

# COMPARING HARMONIC UNBALANCE AT MULTIPLE LOCATIONS TO CHARACTERIZE THE UNBALANCE

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## Abstract

Harmonic emission at any point of connection (PoC) depends on the loads connected there and the background voltage distortion at that point. Under balanced supply voltage and load, the harmonics adhere to the rule that third harmonics are zero sequence, fifth harmonic are negative sequence, etc. However, under unbalance in either load or supply voltage, this assumption is no longer true. A phasor analysis of the voltage and current measured at different locations in a city is presented in this paper. Individual harmonics are decomposed into their sequence components, and the results are presented in terms of polar plots. The components are further quantified in terms of balanced and unbalanced THD, which is an indicator of the impact of unbalance. The harmonics do not follow the defined norm of sequence component in presence of unbalance in the system. The triplen harmonics were shown to have higher unbalanced sequence components than non-triplen harmonics.

## 1 Introduction

Unbalance in a three-phase low voltage network can be due to unsymmetrical supply voltage among the phases, untransposed transmission or distribution lines, and unequal spread of the load over the three phases [1]. A small unbalance of the supply voltage can cause a large unbalance in the current [2]. Many indicators have been proposed to calculate unbalance [3], the symmetrical component method is one of them [4]. The voltages or currents can be decomposed into symmetrical components, and the ratio of negative sequence component to positive sequence component of the fundamental frequency is used to quantify the voltage unbalance. Under balanced supply voltage and load conditions the third harmonic is zero-sequence, fifth harmonic is negative-sequence, seventh harmonic is positive-sequence, etc. However, under unbalance, either load or supply voltage, this rule no longer holds [5]. The existing unbalance in the low voltage grid at different locations, due to the local loads, causes a change in the characteristics of harmonics and their propagation to the upstream network. The presence of a negative sequence component of the fundamental frequency on a three-phase supply leads to the production of odd-triplen harmonics of positive and/or negative sequence. The non-zero-sequence harmonics pass Delta/Y transformers and propagate to the upstream network. Zero-sequence harmonics cause issues like telephone interference [6]. The change in the propagation of harmonics complicates the study of the harmonic power flow [6]. This paper shows the deviation of the harmonics from their characteristic harmonic sequence

in presence of unbalance. The results are presented in the form of polar plots showing the magnitude and phase angle of harmonic phase sequences. The polar plots represent the magnitude and phase of the voltage and current with respect to phase angle of fundamental voltage in this paper. Finally, the balanced and unbalanced total harmonic distortion (THD) is calculated to indicate the impact of unbalance at various locations in an urban area for a three-phase system.

## 2 Harmonic distortion in presence of unbalance

### 2.1. Computing harmonic phase angles

For the fundamental voltage represented by  $V_1(t) = v_1 \cos(\omega_1 t + \varphi_1)$  the discrete Fourier transform (DFT) over the window, starting at upwards zero-crossing gives absolute values  $v_1$  and an angle  $\varphi_1$  for the fundamental. The DFT uses a cosine convention whereas in power engineering normally the sine convention is used. In this paper, a correction of  $90^\circ$  is applied to calculate the phase angle according to the sine convention. The harmonic voltage for the harmonic order  $n$  can be represented by  $V_n(t) = v_n \cos(n\omega_1 t + \varphi_n)$  and the DFT gives the magnitude of harmonic  $v_n$  and phase angle of harmonic  $\varphi_n$ . The phase angle of the harmonic voltage is defined as the angle under the sine convention of the harmonic at the instant of upward zero crossing of the fundamental voltage. Using the sine convention, the harmonic voltage can be represented by (1).

$$V_n(t) = v_n \sin\left(n\omega_1 t + \varphi_n + \frac{\pi}{2}\right) \quad (1)$$

Where  $\omega_1 = \frac{2\pi}{f}$ ,  $f$ = fundamental frequency and  $n$  = order of harmonic.

The phase angle of the harmonic of order  $n$ , at the instant of upwards zero crossing of the fundamental, can thus be calculated as

$$\phi_{nzc} = \varphi_n + \frac{\pi}{2} - n\left(\frac{\pi}{2} + \varphi_1\right) \quad (2)$$

The harmonic voltages are recomputed using the magnitude calculated by DFT and the phase angle calculated by (2). For a three-phase system, the harmonic phase angle for phase L1 is calculated using (2). The reference for harmonic voltages in phases L2 and L3 are calculated using a fundamental supply voltage of their respective phases and the difference in their fundamental phase angle with the fundamental voltage phase angle of phase L1 as shown in (3).

$$\phi_{nzcLj} = \varphi_{nLj} + \frac{\pi}{2} - n\left(\frac{\pi}{2} + \varphi_{1Lj}\right) + n(\varphi_{1Lj} - \varphi_{1L1}) \quad (3)$$

Where,  $\phi_{nzcLj}$  = harmonic phase angle for phase L2 or L3,  $j = 2, 3$ .

This method gives the harmonic phase angles calculated with reference to fundamental and phase L1. The harmonic sequence components are further calculated similarly.

## 2.2. Definition of balanced and unbalanced THD

For the total harmonic distortion (THD) with three-phase measurements, three values for voltage and three values for current can be obtained, one for each of the phase-to-neutral voltages. The three values can be combined into a single THD, which represents the three-phase distortion. The method of computing the three-phase balanced and unbalanced THD proposed by [3] is used in this paper. This method entails the calculation of positive, negative, and zero sequence components for each harmonic order using Fortesque transformation. The harmonic content of a signal is obtained from the DFT applied to the signal. Under an ideal balanced system, the voltage harmonics of the order  $3n-2$ , where  $n = 1, 2, 3, \dots$  behave as a positive-sequence. The voltage harmonics of the order  $3n-1$  and  $3n$  behave as negative-sequence and zero-sequence respectively. The balanced THD is defined as the square root of the signal energy of all balanced components and can be calculated by (4). The unbalanced THD is obtained by the summation over all other components and is represented by (5) [3]. The changes in the sequence components of the harmonics for voltage and currents under unbalanced conditions will be shown in the form of polar plots for different harmonic orders in the results section of the paper.

$$THD_{bal} = \frac{\sqrt{\sum_{n=2}^{H/3} (V_{(3n-2)}^+)^2 + \sum_{n=1}^{H/3} (V_{(3n-1)}^-)^2 + \sum_{n=1}^{H/3} (V_{(3n)}^0)^2}}{V_1^+} \quad (4)$$

$$THD_{ubal} = \frac{\sqrt{\sum_{n=1}^{H/3} A_n^2}}{V_1^+} \quad (5)$$

where, for  $n = 1$ ,  $A_1^2 = (V_2^0)^2 + (V_2^+)^2 + (V_3^+)^2 + (V_3^-)^2$

for  $n > 1$ ,  $A_n^2 = (V_{(3n-2)}^-)^2 + (V_{(3n-2)}^0)^2 + (V_{(3n-1)}^0)^2 + (V_{(3n-1)}^+)^2 + (V_{(3n)}^+)^2 + (V_{(3n)}^-)^2$

$H$  = highest harmonic order,  $n = 1, 2, 3, \dots, H/3$ ,  $V_1^+$  = positive sequence of fundamental

$$U_n = \frac{V_1^-}{V_1^+}, U_0 = \frac{V_1^0}{V_1^+} \quad (6)$$

where,  $V_1^+$  = positive sequence of fundamental,  $V_1^-$  = negative sequence of fundamental,  $V_1^0$  = zero-sequence of fundamental.

The negative and zero sequence unbalance can be obtained by (6). Measurements have been done at three different places in a Swedish city: a residential area, a solar installation in an office building, and a university campus. The measurements have been made for 200ms/10 cycles, with 5 Hz resolution according to IEC 61000-4-30, using class A power quality equipment. Calculations have been made using magnitude and phase angle values of individual harmonics up to 40<sup>th</sup> order for calculating the balanced and unbalanced THD.

## 3 Results

### 3.1. Residential area

Measurements were made for three houses supplied from the same distribution transformer. The supply voltage and the current drawn by individual houses were measured every 10 min for one week in October. The negative-sequence and zero-sequence unbalance for the fundamental voltage is less than 1% for the measured period.

Fig. 1 shows positive, negative, and zero sequence components of the fundamental and odd harmonics for the supply voltage measured at house 1. The fundamental component H1, as expected, is mainly positive-sequence in nature. From the measured samples, the maximum magnitude of zero-sequence and negative-sequence components for fundamental is 2 V and 1.8 V respectively. The harmonic orders 7 and 13 behave mainly as positive-sequence harmonic, whereas the 19<sup>th</sup> harmonic has an approximately equal magnitude of positive and negative sequence component. The 5<sup>th</sup>, 11<sup>th</sup>, 17<sup>th</sup>, and 23<sup>rd</sup> harmonics behave as mainly negative-sequence components. They include positive and zero sequence components as well, due to the unbalance in the system. The 5<sup>th</sup> harmonic has a clear daily pattern, which causes its spread in the third and fourth quadrant. It can be seen from Fig. 1 that the phase angle spread increases with the increase in the harmonic order. The triplen harmonics have higher values of positive and negative sequence components along with zero sequence

components. The presence of other components in triplen harmonics adds to the magnitude of unbalanced THD. For the given period, the voltage unbalanced THD ( $< 0.5\%$ ) is always less than balanced THD ( $< 1.5\%$ ). The 95<sup>th</sup> percentile of unbalanced voltage distortion is 1.18 V (absolute value, the numerator in the expression of total harmonic distortion) whereas balanced voltage distortion is 2.8 V. The zero sequence unbalance for voltage is higher than the negative sequence unbalance. The voltage distortion for non-triplen harmonics at the three houses is similar but differs for triplen harmonics. The current distortion is different for all houses according to the loads connected there. The negative and zero sequence unbalance for fundamental current are approximately equal for the three houses. The pattern of increase and decrease in the value of current balanced and unbalanced THD over time is unique for each house. However, it was observed that if the balanced THD increases the value of unbalanced THD also increases for the three houses. A polar plot representing the sequence components of the current harmonics at the same house is shown in Fig. 2.

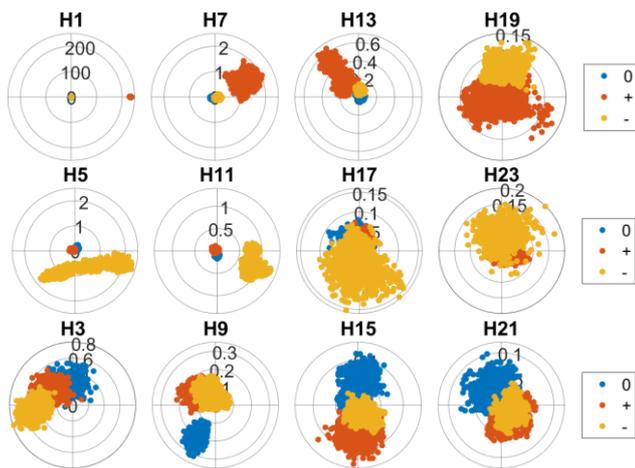


Fig.1. Polar plots of the positive, negative, and zero sequence voltage harmonics for the residential load.

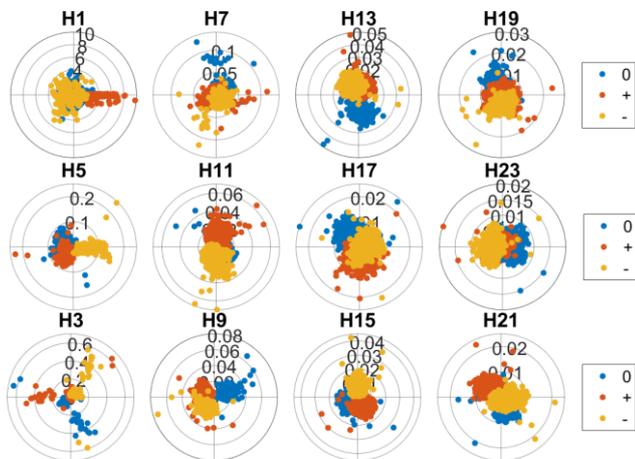


Fig.2. Polar plots of the positive, negative, and zero sequence current harmonics for the residential load.

### 3.2. Office building

Measurements were made at an office building with rooftop solar panels. Two parallel grid-connected solar inverters supply the office building and occasionally export to the grid. The supply voltage at the point of connection to the grid was measured. One snapshot every hour is used for the analysis of the supply voltage. The measurements were done for the whole year, but the results for one week in July, which is a vacation week in summer, and one week in October, which is a working week in autumn, are presented in this paper. In October, the solar power production is lower than in July, due to the lower sun and shorter days. Fig. 3 and Fig. 4 show the positive, negative, and zero sequence voltage harmonics up to order 23 for the vacation week and working week respectively. The highest distortion appears in the 5<sup>th</sup> and 7<sup>th</sup> harmonics. The 5<sup>th</sup> and 7<sup>th</sup> harmonics appear in the first and the second quadrant respectively, whereas for the residential load the 5<sup>th</sup> and 7<sup>th</sup> harmonics are located in the 3<sup>rd</sup> and 4<sup>th</sup> quadrant and the 1<sup>st</sup> quadrant respectively. The office building and the residential area are located in the same city, but electrically not near each other; that is why these two places have different phase angles for 5<sup>th</sup> and 7<sup>th</sup> harmonics. When comparing the vacation week and the working week, the magnitude of the 3<sup>rd</sup>, 5<sup>th</sup>, and 7<sup>th</sup> harmonic increases for the working week. This increase can be attributed to office loads, which are dominant in the working week. Another reason could be the solar inverter. The 13<sup>th</sup>, 17<sup>th</sup>, and 19<sup>th</sup> harmonics have two groups in the balanced component of the harmonic emission. The two groups are in the same quadrant so similar phase angles, but different magnitudes. It was found that the magnitude of the harmonics was less on weekends and increases during weekdays following a daily variation. The magnitude of the balanced component is lower during the night and increases during the day, thus forming two groups. The non-triplen harmonics have a lower magnitude of the unbalanced component. The triplen harmonics on the other hand show both the positive and negative sequence component, showing a more unbalanced behaviour.

The 95<sup>th</sup> percentile value of the balanced and unbalanced voltage THD, as well as negative-sequence, unbalance and zero-sequence unbalance for the fundamental are presented in Table 1.

Table 1: Balanced and unbalanced THD for voltage

Week	Balanced THD (%)	Unbalanced THD (%)	Negative sequence unbalance (%)	Zero sequence unbalance (%)
Vacation	1.78	0.4	0.27	0.162
working	2.26	0.41	0.27	0.32

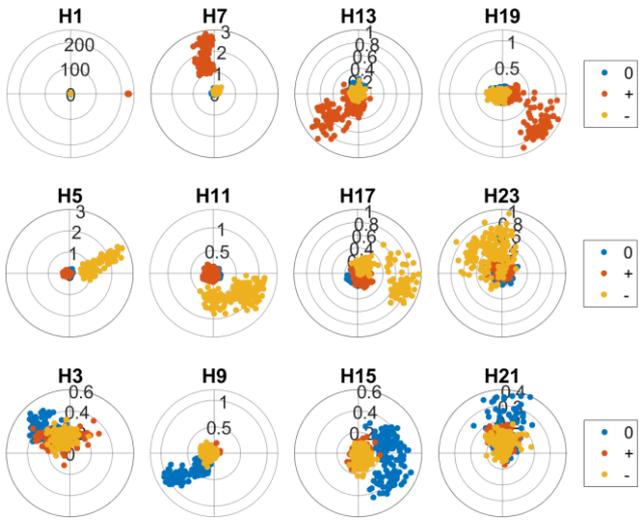


Fig.3 Polar plots of the positive, negative, and zero sequence voltage of harmonics for the vacation week for the office building

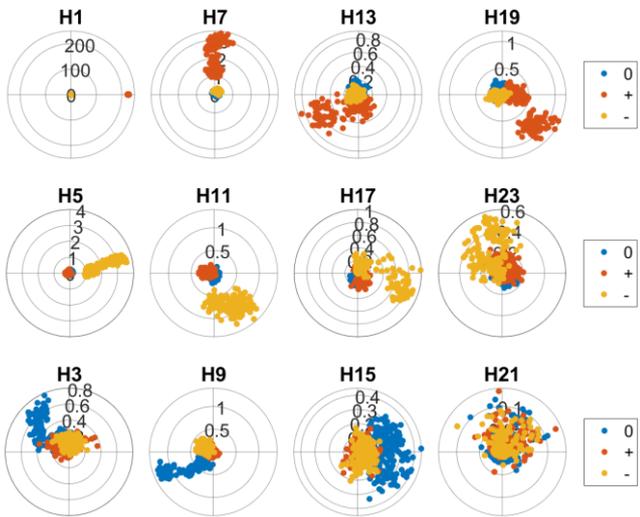


Fig.4. Polar plots of the positive, negative, and zero sequence voltage of harmonics for the working week for the office building.

### 3.3. University campus

Measurements were made at the LV side of the transformer supplying a university campus. One snapshot at noon every day was captured from January to October 2020. The campus has single-phase loads e.g. computers and light. Fig. 5 shows the rms value of the three-phase currents. The weekly and seasonal pattern is seen in the magnitude of the current. The current drawn follows a weekly pattern, with reduced current during the weekends. The value of rms current is least for the vacation months of July and August. The variation in the current drawn affects the voltage balanced THD magnitude; with the voltage balanced THD being higher during weekdays than during weekends. Fig. 6 shows the polar plot for voltage harmonic order up to 23 for the measured period. The 5<sup>th</sup> and 7<sup>th</sup> harmonics have a

higher magnitude of the balanced component when compared with their respective unbalanced component. The 5<sup>th</sup> harmonic is spread over two quadrants. For weekends, it appears in the third quadrant and on weekdays in the fourth quadrant, following the load pattern. The triplen harmonics contain positive and negative sequence components, which are the unbalanced components next to the balanced zero sequence components. The 15<sup>th</sup> and 17<sup>th</sup> order harmonics are spread over two quadrants. This may be due to the load or due to the background voltage distortion. The 95<sup>th</sup> percentile for the balanced THD is 2.49% and for the unbalanced THD, it is 0.46%. The unbalanced THD magnitude is slightly higher than at the two locations presented earlier. The negative and zero sequence unbalance is 0.64 % and 0.33 % respectively. Fig. 7 shows the polar plots for the positive, negative, and zero sequence components of the current. The unbalanced components are higher in current as compared to voltage.

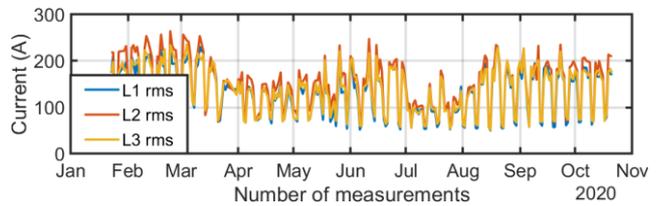


Fig. 5. Current drawn by phase L1, L2, and L3

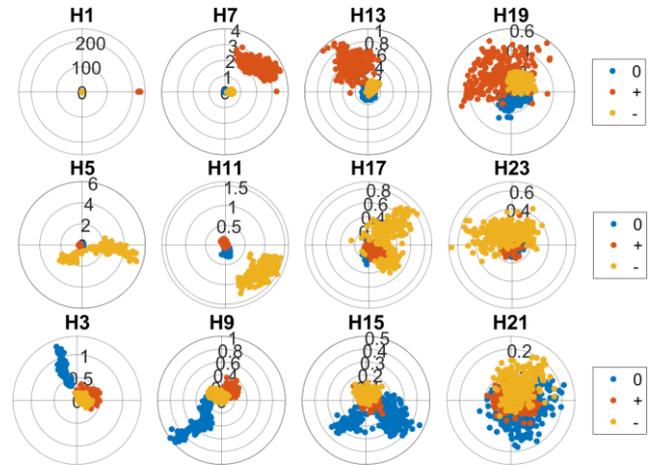
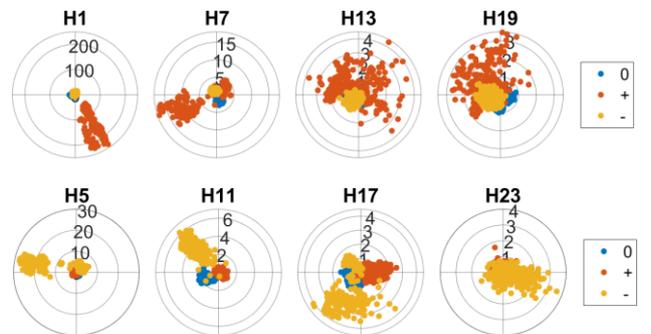


Fig.6. Polar plots representing the positive, negative, and zero sequence voltage of harmonics for the campus



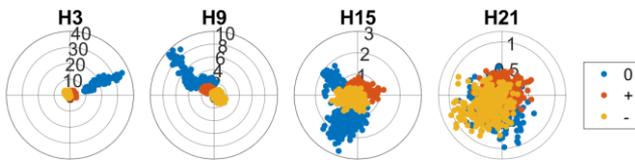


Fig.7. Polar plots representing the positive, negative, and zero sequence current components of harmonics for the campus

#### 4 Triplen and non-triplen harmonics

From the measured supply voltage at the three locations, it can be seen that the triplen harmonics showed higher unbalanced components than the non-triplen harmonics. To highlight the contribution of the triplen harmonics to the balanced and unbalanced THD, the calculated voltage balanced and unbalanced THD are split into triplen and non-triplen harmonic components. The 95<sup>th</sup> percentile values of the voltage balanced and unbalanced THD for triplen harmonics and non-triplen harmonics are presented in Table 2 for the three locations. In the case of residential load, the balanced and unbalanced triplen harmonics are comparable in magnitude. The unbalanced triplen harmonics are 59% of balanced triplen harmonics. For residential loads, this can be expected, as most of the load being supplied is a single-phase load. For the office building and the university campus, the unbalanced triplen harmonics are approximately 30% of the balanced triplen harmonics.

Even though the balanced component of triplen harmonics is higher, the unbalanced part can propagate further or have propagated from elsewhere to the measured location. The two contributions to the unbalanced THD are thus local unbalance in load or spread from unbalance elsewhere. The balanced component of triplen harmonics is due to loads connected at the locations. For non-triplen harmonics, the balanced THD is also higher than the unbalanced THD. Among the three locations, the highest THD occurs at the university campus.

Table 2: Comparison of the balanced and unbalanced THD

Location	Residence	Office building	University
<b>Triplen harmonics</b>			
Balanced	0.22%	0.63%	0.6%
Unbalanced	0.13%	0.18%	0.19%
Ratio(Unbalanced/balanced)	59%	28%	31%
<b>Non-triplen harmonics</b>			
Balanced	1.19%	2.17%	2.459%
Unbalanced	0.5%	0.4%	0.420%
Ratio(Unbalanced/balanced)	42%	18%	17%

#### 5 Discussion

The classical distinction between positive, negative and zero-sequence harmonics is no longer sufficient to study the propagation of harmonics in distribution networks.

Instead, a distinction should be made between balanced and unbalanced components of each harmonic order. In presence of unbalance, each harmonic order contains a dominating (balanced) component and two unbalanced symmetrical components. For example, harmonic order 3, which has a zero sequence dominating component, was shown to have both positive and negative sequence unbalanced component. Values of balanced and unbalanced THD at a given location indicates the amount of inherent unbalance at that location. The ratio of unbalanced and balanced THD can be used as an indicator for the unbalanced distortion.

From a propagation view point, zero-sequence and non-zero-sequence components propagate differently. Each harmonic is partly blocked by a distribution transformer and partly pass through it. The unbalanced components (i.e. the non-zero sequence components) of triplen harmonics can propagate upstream if generated locally or can propagate from elsewhere to the measurement location. From the analysis, it was seen that the triplen harmonics have higher unbalanced components, which are positive and negative sequence components, as compared to non-triplen harmonics.

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