Structural assessment of concrete bridges

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ABSTRACT

The paper summarises the work on concrete bridges performed in the EU project Sustainable Bridges. The work provides enhanced assessment methods that are able to prove higher load-carrying capacities and longer fatigue lives for existing concrete railway bridges, and is implemented in a Guideline [1]. Methods for determination of in-situ material properties and advanced methods for structural analysis were developed. One main focus was non-linear analysis, since these provides the greatest potential for discovering of additional sources for load-carrying capacity. Furthermore, methods to assess the remaining structural resistance of deteriorated concrete bridges with respect to corrosion and fatigue were developed.

Key words: Structural assessment, concrete, railway, bridge, non-linear analysis, fatigue, corrosion.

1. INTRODUCTION

For a sustainable development of Europe, there is a need to at least double the railway transports in the coming 20 years. In order to reach this goal, the residual service lives of existing concrete railway bridges need to be extended, at the same time as they are subjected to higher axle loads, higher railway speeds and heavier traffic intensity. Today, many concrete bridges are replaced or strengthened because their reliability cannot be guaranteed based on the structural assessments made. The objective of the work presented here was to provide enhanced assessment methods that are able to prove higher load-carrying capacities and longer service lives for existing concrete railway bridges.

The work was a part of the EU-project Sustainable Bridges. The results are implemented in the *Guideline for Load and Resistance Assessment of existing European Railway Bridges* [1] that was developed within the project. The guideline is based on the current state-of-the-art, but improved knowledge was developed in a few prioritised areas: material properties in existing bridges, advanced methods for structural analysis and assessment of deteriorated bridges with respect to fatigue and corrosion. This paper focus on the research performed, which is reported more in detail in a background document to the guideline [2]. In the Sustainable Bridges project, guidelines were developed also for inspection and condition assessment [3], for monitoring [4] and for repair and strengthening [5] of railway bridges.

Improved methods for the determination of in-situ material properties in existing concrete bridges were developed, for deterministic as well as for fully probabilistic assessments. To facilitate structural analysis on different levels, recommendations for redistribution of sectional moments and forces from linear finite element (FE) analysis were developed. Furthermore, advanced methods for local resistance analysis were developed, e.g. regarding combined shear, torsion and bending interaction.

To facilitate the usage of non-linear analysis for structural assessment, recommendations for practical use were developed. Non-linear analysis provides the greatest potential to discover any additional sources for load-carrying capacity, and gives a better understanding of the structural response, forming an improved basis for assessment decisions.

Another main objective was to provide methods for assessing the remaining structural resistance of deteriorated concrete bridges. Recommendations on the effect of corrosion on the anchorage capacity of reinforcement were developed. Furthermore, a methodology was presented for improved assessment of the fatigue safety for existing concrete bridges. Here, the emphasis was on evaluation of the remaining fatigue life of short-span bridges and secondary elements, since these often cause fatigue problems for railway bridges.

2. EVALUATION OF MATERIAL PROPERTIES

The purpose of assessment of material properties is to obtain the best possible information about the relevant resistance parameters for a specific bridge. It is also important to describe the uncertainties associated with each parameter e.g. in terms of expected variability. Important bases for evaluation are the material specifications from the original construction as well as testing of current in-situ properties for the materials in the existing bridge structure. For railway bridges dynamic effects on strength and stiffness properties are of interest. Relevant data for modelling of such effects are given in the *Guideline for Load and Resistance Assessment of existing European Railway Bridges* [1], for the materials mentioned above.

A proper description of mechanical properties for concrete as a basis for structural analysis is a complex matter for the following reasons:

- A number of different strength parameters are needed.
- Material properties change with age, due to continuous hardening and deterioration.
- Results from testing of strength depend on size and design of the test specimens used.
- The in-situ strength in the finished structure is different from that obtained by standard specimen testing of the same concrete.

In the guideline [1], recommendations for assessment of concrete properties are presented. A methodology to estimate the properties based on the original strength class specifications combined with the effect of continued hydration was developed. The long-time hydration leads for most bridges to increased strength at higher age, see Figure 1. Recommendations are also given about interpretation of in-situ testing to obtain reliable updated information about strength in the existing structure. The basic reference property for concrete is uniaxial compressive strength. Other properties of interest in non-linear structural analyses are elastic modulus, uniaxial tensile strength, fracture energy, bond strength, ultimate compressive strain and strain at peak compressive stress. These can often be estimated from empirical relations between the property and the compressive strength.

Recommendations on how the yield strength can be estimated given the specified grade of reinforcing steel, usually with rather good precision, are given in the guideline [1]. The variability can be reduced if test results are available for samples taken from the structure, in particular if it is known that the steel in the structure originates from the same producer and/or batch. Other properties of interest for reinforcing steels are tensile strength and strain at ultimate load. These are defined in relation to the yield strength and depend on the ductility class for the steel.

For prestressing steels, recommendations are given on relevant mechanical properties, i.e. tensile strength, proof strength, effective elastic modulus and strain at maximum stress. Nominal strength values are generally specified by manufacturers of prestressing steel products. The variability is different for different types of products such as wires, bars and strands, and such information may in a given case be available from the suppliers. In some cases results from

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Figure 1 – Example of relative compressive strength growth for old concrete qualities, from 28 days to 30 years, for different water cement ratios.

strength tests of the prestressing steels used during construction are available. Such results can be used to estimate the mean values of strength, proof stress and ultimate strain. The variability may also be estimated from such data. Generic information about variability of prestressing steel properties is to some extent given in the guidelines.

3. REDISTRIBUTION OF MOMENTS AND FORCES FROM LINEAR ANALYSIS

FE structural analysis can rationalise and improve bridge assessment and design, in particular for complicated geometries where modelling in three dimensions is required. Due to moving loads and the large amount of different load cases, linear analysis is normally used for bridges. Linear analysis often leads to high stress concentrations, e.g. at point supports or slab-column connections. These are often expressed as concentrated cross-sectional moments and shear forces.

However, the stress concentrations obtained through linear analyses of concrete bridges do often not exists in reality. This is due to the cracking of the concrete, often already for service loads, and the yielding of the reinforcement in the ultimate limit state, leading to even larger re-distribution of moments and shear forces. Another reason for unrealistic stress concentrations in linear analysis can be due to idealisations of the geometry when structural elements like slabs and beams are used to model the bridge.

In this part of the project, recommendations were developed for redistribution of concentrated cross-sectional moments and shear forces, obtained by linear FE analysis, for assessment of concrete bridges. The study focused on slab bridges and on the moments and shear forces at concentrated supports. A state-of-the-art investigation was made and different recommendations regarding distribution of reinforcement in slabs were studied. A typical slab bridge was designed with different methods, following different recommendations for reinforcement distribution, and the structural responses of the different solutions were analysed using non-linear FE analysis, see Figure 2.

It was found that unrealistic moment and shear force concentrations emanating from different sources need to be treated differently. Concentrations due to geometrical idealisations can be overcome by using sufficiently dense FE mesh and by using the cross-sectional forces and moments in the critical cross-sections. The cross-sectional moments and shear forces in a slab will tend to go to infinity, when the element mesh is being refined, if it is supported in a single point. However, in reality, the largest stresses are obtained in the critical cross-sections around the support (or slab-column connection). Moments and shear forces in the critical cross-sections are unaffected by the geometrical simplification if the FE mesh is dense enough. At least two first-order (or one second-order) shell elements between the support point and the critical cross-section was found to be needed. The peak values inside the critical cross-sections can then be ignored since their physical interpretation is unclear and they are not connected to any possible failure mode.

Concentrations of cross-sectional moments and shear forces due to material simplifications, i.e. from the assumption of linear response, need to be redistributed. In the ultimate limit state, the moment distribution in an existing bridge will be governed by the reinforcement provided. When assessing a slab bridge, lateral redistribution of the moment within a slab cross-section is necessary to avoid excessive under-estimation of the load-carrying capacity. The

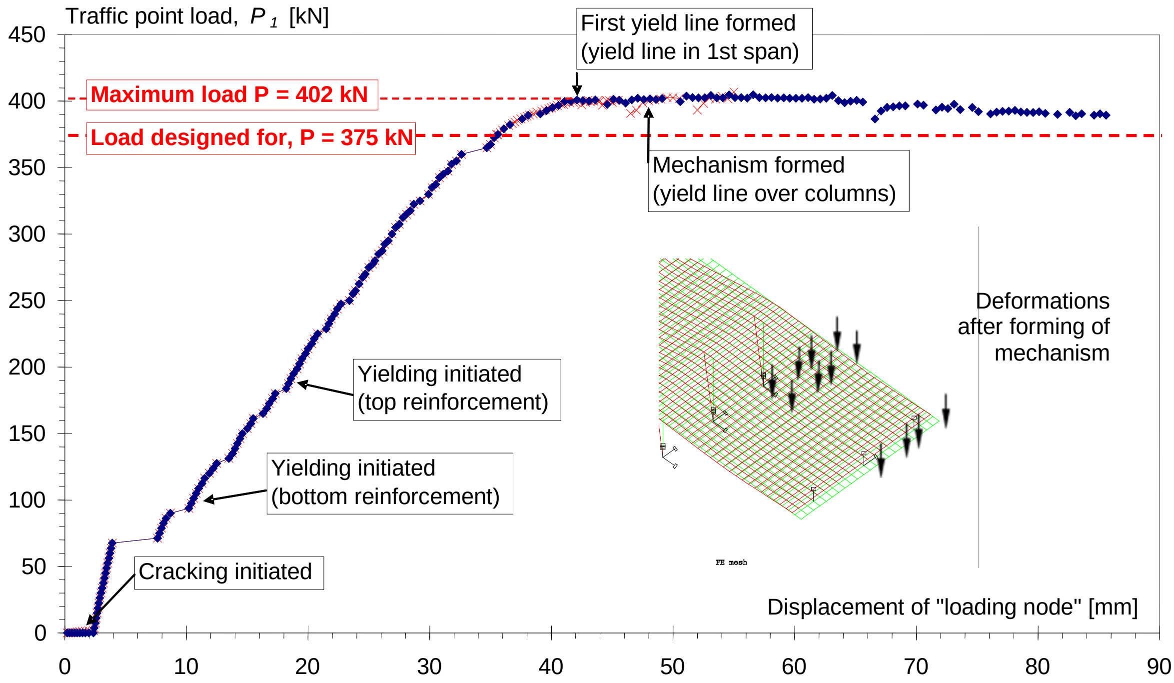

Figure 2 – Example of results from one of the non-linear FE analysis of the slab bridge studied. The graph shows the total of traffic point loads (in addition to dead load and distributed traffic load) versus the displacement of a load distribution node (for distribution of the point loads).

moment should be redistributed so that the moment distribution corresponds to the available reinforcement in the existing bridge. Recommendations in handbooks regarding strip widths for redistributions should not be used, since this would mean that too rigorous limitations on the redistributions are introduced. Instead, the study indicates that substantial lateral redistribution can be allowed compared to the linear distribution, if the slab has sufficient capacity for plastic rotations. Consequently, the ductility should be checked with respect to the requirements in Eurocode 2 [6].

4. BENDING-SHEAR-TORSION INTERACTION

Earlier, bending, torsion and shear was treated as separate actions in the design of a cross section. With the advent of the truss analogy and the modified compression field theory it became clear that the forces interact, see e.g. Collin and Mitchell [7]. This way of thinking is now introduced in the Eurocodes. In the *Guideline for Load and Resistance Assessment of existing European Railway Bridges* [1], recommendations for the use of such methods are given, and several examples of applications for bridge assessment are reported in the background document [2].

Using the theory of plasticity and the assumption of yielding of all longitudinal and transverse reinforcement before concrete compression failure, simple closed interaction surfaces can be obtained, e.g. Elfgren *et al*. [8]. For a common case with compression in the top of a member, an interaction formula may be derived as

$$\frac{M}{M_0} + \left(\frac{V}{V_0}\right)^2 + \left(\frac{T}{T_0}\right)^2 = 1 \qquad (1)$$

Here M, V, and T represent the bending moment, the shear force and the torsion moment respectively, while M_0, V_0, and T_0 are the capacities of a section loaded in pure bending, pure shear or pure torsion respectively.

More detailed results can be obtained with the modified compression field theory where the successive increase of stresses can be studied in a section with the program Response-2000, see Bentz [9]. The torsion stresses are usually added to the stresses of the vertical shear forces. The torsion-bending-shear interaction has been studied for several Swedish bridges, see e.g. Puurula *et al*. [10].

5. ASSESSMENT OF CONCRETE BRIDGES BY NON-LINEAR ANALYSIS

Non-linear analysis is the most realistic method for improved assessment of existing structures. It removes the inconsistency included in standard design approaches where the check of cross-section is done using non-linear material assumptions while the cross-sectional forces are determined based on linear analysis. However, in contrary to linear analysis, it puts higher demand on the engineer and it may require considerable computational resources. For practical applications, numerical computational methods such as the finite element method (FEM) must be used. In the *Guideline for Load and Resistance Assessment of existing European Railway Bridges* [1], recommendations are given for the use of non-linear analysis, but also regarding in which cases non-linear analysis has potential for discovering additional load-carrying capacity of concrete bridges. In the background document [2], validation of the suggested methods and their applicability are given on material as well as on structural level.

Non-linear analysis is a rather general term that encompasses many methods and approaches.

- Geometric non-linearity takes into account large deformation or strains. In most civil engineering structures this is not a dominant source of non-linearity with the exception of various buckling problems.
- Material non-linearity considers the non-linear material response such as: steel and reinforcement yielding, concrete or masonry cracking, and concrete crushing.
- A complete response of a structure to a given imposed loading can be obtained by such an analysis including stages of crack propagation in the pre-peak serviceability state, the failure load and failure mode and the post-peak behaviour. The model can be described on three levels, as shown in Figure 3, each involving certain approximations:

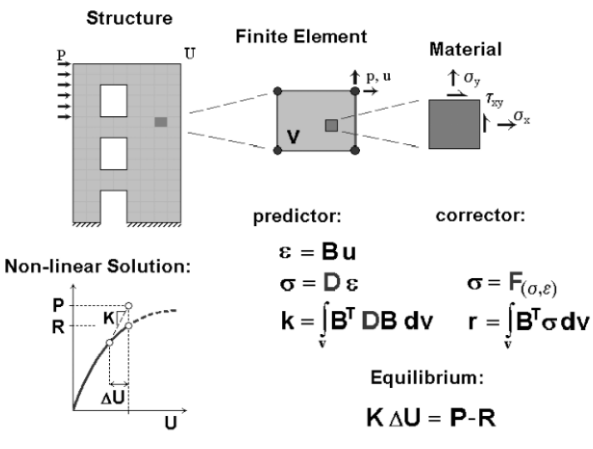

Figure 3 – Main steps of a non-linear analysis.

– Structure. In the stiffness approach the structural geometry is reduced into a system of finite elements, boundary conditions and loading. The structural response is described by the equilibrium matrix equation, where **U** are discrete displacements, **K** is a stiffness matrix and **P** are loading forces.
– Finite element. A shape of the displacement function in terms of nodal displacements (reflected in the matrix **B**) is assumed and used together with the material stiffness **D** to calculate the element stiffness matrix **k**.
– Constitutive relations. They define the behaviour of the material in terms of stress-strain relations, function $\mathbf{F}_{(\sigma,\varepsilon)}$, in a material point and corresponding material stiffness **D**. They reflect the non-linear material effects and failure, such as the concrete cracking or the reinforcement yielding.

The above formulation is typically incremental. The forces, displacements, strains and stresses are linearized increments within each load step.

The first two levels of the structural model in Figure 3 are well known from applications in other fields of engineering, and can be solved with required accuracy, just providing sufficiently fine meshes and adopting reasonable shape functions in finite elements. The third level, the constitutive modelling of specific properties of reinforced concrete, especially their derivation from experiments, represents a difficult task, because material behaviour can not be easily separated from its structural context. In order to verify the validity of nonlinear models, the performance of programs is often confronted with experiments in bench mark tests, e.g. Bonnard & Gardel [11], Margoldova *et al*. [12].

Since cracking is the most important property of brittle materials such as masonry, concrete or rock, a variety of crack models have been proposed: the discrete crack, the embedded crack and the smeared crack model. The smeared crack model is present in some form in most commercial finite element codes. A real discrete crack is simulated by a band of localized strains as illustrated in Figure 3. Due to the energy formulation, this model is objective and its dependency on the finite element mesh size is substantially reduced, Cervenka *et al*. [13]. This was confirmed by numerous studies, for example by those about shear failure published in Cervenka [14]. Nowadays, nonlinear analysis represents a powerful tool for the estimation of remaining load-carrying capacity of existing structures. Typical result from such a non-linear finite element analysis with localized failure zone is shown in Figure 4.

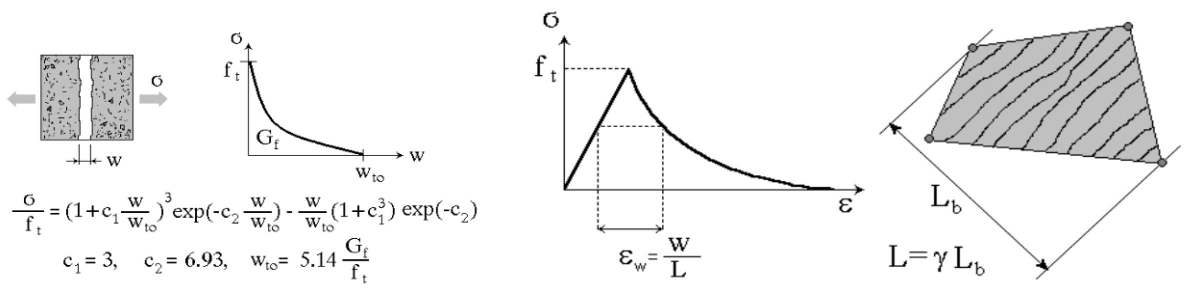

Figure 3 – Crack opening law (left). Strain softening law (middle). Crack band L (right).

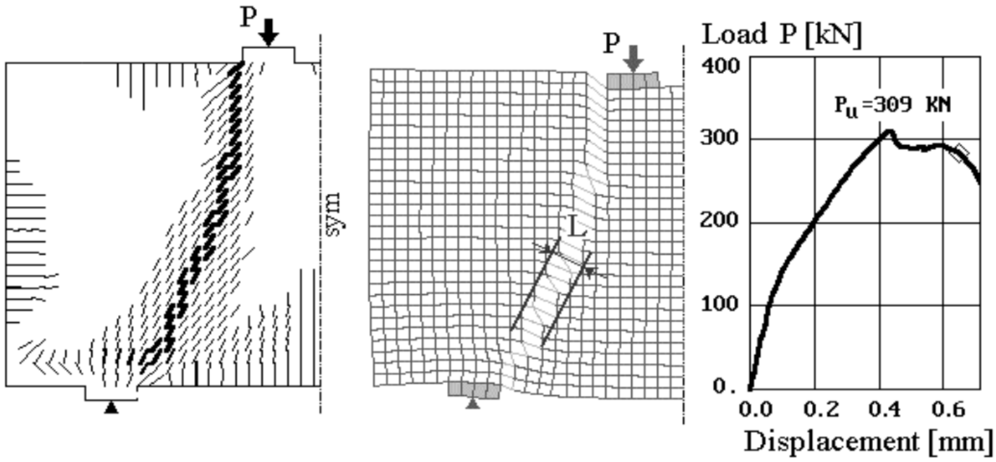

Figure 4 – Example of a typical crack localization in a non-linear finite element analysis.

6. EFFECT OF REINFORCEMENT CORROSION ON BOND AND ANCHORAGE

The volume increase that takes place when reinforcement in concrete corrodes causes splitting stresses in the concrete. Thereby, the bond between the reinforcement and the concrete is influenced. This effect has been studied both experimentally and theoretically by many researchers. In this project, the effect of corrosion on the bond between reinforcement and concrete was investigated and described in a systematic way. Literature studies of experimental work were combined with axisymmetric finite element analyses, for details see Lundgren[15]. A frictional model for the bond between reinforcement and concrete was used, together with a model describing the volume increase due to the corrosion, see Lundgren [16], [17].

The same basic bond mechanisms are active for both ribbed and smooth bars. However, they are of different magnitude, and therefore different mechanisms determine the behaviour. Generally, the bond capacity of smooth bars is smaller, mainly since smooth bars have limited ability to generate normal stresses at slip. Therefore corrosion, as long as it does not crack the cover, can increase the bond capacity of smooth bars to about the level of ribbed bars. For ribbed bars, corrosion might increase the bond capacity, but only to a minor extent. High corrosion levels will damage the bond, especially if there is no confining transverse reinforcement.

Based on the conclusions from the overview made, recommendations for maintenance of bridges were developed. These are included in the *Guideline for Load and Resistance Assessment of existing European Railway Bridges* [1], and are summarised below:

- Ribbed bars without confining transverse reinforcement are most sensitive to corrosion. For bridges where the anchorage capacity may be critical for the load carrying capacity, it is advised to monitor the risk of corrosion and to take measures when there is any corrosion risk.
- Smooth bars will normally have end hooks. However, smooth bars without end hooks oriented into the structure, and without transverse reinforcement, will also be critical. If the anchorage capacity may become critical for the load carrying capacity, it is advised to detect the onset of corrosion, and take measures when there is any indication of corrosion.

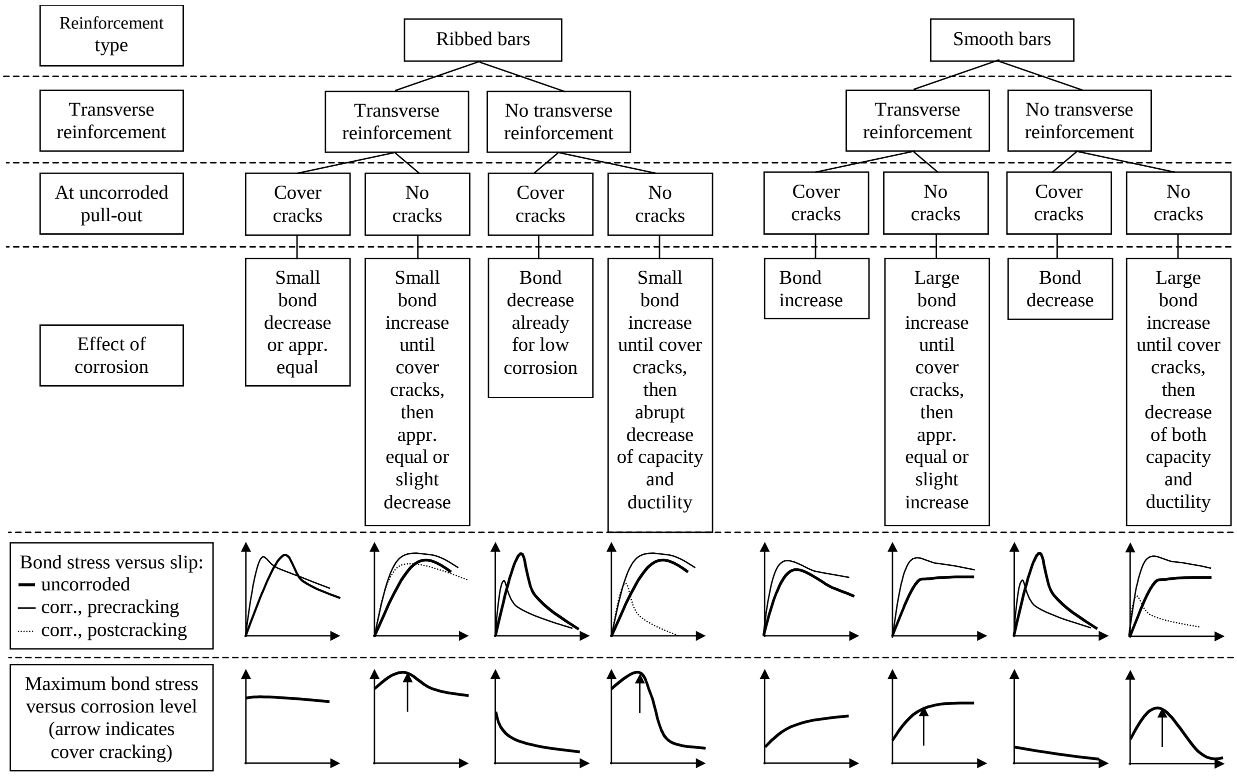

Figure 5 – Overview of the effect of corrosion on bond. The scales in the bond-slip curves are varying, to make all graphs clearly visible. However, the scales in the maximum bond stress versus corrosion level graphs are the same, to enable comparisons.

- Ribbed bars with sufficient amount of confining transverse reinforcement are less sensitive. Here, it is advised to do regular inspections, and to take measures before the cover spalls off in order not to lose the anchorage capacity.
- Smooth bars with sufficient and effective transverse reinforcement are not very sensitive, but measures need to be taken when the concrete spalls off.

7. REMAINING FATIGUE LIFE OF REINFORCED CONCRETE BRIDGES

The aim of performing a proof of fatigue safety is to demonstrate that the fatigue effects of (higher) rail traffic loads will not impair the safety of the structure during its intended service life. For railway bridges, proof of fatigue safety is generally required for all structural elements, and in particular for those subjected directly to wheel loads.

Current knowledge of fatigue behaviour of reinforced concrete suggests that the fatigue safety examination of reinforced concrete elements of existing railway bridges includes in principle a fatigue safety check of the steel reinforcement, and existing knowledge in fatigue behaviour of steel structures can be adopted. Fatigue failure of concrete is very unlikely to occur if the concrete is in good condition, i.e. concrete is not suffering from any deterioration mechanism (cracking) due to bar corrosion, frost or alkali-aggregate reaction.

Consequently, a rational methodology for the assessment of fatigue safety was developed and implemented in the *Guideline for Load and Resistance Assessment of existing European Railway Bridges* [1]. The methodology is based on the three following study areas taking advantage of the fact that the bridge is existing: (1) study of the bridge structure and evaluation of reinforcement detailing, (2) inspection of the existing bridge and study of the past performance, and (3) fatigue safety check. In the following, each of the three study areas is briefly discussed.

If the principles of good fatigue design practice were followed when the bridge was built and if the bridge is in good condition, then the check of structural safety will be usually determinant. However, in cases where low fatigue strength can be expected for the steel reinforcement, this is possibly not the case, and a fatigue safety check may limit the use of the bridge.

The main objective of the study of the bridge structure and the detailing of the reinforcement is thus to detect fatigue vulnerable spots. Such fatigue vulnerable spots are predominantly pre-sent at locations where the rules of “good” fatigue resistant design have not been respected.

Grouping types of reinforcement into fatigue categories in accordance with code provisions allows recognizing types of reinforcement with low fatigue strength. Fatigue vulnerable reinforcement details include, for example, all welded reinforcement, mechanically connected reinforcing bars, anchorages for and coupler between prestressing elements or reinforcement bars showing significant corrosion.

Fatigue fracture of reinforcement bars may be preceded by cracking of the concrete cover. For example, the fatigue failure of a deck slab is characterized by a distinct crack pattern that is formed depending on the state of the fatigue damaged reinforcement bars. Also, the deflection of fatigue damaged reinforced concrete elements may significantly increase when important fatigue damage has occurred. As a consequence, bridge inspection and monitoring of fatigue vulnerable elements should focus on the detection of crack patterns and deformations.
The fatigue safety of a structure is proven if the following condition is satisfied:

$$n = \frac{R_{d,fat}}{E_{d,fat}} \geq 1.0 \qquad (2)$$

where n is the fatigue safety index, $R_{d,fat}$ is the examination value for the fatigue resistance (including a partial safety factor), and $E_{d,fat}$ is the examination value for the fatigue action effect (without partial safety factor). The fatigue safety check is made separately for reinforcing steel and concrete, but may also be performed using the overall structural response of a fatigue vulnerable element.

Proof for reinforcing steel: The fatigue safety check is performed first with respect to the fatigue limit and then with respect to the equivalent stress range.

Proof for concrete: In the determination of stresses in concrete due to fatigue loading it must be considered that such calculated stress values only represent an approximation of effective stresses. Also, reliable fatigue damage accumulation method is still lacking. Consequently, it is not possible to perform a rigorous and reliable fatigue safety check for concrete. Fortunately, proof of fatigue safety by calculation is not required for normal stresses in concrete if inspection shows that the concrete is in good condition.

Proof with respect to ultimate load: Fatigue testing revealed that relevant fatigue damage only occurs if the level of fatigue solicitation is beyond 50% and 40% of the ultimate load for predominant bending and shear fatigue loading respectively. From this follows that no fatigue failure of the structural element will occur if the following condition is fulfilled under predominant bending and shear fatigue, respectively:

$$n_{fat} = \frac{0.5 \cdot F_{ult}}{F_{fat,\max}} \geq 1.0 \qquad n_{fat} = \frac{0.4 \cdot F_{ult}}{F_{fat,\max}} \geq 1.0 \qquad \text{(3a, 3b)}$$

F_{ult} is the ultimate load of the structural element as obtained. It is determined by means of a non-linear structural analysis using nominal values of material properties and considering partial safety factors (resistance coefficients).

Proof considering the crack pattern: In bending fatigue tests on slab-like beams, Schläfli *et al.* [18], it was observed that only beams with an *entirely developed crack pattern* failed under cyclic loading. If the distances between the cracks are larger or even a multiple of the upper bound value given below, the cracking is not stabilised yet and *no fatigue effective stress ranges* occur in the reinforcement and will not occur as long as the crack pattern does not change. The degree of cracking is a function of the average observed crack distance. An upper bound for the crack distance s_r for a stable crack pattern may be assessed according to Oehlhafen [19]:

$$s_r = 2l_r \qquad (4)$$

Here, $l_r = 60 + \lambda \cdot \kappa \cdot s$ [mm] is the transition length for the force transfer between rebar and concrete. $\lambda = 0.25(1+2c/\phi)$ is a factor depending on the ratio between the rebar cover c and diameter ϕ. κ is a coefficient which depends on the nature of the solicitation (it is equal to 1.0 for pure traction and 0.5 for pure bending), and s is the spacing between the reinforcement bars.

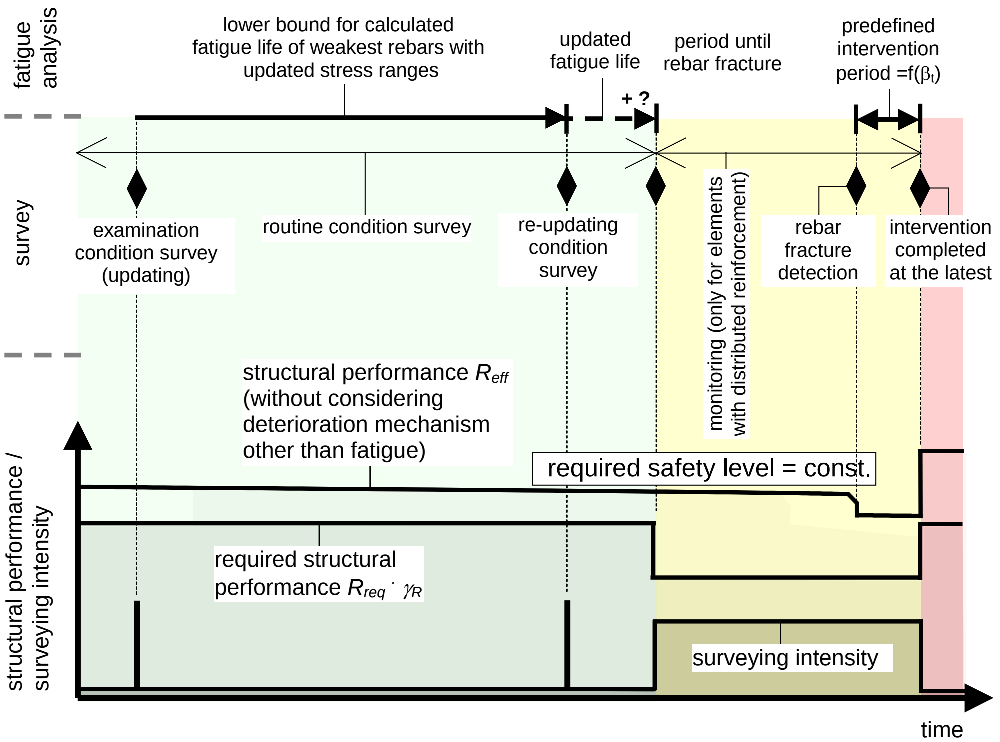

Figure 4 – Scheme illustrating the fatigue safety concept for existing reinforced concrete bridges (monitoring may be considered only for elements with distributed reinforcement).

The fatigue safety concept shown in Figure 4 is based on fatigue life calculation. It considers the extension of the service life with safe operation through monitoring by periodic visual inspections and/or continuous measurements of structural behaviour. Reinforcement is triggered only in case the monitoring results show the beginning of significant fatigue damaging.

8. CONCLUSIONS

The research activities summarised here were performed as a part of the European research project Sustainable Bridges in order to obtain an improved basis for the *Guideline for Load and Resistance Assessment of existing European Railway Bridges* [1], which was developed within the project. The work provides methods for enhanced assessment of existing railway bridges. The use of more advanced analysis methods, such as non-linear analysis, will lead to that higher load carrying capacities can be proven, but also to an improved understanding of the structural response, forming a better basis for decisions in the assessment. Recommendations are given for assessment of corroded concrete bridges, and the improved methods for fatigue assessment will lead to increased remaining service lives.

REFERENCES

1. Sustainable Bridges (2007): *Guideline for Load and Resistance Assessment of existing European Railway Bridges.* Report D4.2. Prepared by Sustainable Bridges - a project within EU FP6. Available from: www.sustainablebridges.net.
2. Sustainable Bridges (2007): *Non-Linear Analysis and Remaining Fatigue Life of Reinforced Concrete Bridges*. Report D4.4.2. (Background document to “Guideline for Load and Resistance Assessment of Railway Bridges”). Prepared by Sustainable Bridges - a project within EU FP6. Available from: www.sustainablebridges.net.
3. Sustainable Bridges (2007): *Guideline for Inspection and Condition Assessment of Railway Bridges.* Report D3.15. Prepared by Sustainable Bridges - a project within EU FP6. Available from: www.sustainablebridges.net.
4. Sustainable Bridges (2007): *Monitoring Guidelines for Railway Bridges.* Report D5.2. Prepared by Sustainable Bridges - a project within EU FP6. Available from: www.sustainablebridges.net.
5. Sustainable Bridges (2007): *Repair and Strengthening of Railway Bridges - Guideline.* Report D6.1. Prepared by Sustainable Bridges - a project within EU FP6. Available from: www.sustainablebridges.net.
6. EN 1992-2 (2004): Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings. European Standard, Brussels: CEN.
7. Collins, Michael P and Mitchell, Denis (1991): *Prestressed Concrete Structures.* Prentice Hall, Englewood Cliffs, N.J., USA, 1991, 766 pp. ISBN 0-13-691635-x. Reprinted by Response Publications, Toronto 1997, 766 pp, ISBN 0-9681958-0-6.
8. Elfgren, Lennart, Karlsson, Inge and Losberg, Anders (1974): Torsion - bending - shear intertaction for reinforced concrete beams. *Journal of the Structural Division,* American Society of civil Engineers (ASCE), Vol 100, No ST 8, Proc Paper 10749, New York, August 19784, p. 1657-1676
9. Bentz, Evan C. (2000): *Sectional Analysis of Reinforced Concrete Members*. A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy, Graduate Department of Civil Engineering, University of Toronto, Toronto 2000, 187 + 118 pp. www.ecf.utoronto.ca/~bentz

10. Puurula, A. (2004): *Assessment of Prestressed Concrete Bridges Loaded in Combined Shear, Torsion and Bending*. Licentiate Thesis 2004:43, Luleå: Division of Structural Engineering, Luleå University of Technology, 103 + 144 pp. Available from: http://epubl.ltu.se/1402-1757/2004/43/index.html [cited 31 November 2006].
11. Bonnard and Gardel (1994): *Bench Mark on Numerical Analaysis of Concrete Structures*. Bonnard&Gardel, Neuchatel, Switzerland, 1994.
12. Margoldova J., Cervenka V. and Pukl R. (1998): Applied Brittle Analysis. *Concrete Engineering International* 8 (2) 1998, 65-69.
13. Cervenka V. and Margoldova J. (1995): Tension Stiffening Effect in Smeared Crack Model. In: *Engineering Mechanics*, Ed. S. Sture, ACSE, New York, USA, ISBN 0-7844-0083-0,1995, 655-658
14. Cervenka V. (1998): Simulation of shear failure modes of R/C structures. In: *Computational Modelling of Concrete Structures* (Euro-C 98), eds. R. de Borst, N. Bicanic, H. Mang, G. Meschke, A.A.Balkema, Rotterdam, The Netherlands, 1998, 833-838.
15. Lundgren, K. (2007): Effect of corrosion on the bond between steel and concrete: an overview. *Magazine of Concrete Research*, Vol. 59, No 6, pp. 447-461.
16. Lundgren K. (2005a): Bond between ribbed bars and concrete. Part 1: Modified model. *Magazine of Concrete Research*, Vol. 57, No. 7, September, pp. 371-382.
17. Lundgren K. (2005b): Bond between ribbed bars and concrete. Part 2: The effect of corrosion. *Magazine of Concrete Research*, Vol. 57, No. 7, September, pp. 383-396.
18. Schläfli M. (1999) *Ermüdung von Brückenfahrbahnplatten aus Stahlbeton* ("Fatigue of Bridge Deck Slabs of Reinforced Concrete", Doctoral thesis. In German), Thèse N° 1998, Ecole Polytechnique Fédérale de Lausanne.
19. Oehlhafen U. (1984) *Rissnachweis*, Conférence aux journées d'études des 12 et 13 octobre 1984 à Lausanne sur le thème « principes et conception de la nouvelle norme SIA 162 » Documentation SIA No. 77 (in German).