The Swebrec[®] function: linking fragmentation by blasting and crushing

Finn Ouchterlony

A new three-parameter fragment size distribution function has been found that links rock fragmentation by blasting and crushing. The new Swebrec[®] function gives excellent fits to hundreds of sets of sieved fragmentation data with correlation coefficients of 0.997 or better ($r^2 > 0.995$) over a range of fragment sizes of 2–3 orders of magnitude. A five-parameter version reproduces sieved fragmentation curves all the way into the $-100\text{ }\mu\text{m}$ range and also handles ball mill grinding data. In addition, the Swebrec[®] function: (i) can be used in the Kuz–Ram model and removes two of its drawbacks – the poor predictive capacity in the fines range and the upper limit cut-off to block sizes; (ii) reduces the JKMRC one-family description of crusher breakage functions based on the t_{10} concept to a minimum; and (iii) establishes a new family of natural breakage characteristic (NBC) functions with a realistic shape that connects blast fragmentation and mechanical comminution and offers new insight into the working of the Steiner's OCS sub-circuits of mechanical comminution. It is suggested that the extended Kuz–Ram model, with the Swebrec[®] function replacing the Rosin–Rammler function, be called the KCO model.

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BLAST FRAGMENTATION

In an area as complicated as rock fragmentation by blasting, Cunningham's Kuz–Ram model^{4,5,13} has for 20 years done an invaluable job in fostering a structured approach to what can be done to change the fragmentation pattern. Much experience has gone into the equations that describe the defining parameters, the median or 50% passing size x_{50} and the uniformity exponent n of the underlying Rosin–Rammler cumulative fragment size distribution in Figure 1:

$$P(x) = 1 - e^{-\ln 2 (x/x_{50})^n} = 1 - 2^{-(x/x_{50})^n} \quad (1)$$

The Kuz–Ram model does not cover all aspects of blasting and was never meant to do so. Timing is one such area, the fines part of the muckpile another. On a more detailed level, the effect of single row blasts is included by a constant prefactor (1.1) and the finite boulder limit is taken care of by a cut-off in the infinite distribution.

The defining equations $x_{50} = h_1$ (rock mass, scale, explosive and specific charge) and $n = h_2$ (geometry) decouple geometry from the other quantities that describe the blast, which is pedagogically attractive. A recent finding²⁴ suggests, however, that the expression x_{50} should involve the prefactor $(\ln 2)^{1/n}/\Gamma(1 + 1/n)$, where Γ is the gamma function. Its effect is to raise the predicted amount of fine material in well-graded muckpiles, i.e. when n is small.

It has gradually become clear that very few sieved fragment size distributions follow Equation (1), especially in the fines range. The JKMRC has come up with two models that address this problem – the crush zone model⁹ (CZM) and the two-component model⁷ (TCM).

In the CZM, the fragment size distribution is made up of two parts, a coarse one given by the Kuz–Ram model that corresponds to tensile fracturing and a fines part that is derived from the compressive crushing around a blast-hole. The fines distribution is also of Rosin–Rammler type but with different values primarily for n and the characteristic size x_c (x_{50} now refers to the combined fine and coarse parts).

The two parts are mutually exclusive so the resulting fragment size distribution has a knee at the grafting point (see Fig. 2) and the CZM refers all -1 mm fines to the blast-hole region. In this way, the effects of quantities like the rock's compressive strength, blast-hole pressure and VOD enter the prediction equations.

In the TCM,⁷ the fragment size distribution is also made up of two Rosin–Rammler distributions that represent the tensile and compressive fracture modes. The two populations co-exist over the whole range so the result is a smooth curve (see Fig. 2). Again, the Kuz–Ram model gives the coarse part but the fines part is obtained, for example, from scaled-up model blasts.

The effect is that the TCM is a five-parameter model; two sets of x_c and n and one parameter that determines the ratio of the two populations. The CZM is a four-parameter model. This makes their predictions more realistic than the Kuz–Ram model at the price of

1 Fragment size distribution of blasted hornfels from Mount Coot-tha quarry¹⁰ with Rosin–Rammler curve fit.²¹ Data range = 0.35–2000 mm. Curve fit parameters $x_{50} = 115.8$ mm, $n = 0.572$. Coefficient of determination $r^2 = 0.9958$

complexity. The JKMRC prefers the CZM and they have used it successfully in a number of fragmentation projects related to their Mine-to-mill concept.^{8,23}

Meanwhile, experimental evidence has emerged²² that clearly contradicts the idea that in massive rock all but a negligible amount of fines are generated in a crushed zone around a blast-hole. In one case,²⁷ Ø 300-mm diameter, 100-kg mortar specimens with concentric coloured layers were shot to produce 2 kg of –1 mm fines, 1 kg from the Ø 120-mm diameter inner layer and as much from the outer layer. The percentage of fines is, of course, higher in the inner layer but what matters in practice is the total

amount. More evidence to this effect is provided in the work of Moser et al.¹⁷ and Micklautsch.¹⁴

CRUSHING FRAGMENTATION

Traditionally, a quarry or mine produces rock on the ground for a plant to process. An eye is kept on hauling and oversize with much less focus on the rest of the blasted fragment-size distribution. This is changed with the Mine-to-mill or drill-to-mill approaches. Still, what blasting and crushing fragmentation or comminution have in common has been neglected.

2 Comparison of fragment size distributions in CZM⁹ and TCM.⁷ Note different characters of curves but same linear behaviour in $\log P$ versus $\log x$ space in fines range

3 Fragmentation curves from crushing and grinding of amphibolite in OCS sub-circuits^{3,17}

Steiner's^{25,26} approach to mechanical comminution is based on the concept that a material, which is fractured under 'pure' conditions exhibits a material-specific 'natural breakage characteristic' (NBC). Some of the tenets are that:

- Rock broken in the crushing and grinding sub-circuits of an 'optimum comminution sequence' (OCS) has the steepest possible cumulative fragmentation curve $P_{NBC}(x)$.
- When the sub-circuit product streams are classified, the fragmentation curves are shifted vertically upward as the comminution progresses.
- When plotted in log-log space, this basically becomes a parallel shift so that the local slope depends only on the fragment size (Fig. 3).

Steiner's approach also contains the energy register concept, which says that when the specific surface ($\text{m}^2 \text{kg}^{-1}$) created by an OCS is plotted versus the energy consumed (J kg^{-1}), the points fall more or less on a material specific straight line. The slope R ($\text{m}^2 \text{J}^{-1}$) is

equal to the Rittinger coefficient of comminution (Fig. 4). Points that represent practical comminution circuits tend to fall below this line.

This concept was recently expanded to cover model scale blasting with different specific charges,¹⁶ with further support coming from the work in the Less Fines project¹⁵ (Fig. 5). It may even carry over to boulder blasting and full-scale bench blasts (Fig. 6).^{18,20}

The JKMRC¹⁹ engineering-oriented approach to crushing fragmentation describes comminution circuits and associated individual crushing and grinding breakage functions. The approach uses a matrix description of the product flow through the system and a one-family description of breakage functions based on the t_{10} concept. t_{10} is that part of a given size fraction of material subjected to a drop weight crushing test, which afterwards is smaller than a tenth of the original feed material's size.

An exponential curve with two material parameters relates the measure t_{10} to the drop energy. This is, in

4 Energy register curves^{3,15} for limestones and the Hengl amphibolite of Figure 3

5 Comparison of fragmentation curves from OCS comminution in Figure 3 and model-scale blasts on same amphibolite¹⁵

principle, equivalent to the Kuz–Ram model equation, which equates x_{50} with the explosive charge as a source of fragmentation energy. The family of breakage functions is usually plotted as in Figure 7; but, from vertical lines of constant t_{10} values, the associated fragment size distributions may be extracted (Fig. 8).

The shape of the curves in Figure 8 looks similar to the curves for model blasting tests and full-scale blasting in Figures 5 and 6, apart from the dips of the latter in the –500 μm range. The NBC crushing curves in Figures 3 and 5 have a different character in that they do not approach the 100% level smoothly at a tangent.

How does all this tie together?

THE SWEBREC© FUNCTION

When analysing the data from the Less Fines project,¹⁵ it was realised that the following fragment size distribution does a very good job of fitting sieved fragmentation data.²¹ The transformation:

$$P(x) = 1/[1 + f(x)]$$

with $f(x_{max}) = 0, f(x_{50}) = 1$ (2a)

ensures that x_{max} and x_{50} are fixed points on the curve and a suitable choice for $f(x)$ is:

$$f(x) = [\ln(x_{max}/x) / \ln(x_{max}/x_{50})]^b$$
 (2b)

6 Comparison of fragmentation curves from model- and full-scale blasts of Bårarp granitic gneiss¹⁸

7 Family of breakage functions for crusher based on the t_{10} concept¹⁹

Like the Rosin–Rammler function, it uses the median or 50% passing value x_{50} as the central parameter but it also introduces an upper limit to the fragment size, x_{max} . The third parameter, b , is a curve-undulation parameter. Unlike the Rosin–Rammler or the CZM/TCM functions, the asymptotic properties of $f(x)$ for small fragments is logarithmic, not a simple power of x .

Figure 9 shows sieved data from a 500-t bench blast with Ø 51-mm diameter blast-holes on a 1.8 × 2.2-m pattern and a specific charge of about 0.55 kg m⁻³. At the Bårarp^{18,20} dimensional stone quarry, 7 single-row test rounds with constant specific charge and an accurate EPD inter-hole delay of 25 ms were shot. The hole diameters ranged from Ø 38 to 76 mm. The muckpiles were sieved in three steps, all of the 25–500

8 Fragment size distributions extracted from the family of breakage functions in Figure 7

mm material and quartered laboratory samples (0.063–22.4 mm). Figure 9 shows round 4. The 1000-mm value is a boulder counting estimate.

The Swebrec function fit is excellent in the range 0.5–500 mm. The average goodness of fit for the seven rounds is $r^2 = 0.997 \pm 0.001$ (mean ± SD). The parameter statistics became $x_{50} = 490 \pm 70$ mm, $x_{max} = 1720 \pm 440$ mm and $b = 2.46 \pm 0.45$ (Table 1). (Such a high x_{50} -value would probably give hard digging in an aggregate quarry but this was a 500-t test blast.) The corresponding Rosin–Rammler fits have a goodness of fit of about 0.98 and the curves start to deviate from the data from 20 mm fragments and below.

Interestingly, the coarse fractions seem to contain information about the fines. Using the +90 mm data

9 Fragment size distribution for Bårarp round 4 with best fit Swebrec function.²¹ Data range 0.5–500 mm. Curve fit parameters: $x_{50} = 459$ mm, $x_{max} = 1497$ mm and $b = 2.238$ (Table 1). The value for $x = 1000$ mm is based on oversize counting, not a sieved value

Table 1 Curve fit data for Bårarp rounds^{18,20}

Round no.	Blast-hole diameter Ø (mm)	x_{50} (mm)	x_{max} (mm)	b	Range (mm)	r^2 (mm)	Residuals (% of scale)
1	51	468	1090	1.778	0.5–500	0.9966	< 2.4
2	51/76 (decoupled)	629	2011	2.735	0.5–500	0.9976	< 0.9
3	76	529	2346	3.189	0.5–500	0.9969	< 1.8
4	51	459	1497	2.238	0.5–500	0.9973	< 1.8
5	38	414	1517	2.398	0.5–500	0.9977	< 2.2
6	64	422	2076	2.651	0.5–500	0.9977	< 2.2
7	76	511	1509	2.261	0.5–500	0.9968	< 1.9

and fitting a Rosin–Rammler function gives entirely different results than fitting the Swebrec function (Fig. 10). The filled symbols denote the data used for the fitting. For a typical Swedish aggregate quarry with marketing problems for –4 mm material, the Rosin–Rammler fit to the +90 mm material predicts 0.3–0.4% of fines whereas the Swebrec function predicts 2%, which is much closer to the measured value 2.5%.

The Swebrec and Rosin–Rammler curves are very similar for fragment sizes around x_{50} . Equating the slopes at x_{50} makes it possible to compare the parameter values:

$$n_{equiv} \approx \frac{b}{\left[2 \cdot \ln 2 \cdot \ln \left(x_{max} / x_{50}\right)\right]}$$

(3)

Furthermore, the Swebrec function has an inflection point in $\log P$ versus $\log x$ space at:

$$x / x_{max} = \left(x_{50} / x_{max}\right)^{(b-1)^{1/b}}$$

or

$$x / x_{50} = \left(x_{50} / x_{max}\right)^{(b-1)^{1/b} - 1}$$

(4)

When $b \rightarrow 1$, the inflection point tends to $x = x_{max}$. For increasing values, it moves towards $x = x_{50}$, which is reached when $b = 2$. When b increases further, the

inflection point moves to smaller values of x and then moves back towards $x = x_{50}$. The inflection point and hence the undulating character of the Swebrec function is always there and this makes it possible to pick up the fines behaviour from the coarse fraction data.

Start instead with a sieved sample with fragments in the range 1–22.4 mm from Bårarp round 4. This data set was obtained after quartering of the –25 mm fraction from an Extec sizer, which sieved all 200-mm material. If we know the percentage of the 22.4-mm fraction and make the guess that $x_{max} \approx B = 1800$ mm because the rock is massive, then a curve fit with the Swebrec function yields the result in Figure 11. The filled symbols again denote the data used for the fitting.

The curve runs remarkably well through the missing coarse fraction data and provides an excellent estimate of x_{50} . It seems that limited portions of the fragment size distribution contain relatively accurate information about the missing mass fractions.

Taking samples from a crusher product stream where the percentage of say –22.4-mm fines is better known than in a muckpile and using the closed-side setting and fragments shape to estimate x_{max} is another

10 Comparison of Swebrec and Rosin–Rammler fits to coarse fraction data +90 mm and extrapolation to fines range

11 Using Bårarp sample data in range 1–22.4 mm plus estimate of x_{max} to make Swebrec prediction of x_{50} and coarse fractions

example of where missing mass fractions might be successfully determined. Figure 12 gives an example from a granite quarry where the fines percentage was known to be 18–20% and the largest crushed pieces were 250–300 mm.

The final dip in the Bårarp round 4 fragment size distribution in the $-500\ \mu\text{m}$ range in Figure 9 may be taken care of by adding a second term to $f(x)$ in the Swebrec function (Fig. 13):

$$f(x) = a \left[\ln(x_{max}/x) / \ln(x_{max}/x_{50}) \right]^b + (1-a) \left[(x_{max}/x - 1) / (x_{max}/x_{50} - 1) \right]^c \quad (5)$$

This extended Swebrec function has 5 parameters and is able to fit most fragment size distributions with extreme accuracy.

The Swebrec function has been fitted to hundreds of sets of sieved blasting, crushing and grinding data from a large number of sources,²¹ including:

- (i) Bårarp full-scale and model blasts.^{18,20}
- (ii) Less Fines project model blasts on 5 types of limestone and an amphibolite.¹⁵
- (iii) Norwegian model and full-scale bench blasts in different rocks with different explosives.¹¹
- (iv) Bench blast samples before and after crushing of gneiss and dolerite.

12 Using 0.5–22.4 mm fraction data from jaw crusher sample of granite plus closed-side setting estimate of $x_{max} \sim 300$ mm to predict missing fractions. Curve fit parameters: $x_{50} = 77$ mm and $b = 2.33$

13 Bårarp round 4 data with best fit extended Swebrec function.²¹ Data range 0.075–500 mm. Curve fit parameters: $x_{50} = 459$ mm, $x_{max} = 1480$ mm, $b = 2.224$, $a = 0.99999812$ and $c = 2.0$. $r^2 = 0.9976$. Note magnitude of prefactor (1-a) in Equation (5)

- (v) Blasting of iron ore oversize.

(vi) South African reef and quarry blasts.

(vii) US bench blasts in dolomite.

(viii) Blasting of magnetite concrete models.

(ix) Blasting of layered mortar models.²⁷

(x) Feed and product streams from gyratory, cone and impact crushing of andesite.
- (xi) Product stream samples from roller crushing of limestone.

(xii) Single particle roll mill crushing.

(xiii) Ball mill grinding of limestone.

(xiv) High energy crushing test (HECT) crushing of seven rock types.

(xv) Drill cuttings from the Christmas mine, *etc.*

Table 2 Curve fit data for crusher product samples at Nordkalk,¹⁵ x_{max} free or fixed, excluding $x = 200$ mm points

Sample number	x_{50} (mm)	x_{max} (mm)	b	Range	r^2 (mm)	Residuals (% of scale)
Free x_{max}						
2-01	75.9	300	2.558	8–300	0.9990	< 1.9
2-02	64.9	508	2.479	8–300	0.9950	< 3.3
2-03	86.7	300	2.211	8–300	0.9997	< 0.9
2-04	48.5	376	2.712	8–300	0.9995	< 1.2
2-05	82.8	300	2.178	8–300	0.9992	< 1.6
2-06	57.3	488	2.681	8–300	0.9987	< 1.9
2-07	84.7	320	2.195	8–300	0.9998	< 0.6
2-08	78.4	300	2.297	8–300	0.9998	< 0.6
2-09	75.7	377	2.218	8–300	0.9985	< 1.5
2-10	86.8	345	2.518	8–300	0.9995	< 1.3
2-11	95.9	300	2.198	8–300	0.9986	< 2.0
2-12	76.3	360	2.142	8–300	0.9991	< 1.1
2-13	64.9	300	2.333	8–300	0.9995	< 1.0
Mean ± SD	75.3 ± 13.2	352 ± 72	2.36 ± 0.20		0.9989	
Fixed x_{max}						
2-01	75.9	315	2.625	8–300	0.9989	< 1.9
2-02	66.0	315	1.984	8–300	0.9930	< 3.2
2-03	86.8	315	2.270	8–300	0.9997	< 0.9
2-04	48.9	315	2.497	8–300	0.9993	< 1.7
2-05	82.8	315	2.234	8–300	0.9992	< 1.6
2-06	58.2	315	2.200	8–300	0.9977	< 2.8
2-07	84.7	315	2.175	8–300	0.9998	< 0.6
2-08	78.4	315	2.356	8–300	0.9997	< 0.7
2-09	75.9	315	2.027	8–300	0.9982	< 1.4
2-10	86.7	315	2.400	8–300	0.9994	< 1.4
2-11	96.1	315	2.254	8–300	0.9985	< 2.0
2-12	76.4	315	2.002	8–300	0.9989	< 1.3
2-13	64.9	315	2.386	8–300	0.9994	< 1.1
Mean ± SD	75.5 ± 13.0		2.26 ± 0.19		0.9986

14 Fragment size distribution for gyratory crusher product 2 (CSS = 1.5'') of andesite with Swebrec fit.²¹ Data range 0.425–63 mm. Curve fit parameters: $x_{50} = 35.6$ mm, $x_{max} = 68$ mm and $b = 1.531$. $r^2 = 0.9961$

Two examples of crusher product size distributions are shown in Figures 14 and 15. For nearly all these sets of sieved data, the 3-parameter Swebrec function gives a better fit than the Rosin–Rammler function.

The experience²¹ is that good-to-excellent fit to different kinds of fragmentation data is obtained with coefficients of determination usually $r^2 = 0.995$ or better over a range of fragment sizes of 2–3 orders of magnitude. This range is at least one order of magnitude larger than the range covered by the Rosin–Rammler function.

Of the three parameters, the central median measure (*i.e.* the size of 50% passing x_{50}) shows the most stable behaviour. The maximum fragment size, x_{max} , will be physically related to the block size *in situ* in blasting; however, as a fitting parameter it varies widely. On the other hand, in crushing it is more or less given by the closed-side setting. Fixing the value of x_{max} has little effect on the goodness of fit and the values of x_{50} or b (Table 2).

The parameter b normally accepts values in the range 1–4. Only in special cases, like model blasting

15 Fragment size distribution for belt sample 2-04 of Nordkalk limestone.²¹ Swebrec function fit with fixed $x_{max} = 315$ mm. Data range 0.25–300 mm. Parameters: $x_{50} = 48.8$ mm and $b = 2.451$. $r^2 = 0.9994$ (Table 2)

Table 3 Curve fit data for block samples blasted with different explosives¹¹

	Explosive	x_{50} (mm)	x_{max} (mm)	b	Range (mm)	r^2	Residuals (% of scale)
Syenite	Extra dynamite	32.9	113	2.441	0.5–100	0.9976	< 2.0
	Extra dynamite	33.2	135	2.635	0.5–100	0.9969	< 2.0
	Extra dynamite	33.8	124	2.437	0.5–100	0.9970	< 2.0
	Dynamite	62.9	167	2.053	0.5–150	0.9955	< 3.1
	Dynamite	76.1	168	1.813	0.5–150	0.9938	< 2.8
	Dynamite	72.1	184	1.976	0.5–150	0.9936	< 4.6
Granite	Extra dynamite	49.9	143	2.233	1–125	0.9984	< 0.8
	Extra dynamite	50.5	136	2.102	1–125	0.9994	< 1.5
	Extra dynamite	46.6	142	2.198	1–100	0.9996	< 0.6
	Dynamite	66.1	127	1.657	1–125	0.9996	< 0.8
	Dynamite	74.2	131	1.607	1–125	0.9994	< 1.6
	Dynamite	78.1	133	1.592	1–125	0.9988	< 2.0
Gneiss	Extra dynamite	60.8	161	2.465	1–125	0.9990	< 2.1
	Extra dynamite	62.5	146	2.279	1–125	0.9988	< 1.6
	Extra dynamite	66.8	168	2.537	1–125	0.9958	< 5.1
	Dynamite	95.3	330	2.939	1–100	0.9985	< 0.9
	Dynamite	103	464	3.559	1–100	0.9995	< 0.4
	Dynamite	106	408	3.261	1–100	0.9989	< 0.7

near the critical burden, does the fitting give a value of $b < 1$. When the sieved fragmentation curve becomes Rosin–Rammler like, both b and x_{max} tend to drift during the fitting procedure and can become unnaturally large. This tendency may be suppressed by choosing a pair of coupled b and x_{max} values that are related through Equation (3) for n_{equiv} . In these cases, the value of n_{equiv} tends to vary less than the b -value. Otherwise, in the majority of cases, b is more constant than n_{equiv} .

Often, b remains constant for a given material even when the fragmentation conditions change. A couple of data sets show, however, that b also depends on the explosive used (Table 3), on the charge concentration,

and on the size of blast (model scale or full-scale, for example). There is thus no basis for considering b as a material property or as depending only on the specimen geometry as the Kuz–Ram model suggests be the case for the uniformity index n .

The Swebrec function is also an improvement over the CZM and TCM. Both JKMRC models show linear behaviour in the fines range in log–log space where a vast majority of the data sets are clearly non-linear. The Swebrec function does not rest on an assumption of the origin of the blasting fines that has been disproved by tests.

The extended Swebrec function with 5 parameters shows the capacity of reproducing sieved fragmentation

16 Laboratory ball mill data for limestone after 6 min grinding, with extended Swebrec function fit.²¹ Data range 0.063–3.36 mm. Curve fit parameters: $x_{50} = 1.00$ mm, $x_{max} = 3.36$ mm, $b = 1.010$, $a = 0.9911$ and $c = 1.753$. $r^2 = 0.9995$

17 JKMRC breakage function for crusher¹⁹ at $t_{10} = 30\%$, with Swebrec function.²¹ Size fraction t/x data range 0.025–1. Curve fit parameters: $\tau_{50} = 0.197$, $\tau_{max} = 1.0$ and $b = 2.431$. $r^2 = 0.9985$

curves all the way into the super fines range ($x < 0.1$ mm) and also of reproducing laboratory ball mill grinding data (Fig. 16).
It could thus be said that the Swebrec function gives the fragment size distributions from blasting and crushing a common form.

CRUSHER BREAKAGE AND NBC CONNECTIONS

A comparison of Figure 8 and the presented size distributions from blasting and crushing shows that

the general curve-forms look alike. Some mass passing versus non-dimensional fragment size data for crusher and AG/SAG mill breakage functions from the JKMRC¹⁹ were matched against the non-dimensional version of the Swebrec function:

$$P(\tau) = 1 / \left\{ 1 + \left[\ln(\tau) / \ln(\tau_{50}) \right]^b \right\}$$

with $\tau = x/t$ and $\tau \leq \tau_{max} = 1$ (6)

One example from the data sets behind Figures 7 and 8 is given in Figure 17. In fact, the whole t_{10} family of crusher curves given by Napier-Munn *et al.*¹⁹ (their

18 Mass passing isolines for constant degrees of size reduction, plotted as function of the breakage index t_{10} . Full lines denote predictions from Equations (7) and (8)

Table 6:1) may be reduced to the following equation:

$$t_n = 100 / \left\{ 1 + (100/t_{10} - 1) [\log(n)]^{b(t_{10})} \right\} \tag{7}$$

where, in this case, *n* is the size reduction ratio. When *n* = 10 e.g. (log(*n*) = 1), Equation (7) reduces to the straight line *t_n* = *t₁₀*. An approximate expression for *b*(*t₁₀*) was obtained from the data fitting:²¹

$$b = 1.616 + 0.02735t_{10} \tag{8}$$

and the results are plotted in Figure 18. The full lines were obtained using Equation (8), the dashed ones for *n* = 2 and 75 using the value *b* = 2.174 valid for *t₁₀* = 20%. It is seen that the variations in the *b*-value do make a difference. The data are well represented by the isolines, differing by at most 3–4%.

Excellent fits were also obtained to the data in Tables 4:7 and 4:9 of Napier-Munn *et al.*¹⁹ This is an indication that the whole set of spline functions used to describe the breakage functions could be replaced by two simple equations – Equation (7) and a form of Equation (8).

The connection with the NBC theory rests on an observation of the asymptotic behaviour of the Swebrec function. The parallel shift property in log*P* versus log*x* space reduces to the statement that *P*'(*x*)/*P*(*x*) = constant, independent of some parameter that describes the shift. This is not met by the Swebrec function itself but the behaviour when *x* → 0:

$$P(x) \approx 1/f(x) \text{ yields}$$

$$P'(x)/P(x) \approx -f'(x)/f(x) = b/x \ln(x_{max}/x)$$

which is independent of *x₅₀*. To retain the meaning of *x_{max}* as the maximum allowable and *x₅₀* as the median fragment size, make the substitutions *x_{max}* → *x_c* and *x₅₀* → *x_{max}*. Now *x_c* denotes a characteristic size value for the distribution, which lies outside the acceptable range of *x*-values, 0 → *x_{max}*.

Then the following function has NBC properties:

$$P_{NBC}(x) = \left[\ln(x_c/x_{max}) / \ln(x_c/x) \right]^b \tag{9}$$

$$\text{with } x_{50} = x_{max} / (x_c/x_{max})^{2^{1/b} - 1}$$

Equation 9 describes a set of parallel shifted curves when the value of *x_{max}* is changed but *x_c* and *b* are kept constant. *P_{NBC}*(*x*) is always concave upwards when *x* < *x_{max}*, which is the behaviour of the OCS sub-circuit curves in Figure 3 except in the super fines range.

The simplest description of an OCS sub-circuit is a sharp sieve that lets the fines bypass the comminution chamber, which in turn processes the coarse material retained by the sieve. When the derivative of *P_{NBC}*(*x*) is used to describe the breakage function of the chamber, the following results emerge for the combined product stream:²¹

- (i) If the entire feed stream passes the chamber, then the product stream has NBC properties irrespective of the feed stream properties.
- (ii) If parts of the feed stream bypass chamber then if the bypass stream has NBC properties then so does the combined product stream.

Otherwise the combined product stream does not have NBC properties.

This has a direct bearing on the interpretation of Figures 3 and 5. As long as the size reduction ratio in an OCS sub-circuit is large enough to make the bypass flow have NBC properties, then the product stream has NBC properties no matter what the size distribution of the processed part looked like beforehand.

The fragment size distributions for OCS crushed or milled rock and rock models blasted with different levels of specific charge look similar in the –20-mm range in Figure 5. Hence, we may conclude that if the NBC curve from mechanical comminution indeed is material specific, then the model blasting tests produce material specific results too in that range. The relation between the blasting curve and the NBC

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curve is:

$$P(x) = 1 / \left[1 + 1 / P_{NBC}(x) \right] \quad (10)$$

Similarly, Equation (10) contains a potential correspondence between the OCS sub-circuits and the JKMRC crusher models.

THE KUZ–RAM CONNECTION

In connection with the Kuz–Ram model, Equation (3) offers the possibility of simply replacing the original Rosin–Rammler function in Equation (1) with the Swebrec function of Equation (2a,b). Thus, we arrive at an extended Kuz–Ram model (or KCO model, see below) based on the prediction formulae shown as boxed text (see equation 11a–d).

The factor $g(n) = (\ln 2)^{1/n} / \Gamma(1 + 1/n)$ essentially shifts the fragment size distribution to smaller values of x_{50} or to predicting more fines.²⁴ For expedience, call the original Kuz–Ram model with $g(n)$ added, the shifted Kuz–Ram model. The shifting factor $g(n)$ could be incorporated into the extended model too, if experience proves that this is an advantage.

CZM⁹ and TCM⁷ have slightly different algorithms for determining A and n and Cunningham⁶ has a new version of A as well. As the Swebrec function has a built-in fines bias, it remains to be seen whether the

factor $g(n)$ in the x_{50} expression is really needed. The expression for x_{max} is tentative and would be replaced when a better description of blasting in a fractured rock mass becomes available.

Despite these uncertainties, the extended Kuz–Ram or KCO model overcomes two important drawbacks of the previous version – the poor predictive capacity in the fines range and the infinite upper limit to block sizes.

Use Bårarp round 4 as an illustration.^{18,20} The rock is hard and weakly fissured so we may try $A = 13^{4,13}$ to make things simple. For the determination of x_{50} , we further need $Q = 9.24$ kg, $q = 0.55$ kg m⁻³ and $s_{ANFO} = 70 \times 4/4.5 = 62.2\%$. Insertion into Equation (11b) yields $x_{50} = 44.8$ cm or 448 mm, which is very close to the 459 mm value given in Table 1.

For the determination of n , we need the geometry of the blast: $\varnothing = 51$ mm, $B = 1.8$ m, $S = 2.2$ m, $H = 5.2$ m, $L_{tot} = 4.2$ m or 3.9 m above grade, $L_b = L_{tot}$ and $L_c = 0$ and $SD \approx 0.25$ m. Insertion into Equation (11c) yields $n = 1.17$. The shifting factor $g(n) = 0.659$. Using the estimate $x_{max} = \sqrt{(B \cdot S)} \approx 2.0$ m = 2000 mm yields $b = 2.431$, which is also close to the value 2.238 given in Table 1.

Figure 19 shows the sieved data together with the three prediction equations in log P versus log x space. Figure 20 does the same in P versus log x space. Since the original x_{50} prediction incidentally was nearly

$$P(x) = 1 / \left\{ 1 + \left[\ln(x_{max}/x) / \ln(x_{max}/x_{50}) \right]^b \right\} \quad (11a)$$

$$x_{50} = \left[g(n) \cdot A \cdot Q^{1/6} \cdot (115/s_{ANFO})^{19/30} / q^{0.8} \right] \quad \text{with } g(n) = 1 \quad \text{or} \quad (\ln 2)^{1/n} / \Gamma(1 + 1/n) \quad (11b)$$

$$b = \left[2 \ln 2 \cdot \ln(x_{max}/x_{50}) \right] \cdot n$$

$$= 2 \ln 2 \cdot \ln(x_{max}/x_{50}) \cdot (2.2 - 0.014 B/\varnothing_h) \cdot (1 - SD/B) \cdot \sqrt{\left[(1 + S/B)/2 \right]} \cdot \left[|L_b - L_c| / L_{tot} + 0.1 \right]^{0.1} \cdot (L_{tot}/H) \quad (11c)$$

$$x_{max} = \min(\text{in situ block size, } S \text{ or } B) \quad (11d)$$

where:

x_{50} = median or size of 50% passing (cm)

Q = charge weight per hole (kg); q = specific charge (kg m⁻³)

s_{ANFO} = explosive's weight strength relative to ANFO (%)

B = blast-hole burden (m); S = spacing (m)

$\varnothing_h$ = drill-hole diameter (m)

L_b = length of bottom charge (m)

L_c = length of column charge (m)

L_{tot} = total charge length, possibly above grade level (m)

H = bench height or hole depth (m)

SD = standard deviation of drilling accuracy (m)

Further, the rock mass factor A is given by:

$$A = 0.06 (RMD + RDI + HF) \quad (11e)$$

where:

RMD = rock mass description = 10 (powdery/ friable), JF (if vertical joints) or 50 (massive)

JF = joint factor = $JPS + JPA$ = joint plane spacing + joint plane angle

JPS = 10 (average joint spacing $S_j < 0.1$ m), 20 (0.1 m-oversize x_o) or 50 (> oversize)

JPA = 20 (dip out of face), 30 (strike perpendicular to face) or 40 (dip into face)

RDI = rock density influence = $0.025 \times \rho$ (kg m⁻³) – 50

HF = hardness factor, uses compressive strength σ_c (MPa) and Young's modulus E (GPa)

HF = $E/3$ if $E < 50$ and $\sigma_c/5$ if $E > 50$.

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exact, the shifting adds nothing in this case. It may well in other cases but neither the original nor the shifted Kuz–Ram model captures the real behaviour of the fragment size distribution in the fines range as well as the extended Kuz–Ram or KCO model does.

Figure 20 focuses the perspective more on the coarse fractions. The slope equivalence between the Rosin–Rammler function and the Swebrec function at x_{50} , which is expressed by Equation (3), is clear. There is only one data point for $x > x_{50}$ in Figure 20. Figure 9 contains a value for $x = 1000$ mm. The value $P(1000 \text{ mm}) = 98.3\%$ obtained from boulder counting is not very accurate but it lies closer to the Swebrec function curve than to the Rosin–Rammler one. The $P(1000 \text{ mm})$ values for the other Bårarp rounds range from 75% to 99%.

The final judgement as to whether the Swebrec function or the Rosin–Rammler function does a better fitting job for the coarse fractions is left open until sufficient sieved muckpile data from full-scale blasts have been studied. The general experience in fitting the Swebrec function to sieved data sets is that it is fully capable of reproducing both the fines and the coarse fractions.

It is hoped that the incorporation of the Swebrec function in the Kuz–Ram model will enhance the tools available to blasting engineers and researchers. Since the underlying size distribution is no longer the Rosin–Rammler function the name might be changed to the KCO (Kuznetsov–Cunningham–Ouchterlony) model.

The description of the effects of initiation delay between blast-holes on fragmentation remains. To do this properly and to account for systematic variations in rock mass properties, for example, good numerical models are required. Model complexity and computation speed are two factors that, for the time being, limit development of such models. Until this is overcome, the KCO model has a role to play.

CONCLUSIONS

A new fragment size distribution function has been found, which ties together rock fragmentation by blasting and crushing, the Kuz–Ram, CZM and TCM models of rock blasting, the JKMRC approach to describing crusher circuits and the original NBC concept of material specific size distribution curves in comminution.

The new Swebrec function has three parameters and gives good-to-excellent fits to different kinds of fragmentation data with correlation coefficients of least 0.997 or better ($r^2 > 0.995$) over a range of fragment sizes of two to three orders of magnitude. Hundreds of sets of sieved data from crushing and blasting have been analysed with excellent results.

The inherent curvature of sieved fragment size distributions is captured by the Swebrec function. Using coarse fractions data to extrapolate into the fines range has the potential of giving accurate fines predictions. Similarly, using samples of fine material from a muckpile or crusher products on a belt and an estimate of x_{max} has the potential to give accurate estimates of the coarser fractions.

The extended Swebrec function with 5 parameters shows capacity of reproducing sieved fragmentation curves all the way into the super fines range ($x < 0.1 \text{ mm}$) and also of reproducing laboratory ball mill grinding data.

The Swebrec© function further gives the following advantages:

- (i) It can be used in the Kuz–Ram model and removes two of its drawbacks – the poor predictive capacity in the fines range and the upper limit cut-off of block sizes.
- (ii) It reduces the JKMRC one-family description of breakage functions based on the t_{10} concept to a minimum.
- (iii) It establishes a new family of NBC functions with a realistic shape that connects mechanical and blasting comminution and offers new

insight into the working of Steiner's OCS sub-circuits of mechanical comminution.

By analogy with the origin of the Kuz–Ram name, it is suggested that the extended Kuz–Ram model be called the KCO (Kuznetsov–Cunningham–Ouchterlony) model. Summing up, it may be said that the Swebrec function helps give a much less 'fragmented' description of rock fragmentation than present models.

On a more philosophical note, it would be useful to explore why such a simple function as the Swebrec function is so successful. Although the three-parameter Swebrec function does a better job than other functions with four or five parameters this cannot, *a priori*, be taken as evidence that these three parameters are physically more relevant. It should, however, motivate further investigation and analysis.

Tests reported here show that geometrically separate regions can not be assigned separate fragment sizes. Assigning such regions separate fracturing modes is misleading too. It is probably also misleading to take the fact that a two term function or bimodal distribution gives a good fit as evidence of two separate fracturing modes. The reverse might be true, however. If two separate fracturing modes were identified and the two associated distribution functions determined, then their merging would yield a compound bimodal fragmentation function. This merger would not be a simple matter if the fracturing modes interact.

The way to go in investigating different fracturing modes could very well be computational physics work.^{1,12} A postulated generic fragmentation model with two distinct mechanisms, a crack branching-merging process that creates a scale-invariant size distribution in the super fines range and a Poisson process that creates an exponential cut-off at system dependent length scale, yields good agreement with our blasting fines data.² Still lacking are the relationships between the model parameters and the microscopic properties of different rock types plus taking the rock mass jointing properly into account.

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