x,y} \nabla h \quad (14)$$

and

$$V_{w_z} = -K_z (\nabla h + 1) \quad (15)$$

where K is the hydraulic conductivity.

The subscript represents the coordinates and z refers to the gravitational direction.

5. Numerical analyses

5.1 Contamination by deposition in areas

To determine ground contamination from deposition of radioactive substances, one begins with the analysis of θC , the mass of radioactive substances in solute phase, using Equation (4). Following this, the mass of radioactive substances adhered per unit time by the solids in the porous medium can be calculated using the relationship of $S=kC$. Finally, deposition D_p of radioactive substances in the contaminated ground can be calculated using Equation (8).

5.2 Discharge of groundwater to contiguous areas

The advective flow of radioactive substances to contiguous areas is a very serious problem, particularly when the contiguous areas include water zones such as rivers, lakes, estuaries, etc. The advective flow can be determined using Equations (9) and (14) to obtain the following relationship:

$$D_{wd} = \int_0^l W q_{md} dz \tag{16}$$

where D_{wd} is the discharge to contiguous water zones and W is the width of discharging border. The upper limit l of the integral refers to the water depth of groundwater at the boundary of the contiguous area and q_{md} refers to the combined diffusive and advective discharge flux, and it is obtained as follows.

$$q_{md} = -\theta D_M \frac{\partial C}{\partial x} + v_H \theta C \tag{17}$$

where v_H refers to the horizon component of free groundwater flow.

The discharge of groundwater due to advective flow [second term of Equation (17)] will be predominant in comparison to the discharge due to diffusive flow [first term of Equation (17)].

6. Discussion

Sometimes, instead of Equation (4), the following equation has been used for assessment of ground contamination.

$$R \frac{\partial(\theta C)}{\partial t} = -\text{div}\mathbf{q}_m - R\lambda\theta C \quad (18)$$

It is understood that this equation was derived from a realization that the disintegration of radioactive substances adhered onto the solids of porous medium is separately and independently treated in the determination of radioactive substances deposition. Equation (4) was derived on the understanding that it should be included as an adhesion term in the sink. Accordingly, the solution for θC obtained from Equation (18) will be smaller by only a portion of an amount of disintegration of radioactive substances adhered onto the solids in the porous medium in comparison to the solution obtained by Equation (4).

7. Conclusions

1. For assessment of environmental radioactive ground contamination Equation (4), which is derived on the basis of transfer of mobile radioactive substances, should be used.
2. The solution for θC obtained from Equation (18) will be smaller by only a portion of an amount of disintegration of the radioactive substances adhered onto the solids of ground in comparison to the solution obtained by using Equation (4), in a case where we use the same value for the retardation coefficient R , that is, the partition coefficient k .

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