

On the deformation of fibrous suspensions

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ABSTRACT

An understanding of the rheology and dynamics of the deformation of fibrous suspension as a multiphase fluid is important in order to be able to fully disclose the flow behaviour from very low to very high shear rates. In this study, a flexible fibre model has been implemented in an open source Computational Fluid Dynamics code. The three-dimensional Navier-Stokes equations which describe the fluid motion are employed while the fibrous phase of the fluid is modeled as chains of fiber segments interacting with the fluid through viscous- and drag forces. The aim of this study is to investigate the fibre dynamics against several orbit classes - i.e. rigid, springy, flexible and complex rotation of the fibres¹⁻³ enabling the model to have all degrees of freedom - translation, rotation, bending and twisting. The simulations are performed using the OpenFOAM open source software.

INTRODUCTION

A complex multiphase material which is a flexible fiber suspension, is lubricating greases^{4, 5}, defined as a dispersion of a thickener agent in a liquid lubricant⁶. Here the liquid lubricant is a base oil which unlike the bulk grease shows a Newtonian rheology. The grease rheology is inherently strongly non-Newtonian due to the fibrous network comprised by the thickener (Fig. 1). For very

low shear rates ($\sim 10^{-2}$ 1/s) the thickener dominates the grease resistance to shear forces, yielding the grease to behave as a visco-elastic material. In the limit of infinite shear rates, the fibre structure is completely disrupted giving the grease the (Newtonian) rheology of the base oil. These properties together with the grease sticky consistency, tackiness and yield stress behaviour makes it an excellent choice as lubricant in various applications such as rolling element bearings and railway lubrication. Compared to oils, greases adhere to surfaces preventing corrosion, and seals the system preventing contaminant particles to enter the lubricated contacts.

With reliable numerical methods disclosing the details of the rheology of the grease through the modelling of the dynamics between the dispersed phase in form of the fibrous network, and the continuous phase (base oil), we would take a large leap in the understanding of the flow dynamics of such flexible fibrous suspensions. This would be of utmost importance from a fundamental scientific point of view, but also enable a renaissance for the industry dealing with such materials.

NUMERICAL METHODS

A Particle-level method based on Ross and Kligenberg⁷ has been implemented in the OpenFOAM open source software.

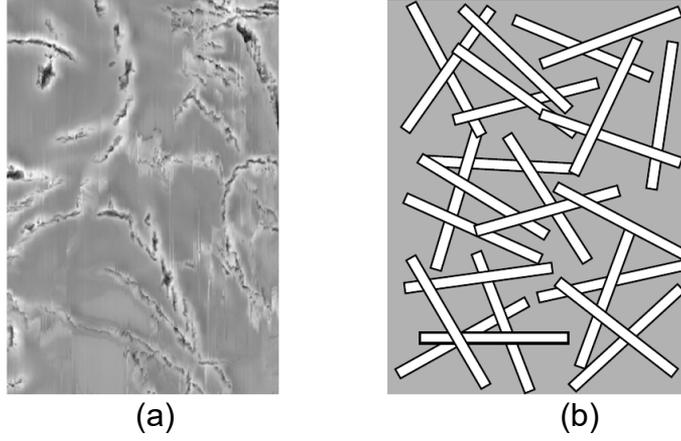


Figure 1: Grease microstructure, (a) AFM (Atomic Force Microscope) image of a lithium complex soap network, (b) Simplification of the fiber network ⁸.

Figure 2 shows a fiber as a series of N rigid bodies. A chain of rigid cylindrical segments was considered to model the flexible fibers and each segment of the fiber is tracked individually through a Lagrangian Particle Tracking method. For each segment, the translational and rotational equations were solved for each fiber segment using Newton's second law. It is worth noting that one-way coupling was considered to model the interaction between the fluid and solid part. In addition, due to small value of the lubrication forces, lubrication forces are neglected in the current study. The fiber extension is also excluded in the governing equations. It needs to be emphasized that to improve the numerical instability, the equations were made dimensionless by choosing proper length, time and force scales.

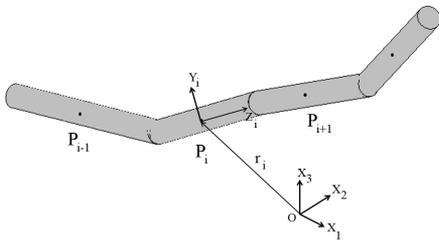


Figure 2. Representation of a flexible fiber as a series of N rigid bodies.

Figure 3a represents a single spheroid segment which is located at \vec{r}_i . The spheroid has a major axis $2a$ and minor axis $2b$. The segments are connected through a ball and socket joint as illustrated in Figure 3b. The connectivity of the fibres satisfies Eq. 1.

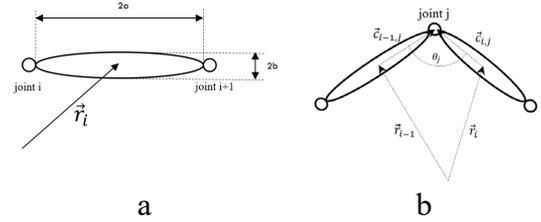


Figure 3. a) single spheroid as a model of a single segment. b) Connectivity of two segments of a fiber.

$$\begin{aligned} (\vec{r}_{i-1} + \vec{c}_{i-1,j}) - (\vec{r}_i + \vec{c}_{i,j}) = \\ 0 \quad (i, j = 1, \dots, N) \end{aligned} \quad (1)$$

Where the vector components are recognized by Figure 3b.

The hydrodynamic force (\vec{F}_i^h) and torque (\vec{M}_i^h) are computed through Eqns. 2-3 ⁹, where

$$\vec{F}_i^h = \vec{A}_i^h \cdot (\vec{U}_i^\infty - \vec{u}_i) \quad (2)$$

$$\vec{M}_i^h = \vec{C}_i^h \cdot (\vec{\Omega}_i^\infty - \vec{\omega}_i) + \vec{H}_i^h : E_i^\infty \quad (3)$$

Where \vec{U}_i^∞ , $\vec{\Omega}_i^\infty$ and E_i^∞ are the velocity, vorticity and the strain rate, respectively. \vec{u}_i and $\vec{\omega}_i$ are velocity and angular velocity of segment i of the fibre. \vec{A}_i^h , \vec{C}_i^h and \vec{H}_i^h are resistance tensors⁷.

For a single segment of fiber, the free-body diagram is shown in Figure 4. The linear and angular momentum equations are solved for each segment i.e. Eq.'s 4-5:

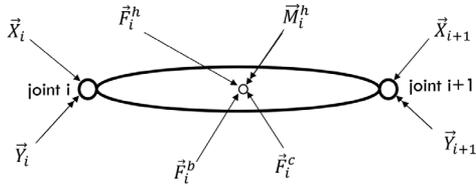


Figure 4. Free body diagram for a spheroid i

$$\vec{A}_i^h \cdot (\vec{U}_i^\infty - \vec{u}_i) + \vec{F}_i^c + \vec{F}_i^b + S_{ij} \cdot \vec{X}_j = m_i \ddot{\vec{r}}_i \quad (4)$$

$$\vec{C}_i^h \cdot (\vec{\Omega}_i^\infty - \vec{\omega}_i) + \vec{H}_i^h : E_i^\infty + S_{ij} \cdot (\vec{c}_{ij} \times \vec{X}_j + \vec{Y}_j) = \dot{H}_i \quad (5)$$

In Eq. 4, \vec{F}_i^c , \vec{F}_i^b and \vec{X}_j are contact force, body force (i.e. ≈ 0) and internal force, respectively. S_{ij} is connectivity matrices. In Eq. 5, \vec{Y}_j is internal momentum and \dot{H}_i is time rate of angular momentum.

Here, the governing equations have been briefly presented and we avoid repeating the full description of the fibre model as we followed⁷.

RESULTS

To evaluate the results a dimensionless shear rate was defined according to the viscosity (η), fibre segment maximum thickness (b) and the bending constant ($K_B = \frac{EI}{a}$), Equation 6:

$$\gamma^* = \pi \eta b^3 \dot{\gamma} / K_B \quad (6)$$

The effect of dimensionless shear rate (γ^*) on the flexibility of a fibre with 7 segments are shown in Figure 5. Three different shear rates of 3.2×10^{-3} , 7.2×10^{-4} and 1.43×10^{-4} respectively, were selected and the flexibility and rotational motion of the fibres are investigated.

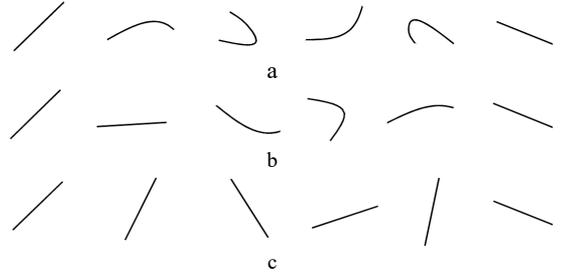


Figure 5. Time sequence of images from simulation for various dimensionless shear rate a) $\gamma^* = 3.2 \times 10^{-3}$, b) $\gamma^* = 7.2 \times 10^{-4}$, c) $\gamma^* = 1.43 \times 10^{-4}$.

Regarding the validation of the numerical implementation, more numerical simulations are required to make it as a benchmark for the future simulations.

CONCLUSION

The model of Ross et. al. (1997) was implemented in Open FOAM software and the motion of a fibre was determined by solving the translational and rotational equations of motion for each spheroid segment. Translation, rotation, bending and twisting were observed in the single flexible fibre motion. The model has to be validated against other benchmarks and will be employed further to capture the rheological properties of the grease lubricant as a fibrous suspension.

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