Proportional Hazards Modeling of Time-Dependent Covariates Using Linear Regression: A Case Study

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Key Words — Proportional hazards model, Time-dependent covariate, Linear regression model

Summary & Conclusions — In the proportional hazards model the effect of a covariate is assumed to be time-invariant. In this paper a graphical method based on a linear regression model (LRM) is used to test whether this assumption is realistic. The variation in the effect of a covariate is plotted against time. The slope of this plot indicates the nature of the influence of a covariate over time. A covariate is time-dependent if a drastic change in the slope of the plot is found and the time-point, at which this drastic change occurs provides a guideline in redefining a time-dependent covariate into two or more time-independent covariates. This method is applied to failure data of cables used for supplying power to electric mine loaders. The results obtained by applying only the proportional hazards model were misleading as the graphical method based on the LRM showed that one covariate was highly time-dependent. This graphical method should be used to supplement the proportional hazards model, not as a separate method. This avoids misinterpretation of the influence of a time-dependent covariate in the proportional hazards model. The proportional hazards model should be used to identify the most important covariates, while the LRM should be used as an explanatory tool to check the consistency of the influence of the covariates. The LRM involves matrix computations which can be quite time consuming for large data-sets. Also, tests for the statistically significant effect of a covariate are not yet well established in the model.

1. INTRODUCTION

Acronyms¹

LHD	load-haul-dump
PHM	proportional hazards model
LRM	linear regression model
TBF	time between failure
TST	test statistic (for the LRM).

The reliability of a system is influenced not only by the operating time, but by other factors. These influencing factors can include:

- operating environment, *eg*, temperature, pressure, humidity, dust;

- operating history, *eg*, overhauls, effect of repair or preventive maintenance;
- type of design or material used.

They are generally referred to as covariates, concomitant variables, or explanatory variables.

To model the effect of covariates on system reliability, regression methods can be used. PHM, introduced by Cox [5], is a major framework for survival analysis, especially in the medical field. A disadvantage of PHM is that the influence of a covariate is assumed to be time-invariant. In many real situations, some covariates are time-dependent. For example, a particular system-design might have better performance during a certain operating time interval. For time-dependent covariates, estimates of the parameters in PHM might be *s*-biased [8]. Therefore, it is better to supplement the results of PHM with a test for time-dependence of the covariates. The general approach is to partition the time-axis into intervals during which the influence of a covariate is time-invariant. For this purpose, both analytic & graphic methods can be used. Analytic methods include: partial likelihood ratio test [4], a step function approach [6], a suitable statistic that is based on a score-test [19], and a weight-function based approach [13]; they have been used to partition the time-axis appropriately. Alternatively, a penalized partial likelihood method [24], or a sieve estimation method [20] can be used to estimate the influence of a time-varying covariate. Graphical methods include plotting the: cumulative regression functions [1, 2], the partial residuals [22], and/or smooth partial residuals [21]; these plots are used to define the time intervals during which the influence of a covariate is time-invariant. We study the graphical method suggested by Aalen [1, 2] because it provides more information about the nature of the influence of a covariate. Aalen's LRM has been applied mainly in the medical field [3, 15, 16].

Notation

t_i	time between failures $i-1$ and i
n	sample size
k	number of observed failure times
z	row vector consisting of the covariates
β	column vector consisting of regression parameters
$h_j(t)$	hazard rate for item j
$h(t)$	vector of the $h_j(t)$
$h(t; z)$	hazard rate in presence of z
$h_0(t)$	baseline hazard rate
$L(\beta)$	likelihood function for β
$NF(t_i)$	risk set just before t_i
m	number of failure times in $NF(t_i)$
d_i	number of tied failure times at t_i
q	number of covariates
$Y(t)$	$n \times (q+1)$ matrix

¹The singular & plural of an acronym are always spelled the same.

- $\alpha(t)$ vector of regression functions
 $A(t)$ vector of cumulative regression functions
 $X(t)$ any generalized inverse of $Y(t)$
 U weighted combination of the summands of $A^*(t)$
 U_i are the elements of U
 K weight function
 CV covariance matrix of U
 $\mathcal{G}(\cdot)$ indicator function: $\mathcal{G}(\text{True})=1$, $\mathcal{G}(\text{False})=0$
 I_j indicator vector: $\text{cell}_j = \mathcal{G}(t_j \text{ is an observed failure time})$
 $*$ implies an estimate.

Other, standard notation is given in "Information for Readers & Authors" at the rear of each issue.

Assumptions

1. The failure times are i.i.d.
2. A repairable system is good-as-new after a repair.
3. Repair times are either negligible or not taken into account.

2. PROPORTIONAL HAZARDS MODEL

For n ordered failure times, $k \leq n$ are observed, and $n-k$ are censored. These failure times can be observed from n non-repairable systems or less than n repairable systems with multiple failures. This paper studies a repairable system.

In PHM, it is generally assumed that the hazard rate of a system is the product of an unspecified $h_0(t)$, dependent on time only, and an exponential function basically incorporating time-independent effects of covariates. Thus,

$$h(t; z) = h_0(t) \cdot \exp(z \cdot \beta), \quad (1)$$

The z is associated with the system and β defines the effects of the covariates. Roughly, the baseline hazard rate is the total hazard rate when the covariates have no influence on the total hazard rate. A parametric form such as the hazard rate from the Weibull distribution [7, 14], can be assumed for the baseline hazard rate, but it is more common to adopt a distribution-free approach [10]. The β can be estimated by maximizing [8]:

$$L(\beta) = \prod_{i=1}^k \frac{\exp(z_i \cdot \beta)}{\left[\sum_{m \in \text{NF}(t_i)} \exp(z_m \cdot \beta) \right]^{d_i}}. \quad (2)$$

For PHM, various goodness-of-fit tests are available, and many computer programs deal with PHM analysis. For a thorough description of goodness-of-fit tests, see Kay [9], and for a general review on PHM (including available computer programs) see Kumar & Klefsjö [10].

3. AALEN LINEAR REGRESSION MODEL

This model was originally suggested by Aalen [1]. Further theoretical investigations have been made by McKeague [17] and McKeague & Utikal [18]. The model has been used to check time-dependence of covariates in the medical field, see eg, Mau [15, 16] and Andersen & Væth [3]. Recent advances are summarized and the model is discussed in a broader context in Aalen [2].

3.1 Model Description

The LRM for $h(t)$ is considered,

$$h(t) = Y(t) \cdot \alpha(t). \quad (3)$$

The rows of $Y(t)$ consist of,

$$z^j(t) = (1, z_1^j(t), \dots, z_q^j(t)), \quad (4)$$

z_i^j = (possibly time-dependent) covariate values, corresponding to those failure times that are in the risk set at time t_j .

The risk set consist of those failure times that have not happened (observed or possibly censored) until t_j . At t_j , the rows in $Y(t)$ corresponding to the times $t_{j-1}, t_{j-2}, \dots, t_1$ consist of 0's. For example, at t_3 , a $Y(t)$ might look like,

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix} \quad (5)$$

In —

$$\alpha(t) = (\alpha_0(t), \alpha_1(t), \dots, \alpha_q(t)), \quad (6)$$

$\alpha_0(t)$ is the baseline parameter function,

$\alpha_i(t)$, $i = 1, 2, \dots, q$, are regression functions, defining the influence of different covariates.

Since it is more difficult to determine the regression functions themselves than the cumulative regression function, we consider the estimate of,

$$A(t) = (A_0(t), A_1(t), \dots, A_q(t))', \quad (7)$$

$$A_i(t) = \int_0^t \alpha_i(s) ds, \quad i = 0, 1, \dots, q. \quad (8)$$

$A_0(t)$ is calculated along with the other parameters. It is analogous to the baseline hazard rate in PHM. The regression parameters in PHM can be estimated without estimating the baseline hazard rate. A reasonable estimator of $A(t)$ is [1],

$$A^*(t) = \sum_{t_j \leq t} X(t_j) \cdot I_j. \quad (9)$$

A simple choice for $X(t_j)$ is based on the least-square principle,

$$X(t_j) = [Y(t_j)' \cdot Y(t_j)]^{-1} \cdot Y(t_j)'. \quad (10)$$

We must calculate $X(t_j)$ for each $t_j \leq t$ in order to calculate $A^*(t)$. $A^*(t)$ can be calculated as long as $Y(t)$ has full rank.

$A^*(t)$ is calculated in order to provide information about the nature of influence of a covariate over time. To study the time-varying effect of covariate i , plot $A_i^*(t)$ vs failure-times. The slope of this plot represents the variation in the influence of a covariate as illustrated in figure 1. If the covariate has a constant influence over time, the plot is approximately a straight line. If the slope is positive (negative), the effect of a covariate is to increase (decrease) the hazard rate. If the slope is zero, the covariate has almost no influence on the hazard rate. If the plot is a curve with an increasing (decreasing) slope, it indicates an increase (decrease) in the magnitude of influence of a covariate.

curve_1 $\Rightarrow$ time-independent effect
 curve_2 $\Rightarrow$ no effect at all
 curve_3 $\Rightarrow$ decreasing effect
 curve_4 $\Rightarrow$ increasing effect of a covariate over time

Figure 1. Illustration of Various Slopes of the Cumulative Regression Function

3.2 Testing the Model

For testing whether covariate i has any influence on the distribution of life times, use the null hypothesis:

$$H_i: \alpha_i(t) = 0, \text{ for all } t. \quad (11)$$

One choice as a TST is,

$U_i \cdot CV_{i,i}^{-1/2}$, which has an asymptotic standard s -normal distribution,

$$U = \sum_{t_j} K(t_j) \cdot X(t_j) \cdot I_j. \quad (12)$$

This is a weighted combination of the summands of $A^*(t)$, where the sum is over all observed failure times. A possible definition of $K(t)$ is:

$$K(t_j) = [\text{diag}\{(Y(t_j)' \cdot Y(t_j))^{-1}\}]^{-1}, \quad (13)$$

$\text{diag}\{B\}$ = diagonal matrix having the same main diagonal as matrix B .

$K(t)$ must be calculated for each t_j .

The covariance matrix of U can be estimated by:

$$CV = \sum_{t_j} K(t_j) \cdot X(t_j) \cdot I_j^D \cdot X(t_j)' \cdot K(t_j), \quad (14)$$

I_j^D = diagonal matrix with diagonal I_j , viz, the only element in the matrix that is not zero is the diagonal element that corresponds to t_j if it is an observed failure time.

Nothing is added to U and CV for censored failure times.

4. THE DATA

In the LKAB Kiruna Iron Ore Mine (Kiruna, Sweden), diesel powered load-haul-dump (LHD) machines have been replaced by electric ones, primarily to improve the working environment, and to reduce ventilation costs. The electric power is transmitted to the LHD machine through a cable. The cable length varies from 275 to 325 meters and weighs 2.9 kg/m. The cross-sectional area is $4 \times 50 \text{ mm}^2$. The cable is wrapped around a cable reel mounted on the LHD machine and one end is connected to a supply source. The cable is wrapped & unwrapped frequently on this reel as the distance between the electrical source point and the operational area of the machine decreases or increases. Frequent wrapping & unwrapping causes failure of the cable due to over-heating or mechanical failure. The part of the cable which has been badly damaged is replaced by a new piece of cable. In most cases of disjoint, it is simply welded.

These cables are mainly of two types and characterized by two manufacturers with some differences in the design of the cable material; these two cables are referred to as cable type A and cable type B. There were 100 observations together on both cable types, listed in the appendix. The average TBF corresponding to failure numbers 1 - 6 is given in figure 2 which indicates that the average TBF of failure #1 is much higher than that of higher order failure numbers.

Figure 2. Comparison of average TBF for failures #1 - #6 [The average failure time for failure #1 is the largest compared to the other failure numbers.]

5. ANALYSIS²

In Kumar *et al* [11], the cable failure time data described in section 4 were analyzed using the PHM to identify which covariates influence the hazard rate. In that analysis, cable type and first failure number were *s*-significant. In this analysis we consider the two covariates cable-type and first-failure-number previously found *s*-significant, and add a new covariate, the repair type new-welded-joint. They are denoted by indicator covariates z_1 , z_2 , and z_3 , respectively.

$z_1 = \mathcal{I}(\text{failure time corresponds to cable type A}),$

$z_2 = \mathcal{I}(\text{failure time is the first failure of a cable}),$

$z_3 = \mathcal{I}(\text{for failure } j \text{ of a cable if a new welded joint was provided after failure } j-1).$

We do not consider failures #2 - #6 in this analysis because only failure #1 was *s*-significant in [11] and figure 2 indicates that the average TBF of failure #1 is much higher than that of higher order failure numbers.

The calculations were done using the software S-plus [23]. The β in the PHM was estimated using (2), and the LRM TST was estimated as discussed in section 3.2. The estimated values are listed in table 1. All three covariates are *s*-significant under,

$H_0: \beta = 0$, at 5% 1-tailed p-value.

The LRM TST agrees with the PHM t-statistic.

Table 1. Results of PHM & LRM Analyses [of cable failure-time data]

Covariate	Cox regression coefficient (StdDev)	Cox Model t-statistic (p-value)	LRM TST (p-value)
z_1 : Cable type	0.625 (0.209)	2.99 (0.00)	2.95 (0.00)
z_2 : Failure #1	-0.593 (0.284)	-2.09 (0.02)	-2.39 (0.01)
z_3 : New welded joint	0.419 (0.233)	1.80 (0.04)	1.76 (0.04)

The results imply that the cable hazard rate is best modeled in the PHM by:

$$h(t; z) = h_0(t) \cdot \exp(0.625z_1 - 0.593z_2 + 0.419z_3), \quad (15)$$

$$\beta_1^* = 0.625,$$

$$\beta_2^* = -0.593,$$

$$\beta_3^* = 0.419.$$

The estimated effect of the covariate z_1 implies that cable type A has a hazard rate that is, on average, 87% [$\exp(0.625) = 1.868$] higher than that of cable type B, provided z_2 & z_3 have fixed values.

The estimated effect of a covariate can be *s*-biased in the PHM if the nature of influence varies over time. For example, the influence of a covariate might be to increase the hazard rate for a certain range of time and to decrease it for another range of time. Time-dependence of a covariate is not easily detected in the PHM. The graphical methods of goodness-of-fit tests such as cumulative hazard plots and residual plots [8, 10] do not clearly reflect time-dependence of a covariate.

Time-dependence of a covariate can be represented graphically by plotting the cumulative regression functions (estimated as (9)) vs time. These plots are shown in figure 3 for z_1 , z_2 , z_3 . The plotted curve for z_1 (cable type) indicates a drastic change in influence at approximately 138 hours, where the slope of the curve changes from a positive to negative. This implies that cable type A has higher hazard rate compared to cable type B only up to 138 hours, not for the whole time range, in contrast to the conclusion drawn in the PHM analysis. After 138 hours, the negative slope of the plot indicates that cable type B has higher hazard rate than cable type A. This indicates that z_1 is time-dependent. The plotted curve for z_2 (first failure number) has a negative slope and the plotted curve for z_3 (new welded joint) has a positive slope. This indicates that z_2 & z_3 are time-independent. We then conclude that the results of the PHM analysis are not valid since z_1 is time-dependent.

Figure 3a shows a change in the nature of the influence of z_1 — there is a drastic change in the slope. In figures 3b & 3c, the slopes are almost constant — showing time-independent effects of z_2 and z_3 , respectively.

²The number of significant figures is not intended to imply any accuracy in the estimates, but to illustrate the arithmetic.

Figure 3. Estimated Cumulative Regression Functions vs Time

Table 2. z_1 reformulated as $z_{1,1}$ and $z_{1,2}$
[on the basis of the + and - slope of the curve in figure 3]

Cable type	For all TBF	For TBF $\leq$ 138 hours		For TBF $>$ 138 hours	
	z_1	$z_{1,1}$	$z_{1,2}$	$z_{1,1}$	$z_{1,2}$
A	1	2	0	0	2
B	0	1	0	0	1

Table 3. Results of PHM Analysis #2
[considering time-dependence of z_1]

Covariate	Cox regression coefficient (StdDev)	Cox Model t-statistic (p-value)
INITIAL RESULTS		
$z_{1,1}$	1.429 (0.276)	5.18 (0.00)
$z_{1,2}$	-0.794 (0.327)	-2.43 (0.01)
z_2	-0.111 (0.299)	-0.37 (0.36)
z_3	0.685 (0.243)	2.82 (0.00)
FINAL RESULTS		
$z_{1,1}$	1.439 (0.275)	5.23 (0.00)
$z_{1,2}$	-0.794 (0.326)	-2.43 (0.01)
z_3	0.722 (0.223)	3.24 (0.00)

To avoid misinterpreting the effect of z_1 , it is necessary to define two separate covariates in place of z_1 :

$z_{1,1}$, for TBF $\leq$ 138 hours

$z_{1,2}$, for TBF $>$ 138 hours.

Table 2 shows the formulation of these new covariates.

Table 3 gives the results of the PHM analysis considering $z_{1,1}$, $z_{1,2}$, z_2 , z_3 ; no TST value is given since a matrix becomes singular when at least 1 row and 1 column consists of 0's. This happens at time 138 hours, as a consequence of the covariate formulation in table 2. The initial results indicate that $z_{1,1}$, $z_{1,2}$, z_3 are s -significant. In the final analysis, these covariates were considered.

Based on the final PHM analysis, the cable hazard rate is:

$$h(t; z) = h_0(t) \cdot \exp(1.439z_{1,1} - 0.794z_{1,2} + 0.722z_3).$$

(16)

These results imply that, if z_3 stays constant, on average the cable type A has 4.22 [$\exp(1.439) = 4.216$] times the hazard rate of cable type B up to the time 138 hours. After 138 hours, cable type B has a hazard rate which is 2.21 [$\exp(0.794) = 2.212$] times that of cable type A. Further, given the same cable type, the hazard rate for the repair type, new-welded-joint, is on average 2.06 times [$\exp(0.722) = 2.059$] the hazard rate for other repair types. This result contradicts those presented in (15), where cable type B has lower hazard rate than cable type A during the whole time range. The conclusions drawn on the basis of the graphical representation of the influence of z_1 (cable type) in figure 3 are confirmed.

This analysis should be complemented with tests of goodness-of-fit, such as residual plotting and log-likelihood test [8, 9, 12]. The results of these tests are not given, since the aim of this paper is mostly introduction of a new method to model time-dependent covariates.

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APPENDIX

The data set that has been analyzed is given here. The numbers in each line corresponds to chronologically ordered TBF of a particular cable. Column #j corresponds to failure #j, $j=1, \dots$. A* corresponds to the repair type new-welded-joint, which was carried out due to breakage in two pieces of the cable at the previous failure.

Cable Type A

225	550*	5*	388	64					
155	4	138	57	160	67*	130	120*		
618	99*	62	322						
42	88*	201*	1	4	97	55			
212	13*	45*	70*	189	123	90	61*	62*	10
135	79*	57*							42
589	27*	2*	92	10*					
290									
59	78*	130	41*	7					
267	96*	100	68*	392	97	455*			

Cable Type B

714	13*	178*	250*						
176	246*	276*	112*						
193	75*	1011	218*						
802	99*								
488	101*	210*	226	79					
279	273*	115*	49*	130	146*	255*	479		
384									
253									
312	93*	95	71*	187					
85	170*	201	88*	52*					
139	156*	364	145	362					

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