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Pulsed Electromagnetic Excitation of a Thin Wire – An Approximate Numerical Model Based on the Cagniard-DeHoop Method of Moments

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***Abstract*—An approximate computational model of an electromagnetic (EM) pulse excited thin-wire antenna is developed. The presented solution methodology is based on the Cagniard-deHoop method of moments (CdH-MoM) and Hallén's approximation of the thin-wire model. It is shown that the proposed time-domain (TD) solution leads to an inversion-free, efficient updating procedure that mitigates the marching-on-in-time accumulation error. An illustrative numerical example demonstrates the validity of the proposed model.**

***Index Terms*—wire antenna, transient waves, time-domain (TD) analysis, integral equations, Cagniard-DeHoop method of moments (CdH-MoM).**

I. Introduction

Electromagnetic (EM) radiation and scattering by a thin-wire antenna can be evaluated through the electric-current distribution along its axis [1, Ch. 4]. The current distribution can be obtained from the corresponding integral equation (IE) either numerically via MoM [2, Ch. 4] or approximately using analytical techniques (e.g., [3] and [4, Ch. 8]).

While powerful TD-IE techniques for the transient analysis of thin-wire structures are widely available (e.g., [5]–[10]), approximate expressions for the antenna current are dominantly limited to time-harmonic EM fields with the triangular and sinusoidal spatial distributions being the most popular approximations [1, Eqs. (4.20)a and (4.20)b]. A rare exception in this respect is the traveling-wave solution based on the first-order Hallén's approximation [11]. In contrast to more widespread frequency-domain approaches, TD techniques account for transient EM phenomena, thus describing the physics in its entirety over a broad range of frequencies [12].

In this letter, the incorporation of Hallén's approximation in CdH-MoM [13] is investigated. It is demonstrated that this approach leads to a tridiagonal TD impedance array elements of which are described via straightforward closed-form analytical expressions. Moreover, the pertaining marching-on-in-time solution procedure mitigates the error accumulation and does not require, for a special choice of the time step, the impedance matrix inversion.

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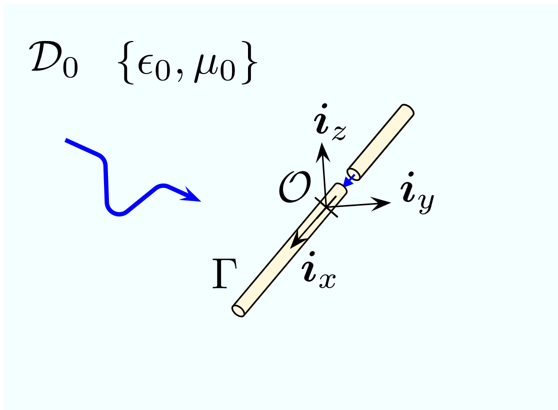

Fig. 1. A pulse-excited thin-wire segment.

II. Problem Description

To localize position in the problem configuration (see Fig. 1) we employ the Cartesian coordinates $\{x, y, z\}$ with respect to an orthogonal Cartesian reference frame that is defined by its origin $\mathcal{O}$ and the base vector $\{\boldsymbol{i}_x, \boldsymbol{i}_y, \boldsymbol{i}_z\}$. The time coordinate is denoted by t. The Heaviside unit-step function is denoted $\mathrm{H}(t)$ ($\mathrm{H}(t) = 1$ if $t > 0$, $\mathrm{H}(t) = 1/2$ if $t = 0$ and $\mathrm{H}(t) = 0$ if $t < 0$). The Dirac-delta distribution is $\delta(t)$. The convolution operator is denoted by $*$, which is supplemented by the pertaining subscript (e.g., $*_t$ denotes the time convolution operator). In a similar fashion, partial differentiation is denoted by ∂ (e.g., ∂_x then denotes the differentiation with respect to x). Finally, the time integration operator is defined as $\partial_t^{-1} f(t) = f(t) *_t \mathrm{H}(t)$.

We shall analyze the space-time electric-current distribution induced along a pulse-excited, perfectly electrically conducting (PEC) straight wire segment. The wire is located in a homogeneous, isotropic and lossless medium that occupies an unbounded domain $\mathcal{D}_0$. Its EM properties are described by (real-valued and positive) electric permittivity, ϵ_0, and magnetic permeability, μ_0. The corresponding wave speed and wave impedance is $c_0 = (\epsilon_0\mu_0)^{-1/2} > 0$ and $Z_0 = (\mu_0/\epsilon_0)^{1/2} > 0$, respectively.

The axis of the wire segment extends along $\Gamma = \{-\ell/2 < x < \ell/2, y = 0, z = 0\}$, where $\ell > 0$ denotes its length.

Its radius, $a > 0$, is assumed to be relatively small such that the thin-wire approximation applies. The wire is supposed to be excited by either an external pulsed-EM source or via a narrow voltage gap source. The effect of the excitation field is incorporated via the axial component of the (incident) electric-field strength denoted by $E_x^{\rm i}(x,t)$. The difference between the total EM field in the configuration and the incident EM field is defined as the scattered EM field (to be denoted by superscript $^{\rm s}$).

The EM antenna problem is formulated here with the aid of the EM reciprocity theorem of the time-convolution type (see [14, Sec. 28.2] and [15, Sec. 1.4.1]). Hence applying the theorem to the scattered and testing (denoted by superscript $^{\rm T}$) states in the unbounded domain exterior to the wire segment, we end up with

$$\int_{x=-\ell/2}^{\ell/2} E_x^{\rm T}(x,t) *_t I_x^{\rm s}(x,t)\mathrm{d}x = \int_{x=-\ell/2}^{\ell/2} E_x^{\rm s}(x,t) *_t I_x^{\rm T}(x,t)\mathrm{d}x, \tag{1}$$

where $E_x^{\rm s}(x,t)$ represents (the axial components of) the scattered electric field along the wire. Consequently, $E_x^{\rm s}(x,t) = -E_x^{\rm i}(x,t)$, for all $t > 0$ along the PEC wire. Furthermore, $I_x^{\rm s}(x,t)$ denotes the (unknown) induced current in the wire, and $I_x^{\rm T}(x,t)$ is the testing current to be associated with a set of sub-domain functions of rectangular and impulsive shape in space and time, respectively (see (14)). The testing electric field on Γ, $E_x^{\rm T}(x,t)$, is related to $I_x^{\rm T}(x,t)$ via [14, Sec. 26.9]

$$E_x^{\rm T}(x,t) = -\mu_0\partial_t A^{\rm T}(x,t) + \epsilon_0\partial_t^{-1}\partial_x^2 A^{\rm T}(x,t) \quad \text{for } \{-\ell/2 \le x \le \ell/2\} \text{ and } t > 0, \tag{2}$$

where $A^{\rm T}$ denotes (the x-component of) the magnetic vector potential. The first-order approximation of Hallén is arrived at by using [11], [16]

$$A^{\rm T}(x,t) = I_x^{\rm s}(x,t) *_t *_x g(x,t) \simeq I_x^{\rm s}(x,t)\Omega(x,a)/4\pi, \tag{3}$$

where we neglected the propagation time over the cross-section of the wire, $a/c_0 \downarrow 0$, $g(x,t) = \delta(t-\varrho/c_0)/4\pi\varrho$, with $\varrho = (x^2+a^2)^{1/2} > 0$, is the Green's function pertaining to the (unbounded, isotropic, homogeneous and lossless) surrounding medium and

$$\Omega(x,a) = \sinh^{-1}\left(\frac{x+\ell/2}{a}\right) - \sinh^{-1}\left(\frac{x-\ell/2}{a}\right). \tag{4}$$

For the convenience of the solution methodology that follows, we shall further replace $\Omega(x,a)$ in Eq. (3) with the average of $\Omega(0,a)$ and $\Omega(\pm\ell/2,a)$. Consequently, we write

$$A^{\rm T}(x,t) \simeq I_x^{\rm s}(x,t)\Omega_0/4\pi, \tag{5}$$

where

$$\Omega_0 = \tfrac{1}{2}\left[2\sinh^{-1}(\ell/2a) + \sinh^{-1}(\ell/a)\right] = \tfrac{3}{2}\log(\ell/a) + \tfrac{1}{2}\log(2) + o(a/\ell) \;\text{ as }\; a/\ell \downarrow 0. \tag{6}$$

The TD reciprocity relation (1) with (2) and (6) is the point of departure for the CdH-MoM analysis of thin-wire antennas [13, Sec. 2.2]

III. Transform-Domain Representation

The presented solution methodology relies on the CdH technique [17] (see also [18, Ch. 2]). This joint-transform methodology combines a unilateral Laplace transform

$$\hat{E}_x(x,s) = \int_{t=0}^{\infty} \exp(-st)E_x(x,t)\mathrm{d}t \quad \text{for } \{s \in \mathbb{R}; s > 0\}, \tag{7}$$

with the wave slowness representation in the axial direction, i.e.

$$\hat{E}_x(x,s) = \frac{s}{2\pi\mathrm{i}}\int_{\kappa=-\mathrm{i}\infty}^{\mathrm{i}\infty} \exp(-s\kappa x)\tilde{E}_x(\kappa,s)\mathrm{d}\kappa. \tag{8}$$

In fact, (8) represents an inverse spatial Fourier transform with respect to x, where the (real-valued and positive) Laplace-transform parameter, s, plays the role of a scaling parameter. Through Eqs. (7) and (8), the TD reciprocity relation (1) can be expressed in terms of complex slowness integrals

$$\int_{\kappa=-\mathrm{i}\infty}^{\mathrm{i}\infty} \tilde{E}_x^{\rm T}(\kappa,s)\tilde{I}_x^{\rm s}(-\kappa,s)\mathrm{d}\kappa = \int_{\kappa=-\mathrm{i}\infty}^{\mathrm{i}\infty} \tilde{E}_x^{\rm s}(\kappa,s)\tilde{I}_x^{\rm T}(-\kappa,s)\mathrm{d}\kappa. \tag{9}$$

where the (approximate) transform-domain testing field is given by

$$\tilde{E}_x^{\rm T}(\kappa,s) \simeq -(s/c_0)\, Z_\Gamma\, c_0^2\gamma_0^2(\kappa)\tilde{I}_x^{\rm T}(\kappa,s), \tag{10}$$

where $Z_\Gamma = (Z_0/4\pi)\Omega_0$ (see (6)) and $\gamma_0^2(\kappa) = c_0^{-2} - \kappa^2$.

IV. Time-Domain Solution

Following next the solution strategy presented in [10, Sec. IV], the space-time solution domain is discretized uniformly. Accordingly, the wire's axis is divided into $N+1$ segments of constant length $\Delta_x = \ell/(N+1)$, thus enabling to represent Γ by its partition $\Delta\Gamma = \{x_n = -\ell/2 + n\Delta_x, y = 0, z = 0\}$ with $n \in \{1, \cdots, N\}$. The time axis is discretized in a similar manner by representing $t > 0$ by $\{t_k = k\Delta_t\}$ with $k \in \{1, \cdots, M\}$ and $\Delta_t > 0$ being the time step. The space-time electric current induced along the wire is then supposed to be a piecewise linear function of x and t according to

$$I_x^{\rm s}(x,t) = \sum_{n=1}^{N}\sum_{k=1}^{M} i_k^{[n]}\Lambda^{[n]}(x)\Lambda_k(t), \tag{11}$$

where $i_k^{[n]}$ (in A) denotes the electric-current coefficients to be computed. Next, $\Lambda^{[n]}(x)$ and $\Lambda_k(t)$ represent the triangular expansion functions that can be defined via

$$\Lambda^{[n]}(x) = \begin{cases} 1 + (x - x_n)/\Delta & \text{for } x \in [x_{n-1}, x_n] \\ 1 - (x - x_n)/\Delta & \text{for } x \in [x_n, x_{n+1}], \end{cases} \tag{12}$$

$$\Lambda_k(t) = \begin{cases} 1 + (t - t_k)/\Delta t & \text{for } t \in [t_{k-1}, t_k] \\ 1 - (t - t_k)/\Delta t & \text{for } t \in [t_k, t_{k+1}]. \end{cases} \tag{13}$$

As the testing current, we choose

$$I_x^{\rm T}(x,t) = \Pi^{[S]}(x)\delta(t), \tag{14}$$

for all $S = \{1, \cdots, N\}$, where $\Pi^{[S]}(x) = 1$ if $x \in [x_S - \Delta_x/2, x_S - \Delta_x/2]$ and $\Pi^{[S]}(x) = 0$ elsewhere.

The use of the transform-domain counterparts of (11) and (14) in the (transform-domain) reciprocity relation (9) leads to a system of equations in the s-domain that can be readily transformed to the TD analytically via the CdH technique. We then arrive at the following time-convolution system of equations

$$\sum_{k=1}^{m} \Delta^2 \underline{\boldsymbol{Z}}_{m-k} \cdot \boldsymbol{I}_k = \boldsymbol{V}_m, \tag{15}$$

where $\underline{\boldsymbol{Z}}_k = \underline{\boldsymbol{Z}}(t_k)$ denotes a 2-D $[N \times N]$ TD impedance array at $t = t_k$, and Δ^2 represents the second-order central-difference operator [19, 25.1.2]

$$\Delta^2 \underline{\boldsymbol{Z}}_n = \underline{\boldsymbol{Z}}_{n+1} - 2\underline{\boldsymbol{Z}}_n + \underline{\boldsymbol{Z}}_{n-1}. \tag{16}$$

Furthermore, $\boldsymbol{I}_k$ is a 1-D $[N \times 1]$ array of the electric-current coefficients, $i_k^{[n]}$, and, $\boldsymbol{V}_m$ represents a 1-D $[N \times 1]$ excitation-voltage array at $t = t_m$.

Owing to the relatively simple structure of (the second-order difference of) the (approximate) TD impedance array, the system of equations can be solved at once as

$$\begin{aligned} \boldsymbol{I}_m &= (\underline{\boldsymbol{\Gamma}} + \underline{\boldsymbol{\Lambda}})^{-1} \cdot \big[\boldsymbol{V}_m - \boldsymbol{V}_{m-1} \\ &\quad + \underline{\boldsymbol{\Lambda}} \cdot (2\boldsymbol{I}_{m-1} - \boldsymbol{I}_{m-2})\big], \\ &\quad \text{for all } m = \{3, \cdots, M\} \text{ with} \\ \boldsymbol{I}_1 &= (\underline{\boldsymbol{\Gamma}} + \underline{\boldsymbol{\Lambda}})^{-1} \cdot \boldsymbol{V}_1, \\ \boldsymbol{I}_2 &= (\underline{\boldsymbol{\Gamma}} + \underline{\boldsymbol{\Lambda}})^{-1} \cdot (\boldsymbol{V}_2 - \boldsymbol{V}_1 + 2\underline{\boldsymbol{\Lambda}} \cdot \boldsymbol{I}_1), \end{aligned} \tag{17}$$

where $(\cdot)^{-1}$ denotes the matrix inverse. Apparently, if $c_0\Delta_t = \Delta_x/2\sqrt{2}$ (see (26)–(30)), the inverse can be carried out at once

$$(\underline{\boldsymbol{\Gamma}} + \underline{\boldsymbol{\Lambda}})^{-1} = -[1/(2\sqrt{2}Z_\Gamma)]\mathbb{I}, \tag{18}$$

where $\mathbb{I}$ denotes the $[N \times N]$ identity matrix. It is interesting to note that the condition for the explicit solution bears a resemblance with the Courant stability limit, $c_0\Delta_t = \Delta/\sqrt{n}$, applying to an n-dimensional uniform lattice [20, p. 651]. It is further stressed that the electric-current array at $t = t_m$ is evaluated via its past values at $t = t_{m-1}$ and $t = t_{m-2}$ only, thereby mitigating the marching-on-in-time accumulation error [21, Sec. 3.2]

V. Illustrative Numerical Examples

In the numerical examples that follow, the wire antenna of length $\ell = 0.10\,(\mathrm{m})$ with $a/\ell = 1/500$ is excited by a voltage delta-gap source that is described by

$$E_x^{\mathrm{i}}(x,t) = V_0(t)\delta(x), \tag{19}$$

where $V_0(t)$ represents the excitation voltage (causal) pulse that has the power-exponential shape [22]

$$V_0(t) = V_{\mathrm{m}}(t/t_{\mathrm{r}})^{\nu} \exp[-\nu(t/t_{\mathrm{r}} - 1)]\mathrm{H}(t), \tag{20}$$

where we take the unit amplitude $V_{\mathrm{m}} = 1.0\,\mathrm{V}$ with $\nu = 11$, which yields $t_{\mathrm{r}} \simeq 1.3132\,t_{\mathrm{w}}$ (see Fig. 2). In the example that follows we take $t_{\mathrm{w}} = 5\ell/c_0 \simeq 1.67\,\mathrm{ns}$. We shall observe the electric-current transient responses at $x = 0$ within the finite time window $\{0 \le c_0 t_{\mathrm{w}} \le 30\ell\}$. The (approximate) CdH-MoM model of the wire is discretized uniformly into $N + 1 = 10$ segments. The time step is chosen such that $a/c_0\Delta_t = 1/5$. Figure 3 shows the corresponding transient electric-current responses as calculated using the proposed approximate CdH-MoM model based on Hallén's approximation, its full rigorous CdH-MoM version [10] and the finite integration technique (FIT) as implemented in CST Microwave Suite®. Despite the fundamental difference between the FIT and CdH-MoM computational models, the resulting signal correlate well. The relatively small and bounded oscillations in the late-time part of the TD response can dominantly be attributed to the approximation introduced by Eq. (5). It has been observed that their amplitude can be reduced by choosing a smoother excitation pulse.

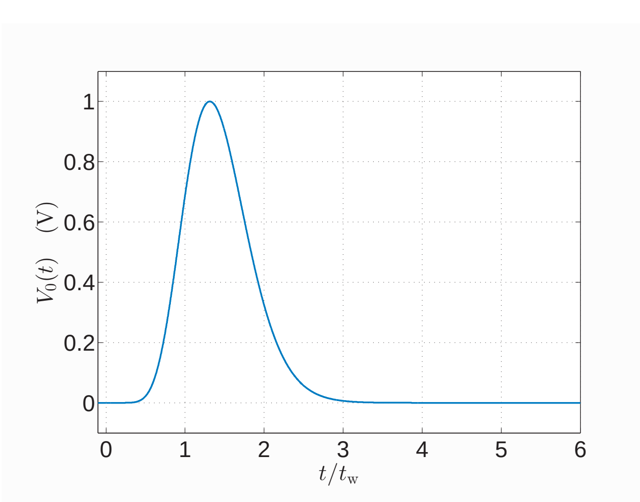

Fig. 2. The excitation power-exponential pulse shape.

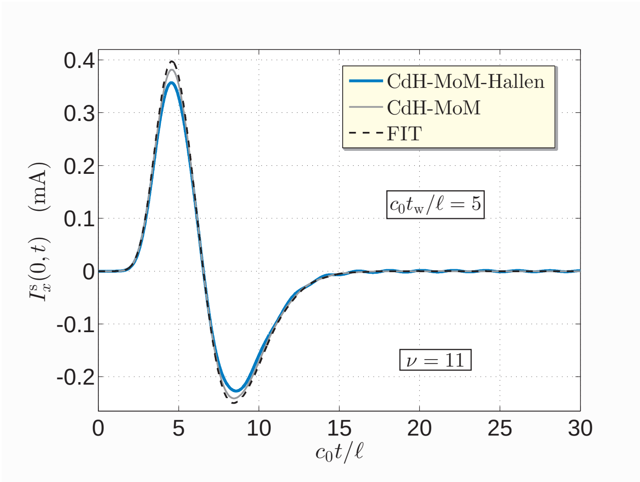

Fig. 3. The electric-current responses as computed using the proposed approximate and rigorous CdH-MoM models and FIT.

VI. Conclusion

We have introduced an approximate numerical model for the transient EM analysis of a straight wire antenna. The antenna problem has been formulated using the TD Lorentz

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reciprocity theorem and solved with the aid of the classic CdH transform method. It has been demonstrated that under the first-order Hallén approximation, the TD impedance array pertaining to a uniformly discretized wire segment can be readily expressed analytically in terms of elementary functions only. Consequently, the evaluation of the impedance array is computationally effortless. In addition, its simple structure makes it possible to express the transient antenna response via an efficient updating scheme that mitigates the marching-on-in-time accumulation error. The introduced computational model can be integrated in more general TD-IE frameworks. The proposed model has been validated with the aid of CST Microwave Suite® and a rigorous TD-IE approach. Finally, it is anticipated that a generalization of the presented methodology to curved thin wires is feasible by incorporating further spatial dimensions in the computational domain as previously applied to the CdH-MoM analysis of a strip antenna in [13, Ch. 14].

Appendix
Time-Domain Impedance Array

The elements of the TD impedance array can be generally expressed as

$$\begin{aligned} Z^{[S,n]}(t) = (Z_\Gamma/c_0\Delta_t\Delta_x)\big[&R(x^{[S,n]}+3\Delta_x/2,t) \\ &- 3R(x^{[S,n]}+\Delta_x/2,t)+3R(x^{[S,n]}-\Delta_x/2,t) \\ &- R(x^{[S,n]}-3\Delta_x/2,t)\big] \\ &\text{for all } S,n\in\{1,\cdots,N\} \text{ and } t>0, \end{aligned} \tag{21}$$

where we used $x^{[S,n]} = x_S - x_n$ and recall that $Z_\Gamma = (Z_0/4\pi)\Omega_0$. The TD function, $R(x,t)$, follows upon carrying out the inverse Laplace transform of $\hat{R}(x,s)$ that is given through the complex slowness integral

$$\hat{R}(x,s) = -\frac{c_0^2}{2\mathrm{i}\pi}\int_{\kappa\in\mathbb{K}_0}\gamma_0^2(\kappa)\frac{\exp(s\kappa x)}{s^3\kappa^3}\mathrm{d}\kappa, \tag{22}$$

where the integration path, $\mathbb{K}_0$, runs along $\mathrm{Re}(\kappa)=0$ and is indented around the origin as shown in Fig. 4. If $x>0$, then $\mathbb{K}_0$ can be closed with an infinite semicircle in the left of the κ-plane. The thus obtained closed contour is subsequently deformed to a small circle around the origin, $\mathbb{C}_0$, the integration around which yields

$$\hat{R}(x,s) = (c_0^2/2s^3)(2-s^2x^2/c_0^2)\mathrm{H}(x), \tag{23}$$

where we have incorporated $\mathrm{H}(x)$ as the integral for $x<0$ yields the zero value. The inverse transform of (23) is straightforward and gives the desired expression

$$R(x,t) = \tfrac{1}{2}(c_0^2t^2 - x^2)\mathrm{H}(x)\mathrm{H}(t). \tag{24}$$

The resulting TD impedance array is a tridiagonal array the elements of which follow as

$$\begin{aligned} Z^{[S,n]}(t) = &-(Z_\Gamma/c_0\Delta_t\Delta_x)\left(c_0^2t^2+\tfrac{3}{4}\Delta_x^2\right)\mathrm{H}(t)\delta_{S-n,0} \\ &+\tfrac{1}{2}(Z_\Gamma/c_0\Delta_t\Delta_x)\left(c_0^2t^2-\tfrac{1}{4}\Delta_x^2\right)\mathrm{H}(t)\delta_{|S-n|,1} \\ &\text{for all } S,n\in\{1,\cdots,N\}. \end{aligned} \tag{25}$$

In the numerical procedure, the impedance array is evaluated along the (uniformly) discretized time axis. Accordingly, upon

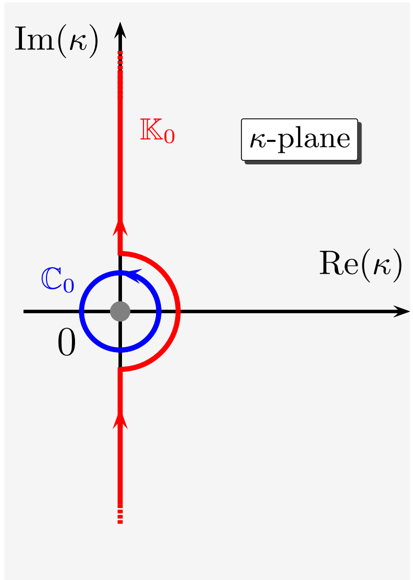

Fig. 4. Complex κ-plane with the integration paths $\mathbb{K}_0$ and $\mathbb{C}_0$.

replacing t with $t_m = m\Delta_t$, the second-order difference (see (16)) follows as

$$\Delta^2 Z^{[S,n]}(t_m) = \Gamma^{[S,n]} + \Lambda^{[S,n]}\,(\delta_{m,0}-\delta_{m,1}), \tag{26}$$

where

$$\Gamma^{[S,n]} = -2\alpha\,\delta_{S-n,0} + \alpha\,\delta_{|S-n|,1}, \tag{27}$$

$$\Lambda^{[S,n]} = 6\gamma\,\delta_{S-n,0} + \gamma\,\delta_{|S-n|,1}, \tag{28}$$

with

$$\alpha = Z_\Gamma(c_0\Delta_t/\Delta_x), \tag{29}$$

$$\gamma = -Z_\Gamma(\Delta_x/8c_0\Delta_t). \tag{30}$$

The simple structure of the TD array (26) makes its filling computationally effortless. Additionally, if $\alpha+\gamma=0$ the system of equations (15) can be solved via a stable, inversion-free updating procedure (see (17) and (18)).

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