Automated Online Measurement of Limestone Particle Size Distributions using 3D Range Data

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Abstract

Fully automated online measurement of the size distribution of limestone particles on conveyor belt is presented based on 3D range data collected every minute during 13 hours of production. The research establishes the necessary measurement technology to facilitate automatic control of rock crushing or particle agglomeration processes to improve both energy efficiency and product quality. 3D data from laser triangulation is used to provide high resolution data of the surface of the stream of rocks. The 3D data is unaffected by color variation in the material and is not susceptible to scale or perspective distortion common in 2D imaging. Techniques are presented covering; sizing of particles, determination of non-overlapped and overlapped particles, and mapping of sizing results to distributions comparable to sieving. Detailed variations in the product sieve-size are shown with an abrupt change when the size range of the limestone particles was changed.

Keywords: image segmentation, particle size measurement, range data, classification, particle delineation

1. Introduction

In the mining and aggregate industries a great deal of effort goes into measuring or estimating the size distribution of particulate material. One reason is that suppliers of particulate material are typically paid to supply a specific size range of material. For both industries there is also a key desire for energy efficiency and size quality that is relevant throughout the mining process in blasting, crushing, and aggregating processes.

Processing plants want to control their value-added products such as baked limestone and iron ore pellets. Fast feedback of the product size range would allow for control optimisations to achieve both energy efficiency and an optimized product size range.

Mine and quarry operators want to measure the particle sizing results of all of these activities but sieving is typically impractical as a routine assessment tool due to slow feedback, inconsistent measurement, and time consuming interruption.

As a result there is an opportunity for online, non-contact, fully automated machine vision systems for measurement of particle size that can provide the necessary accuracy and fast feedback to facilitate process control and allow automatic control optimisations for both product size

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Preprint submitted to Process Control

November 15, 2010

and energy efficiency. This is relevant to a vast range of processes that modify particle size such as rock blasting, crushing or agglomeration of particles, and baking processes in ovens and kilns.

There are however, a number of sources of error relevant to techniques that measure only what is visible on the surface of a pile as follows;

Segregation and grouping error, more generally known as the brazil nut effect [1], describes the tendency of the pile to separate into groups of similarly sized particles. It is caused by vibration or motion (for example as rocks are transported by truck or conveyor) with large particles being moved to the surface. It is advisable to measure at a point early on the conveyor before the material has been subjected to excessive vibration and segregation.

Overlapped particle error, describes the fact that many particles are overlapped (see figure 1) and only partially visible and a large bias to the smaller size classes results if they are treated as small non-overlapped and sized using only their visible profile. This error can be overcome in piles of particulate material using classification algorithms based on 3D range data [2] successfully providing 82% classification accuracy [3].

Figure 1: Illustration of overlapped and non-overlapped particles.

Capturing error, describes the varying probability based on size, that a particle will appear on the surface of the pile. In simple terms, the larger a particle is, the more likely one is to be able to see some part of it on the surface. For example, if a single particle is as large as the height of the pile of material, then it will always be visible, whereas a very fine particle is almost certainly not visible. Thurley [4] has explored capturing error in laboratory rock piles but it remains a source of error in this application.

Profile error, describes the fact that only one side (a profile) of an entirely visible particle can be seen making it difficult to estimate the particles size. However, if the particle is not overlapped, best-fit-rectangle [5] has been demonstrated as a suitable feature for size classification based on the visible profile, that correlates to sieve-size.

In addition to these errors, we note that size measurement using imaging identifies how many particles are observed of various size classes, but manual sieving measures the weight of particles in each size class. Therefore it is necessary to have a method of mapping from numbers of

particles to weight of particles in order to provide a measurement of size that industry understands and can use. A technique based on physical sampling and sieving results is used in this work and explained in detail subsequently.

Particle size measurement using vision has been the subject of research and development for over 25 years [6] with a legacy of predominantly photographic based systems with widely varying degrees of success and no general solution available on the market.

Photographic based 2D imaging systems are subject to bias due to uneven lighting conditions, excessive shadowing, color and texture variation in the material, and lack of distinction between overlapped and non-overlapped particles.

In their review of a commercial photographic based 2D system Potts and Ouchterlony [7, pg. vi, viii] report that for their application the system erroneously assumes the resultant size distribution is unimodal and they conclude by expressing strong reservations saying 2D “imaging has a certain but limited usefulness when measuring the fragment size distribution in a muckpile or from a belt in an accurate way. It could probably detect rough tendencies in fragmentation variations, if the lighting conditions do not vary too much, and if cover glasses for camera lenses are kept clean”.

There are a number of publications relating to 3D size measurement, Noy [8, rocks], Frydendal and Jones [9, sugar beets], Kim et al. [10, river rock] Lee et al. [11]. However, Frydendal Frydendal and Jones [9], and the presenting author [2] are the only publications (2D or 3D) to remove the bias resulting from overlapped particles. For conveyor belt applications some publications recommend installing a mechanical vibration feeder [10, 11] to separate rocks and prevent particle overlap. If there exists the possibility to install a vibration feeder at full-scale in the process it would simplify image analysis, and may improve results but with the added cost and maintenance of the additional equipment.

We note also that it is possible to automatically sample particulate material and separate the material into a single layer for imaging, however this requires additional material handling and additional equipment such as conveyors and vibrating feeders. The focus of this research is on systems for on-line measurement of the particle flow that are non-contact, that is they do not require any additional material handling. If non-contact measurement is required then one must consider overlapped particle error and account for overlapped and non-overlapped particles. Furthermore, in some circumstances, such as examination of rocks in in-production excavators [12], there is no other option than to account for overlapped and non-overlapped particles.

Frydendal [9] used graph theory and average region height to determine the entirely visible sugar beets but this relied on the regular shape and size of the beets. Only the presenting author has made this distinction between overlapped and non-overlapped particles using the advantages of 3D range data and in a manner that does not presume constraints on size or shape [2].

We use an industrial measurement system on conveyor belt based on laser triangulation (a projected laser line and camera at an offset angle) collecting highly accurate 3D profiles of the laser line at about 3000 Hz. This high speed ensures we have a high density of 3D point data at a spacing between consecutive points in the direction of the belt of approximately 1 mm as the belt is running at 3 m/s. The imaging system is installed at a limestone quarry on the conveyor belt used for ship loading and measures the material on the belt during loading every minute. Figure 2 shows some images of the installed laser and camera system.

Using the same laser triangulation technology it is also possible to collect measurement points at 0.1 mm spatial resolution and 0.005 mm depth resolution as we have done on steel slabs (not yet published). Using such a setup it should be possible to confidently measure particles with a lower size limit of 1 mm and be able to detect overlapped and non-overlapped particles on

Figure 2: Installation of the measurement hardware above the conveyor belt.

conveyor belt. An equivalent reduction in measurement speed (one tenth conveyor belt speed) is necessary such as 0.3 m/s but it is theoretically possible to improve this using higher power lasers (beyond class 3B) but we have not tested this.

The presented analysis algorithms in this paper are not dependant on the laser triangulation measurement technology. Any other technique for capturing 3D surface data of a particle pile, such as stereo photogrammetry or time-of-flight 3D cameras could also be used.

The computational speed of the analysis process is approximately 53 seconds on a 2 GHz Mobile Pentium 4 processing a data set of 590,000+ 3D points (2 m long section of the belt). Furthermore, multi-core CPUs offer an almost linear increase in the rate at which data could be sampled from the conveyor and processed. Further advances in both algorithmic efficiency and hardware are both available to improve computational time as the presented morphological image processing operations are readily parallelisable. Computation time could be reduced down to the order of few seconds or less for rapid automatic control applications. Once the analysis can be performed in under a second then it will become possible to measure and analyse the entire surface of the material passing under the measurement system.

There are a number of factors to consider regarding frequency of measurement and the length of the measurement sample. Computational speed is linearly dependant on the number of measured 3D points and therefore defines how fast measurement samples can be analysed, but it is also necessary to consider what is the largest particle size that should be measured and how many of these particles need to be measured to get a statistically valid measurement size. Given the circumstances in this application the 2 m long measurement sample is more than sufficient given the material top size of approximately 90 to 100 mm. Furthermore, the measurement time of under a minute is again sufficient given that ship loading takes place over several hours and it is sufficient to detect and confirm particles out of the expected size range within several minutes.

2. Research Background

The presented research builds upon a series of achievements and research developed on both laboratory rock piles and industrial application.

We have previously implemented an industrial measurement system on conveyor belt for iron ore pellets [13] using the same laser triangulation measurement technology. The high speed camera system ensures we have a high density of 3D point data at a spacing between consecutive points in the direction of the belt of approximately 0.5 mm. This high data density has at least two advantages. Firstly it allows us to detect small sliver regions or crescent-like regions of overlapped particles and ensure that they are not merged into other regions. And secondly, it has ensured that we could detect a very high resolution when it came to measuring the size of each iron ore pellet allowing a size distribution with very fine spacing of 5, 8, 9, 10, 11, 12.5, 13, 14, and 16+ mm size classes.

One of the key criteria for particle size measurement is therefore high data density as it defines the capacity to detect small overlapped particles, the lower limit on particle size that can be reliably detected, and the resolution of size classes detectable.

Iron ore pellet production is a process with cyclic feedback and surging. Changes in the rotational speed of the pelletising disk tend to be observed on the conveyor belt after 5–10 minutes, so measurement every minute for the pellet system [13] is more than adequate.

In addition we have performed a demonstration project for size measurement of rocks in underground LHD excavator buckets [12]. A 3D vision system based on laser scanners was installed on the tunnel roof in a production area of an underground iron ore mine with 3D surface data of the bucket contents being collected as the LHD unit passes beneath. The project successfully demonstrated fragmentation measurement of the rocks in the bucket, identifying overlapped

rocks, non-overlapped rocks, areas of fine material, estimating the sieve-size of the visible rocks only, and the calculating the proportion of the surface that was identified as fine particles below the observable resolution of the laser scanner.

3. Analysis of 3D Range Data

In the presented application for limestone particles the lower limit of particle size is 10 mm and 3D data points are collected at a spatial resolution of 1 mm.

The following analysis strategies apply morphological image processing techniques. These techniques are particularly useful for identifying spatial structure in image data. Morphological image processing operations typically apply a structuring element, which is a spatial probe similar to a neighbourhood operation, to the image. The reader is referred to the text Hands-On Morphological Image Processing [14] for a very clear explanation of this topic, or to any other good text on image processing if they require more information on these morphological image processing operations.

3.1. Segmentation

The first step is to perform the detailed image segmentation to identify the individual rock particles. These techniques have been applied to laboratory rock piles [15] and in an industrial pellet measurement system [13] and were adapted to this application. The technique is predominantly based on morphological image processing [14], based largely on various edge detection techniques to facilitate seed formation for the watershed segmentation algorithm [14, watershed transform].

Figure 3 shows a closeup image of rocks on the conveyor. The raw 3D data comprises points spaced 1 mm apart along the rows, and irregularly spaced (approx 1 mm spacing) down the columns. Therefore the data has been resampled down the columns so that it has a 1 mm spacing.

Figure 3: Rocks on conveyor, limestone aggregate, 40–70 mm product.

A key task of the automated segmentation is to find good edges for the particles from which seed regions can be constructed for use in the watershed segmentation. A strategy that uses two sources of edge information is presented where careful attention is applied to reducing the presence of spurious edge information.

3.1.1. Edge Detection

The first source of edges result from occlusions in the 3D data. One of the advantages of a measurement system using two points of view, in this case a laser and a camera, is that occlusions are good indications of edges. Occlusions are areas of missing data where the camera cannot see the laser line as it is occluded from view by a rock. This occurs at the edges of rocks.

However, not all areas of missing data indicate an edge, particularly if the data is noisy. Therefore we find occlusions in the following way with the result shown in figure 4.

- Identify all points in the data next to an occluded (missing) data point
- Ignore small occlusions - If the occlusion indicates the edge of a rock then it will likely be a reasonably long occlusion comprising many consecutive missing points. We therefore ignore points that are next to gaps of 5 mm or less (5 data points along the rows, or along the columns).
- Fill in discontinuities - The result now comprises a network of lines of data points at the edge of occluded areas. We fill in single data point discontinuities in the lines by performing a morphological closing [14] using a square structuring element of side length 3 mm (3 data points).
- Remove isolated points - Remove data points with only 1 other neighbour in an 8 connected neighbourhood.
- Reconstruct connected components - Perform a morphological reconstruction [14] using a square of side length 3 mm (3 data points) to restore connected components that were removed by the *Remove isolated points* step.

Figure 4: Filtered Occluded Edges

The second edge detection strategy is a morphological edge detection strategy to detect edges in the structure of the data. Again careful attention is applied to obtaining edges without noise and spurious edge points in the middle of the rock particles. We find edges in the following way with the result shown in figure 5.

- Smoothing - Perform a morphological smoothing, a morphological closing followed by a morphological opening [14], on the rock data using a square structuring element of side length 3 mm (3 data points).
- Edge detection - Perform a morphological black-top-hat [14] using a circular structuring element of diameter 7 mm (7 data points).

- Threshold - Retain only the strongest edges by removing all edges less than a threshold of 0.42 mm. This is a conservative small threshold so many spurious edges remain, but it is sufficient to remove many weak and therefore insignificant edges.
- Remove spurious edges - Perform a morphological opening [14] with a plus shaped structuring element (an origin and its 4 connected neighborhood).
- Reconstruct connected components - Perform a morphological reconstruction [14] using a square of side length 3 mm (3 data points) to restore connected components that were removed by the *Remove spurious edges* step.

Figure 5: Filtered Black-Top-Hat Edges

Combining these two edge detection strategies yields the edge image shown in figure 6 that we use in subsequent steps.

Figure 6: Combined Edges: Filtered Black-Top-Hat Edges added to Filtered Occluded Edges

3.1.2. Seed Formation

In this section we use the edge information to construct seed regions that we use in the watershed segmentation algorithm.

We first apply a geodesic distance [14] transform to the combined edge data to calculate the distance from each data point to the nearest edge. The geodesic distance transform is a morphological approximation to the distance transform which is substantially quicker. We use a circular structuring element of diameter 5 mm with the results shown in figure 7.

The resultant distance measures are a multiple of the radius of the structuring element but this produces an overestimation largely due to the structuring elements discrete approximation to a perfect circle. Distances along a row or column can be measured quite accurately by this

approximation but diagonal distances are overestimated in a way similar to the *city-block distance* measure [16] where distance is measured as the sum of the distance along the rows and the distance along the columns in the data grid. For example, a point 10 rows and 10 columns away would have a city-block distance of 20, whereas the straight line distance is $10\sqrt{2}$. We therefore mitigate this overestimation by dividing the resultant distances by $\sqrt{2}$ to ensure that no over-estimations remain.

Figure 7: Geodesic Distance Transform: representing the distance to the nearest edge

We now apply a strategy to use the geodesic distance transform shown in figure 7 to build seed regions for the watershed segmentation. The resultant seeds are shown in figure 8.

- Find local maxima - We find the apex of each region in the geodesic distance transform in figure 7 by using the morphological *h*-maxima operation [14, 6.8 Image Extrema]. This comprises an intensity (z axis) subtraction of 1.5 mm followed by a morphological reconstruction using a square structuring element of side length 3.
- Construct seeds - Use the local maxima and the distance transform to construct seed regions from circles centered at the local maxima [15]. Figure 8 shows the resultant seed regions.

Figure 8: Seed Regions

3.1.3. Watershed Segmentation

The final step is to perform the watershed segmentation algorithm to detect the individual rock particles. The algorithm takes the seeds regions shown in figure 8 to define the initial segmentation. Additionally it requires a surface in which to perform the segmentation and for this we use the morphological gradient [14, 6.1] of the rock data shown in figure 3. Figure

9 shows a closeup image of rocks on the conveyor and the automated segmentation result co-located so that it is easier for the reader to visually evaluate the result.

Figure 9: Rocks on conveyor and automated segmentation for limestone aggregate, 40–70 mm size range.

3.2. Non-overlapped Particles

In order to explain the determination of overlapped and non-overlapped particles it is necessary to introduce a boundary following algorithm developed by Thurley and Ng [2] for the purpose of overlapped particle detection.

The algorithm begins with a segmentation of a pile of particulate material and examines each region in the segmentation. For each region in the segmentation we identify a series of prominent points equally spaced around the perimeter of the region.

Looking at figure 10 we see a representation of two regions (one overlapping the other) with a series of prominent points marked for region R_i .

We then use these points to define small local areas in which we examine the height of the regions in that local area. In the magnified section of figure 10 you can see a circular area about point g , we examine this area, denoted N_g and calculate the average height of all of the points in the region R_i that intersect area N_g , and we calculate the average height of all of the points in the region R_n that intersect region N_g . By comparing these two average height values we can evaluate whether in the local neighbourhood of point g , region R_i is either above or below region R_n . This calculation is used to determine overlapped/non-overlapped particles by counting the ratio of points g where region R_i is above region R_n .

This distinction allows us to overcome misclassification of overlapped rocks are if they were smaller non-overlapped rocks. Refer to Thurley and Ng [2] for an experimental validation of the algorithm.

To summarise, the visibility ratio is a measure of how much a particle overlaps or is overlapped by neighbouring particles and is calculated by analysing the boundary of each particle in the 3D point data.

An equivalent strategy to that applied on laboratory rock piles is used with the visibility ratio [2] as a classifier to determine overlapped and non-overlapped rocks. In this application a

Figure 10: Boundary following algorithm

threshold of 0.65 is used, values above this will indicate non-overlapped rocks, and values below this indicate overlapped rocks. This value is considerably more conservative than the value of 0.4583 used by [2] in their laboratory investigation and has been selected manually based on observation of the data with the aim of ensuring that overlapped rocks are not classified as non-overlapped rocks. Of course setting the threshold too high results in insufficient numbers of rocks classified as non-overlapped so a balance needs to be found. Figure 11 shows only the rocks that were classified as non-overlapped from figure 9.

We will perform sizing on the non-overlapped rocks for which we can make a confident size prediction, and we will ignore overlapped rocks.

Figure 11: Non-overlapped rocks automatically identified from figure 9, 40–70 mm size range.

3.3. Sizing

Sizing begins by calculating a feature called best-fit-rectangle area for each of the regions identified as non-overlapped particles from the segmentation (figure 11). The best-fit-rectangle is the rectangle of minimum area that fits around the 2D projection of each region. We ignore the height component of the points in a region and calculate an arbitrarily rotated best-fit-rectangle. We have used this feature previously [2, 13, laboratory rock piles, iron ore pellets] and demonstrated its suitability for estimating sieve-size of non-overlapped particles.

In order to classify best-fit-rectangle areas into sieve-size-classes however, we require a series of decision boundaries. In effect, thresholds that define which area values separate one size class

from another. To determine these values we use some sieving data that corresponds to some of our image analysis results.

During the measurement campaign a 3D data set of the surface of the rock stream was collected every minute during approximately 13 hours of loading. About 23% of the time material was not being loaded resulting in 606 3D data measurement sets of limestone on the conveyor belt being measured, and over 140,000 non-overlapped rocks being automatically identified and sized. In addition, the mining company obtained some manual samples from the conveyor using an automated sampler, and subsequently sieved these in the laboratory. This automated sampler lowers a scoop head-on into the moving flow of material to extract a section of material off the moving belt. Table 1 shows these sieve results. The result show that two distinct product ranges of material were being loaded at different times. Firstly the 40–70 mm product, then the 20–40 mm product, before changing back to the 40–70 mm. For a given sieving, most of the material is within the product size range, but there is always some small proportion above and below this range. Note also that unfortunately the configuration of sieve decks was changed between the two different products. This change significantly complicated the subsequent analysis. Additional trials will be conducted using a consistent higher resolution set of sieve decks (more decks with closer spacing).

Table 1 shows that at time 21:20, 5.83% of the material (by weight) is less than the 40 mm size class when the 40-70 mm product is being loaded. Similarly, at time 23:05 approximately 13% of the material (by weight) is larger than 40 mm when the 20-40 mm product is being loaded. The product name does not indicate the actual limits on the size of material, but the size of the majority of material in that product.

Table 1: Sieving results (cumulative %)						
Sieve Size (mm)	Product					
	40-70mm		20-40 mm		40-70 mm	
	21:20		23:05		00:15 06:50 09:45 11:40	
> 75	100	100	100	100	100	100
< 75	100	100	100	100	95.2	89.9
< 63	77.8	100	100	76.2	78.1	59.6
< 50	45.9	100	100	40.8	40.9	28.1
< 45	–	98.7	100	–	–	–
< 40	5.83	87.1	86.8	6.56	4.79	3.65
< 37.5	2.43	–	–	2.82	1.86	1.57
< 31	–	44.2	36.3	–	–	–
< 25	0	14.6	10.6	0	0	0
12–20	0	0.736	0.762	0	0	0

We select two data sets from each product range (times 23:05, 00:15, 09:45, and 11:40) and use the sieving data and corresponding image analysis results to calculate decision boundaries for mapping best-fit-rectangle area into sieve-size-class.

Nelder-Mead [17] iterative error minimisation process is used to calculate decision boundaries with minimised error. This optimisation method is available both in Matlab using function `fminsearch`, or in the statistical package R using function `optim` which was used here. Given an initial estimate of the decision boundaries, Nelder-Mead minimisation was used to produce a final set of boundaries.

The Nelder-Mead optimisation takes an estimate of the decision boundaries and calculates

the error at that estimate (we define this error function in more detail in the next section). The optimisation modifies the decision boundaries slightly following a range of internal rules before repeating the process. Changes that reduce the error are kept, while changes that increase the error are typically discarded. We first calculate optimum decision boundaries for each set of sieving data and then calculate the median values as the initial estimates for the Nelder-Mead and generate the error minimised result. The Nelder-Mead decision boundary values are shown in table 2.

Table 2: Decision boundaries (mm^2) to convert best-fit-rectangle area into sieve-size-classes

	Sieve Deck (mm)									
	75	63	50	45	40	37.5	31	25	20	12
Boundary (mm^2)	7074	4923	2535	1935	1701	999.8	630.8	347.9	252.1	69.69

Figure 12 shows the distribution of best-fit-rectangle (BFR) area measurements for the non-overlapped rocks calculated by the system for the two different products from all 606 3D measurements. The figure illustrates that when the smaller product was on the belt, smaller values of the BFR area are generally calculated. The figure also shows the location of the decision boundaries for each sieve-size-class on the bottom axis as calculated using the Nelder-Mead optimisation. These decision boundary intervals have been alternatively presented in white and grey shading to improve their presentation.

Figure 12: Best-fit-rectangle areas results for the Non-overlapped rocks by product size range.

3.4. Mapping from Number to Weight

One of the complications for image analysis systems is that they measure the number of observable particles, and using classification strategies for size can calculate number of particles in each size class. However, what is required is the estimated weight of the material in each size class as this percentage by weight is what is measured by sieving and required by industry. Therefore it is necessary to convert number of particles by size class into an estimated weight by size class. We do this by collecting sieving results and counting the number of particles in each size class in order to determine an average weight of a fragment by size class as shown in table 3.

Table 3: Mean Particle Weight by Size Class

Sieve Size (mm)	Mid Size (mm)	Mean Particle Weight (g)						$f(x)$ ★
		Time of Sieving						
		21:20	23:05	0:15	6:50	9:45	11:40	(g)
> 75	80	—	—	—	—	793	719	757
63–75	69	540	—	—	480	504	509	505
50–63	56.5	289	—	—	276	285	311	296
45–50	47.5	na	168	—	na	na	na	188
40–50	45	164	na	na	172	172	187	164
40–45	42.5	na	115	117	na	na	na	142
37.5–40	38.8	132	na	na	129	105	148	113
31–40	35.5	na	73.7	72.3	na	na	na	91.7
31–37.5	34.3	na	na	na	na	na	na	84.2
25–37.5	31.3	72.5	na	na	80.2	84.3	99.9	68.0
25–31	28	na	42.3	44.0	na	na	na	53.1
20–25	22.5	—	27.1	27.6	—	—	—	33.3
12–20	16	—	9.21	7.98	—	—	—	17.3

★ $f(x)$ is a polynomial line of best-fit for the weight data in equation 1, where x is the mid size of each sieve range

The immediate thing that one notes from table 3 is that different arrangements of sieve decks were used at different times (for the different products) complicating the capacity to compare and combine the results. To provide a consistent estimation of weight by size class a polynomial of best-fit for the sieve-size versus weight was calculated. This provides a continuous estimate of weight for any given size class that we wish to estimate with the imaging.

Based on the data in table 3 we calculate a third order polynomial where x is the mid-size of the sieve range and the weight is given by equation 1. The graph of table 3 and the best-fit polynomial is shown in figure 13.

$$f(x) = 0.001208x^3 + 0.01494x^2 + 0.5306x \text{ with correlation } R^2 = 0.9909 \quad (1)$$

Figure 13 uses a log scale on the weight axis so that variations in the estimated weights can also be seen at the lower end of the size classes. The figure illustrates the difference in the average weight of particles of similar sieve-size-class between the two products. This could be due to a number of factors, bias in the distribution of particle size within a given sieve-size-class, an artifact of the sampling and sieving of an already sieved product, or variations in material shape. For example, in the 40–70mm product the rocks in the 25–37.5 sieve-size-class might actually being closer to 37.5mm and therefore heavier than we expect at the mid point of the sieve size, and from the 20–40mm product the rocks in the 40–45 sieve-size-class might be closer to 40mm in size and therefore lighter than expected.

As the weights are clearly different there will be errors in estimating proportions by weight that match the two products as the estimates will be based on an intermediate weight value provided by the polynomial and not the actual weight from any given sieving.

The mismatch between measuring numbers of particles in imaging and comparing against sieve weights affects how we optimise the decision boundaries for converting imaging results

Figure 13: Average weight by sieve-size-class for both products

into sieve-size classes and a final result of sieve-size distribution by weight. As a result we use the weights calculated using the best-fit polynomial within the Nelder-Mead error minimisation. We express this error function (our cost function for the optimisation) as a sum of squares as shown in equation (2).

When considering a single product it is important for the optimisation function to prioritise heavier size classes as sieving results are percentage distributions by weight focusing on heavier size classes. In a percentage by weight size distribution large variations in the number of fine particles can be largely insignificant due to their small total percentage weight. However, when considering two different products, the question arises as to which individual size classes are most significant in the optimisation function. For the 40-70 mm product the significance of the 35-40 mm size class is quite low (its relatively light), but for the 20-40 mm product this size-class is of highly significant because it is one of the heaviest size classes in that product.

The cost function (equation 2) needs to accommodate this variation in the importance of a given size class and this is achieved using the normalised weight component $\bar{W}_{x,t}$. Size class x is given less significance in the cost function when it is relatively lighter at time t , and more significance when it is relatively heavy at another time. The normalized weight provides a weight of 1 for the heaviest size class at a given time t , with the lighter weights scaled relatively to this value.

Furthermore, as decision boundaries must be monotonically increasing with size class x (the larger the size class the larger the decision boundary), we must further constrain the error-function by using a penalization factor when the decision boundaries are out of order. Specifically we scale the total error by the amount that any consecutive decision boundary is out of order. Without this constraint the error minimization can generate meaningless out of order results.

$$\epsilon = \sum_t \left[\sum_{i=1}^N [(C_{x,t} - S_{x,t}) \widehat{W}_{x,t}]^2 \right] \quad (2)$$

$$\widehat{W}_{x,t} = \frac{f(x)}{\max_t(f)}$$

t	Time of day
x	Sieve-size-class
N	Number of sieve-size-classes
$C_{x,t}$	Calculated value of the size class x at time t
$S_{x,t}$	Sieving result for size class x at time t
$f(x)$	Weight of size class x using polynomial weight function
$\max_t(f)$	Weight of heaviest size class at time t
$\widehat{W}_{x,t}$	Normalized weight

Using the optimised Nelder-Mead decision boundaries for determining the sieve-size-class boundaries (depicted in figure 12) we produce estimates of the sieve-size-distributions for the 606 measurement sets. The results are presented in figure 15 with the sieve data from table 1 overlaid as colored points.

If we look only at the sieving results from times 0:15 and 6:50, and the two imaging measurements that correspond to these times we can plot a typical log-linear cumulative size distribution curve as shown in figure 14.

Figure 14 shows that the results at time 6:50 look reasonably good, whereas the results at 0:15 are shifted to the higher size classes, with proportions in the lower size classes being significantly underestimated.

4. Discussion

The imaging results (figure 15) are shown to clearly trend in the right direction tracking changes in the material size. Furthermore, in the higher size classes, 50, 63, 75, where decision boundaries could be set for the 40–70 mm product independantly of the 20–40 mm product, the imaging results appear to track the sieving results well.

If we examine figure 15 and compare it to the cumulative sieving results in table 1 we can see that the image analysis overestimates the amount of < 40 mm size class in the 40–70 mm product, by about 10%, and underestimates the cumulative amount in the 20–40 mm product by about 25%.

The poor result at time 00:15 in figure 14 is most significantly contributed to by the behaviour of the optimisation function. We have made an attempt to make the cost function in the optimisation sensitive to the importance of size classes of different weights, while reflecting the need to optimise against the final weight based result. However, it is clear that the 40-70 mm heavier product dominates the optimisation. Cost function calculations for the 20-40 mm product have no effect on the larger sizes from the 40-70mm product but cost function calculations for the 40-70mm product do have an impact on the larger sizes from the 20-40 mm product.

Additionally the Nelder-mead optimisation "can lead to inappropriate termination at non-optimal points" [18]. Starting conditions might also be a factor with the larger product badly

Figure 14: Imaging and sieving results for 2 measurements and sieve results at 0:15 and 6:50.

influencing starting conditions for the lower size classes, resulting in non-optimal termination of the Nelder-mead.

Particle weight is also another factor as the optimisation is using weights that in some cases are quite dissimilar to the actual sieve weights being compared against (as shown in table 3). The strategy presented uses an average particle weight by size class and this will work fine if the particles are always uniformly distributed across that size-class. However, an inconsistent set of sieve decks has been used to measure the two products and we have a mean particle weight that differs significantly where the products overlap as shown in figure 13. It seems likely, that we have a non-uniform distribution in some size-classes with larger particles in the 25–37.5 mm size class from the 40–70 mm product, and smaller particles in the 40–45 mm size-class from the 20–40 mm product. Improvements are expected with additional sieving experiments with a much finer stack of sieving meshes in order to better approximate particle weight.

There are a number of issues that require ongoing investigation and improvement;

- optimisation that does not inappropriately suppress lighter products
- optimisation that is less sensitive to starting criteria
- specification of the starting criteria for the optimisation
- estimation of the mean weight by size-class for particles

Figure 15: Imaging and sieving results for 606 measurements over 13 hours of loading.

5. Acknowledgements

Thank you to ProcessIT Innovations for their continuing support in developing and initiating industry projects. Thank you to the MinBaS II research program and Nordkalk for their participation and support, and a thank you to our measurement technology partners, MBV-Systems. In addition we acknowledge the EU INTERREG IVA program for their financial support. None of the funding sources of this research were involved in the collection, analysis or interpretation of the data. The only exception being Nordkalks assistance in collecting sieving data of the rock samples.

6. Biography

Matthew Thurley is an assistant professor in industrial image analysis and founder of the industrial image analysis group at Luleå university of technology. He has a strong focus on machine vision research applied to fully automated online particle size measurement of bulk material which was also the focus of his PhD research at Monash University in Australia. Matthew's focus is on industrial measurement projects and research in morphological image processing, algorithms, and classification.

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