Material parameter estimation for boron steel from simultaneous cooling and compression experiments

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Abstract

In order to increase the accuracy of numerical simulations of the hot stamping process, reliable material data is crucial. Traditionally, the material is characterized by several isothermal compression or tension tests performed at elevated temperatures and different strain rates. The drawback of the traditional methods is the appearance of unwanted phases for some test temperatures and durations. Such an approach is also both time consuming and expensive. In the present work an alternative approach is proposed, which reduces unwanted phase changes and the number of experiments. The isothermal mechanical response is established through inverse modelling of simultaneous cooling and compression experiments. The estimated material parameters are validated by comparison with data from a separate forming experiment. The computed global response is shown to be in good agreement with the experiments.

1. Introduction

The use of ultra high strength steel components has increased in the automotive industry, owing to the need for higher passive safety and reduction of weight. Hot stamping of boron alloyed steels is one technique to produce open section, ultra high strength steel components. This manufacturing process is used in a number of vehicle structure applications, for example, side impact protection beams and for bumper system components. The prediction of the product manufacturing and performance requires a detailed knowledge of the material response during the complete manufacturing process, including deformations during the fast cooling from approximately 900 °C to room temperature. Coupled thermo-mechanical analysis is used to predict the material behaviour during the hot stamping process, see [1].

The reliability of the numerical simulations depends not only on the models and methods used, but also on the accuracy and applicability of the input data. The material model and

Table 1. Chemical composition for boron 02 in weight per cent.

C [%]	Si [%]	Mn [%]	P [%]	S [%]	Cr [%]	B [%]
0.20–0.25	0.20–0.25	1.00–1.30	0–0.025	0–0.015	0.15–0.25	0.0015–0.0050

the related material property data must be consistent with the conditions of the material in the process of interest. In the case of simultaneous forming and quenching, the material is subjected to a temperature history of heating and subsequent cooling. Since the complete thermo-mechanical history will influence the material properties, similar deformation and temperature histories must be used for the material testing. Traditionally, the material is characterized by several tension, compression or torsion tests at elevated temperatures and different strain rates, see, e.g. [2–5]. One drawback with isothermal tests is the increase of unwanted phases for certain test temperatures and durations. For tests with austenite, the time to perform isothermal testing of boron alloyed steel is limited, see, e.g. [6]. In the temperature interval from 600 °C to the martensite start temperature at approximately 400 °C, the time allowed for a complete isothermal compression test is less than 10 s before bainite starts to form. This time for cooling from approximately 900 °C, temperature stabilization and compression is too short for moderate deformation rates.

In the present work an alternative approach is proposed in which unwanted phase transformations are avoided and the number of tests needed can be significantly reduced. The material response during cooling and deformation is studied by inverse modelling of experiments with simultaneous cooling and deformation. With this approach, the mechanical properties are established for a broad temperature range with only a few physical tests. Furthermore, an obvious advantage of the inverse method is also that the inhomogeneous stress and strain conditions in the specimen are taken into account in the evaluation. In [7] a similar technique is used to estimate the material parameters by a single compression test during continuous cooling. The method described here is a development of [7] in order to provide more information for the material characterization process by using several tests.

2. Experimental procedure

The compression tests have been carried out in a Gleeble 1500 thermo-mechanical test equipment. The material used is boron steel from Swedish Steel AB—SSAB, see table 1 for chemical composition. The circular cylindrical specimens have the following dimensions: height 5.5 mm and diameter 4.0 mm. The test procedure can shortly be described by the following steps:

- (a) insertion of the specimen into a vacuum chamber,
- (b) resistive heating to the austenitic temperature and holding for 300 s,
- (c) continuous cooling and compression starting at the desired temperature is maintained until the true axial strain of 0.5 is obtained. The cooling rate and applied axial strain rate are 50 °C s^{−1} and 0.08 s^{−1}, respectively.

During a test, the compression force, compression depth, diameter increase and temperature are measured and logged. The diameter increase is measured using a clip-on type water cooled gage with a linear variable displacement transducer (LVDT). The accuracy of the diametric measurements is specified by the manufacturer to be 0.5% of the full measurement range, ±6 mm, which means that the measurement error is less than 0.06 mm. Direct measurements after the tests with a micrometre caliper give a maximum diameter deviation to the experimental

Figure 1. CCT diagram for the boron 02 steel given in the manufacturer's technical specification. The meaning of the letters used in the figure are as follows: A—Austenite, F—Ferrite, P—Pearlite, B—Bainite and M—Martensite. The Vickers hardness is denoted by HV.

of 0.05 mm. With the current test equipment, the diametric increase for one cross section at the time can be measured, in this case at the specimen mid-height. The temperature is measured using a Pt/PtRh-thermocouple welded on the surface of the specimen (mid-height). Graphite and tantalum foils have been used to reduce the friction and to prevent sticking between the tungsten carbide anvils and the test specimen. For the strain rate 0.08 s^{-1} , three overlapping temperature intervals for the compression tests have been carried out in order to provide a wide range of deformation–temperature combinations. The temperature intervals for the strain rate of 0.08 s^{-1} are (in degree centigrade): *high*—[936,630], *medium*—[788,481] and *low*—[638,350]. Two specimens at each temperature interval have been tested, to assure that the obtained data are reasonable and that the deviations between the tests are small. The continuous cooling diagram (CCT), which indicates the hardenability of the steel, is shown in figure 1. The diagram shows that a cooling rate of 50°C s^{-1} would give a final microstructure containing only martensite.

3. Numerical procedures

The parameter estimation process is based on the inverse approach, see, e.g. [7–10]. Here the FE-method is applied to solve the equations of motions of the direct problem, see section 3.1, and an optimization algorithm is used to find a suitable set of material parameters to a model describing the material hardening.

3.1. The direct problem

The direct problem consists of computing the response of the FE model, which is subjected to variations of the parameters in a model that generates the flow stress. In the present work the explicit FE program LS-Dyna [11] is used for solving the direct problem. An explicit simulation is accomplished without forming a stiffness matrix or solving any equations, thus reducing core storage requirements and avoiding solution failures, see [12]. Complex geometries in industrial applications with contact impact, large deformations, nonlinear materials and

Figure 2. Axisymmetric FE mesh that describes the Gleeble model.

coupled field models can be treated effectively and quickly. Explicit codes are commonly used for large scale coupled thermo-mechanical analyses of metal forming processes in industrial applications.

The material model used is a rate independent thermo-elastic-plastic model described in [1, 13]. The plastic behaviour is described by a von Mises isotropic yield function, an associated flow rule and a nonlinear isotropic hardening law defined as tabulated values of yield stress and effective plastic strain. The axisymmetric FE mesh used in the simulation of the Gleeble experiments is shown in figure 2. The dimensional increase of the FE specimens due to the resistive pre-heating is not accounted for. The mesh, which is a quarter of the actual geometry due to symmetry conditions, consist of 68 nodes and 47 four-node fully integrated quadrilateral elements (volume weighted Galerkin elements, see [11]). Displacement boundary conditions are applied to the anvil, taken from the experiments. The anvil is modelled as a rigid body. Contact between the anvil and the specimen is included, and the friction is assumed to follow Coulomb's friction law.

During each simulation, the temperature variation within the specimen is neglected and assumed to be prescribed and homogeneous. To check the validity of this assumption, coupled thermo-mechanical simulations have been conducted. The results are given in section 4.2.

The hardening model is a physically based model developed for both bcc and fcc metals by Nemat-Nasser [14]. Most parameters of this model have a clear physical meaning. The athermal and thermally activated flow stress originally defined for commercially pure tantalum and oxygen free, high conductivity (OFHC) copper can be written as

$$\sigma_y(\dot{\bar{\epsilon}}, \bar{\epsilon}^{pl}, T) = \sigma^0 \left(1 - \left(-\frac{kT}{G_0} \left(\ln \frac{\dot{\bar{\epsilon}}}{\dot{\bar{\epsilon}}_0} + \ln f(\bar{\epsilon}^{pl}, T) \right) \right)^{1/q} \right)^{1/p} f(\bar{\epsilon}^{pl}, T) + \sigma_a^0 g(\bar{\epsilon}^{pl}, d_g). \quad (1)$$

In equation (1), $\dot{\bar{\epsilon}}$ is the effective strain rate, $\bar{\epsilon}^{pl}$ is the effective plastic strain, T is the temperature, σ^0 is an effective stress value to be determined empirically, k is the Boltzman constant and G_0 is the magnitude of the energy barrier that the dislocations must overcome. $\dot{\bar{\epsilon}}_0$ is a reference strain rate related to the density and the average velocity of mobile dislocations and barrier spacing. In equation (1), the function $f(\bar{\epsilon}^{pl}, T)$ is defined in the sequel for fcc metals, and

$\sigma_a^0 g(\bar{\varepsilon}^{\text{pl}}, d_g)$ is the athermal part of the flow stress (depending on density and the distribution of dislocations, approximated by $\bar{\varepsilon}^{\text{pl}}$ and grain size d_g). The parameters p and q define the shape of the energy barrier. The function represents the ratio of the initial to the current average dislocation spacing. For fcc metals, the dislocation spacing defines the barrier spacing and changes with the dislocation density. As for oxygen free, high conductivity copper (OFHC copper), the following representations are assumed to apply to the boron steel:

$$f(\bar{\varepsilon}^{\text{pl}}, T) = 1 + a(T)\sqrt{\bar{\varepsilon}^{\text{pl}}} \quad (2)$$

and

$$a(T) = a_0 \left(1 - \left(\frac{T}{T_m} \right)^2 \right), \quad (3)$$

where T_m is the melting temperature and a_0 depends on the initial average dislocation spacing. Note that it is assumed that the last term in equation (1), $g(\bar{\varepsilon}^{\text{pl}}, d_g)$, can be approximated by

$$g(\bar{\varepsilon}^{\text{pl}}, d_g) \approx (\bar{\varepsilon}^{\text{pl}})^{n_1}. \quad (4)$$

The used hardening model assumes that after a change in temperature the new flow stress is reached instantaneously, without any delay in time. Transient conditions may cause a delay of the response in many materials. This phenomenon is reported in, for example, [15, 16], where transient strain rate changes of two orders of magnitude is shown to give a delay of the flow stress level before it reaches the stationary value. How the continuous cooling of the specimen affects the instantaneous flow stress is not investigated in this work.

In this model there are 8 material dependent parameters to be determined: $D_1 = \sigma^0$, $D_2 = k/G_0$, $D_3 = \dot{\varepsilon}_0$, $D_4 = a_0$, $D_5 = q$, $D_6 = p$, $D_7 = \sigma_a^0$ and $D_8 = n_1$. The choice of the 8 material parameters follows the description of the hardening model in [14].

3.2. The inverse problem

The solution of the direct problem together with the experimental results gives the necessary input data to the optimization problem. In the Gleeble application, the FE simulation provides output data in the form of an axial reaction force and a diametric increase at mid-height (surface) of the specimen as a function of the model parameters and the friction coefficient. In the present study, three different experiments and corresponding numerical simulations are used for the definition of the objective function.

The objective of the inverse problem is to find the parameters used to generate the flow stress hardening function, which gives the best possible match between the FE solutions and the experimental reference data. An optimization approach for estimating the material parameters D_i where $i = 1, \dots, M$, is stated as

$$\min_{D_i \in \mathbb{R}^M} \varphi_{\text{tot}}(D_i), \quad (5)$$

where

$$\varphi_{\text{tot}}(D_i) = \sum_{k=1}^3 \varphi^k(D_i), \quad i = 1, 2, \dots, M \quad (6)$$

and

$$\varphi^k(D_i) = \frac{1}{2} \sum_{j=1}^{N_k} \left(\frac{f_j^{\text{exp}} - f_j^{\text{fem}}(D_i)}{s_j^f f_j^{\text{exp}}} \right)^2 + \left(\frac{d_j^{\text{exp}} - d_j^{\text{fem}}(D_i)}{s_j^d d_j^{\text{exp}}} \right)^2. \quad (7)$$

Here f_j^{exp} , f_j^{fem} , d_j^{exp} and d_j^{fem} are the experimental and calculated forces and diametric increases for point j on each curve, respectively. The terms $(1/f_j^{\text{exp}})$ and $(1/d_j^{\text{exp}})$ are normalization or weight factors that are applied to avoid adding terms of different orders of magnitude. The minimum of equation (6) gives the closest match in a least square sense between the N_k sample points of each FE solution and corresponding experimental data (index k is used to represent the different temperature intervals). The justification of the least square criterion is based on the hypothesis that the sum of several different contributions will tend to be normally distributed irrespective of the probability distribution of the individual contributions according to the central limit theorem, see e.g. [17]. In the Gleeble tests, the uncertainties associated with the set of measurements are not known in advance, and we assume that all measurements have the same standard deviation ($s_j^f = s_j^d = s$) and that the model fits well. Then we can proceed by first assigning an arbitrary constant value, in this case $s_j^f = s_j^d = s = 1$, to all points. When the material parameters are determined by minimizing equation (6), one can proceed by calculating the standard deviation between measured and calculated data ([18]), see section 4.1.

In searching for the minimum value of equation (6), the subplex algorithm proposed by [19] has been used. The subplex algorithm is a direct search method for unconstrained optimization, i.e. there is no need for calculating the gradient of $\varphi^k(D_i)$. The subplex method is well suited for optimizing noisy and non-smooth objective functions. Consequently, constraints and error handling can rationally be implemented by means of uncomplicated penalty functions. The focus in this work has been on testing and evaluating models in the context of inverse analysis. There are a number of more efficient candidate algorithms available for finding local and global minima of functions. However, the choice of optimization method here emphasizes ease of implementation, robustness and stability rather than computational efficiency.

The inverse problem is solved by a set of customized subprograms: analyser programs, interface processors and a connecting network. The analyser programs are the FE program with a user defined material subroutine and the optimization software. Interface processors are in-house programs for setting up the correct input format for each analyser program. The connecting network is in this case the Linux operating system, which can be programmed through shell programs to control the data flow and to manage the necessary data communication. Figure 3 shows a schematic structure for the optimization process.

3.3. Sensitivity and parameter confidence interval estimation

In this section a procedure for sensitivity and confidence interval estimation is introduced for the different model parameters. The results can be seen in section 4.1 for the estimated parameters with the friction coefficient 0.02. A more detailed description of the procedure can be seen in, e.g. [20, 21].

3.3.1. Sensitivity. An indicator of the quality of the parameter estimation can be obtained from the Gauss–Newton matrix, $\mathbf{G}$, which is a first order approximation of the Hessian matrix for φ_{tot} . The objective function for temperature interval No k can be written as

$$\varphi^k(\mathbf{D}) = \frac{1}{2} \sum_{l=1}^{N_k} (r_{1l}^2(\mathbf{D}) + r_{2l}^2(\mathbf{D})) = \frac{1}{2} (\mathbf{r}_1^T(\mathbf{D})\mathbf{r}_1(\mathbf{D}) + \mathbf{r}_2^T(\mathbf{D})\mathbf{r}_2(\mathbf{D})), \quad (8)$$

where

$$r_{1l}(\mathbf{D}) = (f_l^{\text{exp}} - f_l^{\text{fem}})w_{1l} \quad (9)$$

Figure 3. Basic flowchart of the inverse problem.

and

$$r_{2l}(\mathbf{D}) = (d_l^{\text{exp}} - d_l^{\text{fem}})w_{2l}. \quad (10)$$

In equation (9) and (10), $w_{1l} = 1/f_l^{\text{exp}}$ and $w_{2l} = 1/d_l^{\text{exp}}$ are the weight factors given in equation (7). The nonlinear objective function is then approximated by a second order Taylor series:

$$\varphi^k(\mathbf{D} + \Delta\mathbf{D}) \approx \varphi^k(\mathbf{D}) + \frac{\partial\varphi^k(\mathbf{D})}{\partial\mathbf{D}} \Delta\mathbf{D} + \frac{1}{2} \Delta\mathbf{D}^T \frac{\partial^2\varphi^k}{\partial\mathbf{D}^2} \Delta\mathbf{D}. \quad (11)$$

In equation (11) the term $\partial\varphi^k/\partial\mathbf{D} = \nabla\varphi^k$ is the gradient and $\partial^2\varphi^k/\partial\mathbf{D}^2 = \nabla^2\varphi^k = \mathbf{H}$ is the Hessian. Differentiation of equation (8) gives

$$\frac{\partial\varphi^k(\mathbf{D})}{\partial\mathbf{D}} = \mathbf{J}_1^T(\mathbf{D})\mathbf{r}_1 + \mathbf{J}_2^T(\mathbf{D})\mathbf{r}_2, \quad (12)$$

where $\mathbf{J}$ is the Jacobian matrix that holds the first order partial derivatives. Finally differentiation of the gradient, equation (12), gives the Hessian

$$\mathbf{H} = \mathbf{J}_1^T(\mathbf{D})\mathbf{J}_1(\mathbf{D}) + \mathbf{J}_2^T(\mathbf{D})\mathbf{J}_2(\mathbf{D}) + \mathbf{r}_1^T(\mathbf{D})\mathbf{F}_1(\mathbf{D}) + \mathbf{r}_2^T(\mathbf{D})\mathbf{F}_2(\mathbf{D}), \quad (13)$$

where the vector $\mathbf{F}$ contains the second order partial derivatives. Since $\mathbf{r}$ is minimized in the least square sense, it is often the case that the components of $\mathbf{r}$ are small. This makes a first order approximation of the Hessian matrix possible. This is equivalent to making a linear approximation to the residuals $\mathbf{r}$. In the Gauss-Newton method, the Hessian is approximated by the first order terms of equation (13) according to

$$\mathbf{H} \approx \mathbf{J}_1^T\mathbf{J}_1 + \mathbf{J}_2^T\mathbf{J}_2 = \mathbf{G}, \quad (14)$$

where $\mathbf{G}$ can be calculated as

$$G_{ij} = \left\langle \frac{\partial\varphi_{\text{tot}}}{\partial D_i}, \frac{\partial\varphi_{\text{tot}}}{\partial D_j} \right\rangle = \sum_{k=1}^3 \sum_{l=1}^{N_k} \left((w_{1l})^2 \frac{\partial f_l^{\text{fem}}}{\partial D_i} \frac{\partial f_l^{\text{fem}}}{\partial D_j} + (w_{2l})^2 \frac{\partial d_l^{\text{fem}}}{\partial D_i} \frac{\partial d_l^{\text{fem}}}{\partial D_j} \right)_k. \quad (15)$$

Here N_k is the number of measured points in temperature interval k . To approximate the sensitivity terms $\partial f_l^{\text{fem}} / \partial D_m$ and $\partial d_l^{\text{fem}} / \partial D_m$ the forward difference approach is applied as

$$\frac{\partial f_l^{\text{fem}}}{\partial D_m} \approx \frac{f_l^{\text{fem}}(D_m + \Delta D_m) - f_l^{\text{fem}}(D_m)}{\Delta D_m} \quad (16)$$

and

$$\frac{\partial r_l^{\text{fem}}}{\partial D_m} \approx \frac{d_l^{\text{fem}}(D_m + \Delta D_m) - d_l^{\text{fem}}(D_m)}{\Delta D_m}. \quad (17)$$

To select appropriate step sizes in equations (16) and (17), a method to determine ‘optimal’ steps has been used. The procedure is described in [22,23]. An estimation of the correlation between two derivatives can be calculated by

$$c_{ij} = \cos \left(\frac{\partial \varphi_{\text{tot}}}{\partial D_i}, \frac{\partial \varphi_{\text{tot}}}{\partial D_j} \right) = \frac{G_{ij}}{\sqrt{G_{ii}G_{jj}}}. \quad (18)$$

3.3.2. Confidence interval estimation. To estimate confidence intervals for the individual material parameters, the following expressions, see [21], can be used:

$$I = (D_i^{\text{opt}} - t\sqrt{V_{ii}}, D_i^{\text{opt}} + t\sqrt{V_{ii}}), \quad (19)$$

where the variance can be calculated as

$$\mathbf{V} = \frac{[\mathbf{G}(\mathbf{D}^{\text{opt}})]^{-1} \varphi_{\text{tot}}(\mathbf{D}^{\text{opt}})}{N_{\text{tot}} - M} \quad (20)$$

and $\mathbf{G}(\mathbf{D}^{\text{opt}})$ is the Gauss-Newton matrix calculated in the neighbourhood of the optimal model parameters. N_{tot} is the total number of points in the measured data and t depends on the confidence level, a 95% confidence interval gives $t = 1.960$.

3.4. Validation of estimated stress–strain data

To evaluate the obtained data from the optimization procedure, a separate forming experiment has been carried out. In this section, the experiment and corresponding simulation model are described. Section 4.3.1 summarizes the outcome from the validation.

3.4.1. Validation experiment. The principle of the test is to hot-form a simple hat section with a conventional tool, according to figure 4, from a boron steel strip. The tool material used is SS2242. Prior to forming, the steel strip is heated in an electrical furnace to approximately 930 °C and kept at this temperature for 300 s. Therefore, we can expect a fully austenitized structure. Figure 5 shows the dimensions of the steel strip. The forming test has been carried out in a Roell Amsler HBS500 test machine. During the test the upper tool was given a constant velocity of 10 mm s⁻¹. The process time, forming force, temperature and displacement were recorded during the test. The total time to finish the forming operation was 5 s. Only the force in the actual deformation step is of interest (but not the force when the upper tool hits the bottom and the final cooling and hardening take place). The temperature was measured at several points in the upper/lower tool and at one point on the steel strip edge as illustrated in figure 5.

Figure 4. Forming tool that gives a hat section with a profile height of 0.053 m, all dimensions in metres.

Figure 5. Steel strip used in the experiment, dimensions in metres.

3.4.2. Simulation model. The coupled thermal-mechanical simulation model for LS-Dyna 970 consists of 2746 nodes, 112 shell and 1558 brick elements. The boron steel strip is modelled using four-node Hughes–Liu shell elements, see [11], with 9 through thickness integration points. For the thermal calculations of the steel strip, the shells are treated as twelve-node brick elements to allow heat conduction through the thickness, see [1] for more details. The tool parts are modelled with eight-node brick elements. Only a quarter of the real set-up is modelled due to symmetry. The initial temperature in the steel strip is 827 °C, according to the experiment, and assumed homogeneous. The average temperature, 26 °C, from the experiment measurements is used as a homogeneous initial temperature for the tool. A heat transfer coefficient of 6500 W m⁻² K⁻¹ for the tool-sheet interface is used. The gap on each side of the shell element, when thermal contact is considered, is set to 6.0×10^{-5} m. Radiation heat exchange is considered between the surfaces of the strip and its surroundings. The surrounding temperature is assumed to be 30 °C, and the view factor of the strip is set to 1. An emissivity of 0.6 corresponding to oxidized steel is used. The static and dynamic friction coefficient used between the tool and boron sheet is assumed to be 0.30. The actual forming time is 5 s. A speed-up factor of 50 is used, which gives a simulated time of 0.01 s. Instead of scaling the thermal conductivity, the heat transfer coefficient and the radiation factor, the specific heat for the tool and boron steel is reduced by a factor 1/50 to compensate for the speed-up.

4. Results

When studying the specimens after the test, it can be seen that the bulging effect is quite moderate, indicating that the friction coefficient between the specimen and the foils is low. This is especially prominent for the compression test executed at the lowest temperature interval. For the Nemat-Nasser model, different constant friction coefficients have been tested, and in

Figure 6. Objective function value for different friction coefficients.

one optimization run the friction coefficient has been included as a design variable. Friction between the tungsten carbide anvils, tantalum foil, graphite foil and the boron specimen in the vacuum chamber at elevated temperatures is not readily obtained. The static and dynamical frictional coefficients are nonlinear relations, dependent on relative sliding velocity, contact pressure, temperature, atmosphere and the nature of the surfaces. In experiments performed by Boyd and Robertson [24] at high pressures (up to 2750 MPa) and with solid material as lubricants, very low friction coefficients are found. When using graphite as a lubricant [24], a friction coefficient in the interval 0.036–0.058 was obtained. During the Gleeble tests the mean pressure on the specimens contact surfaces reaches 1100 MPa. Normally at room temperature and normal atmosphere, the friction coefficient in steel to graphite contact is near 0.1, according to Bowden and Tabor [25].

In the following sections the terms *high*, *med* and *low* are used in diagram legends to characterize the different temperature intervals as defined in section 2. In the FE-simulation of the Gleeble compression test, the compression time is limited to 5 s (stop temperature of 389 °C) for the *low* temperature interval to avoid the influence of martensite formation during the cooling/compression cycle.

4.1. Nemat-Nasser model

Five different optimization runs with constant coefficients of friction (0.00, 0.04, 0.08, 0.12 and 0.16) have been conducted. The objective function indicates decreasing values when reducing the friction coefficient. Therefore, an optimization run with the friction coefficient included as an additional design variable has been conducted, resulting in a minima for $\mu = 0.02$. Figure 6 shows the objective function value as a function of the friction coefficient.

The friction and material parameters are not independent in the simulation; a variation in the friction coefficient gives a new set of optimal material parameters (and different ‘bulging’) and therefore, different states of strain and stress within the specimen. The maximum effective plastic strain for the friction coefficients 0.02, 0.08 and 0.16 with corresponding optimized parameters are 0.520, 0.616 and 0.720, respectively.

The obtained compression forces and diametric increase of the specimen for different friction coefficients together with experimental curves are plotted in figures 7–10. The estimated material parameters are specified in table 2. The effect of the friction variations on the obtained stress–strain curves is illustrated in figure 11 for the temperatures 500 and 800 °C.

Figure 7. Measured and calculated compression forces for the friction coefficients (a) $\mu = 0.02$, (b) $\mu = 0.08$ and (c) $\mu = 0.12$ used in the simulations.

Figure 8. Measured and calculated diametric increase— $\mu = 0.02$.

Figure 9. Measured and calculated diametric increase— $\mu = 0.08$.

Table 3 summarizes the calculated standard deviations of measured and calculated data for the different temperature intervals and friction coefficients. For the actual case, the covariance matrix, $\mathbf{c}$, becomes

$$\mathbf{c} = \begin{pmatrix} 1 & -0.95711 & -0.95801 & 0.98106 & -0.99020 & 0.96436 & 0.81035 & -0.62425 \\ & 1 & 0.99996 & -0.91437 & 0.98004 & -0.99573 & -0.92460 & 0.75872 \\ & & 1 & -0.91674 & 0.98063 & -0.99546 & -0.92233 & 0.75407 \\ & & & 1 & -0.95866 & 0.92107 & 0.73500 & -0.49784 \\ & & & & 1 & -0.97815 & -0.84660 & 0.67198 \\ & & & & \text{sym} & 1 & 0.92505 & -0.75651 \\ & & & & & & 1 & -0.91169 \\ & & & & & & & 1 \end{pmatrix}. \quad (21)$$

Figure 10. Measured and calculated diametric increase— $\mu = 0.12$.

Table 2. Estimated parameters for the Nemat-Nasser model.

μ	0.00	0.02	0.04	0.08	0.12	0.16
$D_1 = \sigma^0 10^2$ [MPa]	8.445	8.885	8.396	7.483	7.760	6.818
$D_2 = k/G_0 10^{-5}$ [K $^{-1}$]	1.406	1.267	1.476	1.454	1.524	1.540
$D_3 = \tilde{\varepsilon}_0 10^{10}$ [s $^{-1}$]	2.863	0.470	3.047	2.487	2.154	3.472
$D_4 = a_0$	2.405	2.310	2.622	3.059	3.029	4.028
$D_5 = q 10^{-1}$	7.598	8.008	8.464	7.829	7.714	8.049
$D_6 = p 10^{-2}$	7.690	6.839	10.150	8.002	7.735	8.357
$D_7 = \sigma_a^0 10^2$ [MPa]	1.574	1.375	1.478	1.916	2.325	2.482
$D_8 = n_1 10^{-1}$	1.509	1.370	1.594	2.070	2.605	2.827

Figure 11. Friction influence on obtained stress–strain curves for the temperatures 500 and 800 °C.

When an off-diagonal element (the cosines between two sensitivity vectors) is close to ± 1 , $\mathbf{G}$ is ill conditioned and the confidence intervals become wide. Then the parameters in question are linearly dependent and may be expressed in terms of each other, which means that the influence of two parameters on the data is similar. The three vector combinations that are most

Table 3. Standard deviations—calculated to measured data for different friction coefficients.

μ	Temperature interval					
	High		Medium		Low	
	s_1^f [N]	s_1^d 10^{-5} [m]	s_2^f [N]	s_2^d 10^{-5} [m]	s_3^f [N]	s_3^d 10^{-5} [m]
0.00	267	5.25	209	9.83	158	1.32
0.02	279	3.57	196	8.22	157	1.33
0.04	329	2.21	199	6.83	147	1.95
0.08	387	1.39	252	4.45	146	3.81
0.12	463	2.87	297	2.64	222	5.61
0.16	534	4.38	356	1.70	324	6.89

Table 4. Estimated parameter confidence intervals (95%).

$D_1 \in (737.0, 1040.0)$
$D_2 \in (-2.763 \times 10^{-7}, 2.561 \times 10^{-5})$
$D_3 \in (-1.097 \times 10^{11}, 1.191 \times 10^{11})$
$D_4 \in (1.842, 2.779)$
$D_5 \in (0.775, 0.826)$
$D_6 \in (0.063, 0.074)$
$D_7 \in (122.0, 153.1)$
$D_8 \in (0.113, 0.162)$

closely correlated (close to ± 1) are

$$c_{23} = \cos\left(\frac{\partial \varphi_{\text{tot}}}{\partial D_2}, \frac{\partial \varphi_{\text{tot}}}{\partial D_3}\right) = 0.999\,96,$$

$$c_{26} = \cos\left(\frac{\partial \varphi_{\text{tot}}}{\partial D_2}, \frac{\partial \varphi_{\text{tot}}}{\partial D_6}\right) = -0.99\,573,$$

and

$$c_{36} = \cos\left(\frac{\partial \varphi_{\text{tot}}}{\partial D_3}, \frac{\partial \varphi_{\text{tot}}}{\partial D_6}\right) = -0.99\,546,$$

where $D_2 = k/G_0$, $D_3 = \dot{\epsilon}_0$ and $D_6 = p$ according to table 2. Table 4 shows the estimated parameter intervals.

4.2. Temperature distribution in the cylindrical specimen

Coupled thermal-mechanical simulations have been conducted to verify the validity of using a homogeneous temperature assumption. This is shown to only affect the obtained mechanical response marginally as can be seen in figure 12, making the assumption valid. In these FE simulations the heat transfer coefficient between the anvil and specimen in combination with the temperature of the anvil is adjusted to obtain a temperature drop close to that measured in the experiments. The maximum temperature difference within the specimen for this case is about 30 °C after 0.5 s and decreases to 10 °C at the end of the simulation. Similar temperature differences for the other temperature intervals were obtained. Note that the 30 °C difference is between the contact interface (top/bottom surface) and the measurement point. At a depth of 0.4 mm beneath the contact interface, the maximum temperature difference to the measured temperature is 17 °C, decreasing to 7 °C at the time 6 s. At quarter height, the maximum temperature difference is 7 °C and is decreasing down to 2 °C at the time 6 s. A temperature

Figure 12. Simulations with and without homogeneous temperature in the specimen; (a) diametric increase of specimen mid (increase from initial diameter), (b) compression force.

decrease of 7°C give a rise in flow stress of 1.7% at 900°C and at 600°C , the corresponding rise is 2% (using equation (1) with optimized parameters). A homogeneous temperature assumption can therefore be further justified, because a very limited material volume is deviating in temperature more than 7°C throughout the experiment. If coupled simulations were applied in the inverse modelling, the obtained temperature history response at the specimen mid-height is not guaranteed to follow the measured, therefore we may introduce errors greater than the above estimated.

4.3. Strain rate effects

In this section results from Gleeble compression experiments at higher strain rates are presented together with corresponding FE-simulations. The compression start temperatures correspond to those used for the reference strain rate 0.08 s^{-1} defined in previous sections. The natural axial strain rates used are 0.5 and 5.0 s^{-1} . In corresponding simulations, the Nemat-Nasser model was used to generate the stress-strain data, based on the estimated parameters. The experimentally measured temperature on each tested specimen is used in the simulation as a prescribed temperature. For the strain rate 0.5 s^{-1} the mean measured cooling rate is $50\text{--}57^{\circ}\text{C s}^{-1}$. It was not possible to cool the specimens at the strain rate 5.0 s^{-1} . Here an increase in temperature of $1\text{--}22^{\circ}\text{C}$ as observed (the highest temperature increase for the lowest start temperature). The rise in temperature is due to plastic deformation. When the rate of heat generation is greater than the heat loss to the anvils and the surroundings, the temperature in the material increases. The material model parameters have been estimated from an experiment using a constant strain rate of 0.08 s^{-1} . This might be unsatisfactory from a completeness point of view. Nevertheless, the model seems to be valid when tested using a forming simulation case, see figure 13. The outcome of the test shows that the model with the optimized parameter set can predict the response for strain rates up to 5 s^{-1} with good accuracy.

4.3.1. Validation results. Figure 14 shows the measured and calculated temperature histories in the boron steel strip. At the measurement point, the temperature drop was almost 90°C during the forming operation, leading to a mean cooling rate of $18^{\circ}\text{C s}^{-1}$. The maximum mean cooling rate within the sheet reached $63^{\circ}\text{C s}^{-1}$. Note also that the thermal contact during the forming process was one-sided except when the upper tool approaches its lowest level. Computed and

Figure 13. Measured and calculated compression forces at the true axial strain rate of (a) 0.5 s^{-1} and (b) 5.0 s^{-1} .

Figure 14. Measured and calculated temperature histories on the boron sheet.

measured forming forces are plotted in figure 15. Note that the global force response is in good agreement with the experimental for the simulation with stress–strain data generated by the optimized parameters for the Nemat-Nasser model (legend NN). For comparison purposes, a simulation with material data obtained from conventional isothermal tension tests is added to figure 15 (legend TD). The obtained forming force in the simulation using tension test data, is at least 20% lower than the response obtained using the material data from the proposed method. The maximum effective plastic strain in the simulation reached 0.24 slightly below the upper radius. Maximum effective plastic strain at the upper and lower radius reached 0.2 and 0.1, respectively.

5. Discussion and conclusions

In the current paper a methodology for estimation of material parameters for a boron steel grade using thermo-mechanical compression experiments and inverse modelling is presented. Simultaneous cooling and compression experiments are performed instead of traditional isothermal testing. Three different starting temperatures have been considered. The strategy

Figure 15. Measured and calculated forming forces in the validation test case (NN—stress strain data from optimization, Nemat-Nasser model; TD—stress strain data from iso-thermal tension tests).

has been to use three overlapping temperature intervals in order to give additionally useful information about the temperature dependent mechanical response.

The objective function is a least square functional, containing the measured reaction forces at the anvils and the mid-specimen diameter change. The choice of a least square criteria has been justified on the basis of the central limit theorem, thus assuming that the sum of several different contributions will tend to be normally distributed, irrespective of the probability of the individual contributions. The least square approach also provides an attractive way of computing the confidence with which the parameters are determined. Note that with the here assumed degree of confidence, parameters D_2 and D_3 may take negative values according to table 4. This is, of course, not physically realistic and would corrupt the material model. Nevertheless, it is in agreement with the supposed error distribution. An alternative solution would be to use a non-symmetric distribution instead. In practice this problem is managed by applying physically sound constraints on each parameter. The purpose of the current confidence study is to estimate and picture the uncertainty in the model parameters based on normally distributed errors, not to assert the validity of their values in the interval. Parameter values shall always be examined for their physical soundness before use in simulation models.

One difficulty of the presented method is the influence of friction between the specimen and the anvils during compression. The constant Coulomb friction model used shows that a reduction of the friction coefficient tends to reduce the slope of the flow stress curve. The maximum shift in yield stress is about 40 MPa for a change in the friction coefficient from 0.04 to 0.12 at 500 °C. The corresponding stress shift is 22 MPa at 800 °C. To further improve the proposed method to account for the nonlinear friction dependence on temperature, a similar methodology as described in [26] and shortly summarized below may be employed. In [26] two alternative approaches to estimate the friction coefficient during compression of hollow cylinders (rings) as function of the temperature are presented. In the first approach, the flow stress description is assumed to be independent of the friction coefficient. Then the objective function is formulated as the difference (simulation to experiment) in diameter at several locations on the inner and outer surface of the ring after the experiment to estimate the average of the friction coefficient. The procedure is repeated for three constant temperatures, 800, 900 and 1000 °C. From these results, a linear function is fitted to describe the friction factor as a function of temperature. In approach two, both the difference in measured and calculated compression force and the difference in final shape of the ring are included in the objective

function. Both the constant in the linear friction model and several material parameters are then included as design variables in the inverse modelling. This last approach is showed to further improve the simulated response.

An interesting extension of the proposed model would be to find a suitable method to continuously measure the specimen diameter at many points along the specimen during compression, not only at the mid-height section as in the current work. It is unclear if the friction coefficient in the current work would be accurately represented by a linear function of temperature due to the wide temperature span or if some more complicated friction law need to be employed.

The uniqueness of the solution has been tested by changing initial values of the parameters. Different minima are found, i.e. the final parameter values change slightly when different initial values are used. However, the parameters reach realistic values and the final objective function value changes negligibly. An explanation to this behaviour can be based on the correlation analysis, which reveals that the optimization problem to some extent is ill conditioned.

With the proposed methods, it is shown that the evaluation of material properties can be performed for conditions where traditional experiments and evaluations would be practically impossible to accomplish.

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