On the Background to the Formulae and Accuracy of Rock Fracture Toughness Measurements Using ISRM Standard Core Specimens

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The background to the formulae in the two ISRM Suggested Methods for Determining the Fracture Toughness of Rock Material is presented. The Short Rod and the Chevron Bend are the recommended specimens. Both are easily made from core pieces. Their standard proportions and machining tolerances are justified. Values for stress intensity factors and compliances are given, with linear correction terms for specimens within the tolerance limits, an assessment of transverse tensile failure in the Short Rod arms is made and an expression for crack speed vs controlling displacement rate is derived. The influence of rate dependent material properties is discussed briefly.

A basic brittle fracture toughness evaluation is suggested which only requires load registration. A correction for non-brittle rate independent energy dissipation at the crack tip is also derived and the necessary steps to evaluate it are discussed. It requires displacement measurements to be made during the testing.

Finally specimen calibration and fracture toughness testing work are discussed in the perspective of testing accuracy.

NOTATION

ISRM = International Society for Rock Mechanics;
SM = Suggested Method;
LEFM = Linear Elastic Fracture Mechanics;
CB = Chevron Bend specimen;
SR = Short Rod specimen;
LPD = Load Point Displacement;
 a = (maximum) crack length (m);
 a_0 = initial crack length or chevron tip distance from bottom fibre of CB or from loaded section of SR (m);
 a_1 = chevron base position (m);
 B = crack front length (m);
 w = specimen width (m);
 D = specimen diameter (m);
 S = support span (m);
 A = area (m²);
 2θ = chevron notch angle (θ in SMs);
 t = notch or slot thickness (m);
 F = load on specimen (N);
 K_I = stress intensity factor (thickness average) (N/m^{1.5});
 Y', A, Y^* = non-dimensional stress intensity factors;
 s = superscript, refers to specimen of standard proportions;
 m = subscript, refers to point $a = a_m$ at which $K_I(a)$ has a minimum;
 $\alpha = a/D$, non-dimensional crack length;
 $\beta = a/w$, non-dimensional crack length;

K_{CB}, K_{SR} = fracture toughness values evaluated on small specimens under LEFM assumptions (N/m^{1.5});
 λ_F = physical compliance (m/N);
 δ_F = LPD (m);
 E = Young's modulus (Pa);
 ν = Poisson's ratio;
 $g = \lambda_F \cdot ED$, non-dimensional compliance;
 i = subscript, refers to initially uncracked specimen or to arbitrary point in discrete set of values;
 I = moment of inertia (m⁴);
 $s_m = g_i/g_m$, ratio of compliances;
 σ_t = tensile strength (Pa);
 t = time (sec);
 $\dot{}$ = superposed dot denotes differentiation with respect to time or rate of change;
ref = subscript, refers to reference value;
 n = exponent in crack growth law, equation (35), or superscript referring to SR configuration with non-standard proportions;
 δ_F^r = residual LPD after unloading;
 G = potential energy release rate (J/m²);
 W = work done (J);
 p = degree of plasticity or non-linearity correction factor;
 $\bar{}$ = superposed bar, denotes average;
el = superscript, refers to LEFM evaluation;
 ϕ = angle in F vs δ_F diagram;
 S_{F_0} = initial tangent slope in F vs δ_F diagram ($s_{\text{initial tangent}}$ in SMs);
 S_{F_m} = slope at evaluation point F_m (s_c in SMs);
 F_m = (load at) evaluation point (F_c in SMs);
 F_i = intermediate point (F_i in SMs);
 F_c = interpolation point;
 l = recovered displacement (x_2 in SMs);
 K_{CB}^c, K_{SR}^c = fracture toughness values corrected for non-linearity of F vs δ_F diagrams.

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OUTLINE OF CONTENTS OF THE ISRM SUGGESTED TEST METHODS

A Working Group within the International Society for Rock Mechanics (ISRM) Commission on Testing Methods has finalized two "Suggested Methods (SMs) for Determining the Fracture Toughness of Rock Material". The document [1] specifies the use of two specimens that can be machined directly from pieces of rock core, the Chevron Bend specimen [2] and the Short Rod [3].

Since rock material is usually available in the form of cores, this implies ease of machining and easier acceptance by prospective users than rectangular specimens.

The Chevron Bend specimen has a notch cut perpendicular to the core axis, see Fig. 1, and the notch in the Short Rod is cut parallel to the core axis, see Fig. 2. In both cases, the ligaments of the notched section have the form of a "V" or chevron. This shape results in a relatively long period of stable crack growth under increasing load before the point at which fracture toughness is evaluated. The advantages of using chevron notched specimens are also considered important in metals and ceramics testing, Underwood *et al.* [4].

The use of two specimens with different crack orientations permits an assessment of fracture anisotropy from one core while saving core material, since the bend specimen is long enough for the halves remaining after testing to be used for Short Rod testing. Two levels of testing are offered in each method, giving the prospective user a choice between accuracy and cost. Level I requires

Fig. 1. The Chevron Bend specimen with bend test fixture and basic notation.

Basic notation:

- D = diameter of short rod specimen
- w = length of specimen, 1.45D
- 2θ = chevron angle, 54.6°
- a_0 = chevron tip distance from load line, 0.48D
- a_1 = maximum depth of chevron flanks, in practice not necessarily equal to w .
- t = notch width
- a = crack length
- B = crack front length
- F = load on specimen
- δ_F = load line displacement, LPD
- δ_{CMOD} = crack mouth opening displacement, CMOD = LPD

Fig. 2. The Short Rod specimen with basic notation.

only the registration of maximum load to be made during the test. Level II requires continuous load displacement measurements to be made during the test. While level I is simple and cheap, yielding index like results, level II testing is more complicated. Since level I testing of normal core sizes tends to yield fracture toughness values which are lower than values obtained from larger specimens, but level II testing in some cases yields values that are independent of core size and that correlate well with results from other test methods, level II testing is sometimes justified.

The SMs follow other ISRM Suggested Methods closely [5]. The general contents are:

General Introduction

Technical Introduction: On Fracture Mechanics, On the Concept of Fracture Toughness, On the Use of the Fracture Toughness Values for Rock, Purpose of these Suggested Methods, On the Suggested Methods.

Method 1: Suggested Method for Determining Fracture Toughness Using Chevron Bend Specimens.

Method 2: Suggested Method for Determining Fracture Toughness Using Short Rod Specimens.

Notes

References

Each SM contains detailed specifications of testing, as e.g. in

Method 1 (Chevron Bend Testing),

Scope

Specimen Description

Apparatus: Specimen Preparation Equipment, Testing Machine and Load Fixtures, Specimen Alignment Aids, Displacement Measuring Equipment, Recording.

Procedure: Specimen Selection and Preparation, Calibration, Setting Up, Testing.

Calculations: Calculation of Loading Rate, Calculation of Slope Values, Calculation of Fracture Toughness, Correction of Fracture Toughness for Non-linearity, Calculation of Additional Quantities (Young's Modulus, Energy Rate Resistance and Specific Work of Fracture).

Validity: Specimen Size and Type, Use of Fracture Toughness Values.

Reporting of Results: General Data, Test Sample Data, Specimen Data.

The specifications concerning Method 2 (Short Rod Testing) are of course as identical as possible.

This paper gives a background to the evaluation formulae used in the SMs. Two previous ones [6, 7] represent the drafting stage of the SMs, they too rely on the report [8]. Finally, specimen calibration and fracture toughness testing work, initiated by the working group members and others, is discussed in the perspective of testing accuracy.

RECOMMENDED STANDARD GEOMETRIES AND LEVEL I FORMULAE

The Chevron Bend specimen

The specimen geometry and basic notation are shown in Fig. 1. The suggested standard proportions are those calibrated by Ouchterlony [9, 10]

$$S = 3.33D \pm 0.02D \quad \text{or} \quad |\Delta S| = |S - S^s| \leq 0.02D,$$

$$a_0 = 0.15D \pm 0.10D,$$

$$2\theta = 90.0^\circ \pm 1.0^\circ,$$

and

$$t \leq 0.03D \text{ or } 1 \text{ mm, whichever is greater. (1)}$$

The tolerances are a compromise between ease of machining and the available calibration data. Superscripts refers to the standard configuration.

Using one parameter as a measure of the size of the specimen, the four others describe its uncracked proportions. With a crack present, its length is the major

variable and the crack front length becomes

$$\begin{aligned} B/D &= 2 \tan \theta \cdot (a - a_0)/D \\ &= 2(a - a_0)(Da_1 - a_1^2)^{0.5}/(a_1 - a_0) \end{aligned} \quad (2)$$

inside the chevron. With $2\theta^s = 90^\circ$ then $a_1^s/D = 0.632$.

The K_I representation of Ouchterlony [2, 8–10] uses D as the size measure. It is

$$K_I = 0.25(S/D)Y'_c \cdot F/D^{1/5}$$

where

$$Y'_c = Y'_c(a_0/D, \theta, t/D; a/D) \quad (3)$$

or

$$Y'_c = 2(D/S) \cdot \left\{ \frac{\partial g}{\partial \alpha} \right\} / [(1 - \nu^2)B/2D]^{0.5} \quad (4)$$

in terms of g . Here g is the non-dimensional specimen compliance, $g = \lambda_F \cdot ED$ and $\alpha = a/D$ is a non-dimensional crack length. The advantage of equations (3) and (4) is that first order variations in S are taken care of.

In the fracture toughness testing, the characteristic minimum of K_I at an *a priori* determinable crack length, $a = a_m$ say, is of crucial importance. The desired minimum is

$$\begin{aligned} Y'_c(\dots; a_m/D) &= Y'_{cm} \\ &= 7.34 + 28.6a_0/D + 39.4(a_0/D)^2, \end{aligned} \quad (5)$$

valid when $0.05 \leq a_0/D \leq 0.25$. If we use Barker's representation [3] instead

$$K_I = AF/D^{1.5}$$

where

$$A = A(S/D, a_0/D, \theta, t/D; a/D) \quad (6)$$

then $A = 0.25(S/D)Y'_c$ and

$$A_{\min} = [1.835 + 7.15a_0/D + 9.85(a_0/D)^2] \cdot (S/D)$$

or

$$A_{\min}^s = 10.42. \quad (7)$$

A discussion in [8] suggests that variations in t and θ within the tolerances of equation (1) may be neglected if an error less than 1% is acceptable.

The Short Rod specimen

The specimen geometry and basic notation are shown in Fig. 2. The geometry is a simplification of that used by the ASTM for metals, for which much calibration data exists [11]. The suggested standard proportions are

$$w = 1.45D \pm 0.02D \quad \text{or} \quad |\Delta w| = |w - w^s| \leq 0.02D,$$

$$a_0 = 0.48D \pm 0.02D,$$

$$a_1 - a_0 = 0.97D \pm 0.02D,$$

$$2\theta = 54.6^\circ \pm 1.0^\circ$$

and

$$t \leq 0.03D \text{ or } 1 \text{ mm, whichever is greater. (8)}$$

These tolerances are wider than their ASTM counterparts.

Again four independent parameters describe the uncracked specimen. With a crack present the front length is

$$B/D = 2 \tan \theta \cdot (a - a_0)/D = (a - a_0)/(a_1 - a_0) \quad (9)$$

when $a_0 \leq a \leq a_1$, i.e. inside the chevron.

Barker [3, 12] uses D as the size measure and essentially represents the stress intensity factor for a linearly elastic material as in equation (6) whereby $A = A(w/D, a_0/D, \theta, t/D; a/D)$. Bubsey *et al.* [13] and Shannon *et al.* [14] use w and the representation

$$K_I = Y^* F / (D \sqrt{w})$$

where

$$Y^* = Y^*(w/D, a_0/w, a_1/w, t/w; a/w). \quad (10)$$

With the non-dimensional crack lengths $\alpha = a/D$ or $\beta = a/w$ the stress intensity factor functions become

$$A = \left[\frac{\partial g}{\partial \alpha} / (2B/D) \right]^{0.5}$$

and

$$Y^* = \left[\frac{\partial g}{\partial \beta} / (2B/D) \right]^{0.5} = \left[\frac{1}{2} \frac{\partial g}{\partial \beta} \cdot \frac{\beta_1 - \beta_0}{\beta - \beta_0} \right]^{0.5}. \quad (11)$$

Thus $Y^* = A \cdot \sqrt{(w/D)}$. The desired minima are denoted

$$A(\dots; a_m/D) = A_{\min}(w/D, a_0/D, \theta, t/D) \quad (12a)$$

and

$$Y^*(\dots; a_m/w) = Y_m^*(w/D, a_0/w, a_1/w, t/w). \quad (12b)$$

Thus also $Y_m^* = A_{\min} \cdot \sqrt{(w/D)}$. The present isotropic consensus value for the standard geometry is [11]

$$Y_m^{*s} = 28.9 \pm 0.3 \quad \text{or} \quad A_{\min}^s = 24.0 \pm 0.25. \quad (13)$$

Superscript s refers to the standard parameter set in equation (8), except that $t/D = 0$.

Barker [15] gives corrections for variations in specimen proportions related to a slightly different nominal configuration, $2\theta = 55.2^\circ$ or $\beta_1 = a_1/w = 0.992$,

$$A_{\min} \approx A_{\min}^s \cdot (1 - 0.5475\Delta w/D + 1.0\Delta a_0/D - 0.0214\Delta\theta). \quad (14)$$

Equation (8) keeps $A_{\min} \approx$ within 0.5% of the true value when $|\Delta w| < 0.03D$, $|\Delta a_0| < 0.015D$ and $|\Delta\theta| < 0.75^\circ$. In later work Barker [12] gives data for curved chevron flanks which are believed to represent the nominal configuration with straight notches above accurately. A linear expansion about this configuration and a shift to the standard one yields [8]

$$A_{\min} = A_{\min}^s \cdot (1 - 0.7891\Delta w/D + 1.8340\Delta a_0/D - 0.332\Delta\theta)$$

with

$$A_{\min}^s = 23.61 \quad (15)$$

and a negligible influence of t/D . Equation (15) is considerably more sensitive to variations in specimen

geometry than equation (14) and should be valid up to 1/3 or 1/2 of the latter's tolerance limits.

In the other representation, the influence of t/w is neglected. According to Bubsey *et al.* [13]

$$\begin{aligned} Y_m^*(w/D, \beta_0, \beta_1) &= \left[\frac{\beta_1 - \beta_0}{1 - \beta_0} \right]^{0.5} \cdot Y_m^*(w/D, \beta_0, 1) \\ &= Y_{m1}^* / [2(1 - \beta_0) \tan \theta \cdot w/D]^{0.5} \end{aligned} \quad (16)$$

where

$$\begin{aligned} Y_{m1}^* &= [19.98 - 9.54w/D + 6.80(w/D)^2] \\ &\quad - [118.7 - 125.1w/D - 22.08(w/D)^2]\beta_0 \\ &\quad + [379.4 - 363.6w/D + 84.4(w/D)^2]\beta_0^2, \end{aligned} \quad (17)$$

valid when $0 \leq \beta_0 \leq 0.4$, see [14]. Equation (17) is based on an extensive compliance calibration of Short Rod specimens with different proportions. A linear expansion of equation (17) about the standard configuration, $\beta_0^s = 0.332$ and $w^s/D = 1.45$, logarithmic differentiation of equation (16) and conversion of θ to degrees yields

$$\begin{aligned} Y_m^* &= Y_m^{*s} \cdot (1 + 0.2514\Delta w/D + 1.9834\Delta\beta_0 \\ &\quad - 0.02141\Delta\theta) \end{aligned}$$

with

$$Y_m^{*s} = 29.11. \quad (18)$$

The parameter $\beta_0 = a_0/w$ may be replaced by a_0/D through $\beta_0 = (a_0/D)/(w/D)$. Use of it, $Y_m^* = A \cdot \sqrt{(w/D)}$ and logarithmic differentiation yields

$$\begin{aligned} A_{\min} &= 24.17 \cdot (1 - 0.5475\Delta w/D \\ &\quad + 1.3679\Delta a_0/D - 0.0214\Delta\theta), \end{aligned} \quad (19)$$

which is directly comparable with equations (14) and (15).

The effects of geometry variations predicted by equation (19) lie between those of equations (14) and (15). Equation (19) is believed to be the most accurate one. For practical purposes the consensus value for $A_{\min}^s$ in equation (13) should be used. After rounding then

$$A_{\min} = 24.0 \cdot [1 - 0.6\Delta w/D + 1.4\Delta a_0/D - 0.01\Delta(2\theta)]. \quad (20)$$

A discussion in [8] suggests that equation (20) is sufficiently accurate within the tolerances of equation (8).

Level I fracture toughness

An estimate of the fracture toughness of a brittle material is then obtained as

$$K_{CB} = A_{\min} F_{\max} / D^{1.5} \quad (21a)$$

and

$$K_{SR} = A_{\min} F_{\max} / D^{1.5}. \quad (21b)$$

Here $F_{\max}$, the failure load, is assumed to coincide with $a = a_m$ and $A_{\min}$ from equation (7) or (20) is used.

COMPLIANCE FORMULAE, TRANSVERSE TENSILE FAILURE AND CRACK SPEED ESTIMATES

Chevron Bend compliance

Two compliances are of primary interest, that of the uncracked specimen g_i and that at the K_I minimum, $a = a_m$. Based on the compliance data in [9] one finds $g = g(S/D, \alpha_0, \theta; \alpha)$. Hence

$$g_i = g(S/D, \alpha_0, \theta; \alpha_0) \quad \text{and} \quad g_m = g(S/D, \alpha_0, \theta; \alpha_m) \quad (22)$$

were computed over the parameter range in equation (1), resulting in

$$g_i \approx (20.8 + 19.4\alpha_0 + 142.3\alpha_0^2) \times [1 - (0.79 - 0.3\alpha_0)\Delta S/D] \quad (23a)$$

and

$$g_m \approx (19.2 + 71.9\alpha_0 + 113.7\alpha_0^2) \times [1 - 0.8 \cdot (1 - 0.8\alpha_0 + \alpha_0^2)\Delta S/D]. \quad (23b)$$

The valid α_0 range is 0.05–0.25 and if $|\Delta S| \leq 0.02D$ then $|\Delta g_i|$ and $|\Delta g_m| < 0.016$. Thus the influence of S/D on g_i and g_m can be neglected in most cases. The influence of θ and t can probably be neglected too. For the slope ratio s_m the influence of $\Delta S/D$ nearly cancels out anyway and

$$s_m = 1.05 - 2.15\alpha_0 + 4.21\alpha_0^2 \quad (24)$$

to within 1% over the whole α_0 range.

Short Rod compliance

The most complete set of compliance data for the Short Rod is that of Bubsey *et al.* [13] and Shannon *et al.* [14, 16]. Thus $g = g(w/D, \beta_0, \beta_1 = 1; \beta)$, with data for specific β -values in Table 1 in [11] and fitted polynomials for ranges of β -values in [16]. Hence

$$g_i = g(w/D, \beta_0, 1; \beta_0)$$

and

$$g_m = g(w/D, \beta_0, 1; \beta_m) \quad (25)$$

were computed over the parameter ranges $1.5 \leq w/D \leq 2.0$ and $0.2 \leq \beta_0 \leq 0.5$ [8], resulting in

$$\begin{aligned} g_i = & -[21.88 + 74.25w/D - 49.73(w/D)^2] \\ & -[308.13 - 1558.92w/D + 706.08(w/D)^2]\beta_0 \\ & +[3599.6 - 7944.9w/D + 3335.4(w/D)^2]\beta_0^2 \\ & -[6011.4 - 10625.7w/D + 3763.4(w/D)^2]\beta_0^3 \end{aligned} \quad (26a)$$

and

$$\begin{aligned} g_m = & -[499.19 - 179.12w/D - 89.78(w/D)^2] \\ & +[8278.9 - 6139.9w/D + 963.6(w/D)^2]\beta_0 \\ & -[32245 - 26045w/D + 4175.4(w/D)^2]\beta_0^2 \\ & +[44009 - 39581w/D + 7934.5(w/D)^2]\beta_0^3. \end{aligned} \quad (26b)$$

The parameter range very nearly covers the standard configuration, $w^s/D = 1.45$ and $\beta_0^s = 0.332$.

In [8] it is argued that the influence of β_1 or θ is small, but that of t is not. A simple assessment based on semicircular cantilevers minus slices of area $\Delta A = D \cdot t/2$ yield the moment of inertia $I(A - \Delta A) \approx I(A) \times (1 - 3.282t/D)$ with centroid position $e \approx 0.2122D$ and $I(A) \approx 0.00686D^3$ to be used in the formulae. Then

$$g + \Delta g = \frac{1}{3}a^3D/I(A - \Delta A) + \frac{20}{9}(1 + \nu)aD/(A - \Delta A)$$

or

$$\frac{\Delta g}{g} \approx 3.282 \cdot \frac{(a/D)^2 + 0.0452(1 + \nu)}{(a/D)^2 + 0.1164(1 + \nu)} \cdot t/D, \quad (27)$$

compare e.g. with Barker [3]. With standard configuration values, $a_0^s = 0.48D$, $a_m^s = 0.824D$ and $\nu = 0.2$, equations (26–27) yield

$$g_i = 84.5 \cdot [1 + 2.9\Delta a_0/D + 2.5(t/D - 0.012)] \quad (28a)$$

and

$$g_m = 166.3 \cdot [1 + 1.1\Delta w/D + 1.4\Delta a_0/D + 2.9(t/D - 0.012)], \quad (28b)$$

since the experiments in [13] were conducted on specimens with $t = 0.012D$. The $\Delta w/D$ term in g_i was so small that it was dropped. Still assuming brittle material behaviour, the ratio g_m/g_i can be used to determine where on the test record, F vs LPD, the desired evaluation point F_m , corresponding to $a = a_m$, occurs. The inverted value

$$s_m = g_i/g_m = 0.5 \cdot [1 - 1.0\Delta w/D + 1.5\Delta a_0/D - 0.4(t/D - 0.012)] \quad (29)$$

is the corresponding slope ratio which defines F_m . In practice F_m and $F_{\max}$ hardly coincide.

Transverse tensile failure

The relative compactness of the cantilever arms of the Chevron Bend specimen will seldom induce tensile bending stresses of sufficient magnitude to cause transverse failure of the arms before the evaluation of fracture toughness can be made. The relative slenderness of the Short Rod arms may well however.

A simple beam estimate relates the largest tensile stress, $\sigma = M/[I(A)/e] \approx 30.93Fa/D^3$, to crack length and load. Maximum crack length without transverse failure is thus $a_{\max} = 0.0323\sigma_t D^3/F$ with σ_t a measure of the uniaxial tensile strength. If the crack is to stay in the chevron plane beyond a_m , then $a_{\max} > a_m$ is required and with $K_I = K_{SR}$ and equation (21) there follows

$$D > (30.93/A_{\min} \cdot \alpha_m \cdot K_{SR}/\sigma_t)^2 \approx 1.15(K_{SR}/\sigma_t)^2. \quad (30)$$

This prediction appears to be very accurate for some hard rocks [8].

Barker [17] proposes a Short Rod size limit like equation (30) but effectively using a factor 1.25 instead. Fortunately that limit is less restrictive but for the more slender ASTM Short Rod configuration equation (30) with the factor 1.65 applies and this factor should be relatively sensitive to notch thickness. The desire to avoid transverse tensile failure is obviously in conflict

[1], for which the formula background is given here, now offer one. For it to gain widespread acceptance however, it will have to provide accurate results at acceptable costs.

The concept of accuracy contains elements of precision and bias [27], bias roughly being systematic errors that contribute to the difference between means test results and an accepted true value. The latter may exist, in which case the test is valid, or it may not, depending on how well the tested material follows the assumptions made, e.g. of rate independence, linearity and brittleness. Linearity of the test record and brittleness are however a matter of specimen size. Thus a restrictive indication of validity or appropriateness of the test method is if p were so small that level I uncorrected toughness values and level II corrected ones essentially agree, i.e. if $p \leq 0.05$ say. A more extensive indication of validity is if the corrected fracture toughness values should prove to be independent of specimen size for larger p -values. There is evidence to this effect for rocks such as Indiana limestone [22], Stripa granite [28], Kallax gabbro [29] and Ogino tuff [30]. For Inada granite and Tohoku marble [30] on the other hand, the fracture toughness values increase with increasing specimen size. It is noteworthy that p for all rocks simultaneously seems to level off at a non-zero value.

Two possible causes of bias in the SMs are rock anisotropy and the previously smaller data basis for the Chevron Bend calibration, as compared with the Short Rod [11]. The present Chevron Bend calibration data is extensive, apart from the original work [9, 10], Takahashi *et al.* [31, 32] and Wai and Li [33] have made experimental calibrations and Gerstle [34], Ouchterlony [35] and Hayashi *et al.* [36] have made numerical calibrations. The span of K_I -values about $A_{\min}$ in equation (7) is $\pm 8\%$, but a detailed discussion [35] puts the true, i.e. consensus, value at about only 2–3% lower than in equation (7). Thus this bias factor is believed to be small.

Another way of checking it is to compare Chevron Bend and Short Rod toughness values, when anisotropy and size effects are properly accounted for. The work of Matsuki *et al.* [30] is affirmative regarding the toughness agreement.

The influence of anisotropy was discussed in [23] and is presently being investigated jointly by Tohoku University in Sendai, Japan, and the Swedish Detonic Research Foundation for Iidate granite and Älvdalen sandstone.

Concerning precision, or closeness of agreement between randomly selected individual test results, tolerances and procedure specifications in the SMs are the main safeguard. It is believed that elements like sources of variation and ruggedness, which belong to the first phase of investigating the actual performance of the SMs in practice [37], are relatively well covered. Even the control charting element appears to be covered in that single material results are consistent over a period of time. The next phase then is a broader interlaboratory study with many participants. The performance of the

SMs can then be expressed in terms of repeatability, which gives the scatter in in-house experiments, and reproducibility, which includes the between-laboratory scatter as well.

Such testing could of course cause revisions of the SMs to be made. Basic points like specimen calibration, which was discussed above, and test procedures are believed to be under control as a result of the co-operative efforts of the working group. Concerning the introduction of new specimens, the SMs mention the future addition of a third specimen so that a complete investigation of anisotropy can be made using only one core.

As an extreme, extensive or even complete revision of the SMs may be called for. The work of Hashida *et al.* [32] point to a direction which this might take. They use a J-integral approach and acoustic emission techniques to pin point a critical value, J_{IAE} , for the initiation of crack growth from straight notches in virgin material. Since J_{IAE} is reasonably constant of a wide range of specimen types and sizes for the low modulus Iidate granite, this method would seem to have a potential for a wide variety of rocks, see also Takanohashi *et al.* [38].

The draft recommendations of RILEM for concrete [39], in which the fracture energy G_F is measured, point to another possible direction. The fracture energy is justified by the postulated existence of a finite cohesive line zone at a crack tip, Hillerborg [40]. Such a zone has been shown to exist in rock, Labuz *et al.* [41].

G_F is given by the work per unit area (J/m^2) which is required to completely divide (fracture) an element of material. It is, in principle, measured exactly as $\bar{R}$ in equations (53) and (54). It was shown earlier that $\bar{R}$ for rock can be quite dependent on for example specimen size however, Ouchterlony [20, 42].

The results for concrete show that G_F on average increases by 30% when specimen size increases 3-fold, Hillerborg [43]. The variations in fracture toughness of rock with specimen size in the work related above [22, 28, 32] are generally substantially smaller even when the error doubling effect in equation (52) is included.

A comparison of the two approaches and testing methods is in place and it should at this stage emphasize real material behaviour over a theoretical analysis based on assumed behaviour like the one of Hassanzadeh and Hillerborg [44]. Nor should the J-integral related work of Leung and Li [45] be forgotten in this context.

Acknowledgements—The author expresses his gratitude to the following colleagues in the Working Group, without whose active support these Suggested Methods would never have materialized: Dr John A. Franklin (Canada); Professor Sun Zongqi (P.R. China); Drs Barry K. Atkinson and Philip Meredith (U.K.); Professor Fritz Rummel and Dr Wolfram Müller (F.R. Germany); Professor Hideaki Takahashi and his colleagues at Tohoku University and Professor Yuichi Nishimatsu at University of Tokyo (Japan); Professor Ove Stephansson (Sweden); Dr Laurence S. Costin and Professor Anthony R. Ingraffea (U.S.A.).

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