Fracture Toughness of Stripa Granite Cores

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There is a growing interest in the study of fracture parameters of rock material, but to date no standard testing method for measuring the fracture toughness of rock exists. Three round specimen configurations, namely the notched beam, the compact specimen, and the short rod were used in this work and different fracture toughness measures evaluated for Stripa granite. An approximate fracture toughness, K_m , obtained without pre-cracking the specimens, was analysed statistically to indicate its dependence on notch length. Then a comparison with an apparent fracture toughness, K_{Qm} , obtained from pre-cracked specimens, showed that low K_m values arise because the microcrack growth that precedes crack extension is neglected.

Acceptable K_{Qm} values, in the sense that they are good estimates of K_{Ic} , can be obtained from three point bending tests on 40 mm diameter drill cores with notches of about 12 mm depth. The fracture toughness from short rod tests was found to be size independent in this investigation, and the value consistent with K_{Qm} values from the compact tension and the three point bending tests. The ligament width was found to be a more critical size parameter than the crack length during the testing.

NOTATION

ASTM = American Society for Testing and Materials;
LEFM = Linear Elastic Fracture Mechanics;
LPD = Load Point Deflection or Displacement;
CMOD = Crack (Notch) Mouth Opening Displacement;

a = crack or notch length;
 a_0 = initial notch length;
 b = crack or notch front length;
 B = specimen thickness;
 D = diameter of round specimen;
 a/D = crack length ratio;
 a_0/D = notch length ratio;
 w = specimen width;

$W - a, D - a$ = ligament width;

$a_{min}, B_{min},$
 $(W - a)_{min}$ = minimum required specimen size in LEFM;

L = support span of specimen;
 F = load on specimen;

F_{max} = failure load or largest load specimen can support;

F_Q = evaluation load for fracture toughness determination;

F_s = load used in evaluation of K_{SR}^{pl} ;

E = Young's modulus of material;

ν = Poisson's ratio;

K_I = stress intensity factor (thickness average);

K_{Ic} = plane strain fracture toughness (of metallic materials), originally as per ASTM standard E 399 for LEFM;

K_m = approximate fracture toughness defined in equation (1);

$G_m = (1 - \nu^2)K_m^2/E$ = energy consumption rate version of K_m ;

K_{Qm} = apparent fracture toughness defined in equation (4), modelled on conditional fracture toughness value K_Q of ASTM standard E 399;

K_{SR} = uncorrected short rod toughness defined in equation (7);

K_{SR}^{pl} = plasticity corrected short rod toughness defined in equation (10);

J_{Ic} = energy rate resistance measure of fracture toughness (of metallic materials), originally as per ASTM standard E 813;

$K_{Ic} = (J_{Ic} E / (1 - \nu^2))^{0.5}$ = stress intensity equivalent of J_{Ic} ;

m_0 = initial tangent slope of load vs CMOD or LPD curve;

Δm = slope offset;

p = plasticity correction factor defined in equation (9);

$\Delta x, \Delta x_0$ = displacement measures used in defining p ;

Δa = crack growth increment;

H = slope offset function in equation (14);

w_s = specimen width for which 5% offset secant method is sufficient;

σ_t = uniaxial tensile strength of rock;

σ_c = uniaxial compressive strength of rock;

σ_y = yield stress of metals;

σ_f = homogeneous fracture stress.

INTRODUCTION

A fracture mechanics approach has recently been introduced in rock mechanics. Most investigations have concentrated on the direct measurement of fracture parameters for material properties [1]. The fracture toughness of a material expresses its resistance to crack propagation or the fracture energy consumption rate required to create new surfaces. Some applications of such values for rock are as

- (i) a parameter for classification of rock material,
- (ii) an index of fragmentation processes such as tunnel boring and model scale blasting, and

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(iii) a material property in the modelling of rock fragmentation like hydraulic fracturing, explosive stimulation of gas wells, radial explosive fracturing, and crater blasting as well as in stability analysis, see Ouchterlony and Sun [2] for references.

Unfortunately, no standard method exists for the fracture toughness testing of rock. Two principal approaches may be chosen. In the first, one adheres closely to the ASTM recommendations for metallic materials, Schmidt [3] and Costin [4]. This yields acceptable results for hard rock but requires unreasonably large and impracticably shaped (usually rectangular) specimens, as well as a testing procedure that is not adapted to rock.

The other approach presumes that a core based specimen type is the only viable alternative for rock. In early work, Ouchterlony [5] and Swan [6, 7], the fracture toughness evaluation was performed on notched virgin specimens without pre-cracking, using only the failure load and initial notch depth to yield an approximate fracture toughness value. This makes both specimen preparation and test procedure simple, but it was found that the fracture toughness values depended on notch depth and that they were too low. This paper emphasizes the improvement of test procedures for core based (round) specimens. It is based on parts of a recent thesis [8], and addresses questions concerning specimen size requirements, if one can ignore the pre-cracking of specimens before testing, and what notch depth should be used in the fracture toughness testing of rock. These matters have also been discussed previously [1, 2, 9].

Rock is an inherently variable material and often anisotropic. The use of three different specimen types in this work serves as a check on the consistency of the testing procedures and the material, when results are independent of specimen type. The experiments reported here were made on Stripa granite. Its elastic properties are: $E = 65\text{--}70\text{ GPa}$, $\nu = 0.12$, and $\sigma_c = 208\text{ MPa}$ [10]. Sun's thesis [8] also contains fracture toughness data for other rocks.

EXPERIMENTS AND TEST PROCEDURES

The test procedures presented here are for producing representative plane strain fracture toughness values in stress intensity terms. The three round specimen configurations used were a notched beam bend specimen, a round compact specimen, and the short rod.

Three point bending tests

Three point bending tests are commonly used for fracture toughness testing of rock, see Schmidt [3, 11], Costin [4], Ouchterlony [5] and Swan [6, 7] e.g. The ASTM influenced testing of Schmidt and Costin requires fatigue pre-cracking of the specimens (ASTM Standard Test Method E 399 for Plane Strain Fracture Toughness of Metallic Materials). Ouchterlony and Swan made their tests without pre-cracking and used the failure load, $F_{\max}$, instead of the conditional load, F_Q , and an initial notch depth, a_0 , to obtain an approximate fracture

Fig. 1. Geometry of the single edge crack round bar in three point bending configuration. D = diameter of round bar specimen, 41.6 mm; L = support span, $L/D = 3.33$; a = crack or notch length; b = crack front length; F = load.

toughness value from

$$K_m = K_I(a_0, F_{\max}). \quad (1)$$

The geometry of the single edge notch round bar specimen they used is shown in Fig. 1. The stress intensity factor formula is given by

$$K_I = 0.25(L/D)Y' \cdot F/D^{1.5} \quad \text{where} \quad (2)$$

$$Y' = 12.7527\alpha^{0.5}(1 + 19.646\alpha^{4.5})^{0.5}/(1 - \alpha)^{0.25} \quad (3)$$

is given by Bush [12] and Ouchterlony [5] and $\alpha = a/D$. Y' is valid when $0 \leq \alpha \leq 0.6$.

The diameter of the specimens used here was 41.6 mm with $L/D = 3.33$. Notch depths varied from 1.5 to 20 mm. A clip gauge was mounted across the notch on the beam and provided the feedback signal to the controller of the servo-hydraulic testing machine. The experimental set up is described in more detail by Ouchterlony *et al.* [13] and Sun [8]. A single K_m value was obtained from each notched specimen by loading it to complete failure by increasing the CMOD at the constant rate of 0.05 $\mu\text{m/sec}$.

The second procedure presented uses notched specimens that are pre-cracked by a single load cycle. Each successive cycle then gave an apparent fracture toughness value which was evaluated with the 5% offset secant method of E 399, see Fig. 2, and in analogy denoted by K_{Qm}

$$K_{Qm} = K_I(a_i, F_{Qi}) \quad (4)$$

where a_i is the crack length of load cycle i , and F_{Qi} is the associated secant load. The typical load vs LPD curve in Fig. 2 shows the initial curving (as the crack opens), a linear portion (where the crack is fully open), the curving again (as inelastic behaviour develops at the crack tip zone), followed by stable crack growth as the load drops, and finally unloading, cf. Ouchterlony [9]. Four Stripa granite specimens with $D = 41.6\text{ mm}$ were used in the K_{Qm} testing.

The crack length was determined from the LPD compliance measurements as follows: the first (pre-cracking) load cycle yields the Young's modulus of the

Fig. 2. Load Point Displacement (LPD) curve for three point bend test on specimen NSG 1. The first load cycle is used to pre-crack the specimen and to calculate K_m . Subsequent load cycles are used to calculate K_{Qm} values.

specimen from the initial tangent compliance, the best linear loading compliance of each successive cycle then gave the equivalent crack length ratio a/D at that stage from an inversion of the compliance formula, see Sun [8] for details. The LPD measurement method was developed by Ouchterlony [5] and has proven to yield accurate Young's modulus results when compared with strain gauge measurements, see [13].

Round compact tension tests

The geometry of this specimen is shown in Fig. 3, it has hole sizes and locations of the holes that are only slightly different from the standard geometry proposed in E 399. The diameter was 151 mm and the initial notch depth varied between 53 and 56 mm. Three out of seven tests on Stripa granite were successful. Again the tests

Fig. 3. Geometry of compact tension specimen. D = diameter of round compact specimen, 151 mm; a_0 = initial notch length; w = specimen width, 112 mm; $D/w = 1.35$; d = diameter of holes, 25 mm; $d/w = 0.22$; l = hole offset, $l/w = 0.30$; B = specimen thickness.

Fig. 4. Typical load vs Crack Mouth Opening Displacement (CMOD) record for loading to failure of a compact tension test on Stripa granite.

were run in strain control with a clip gauge spanning the mouth of the notch. The details of the set up are given by Sun [8]. CMOD was increased at the rate of $0.06 \mu\text{m}/\text{sec}$ during loading.

Here both K_m and the apparent fracture toughness K_{Qm} was evaluated, using the single load cycle pre-cracking, the 5% offset secant method, and CMOD (instead of LPD) compliance to measure crack length as described previously. A typical load vs CMOD record is shown in Fig. 4. K_{Qm} was calculated from equation (4), using the K_I formula provided by Newman [14].

$$K_I = f(\beta) \cdot F / (Bw^{0.5}) \quad (5)$$

where

$$f = (2 + \beta) \cdot (0.76 + 4.8\beta - 11.58\beta^2 + 11.43\beta^3 - 4.08\beta^4) / (1 - \beta)^{1.5} \quad \text{and} \quad \beta = a/w. \quad (6)$$

The details of the compliance evaluation, that gives the effective crack length, are given by Sun [8]. As will be seen, the size of these specimens is so close to the LEFM size requirements that the K_{Qm} values obtained should be accurate estimates of K_{Ic} , the plane strain fracture toughness.

Short rod tests

There is a growing interest in the use of chevron notched specimens for fracture toughness testing of brittle materials like rock and ceramics. Barker [15] first introduced this method to test rocks like siltstone and limestone, later Costin [4] used it on oil shale.

The initial crack growth in a chevron notched specimen is stable. Then there is a minimum in the stress intensity curve, $K_I(a, F = \text{constant})$, at a specific crack length, $a = a_c$ say, beyond which the crack extension tends to become unstable in a soft testing machine, provided that the crack extension resistance of the material is constant. The position of a_c is very nearly independent of the material properties. The short rod toughness K_{SR} was originally defined as

$$K_{SR} = K_I(a_c, F_{\max}) = K_{I\min}(F_{\max}) = A_{\min} \cdot F_{\max} / D^{1.5} \quad (7)$$

where the most recent value of $A_{\min}$ is 23.4 for a specimen with $w = 1.45D$ and $a_0 = 0.48D$, Barker [16].

The geometry of the short rod specimen used here is shown in Fig 5. The specimen diameters ranged from 25 to 45 mm, keeping the ratio w/D fixed at 1.5. In this case

Fig. 5. Geometry of short rod specimen. D = diameter of short rod specimen; w = specimen width; $w/D = 1.5$; a_0 = initial crack length (notch depth), measured from line of load application; a_1 = distance from load line to end of chevron; β = chevron half angle; approx. 27° .

the value of $A_{\min}$ is given by

$$A_{\min} = [12.37 + 110.1a_0/w - 285.2(a_0/w)^2 + 338.5(a_0/w)^3]/(w/D)^{0.5} \quad (8)$$

which follows from Bubsey *et al.* [17] when $w/D = 1.5$. Equation (8) is valid when the chevron apex a_0 is in the interval $0.2w$ to $0.5w$. The width of the slot was roughly 2 mm.

The advantages of the short rod specimen are that

- (i) it requires no fatigue pre-cracking since the specimen produces its own pre-crack prior to the evaluation point $F_{\max}$,
- (ii) neither crack length nor displacement measurements seem necessary since the suggested evaluation point a_c depends only on specimen geometry, and
- (iii) the suggested evaluation method requires only knowledge of the specimen diameter and the failure load in a soft testing machine. The inherent monotonic pre-cracking has the effect that K_{SR} values may be somewhat higher than K_Q ones [16].

Barker's work [18] showed that K_{SR} values for Indiana limestone depend on specimen size. He viewed this as an effect of a non-negligible microcrack growth zone at the tip of the crack and developed a "plasticity" correction

procedure to account for it [18, 19]. He defines the degree of plasticity as

$$p = \Delta x_0 / \Delta x \quad (9)$$

and then a plasticity corrected short rod toughness value as

$$K_{SR}^{pl} = \left(\frac{1+p}{1-p} \right)^{0.5} \cdot (F_s/F_{\max}) \cdot K_{SR} \quad (10)$$

where the evaluation load F_s should correspond to a_c . It is thus defined as the average load of that load cycle which spans the secant slope value of $0.5m_0$. The principle is shown in Fig. 6, which gives a typical load vs CMOD curve for Stripa granite from our tests. Again they were run in strain control with a clip gauge across the notch mouth to provide the feedback signal. Note that CMOD and LPD nearly coincide for the short rod.

FRACTURE TOUGHNESS RESULTS

Presentation of results

The K_m values obtained in the present three point bending tests are summarized in Table 1. They show that K_m may depend on notch depth. Due to their inherent scatter they were, however, analysed statistically in order to reach a conclusion, see the Appendix.

This analysis of the Stripa granite data shows that K_m is significantly lower than the mean when the notch length ratio is small, less than 0.1 say. On the other hand, the Ekeberg marble data of Ouchterlony [20] indicates that G_m , the energy consumption rate equivalent of K_m , is significantly lower when a_0/D exceeds 0.4. Both sets of data show an almost constant value of the fracture parameter when $0.2 < a_0/D < 0.4$. Thus, it can be concluded that the approximate fracture toughness K_m varies qualitatively with a_0/D for a given specimen size as depicted in Fig. 7.

To explain these results, the general relation between fracture stress σ_f and crack size for a metal specimen in Fig. 8 is used as an analogy. For long cracks, σ_f varies with crack length as $1/a^{0.5}$. On the other hand, K_I is given by $\sigma_f \cdot (\pi a)^{0.5}$, implying constant critical K_I or K_{Ic} if the crack is long enough.

With decreasing crack length, σ_f increases until it approaches the yield strength and widespread plastic deformation of the structure occurs before failure. It is

Fig. 6. Load vs CMOD from test on short rod specimen SR5 of Stripa granite.

Table 1. Approximate fracture toughness K_m for Stripa granite. From three point bend tests. $D = 41.6$ mm

a_0 [mm]	K_m [MN/m ^{1.5}]	a_0 [mm]	K_m [MN/m ^{1.5}]	a_0 [mm]	K_m [MN/m ^{1.5}]
9.0	1.52	12.0	1.86	14.0	1.57
9.9	1.63	12.0	1.61	15.6	1.49
9.9	1.80	12.0	1.74	16.0	1.68
10.0	1.99	14.0	1.82	16.0	1.82
10.15	1.88	14.0	1.60		

Table 2. Fracture toughness data from three point bend tests on Stripa granite. Values in MN/m^{1.5}. $D = 41.6$ mm

Specimen	NSG 1 ^a	NSG 2 ^a	NSG 3 ^b	NSG 4 ^b
K_m —pre-crack cycle	2.01	2.02	1.60	1.80
K_{Qm} —1st cycle	2.29	2.30	2.05	2.21
K_{Qm} —2nd cycle	2.376	2.343	2.043	2.365
—3rd cycle	2.442	2.487	—	—
—4th cycle	2.374	2.513	—	—
—5th cycle	2.361	2.504	—	—
Mean $\pm$ SD	2.39 $\pm$ 0.04	2.46 $\pm$ 0.08		

^a $a_0 = 12.1$ mm.^b $a_0 = 20.0$ mm.

reasonable to assume a similar fracture stress vs crack length relation based on microcracking for rock [22]. Thus if the notch depth in a rock specimen is small, then a large portion of the specimen will undergo microcracking and thereby increase the effective notch length before the stress in the rock is sufficient to extend the crack. The LEFM based equation to calculate a critical K_I then no longer applies. This would explain why low K_m values were obtained when the initial notch depth was too small.

The reduction in K_m when a_0 becomes larger is due to the small ligament width $D - a$. The proximity of stress-free boundaries significantly affects the “ K -description” of the stress field at the crack tip, tending to invalidate it [23]. Thus, only a K_m value obtained from tests in a certain range of notch lengths can be considered as a representative one for a given specimen size. For Stripa granite, it is 1.74 ± 0.14 MN/m^{1.5}, see equation (A4).

The K_{Qm} results from the three point bend tests are summarized in Table 2. Five values were obtained from each of the specimens with $a_0 = 12$ mm, only two from those with $a_0 = 20$ mm because of the limited range of the compliance equations [8]. The K_m values from the pre-cracking load cycles are also given in Table 2 for comparison.

The K_{Qm} results from the round compact tension tests are summarized in Table 3. Just as for the bend tests, a K_{Qm} value obtained from the first load cycle on a pre-cracked specimen is lower than the values obtained from subsequent loadings. These latter values are averaged and may give a better representation of the actual fracture toughness. Somewhat surprisingly, the K_{Qm} values from the bend tests with $D = 41.6$ mm and $a_0 = 12$ –20 mm are not significantly lower than those from the round compact tension tests for which $D = 151$ mm and $a_0 = 53$ –56 mm. The results from the

Fig. 7. Qualitative variation of fracture toughness K_m with notch ratio, a_0/D , for a given specimen diameter. Actual data refers to specimens with $D = 41$ –42 mm.

Fig. 8. The general relation between fracture stress and crack length for metals. After Irwin [21].

that the results of Ouchterlony [5] relate to LPD compliance rather than CMOD compliance. The pertinent equation reads

$$\Delta m/m_0 = H(a/D) \cdot \Delta a/a \quad (14)$$

where $H(a/D)$ may be expressed in terms of the LPD compliance. Later results yield a similar expression in terms of CMOD compliance [9]. A 5% offset secant in LPD compliance corresponds to 5.9% $\Delta a/a$ when $a/D = 0.3$ and to 2% $\Delta a/a$ when $a/D = 0.5$. With $D = 41$ mm, we thus have $\Delta a = 0.7$ mm in the first case and $\Delta a = 0.4$ mm in the second one. This equation may thus be used together with the variable secant method above to keep Δa constant.

Since the round bend tests presented here cover the crack (notch) length ratio range 0.3 to about 0.5, the 5% offset secant used in the K_{Qm} evaluation overcompensates for short crack lengths. On the other hand, Fig. 2 shows that a small change in the percentage of the offset secant does not change the resulting K_{Qm} value much. This implies that neither the 5% offset secant nor the variable secant need really be used. It would even be possible to consistently use F_{max} instead of F_Q in the evaluation, thereby making it simpler.

Comparison of fracture toughness results

It is very important to note the consistency between the K_{Qm} values obtained from the round three point bending tests and those obtained from the round compact tests, cf. Tables 2 and 3. Since the round compact tests appear to fulfil reasonable LEFM size requirements, this implies that the K_{Qm} values obtained from the round bend tests with a diameter of about 40 mm will be very close to a proper K_{Ic} value. The K_{Qm} values are thus acceptable and to be able to obtain a good estimate of K_{Ic} even for specimens taken from common core sizes is indeed valuable. K_{Qm} for Stripa granite is about 2.2–2.4 MN/m^{1.5}.

The average value of K_m obtained from the round three point bending tests on Stripa granite in Tables 1 and 2 is 1.75 MN/m^{1.5}, that from the round compact tension ones in Table 3 is 1.73 MN/m^{1.5}. These values are also consistent, but more important, they are significantly lower than the corresponding K_{Qm} values. This discrepancy arises despite the fact that F_Q usually is somewhat lower than F_{max} , see e.g. Table 3. It is commonly attributed to the use of the initial notch length a_0 to calculate K_m in a situation where there is substantial microcrack growth preceding crack extension [9].

It is well known that the stress-strain behaviour of rock in tension is often both non-linear and inelastic and that this behaviour arises from the growth of microcracks, see Wawersik [28], Peng [29] and Schmidt and Lutz [26]. The successful application of LEFM to such a material arises from the fact that the fracture testing is made on pre-cracked specimens that have a fully developed microcrack zone at the crack tip [26]. The discrepancy between K_m and K_{Qm} illustrates the importance of taking this zone into account here, but in

order to do that crack length measurements are necessary. Here a compliance technique was used which requires accurate LPD or CMOD measurements.

Turning to a consideration of the short rod tests, the uncorrected K_{SR} values lie 10–20% below the corresponding plasticity corrected ones. The differences between the K_{SR}^{pl} values for specimens of different sizes is very small, however, and they may therefore be considered to represent the same material property. Analysis of the results in Table 4 clearly shows that the plasticity factor p increases with decreasing specimen size. Furthermore, K_{SR}^{pl} is almost identical to K_{SR} when p is small, as shown by sample 45SR2 with 45 mm diameter. Thus, specimen sizes slightly larger than this may give practically zero plasticity and require no correction. Since the K_{SR}^{pl} values discussed here are in good agreement with the previous K_{Qm} values they may also be good measures of K_{Ic} .

The practical implication of this is that no displacement measurements would be needed in order to obtain good estimates of K_{Ic} for rock, if short rod specimens of moderate sizes were used. If a plasticity correction turns out to be necessary, then the relative advantages of the short rod specimen are no longer as pronounced since this requires that both force and displacement measurements be continued well beyond maximum load. This is only feasible in a stiff testing machine or in a servo-controlled one.

The advantage of K_{SR}^{pl} over K_{Qm} would then be 2-fold:

- (i) no pre-cracking required; and
- (ii) in the evaluation of the former the displacement measurements are used in a simple relative fashion whereas for the latter a more involved crack length determination is made. Yet, if energy consumption rate toughnesses and Young's moduli are required then accurate quantitative displacement measurements are necessary in both cases.

CONCLUSIONS

The results presented in this paper are summarized as follows:

(1) A statistical analysis showed that constant K_m values exist for a certain range of notch ratios, which may vary with rock type and specimen size in the round three point bend test. For specimens with about 40 mm diameter, the range is $0.2 < a_0/w < 0.4$ for Stripa granite or slightly wider for Ekeberg marble. Outside this range, the K_m values are invalid because either crack length or ligament width are too small. Only a K_m value within this range can be considered as a representative one for a given size of specimen.

(2) K_m values evaluated from either round three point bend specimens or round compact tension ones without pre-cracking are invariably lower than K_{Qm} values from the same, pre-cracked, specimens. The reason is that K_m is evaluated using an initial notch length which ignores microcrack growth prior to crack extension.

(3) Values of K_m and K_{Qm} from the round three point

bend specimens with 41 mm diameter are in good agreement with the corresponding fracture toughness values from the round compact tension specimens with 151 mm diameter. This shows that the testing procedures are consistent. The latter specimens fulfil the LEFM size requirements with adjusted prefactor whereas the former ones do not.

(4) It is possible to obtain acceptable K_{Qm} values for Stripa granite from round three point bend tests on common core sizes of about 40 mm in diameter, in the sense that they are good estimates of K_{Ic} . Here a notch depth ratio of about 0.3 was used.

(5) The plasticity corrected short rod toughness K_{SR}^{pl} , obtained from Stripa granite specimens of different sizes, are effectively identical and consistent with K_{Qm} from the round compact and three point bend specimen.

(6) In order to obtain consistent acceptable fracture toughness values from different test methods, the non-linear and inelastic behaviour of the rock must be taken into account. One possibility used here was to pre-crack a notched specimen with a single load cycle before the testing. Another is to use a specimen type, here the short rod, which produces its own pre-crack during the testing.

(7) For fracture toughness testing of rock, there are different requirements regarding specimen thickness, crack length and ligament width. The thickness does not affect the fracture toughness. For K_m toughness, the ligament width is more important than crack length. For K_{Qm} toughness, LEFM arguments tend to give equal weight to them. Since the round three point bend tests give acceptable K_{Qm} values, the tendency is again that ligament width is more important than crack length.

(8) The most attractive method, of the ones used here, for fracture toughness testing of rock is the short rod test. No pre-cracking is required and the plasticity corrected toughness K_{SR}^{pl} of Stripa granite from a specimen as small as 25 mm diameter is consistent with that from larger ones. Present short rod testing has the short-coming of relatively complicated specimen preparation. Thus the chevron notched round beam, Ouchterlony [30, 31], which combines the advantages of the bend beam and the short rod should receive more attention in the future. Fracture toughness data obtained from such specimens are now emerging in the literature [2, 32].

These conclusions are based mainly on test results for Stripa granite and their validity to other rocks, especially soft ones, is therefore necessarily restricted.

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APPENDIX

Statistical Analysis of K_m from Three Point Bending Tests

First an analysis of data for Stripa granite is made. Previous data obtained by Swan [6, 7] and Ouchterlony *et al.* [13] were pooled with the present data. Altogether 36 values are given in Table A1, grouped with respect to notch depth ratio.

Data from groups 3-6 are analyzed first. Tukey's method, Box *et al.* [33] is used to compare these four groups of samples. The confidence limits for the difference between group means, $\eta_i - \eta_j$, are given by

$$\bar{y}_i - \bar{y}_j \pm \frac{g(k, v_n, \alpha/2)}{\sqrt{2}} s \cdot \sqrt{\frac{1}{n_i} + \frac{1}{n_j}} = \bar{y}_i - \bar{y}_j \pm \Delta_{ij} \quad (A1)$$

Here $g(k, v_n, \alpha/2)$ is the studentized range for k means at appropriate upper significance level α ; $v_n = N - k$ is the number of degrees of freedom in the estimate s^2 of variance σ^2 ; $\bar{y}_i, \bar{y}_j$ are the averages in groups i or j ; η_i, η_j are the population means for groups i or j ; n_i, n_j are the number of data within the groups i or j ; and k is the number of groups.

In equation (A1) above, the pooled variance for our groups is

$$s^2 = [(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + (n_3 - 1)s_3^2 + (n_4 - 1)s_4^2]/(N - k) \quad (A2)$$

where the group variances are given by

$$s_m^2 = \sum_{i=1}^{n_m} (y_{mi} - \bar{y}_m)^2 / (n_m - 1) \quad (A3)$$

for each $m = 1, 2, 3$ and 4. Here N is the total number of data points.

In our case $k = 4$, $v_n = 15$, $s^2 = 0.221$ or $s = 0.149$, and $\alpha = 0.05$ which corresponds to 95% confidence limits. Thus $g(4, 15, 0.025) = 4.08$. See e.g. *Biometrical Tables for Statisticians*, Cambridge University Press (1970). Hereby $\Delta_{34} = \Delta_{45} = 0.259$, $\Delta_{35} = 0.271$, $\Delta_{36} = \Delta_{56} = 0.313$, and $\Delta_{46} = 0.303$.

Any difference between group averages, $\bar{y}_i - \bar{y}_j$, greater in absolute value than the corresponding Δ_{ij} value, can be considered statistically significant. It is obvious from Table A1, however, that for $i, j = 3, 4, 5$ and 6 this is not the case. Thus, the four group means are considered to represent the same fracture toughness value, of which a pooling of these groups yields a better estimate.

Next the pooled group, denoted by index p , was compared with groups 1 and 2. Group statistics are

$$n_p = 19, \bar{y}_p = 1.736 \quad \text{and} \quad s_p = 0.143. \quad (A4)$$

The relevant parameters become $N = 36$, $k = 3$, $v_n = 33$, and $s^2 = 0.0158$ or $s = 0.126$. With $\alpha = 0.05$ and $g(3, 33, 0.025) = 3.475$ we obtain $\Delta_{12} = 0.157$, $\Delta_{1p} = 0.117$ and $\Delta_{2p} = 0.145$. Further $\bar{y}_2 - \bar{y}_1 = 0.390$, $\bar{y}_p - \bar{y}_1 = 0.518$ and $\bar{y}_p - \bar{y}_2 = 0.128$. Thus, at least group 1 is significantly different from the other two. Group 2 is just barely within limits when compared with the pooled group.

Next an analysis of some fracture toughness data $G_m = (1 - \nu)^2 \cdot K_m^2/E$ for Ekeberg marble, reported by Ouchterlony [20] is made. Altogether 47 values are given in Table A2. The first two steps show that groups 1-3 can be considered to be identical, as can groups 4 and 5, at the same significance level as before. Since the procedure is the same as for Stripa granite, these steps are omitted.

Pooling of groups 1-3, index q , and of groups 4 and 5, index r , yields the new group statistics

$$\left. \begin{aligned} n_q = 27, \bar{y}_q = 22.00 \quad \text{and} \quad s_q = 1.73; \\ n_r = 10, \bar{y}_r = 19.46 \quad \text{and} \quad s_r = 1.52. \end{aligned} \right\} \quad (A5)$$

A comparison of groups q, r and 6 is based on $k = 3$, $v_n = 44$, $s^2 = 2.51$ or $s = 1.58$, $\alpha = 0.05$ and $g(3, 44, 0.025) = 3.432$. Thus, $\Delta_{qr} = \Delta_{q6} = 1.42$ and $\Delta_{r6} = 1.71$. Since Table A2 and equations (A5) show that the values $\bar{y}_q - \bar{y}_r = 2.54$, $\bar{y}_q - \bar{y}_6 = 3.68$ and $\bar{y}_r - \bar{y}_6 = 6.22$ are all larger than their corresponding Δ_{ij} values, all three groups are considered to be independent.

Tables A1 and A2 on facing page (p. 409)

Table A1. Approximate fracture toughness, K_{Ic} , of Stripa granite with group statistics (mean no. tests $\pm$ SD)

Group 1		Group 2		Group 3		Group 4		Group 5		Group 6	
$a_0/D = 0.032-0.058$	K_{Ic}	$a_0/D = 0.115-0.199$	K_{Ic}	$a_0/D = 0.22-0.24$	K_{Ic}	$a_0/D = 0.26-0.29$	K_{Ic}	$a_0/D = 0.33-0.36$	K_{Ic}	$a_0/D = 0.37-0.385$	K_{Ic}
[mm]	[MN/m ^{1/2}]	[mm]	[MN/m ^{1/2}]	[mm]	[MN/m ^{1/2}]	[mm]	[MN/m ^{1/2}]	[mm]	[MN/m ^{1/2}]	[mm]	[MN/m ^{1/2}]
1.5	1.08	4.8	1.71	9.0	1.52 ^b	11.8	1.62	13.8	1.85	15.6	1.49 ^b
1.5	1.20	4.8	1.54	9.0	1.63 ^b	11.8	1.91	14.0	1.82 ^a	16.0	1.68 ^b
1.7	0.99	6.4	1.64	9.9	1.80 ^a	12.0	1.84	14.0	1.60 ^b	16.0	1.82 ^b
1.7	1.21	8.0	1.74	10.0	1.99 ^a	12.0	1.86 ^b	14.0	1.57 ^b		
2.0	1.26	8.0	1.51	10.15	1.88 ^a	12.0	1.61 ^b	14.8	1.76		
2.0	1.32	8.3	1.51			12.0	1.74 ^b				
2.4	1.19										
2.4	1.30										
2.4	1.23										
2.4	1.26										
2.4	1.36										
1.218 ₁₁ $\pm$ 0.106		1.608 ₆ $\pm$ 0.103		1.764 ₅ $\pm$ 0.189		1.763 ₆ $\pm$ 0.128		1.720 ₅ $\pm$ 0.128		1.663 ₃ $\pm$ 0.166	

^a Tests made at SveDeFo [13]. $D = 41.6$ mm.^b Tests made at Luleå University of Technology. $D = 41.6$ mm.Remaining data from Swan [6, 7]. $D = 41.6, 45$ or 52 mm.Table A2. Approximate energy resistance rate, G_{Ic} , for Ekeberg marble with group statistics (mean no. tests $\pm$ SD)

Group 1		Group 2		Group 3		Group 4		Group 5		Group 6	
$a_0/D = 0.13-0.19$	G_{Ic}	$a_0/D = 0.253-0.30$	G_{Ic}	$a_0/D = 0.36-0.39$	G_{Ic}	$a_0/D = 0.4-0.416$	G_{Ic}	$a_0/D = 0.47-0.49$	G_{Ic}	$a_0/D = 0.55-0.59$	G_{Ic}
[mm]	[J/m ²]	[mm]	[J/m ²]	[mm]	[J/m ²]	[mm]	[J/m ²]	[mm]	[J/m ²]	[mm]	[J/m ²]
5.29	22.05	10.47	25.09	15.02	23.79	16.68	19.64	19.29	18.46	23.31	15.94
5.69	21.44	10.52	23.24	15.30	21.76	16.68	20.79	19.51	21.48	23.29	15.74
5.69	23.53	10.72	24.37	16.02	19.89	16.69	17.56	20.20	18.59	23.39	14.53
5.71	20.71	10.72	19.98			16.81	21.64			23.39	17.54
5.71	20.11	11.40	23.30			16.89	20.25			23.52	14.58
5.81	20.92	11.51	23.24			16.89	18.48			23.54	16.44
5.92	20.07	11.62	21.61			17.22	17.69			23.58	17.44
6.19	20.84	11.71	20.52							23.59	16.00
6.39	21.89	11.71	19.07							23.72	15.40
6.81	24.65	11.79	24.80							24.22	14.18
7.90	20.62	11.90	23.40								
		12.00	20.67								
		12.40	22.40								
21.53 ₁₁ $\pm$ 1.44		22.44 ₁₃ $\pm$ 1.92		21.81 ₃ $\pm$ 1.95		19.44 ₇ $\pm$ 1.58		19.51 ₃ $\pm$ 1.71		15.78 ₁₀ $\pm$ 1.16	