Paper D

REDUCING THE FLOW MEASUREMENT ERROR CAUSED BY PULSATIONS IN FLOWS

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Different types of errors are generated by pulsations in flows. Among these errors is the sampling error due to a unadapted time-averaging of the flowrate. An improved model for pulsations in flows including harmonics is derived. The localisation of the harmonics is performed by a detector. The period of the pulsations is estimated. It is then possible to reduce the sampling error by performing a correct averaging. The reduction of the sampling error is confirmed by simulations.

Keywords: Flowmeter, Flow pulsations, Harmogram, Error, Sampling.

D.1 Introduction

Pulsating flows are often encountered in sectors such as petrochemical industries, natural gas distribution, or district heating industries. They are generally generated by pumps, compressors, rotary engines, pressure regulators, etc [1]. But even a single tap, a valve or vibrations of pipes can cause flow pulsations. A pulsating flow is the source of several errors in flow measurement([2],[3]). This

problem is known since the beginning of the twentieth century and remains a major preoccupation. Depending on the type of flowmeter, the possible errors are square-root errors [2], flow velocity profile errors, sampling error [3]. The present paper focuses essentially on the latter. This sampling error has nothing in common with the statistical notion of sampling error nor with the quantization error encountered when sampling a signal. The sampling error discussed here is the estimation error when estimating the time-average of a mean flow velocity. A time-averaged value is often preferred for computations where the mean flow velocity is needed. In district heating applications, the estimation of the energy transferred by the heat exchanger requires a value of the mean flow velocity every second only whereas most of the flowmeters are capable to provide a value at much higher rate. The error of a flowmeter is easily bounded when the statistical properties of the flow are steady. Standards like EN 1434-2 [4] determine a maximum permissible error for each class of flowmeters. However, these standards are not suitable when pulsations are present in the flow. Amplitude thresholds have been found and reported in [1] for most of flowmeter types. But these thresholds do not give any model for the flow pulsations. A parametrization of the flow pulsations might allow a compensation and a reduction of the error generated. In the theoretical part, the spatial mean of the flow velocity (fig.D.1) inside the flowmeter (simply called mean flow velocity from now) is modelled as the sum of a constant velocity and fluctuations. A third term describing periodic fluctuations is added for the case of a pulsating flow. The signal observed by the flowmeter is an estimation of the time-averaged mean flow velocity (fig.D.1). An upper bound of the sampling error was found in [3]. That upper bound is now improved and that allows a considerable reduction of the sampling error.

D.2 Theory

D.2.1 Model of a stationary flow

Keeping the same notations as in [3], the mean flow velocity of a stationary flow passing through a flowmeter can be modelled as the sum of a constant mean flow velocity u_{mean} and the fluctuations around u_{mean} :

$$u(t) = u_{mean} \left(1 + \sqrt{2}a_n h(t) \right), \quad (D.1)$$

where $h(t)$ is the normalized function representing the fluctuations over u_{mean} and a_n is the relative amplitude of the fluctuations. In a fully developed turbulent flow, the root mean square amplitude a_n ranges between 1% and 20% of u_{mean} [3].

a\ laminar flow

b\ turbulent flow

Figure D.1: The appellation mean flow velocity denotes the average velocity of the fluid elements inside the flowmeter body. In the laminar case (a), the mean flow velocity is equal to half the max flow velocity, and in the turbulent case (b), the distribution of the velocity is almost uniform and nearly equal to the mean flow velocity.

Figure D.2: The ultrasonic flowmeter estimates the part of the mean flow velocity that is constant over the time-period of any pulsations. The input signal $U(t)$ is the continuous flow velocity inside the flowmeter. The output signal $\hat{U}_{mean}(nT_s)$ is the estimation of the time-constant part of the mean flow velocity at times nT_s , where T_s is the sampling period.

D.2.2 Model of a pulsating flow

When flow pulsations are involved, a third term is added to expression (D.1). In [3], the term representing the flow pulsations is modelled by a sinus form:

$$u(t) = u_{mean} \left(1 + \sqrt{2} (a_{r.m.s.} \sin(2\pi f_{puls} t) + a_n h(t)) \right), \quad (D.2)$$

where $a_{r.m.s.}$ is the root mean square amplitude of the pulsations, supposed to be sinusoidal, and f_{puls} is the their frequency. In fact, such a model is too naive to be representative of flow pulsations. In [5] and [6], a series of harmonics is observed at frequencies multiple of the pulsations' frequency in the Fourier transform of a pulsating flow. This leads to the conclusion that a pulsating flow is periodic but that the sinusoidal model is rather too simple. In fact, the mean flow velocity of a pulsating flow would be better modelled by a trigonometric sum:

$$u(t) = u_{mean} \left(1 + \sqrt{2} \left(\sum_{k=1}^P a_k \sin(2k\pi f_{puls} t + \phi_k) + a_n h(t) \right) \right), \quad (D.3)$$

where $\{\phi_k, k \in [1, P]\}$ is the set of the phases at time $t = 0$ and P is the number of harmonics used for the model. The output of the flowmeter at time t is simply the estimate of the flow velocity at time t . The flow velocity is sampled at rate $f_s = 1/T_s$ (T_s is called sampling time). Typical values for the sampling frequency f_s are between 10 Hz and 100 Hz. According to Nyquist's sampling

theorem, the frequency of the highest harmonic should be less than half the sampling frequency:

$$P f_{puls} < f_s/2. \quad (D.4)$$

An observation noise $b(t)$ is added to the flow velocity signal, so that expressions (D.1) and (D.3) become respectively:

$$\hat{u}(t) = u_{mean} \left(1 + \sqrt{2} a_n h(t) \right) + b(t) \quad (D.5)$$

$$\hat{u}(t) = u_{mean} \left(1 + \sqrt{2} \left(\sum_{k=1}^P a_k \sin(2k\pi f_{puls} t + \phi_k) + a_n h(t) \right) \right) + b(t). \quad (D.6)$$

The observation noise $b(t)$ is supposed to be Gaussian and centered. More generally, the sum $a_n h(t) + b(t)$ is assumed to be Gaussian and centered.

D.2.3 The error of estimation

As the flowmeter is often a part of measurement system, it is more suitable to have the constant term u_{mean} as the output of the flowmeter (fig.D.1). For example, in district heating applications, an integrator computes the energy supplied to a house with help of the estimation of the flow velocity together with incoming and outgoing temperature measurements. An estimation of the supplied energy is required approximately every second ([7], [8], [9]). It would be therefore unnecessary for the flowmeter to provide a value at a higher rate. That is why the time-average of the mean flow velocity is preferred to the instantaneous value of the mean flow velocity as an output for the flowmeter. The time under which the samples of the flow velocity are averaged is called the integration time T_i . A correct estimation of u_{mean} is made if the integration time T_i is equal to a multiple of the pulsation's period $T_p = 1/f_p$, since the integral of a T_p -periodic function on an interval of length T_p is zero:

$$\forall m \in N, \hat{u}_{mean} = \int_t^{t+mT_p} u(t) dt. \quad (D.7)$$

By now, T_i is chosen such that:

$$\exists m \in N, T_i = mT_p. \quad (D.8)$$

An unbiased estimate of u_{mean} becomes then:

$$\hat{u}_{mean} = \int_t^{t+T_i} u \simeq \frac{1}{q} \sum_{k=1}^q \hat{u}(kT_s + t), \quad (D.9)$$

where q is the greatest integer inferior to T_i/T_s . The integer q denotes the number of samples collected during T_i . But, so far, the length of the integration time T_i is chosen independently from eventual flow pulsations. The probability

for T_i to be a multiple of the period of any eventual flow pulsations is then very small. The error induced by keeping (D.9) as an estimate of u_{mean} is introduced in [3] and is equal to:

$$E_p = \frac{\hat{u}_{mean} - u_{mean}}{u_{mean}}. \quad (D.10)$$

Inserting (D.6) in (D.9) leads to:

$$\hat{u}_{mean} = u_{mean} \left(1 + \sum_{k=1}^P \frac{a_k}{\sqrt{2k\pi f_{puls} T_i}} [\cos(2k\pi f_{puls}(t + T_i)) - \cos(2k\pi f_{puls}t)] + \sqrt{2} \int_t^{t+T_i} a_n h \right) + \int_t^{t+T_i} b. \quad (D.11)$$

As the observation noise $b(t)$ is centered, the last term of (D.11) is zero. Moreover, it is assumed that the amplitude of the fluctuations due to the turbulence present in the flow can be neglected compared to the flow pulsations:

$$\forall k \in [1, P], a_n \ll a_k. \quad (D.12)$$

The expression of the error E_p then becomes:

$$E_p = \sum_{k=1}^P \frac{a_k}{\sqrt{2k\pi f_{puls} T_i}} [\cos(2k\pi f_{puls}t) - \cos(2k\pi f_{puls}(t + T_i))]. \quad (D.13)$$

In [3], it is suggested to use the real number 2 as a upper bound for $\cos(2k\pi f_{puls}t) - \cos(2k\pi f_{puls}(t + T_i))$. This leads to an upper bound of E_p that is:

$$|E_p| \leq \frac{\sqrt{2}}{\pi f_{puls} T_i} \sum_{k=1}^P \frac{a_k}{k}. \quad (D.14)$$

This upper bound is then a hyperbolic function of the integration time T_i (fig.D.2.3). That upper bound does not put into evidence the fact that E_p is equal to zero if the integration time T_i is a multiple of the flow pulsations' period T_p . However, a better upper bound can be found by using the trigonometric identity:

$$\cos(\alpha) - \cos(\beta) = -2 \sin\left(\frac{\alpha - \beta}{2}\right) \sin\left(\frac{\alpha + \beta}{2}\right). \quad (D.15)$$

The expression of E_p presented in (D.13) is then transformed to:

$$E_p = \sum_{k=1}^P \frac{\sqrt{2}a_k}{k\pi f_{puls} T_i} \sin(k\pi f_{puls} T_i) \sin(k\pi f_{puls}(2t + T_i)). \quad (D.16)$$

It permits to put 1 as an upper bound of the latter sinus. A new upper bound of the error E_p becomes:

$$|E_p| \leq \sqrt{2} \text{sinc}(\pi f_{puls} T_i) \sum_{k=1}^P \frac{a_k}{k}. \quad (D.17)$$

Figure D.3: Two possible upper bounds for the error E_p . The curve plotted in solid style represents the upper bound found in [3] and written in (D.14). The curve plotted in dash-dot represents the new upper bound found in (D.17). The flow pulsations are defined by $a_1 = 1$, $a_2 = 1/2$, $a_3 = 1/4$, and $f_{puls} = 4 \text{ Hz}$. One can see that the error E_p is considerably reduced if T_i takes the value of the one of the zeros of the cardinal sinus.

This upper bound found in (D.14) is the hull of the new upper bound found in (D.17). The new upper bound shows that the error E_p is equal to zero when the integration time T_i is a multiple of T_p (c.f. fig.D.2.3). With the upper bound presented in (D.14), the only way to reduce the error was to increase considerably the integration time T_i . With the upper bound presented in (D.17), it is now possible to choose correct values for the integration time such that the error E_p decreases to zero. As it has been shown before, the integration time T_i just has to be a multiple of the period T_p of flow pulsations. Obviously, the error E_p is not the only error caused by pulsating flows. There are also errors induced by the variations of the flow profile, by the perturbations of the propagation media, etc. Nevertheless, reducing E_p can be considered as a first step of the reduction of the total error.

Figure D.4: The principle of the system used to detect and reduce the effects of flow pulsations. The harmogram estimates the period T_p eventually present in the mean flow velocity $u(t)$. If there are flow pulsations, then that estimation is used as the new integration time in order to reduce the error E_p .

Figure D.5: The P.S.D. (power spectral density) S_u of the mean flow velocity is estimated via the periodogram method. The P.S.D. $S_{a_n h+b}$ of the background noise present in the signal is estimated by applying a median filter (non-linear) to $\hat{S}_u$. The estimation of the P.S.D. $\hat{S}_u$ is then divided by the estimation $\hat{S}_{a_n h+b}$. If there is no pulsating flow (H_0), the random variable constituted by the output follows a χ^2 distribution. Otherwise (H_1), some harmonics are present in the spectrum and emerge from the background noise. If the maximum of the output exceed the value of the threshold, the hypothesis H_1 is chosen.

D.2.4 Detection of flow pulsations

The next step consists in correctly estimating the period T_p of the flow pulsations. The problem of finding a hidden periodicity in a signal is simply enunciated but rather hard to solve. However, different solutions exists, and among them is the harmogram found by Hinich [10]. It belongs to the group of methods for which an estimation of the background noise is necessary [11]. Indeed, the harmonics present in the spectrum can emerge from the background noise if the latter is estimated (fig.D.2.4). As in every detection problem, the decision space has to be defined. The models adopted in (D.5) and (D.6) correspond to hypotheses H_0 and H_1 respectively. The noll-hypothesis H_0 is verified when there are no flow pulsations. The presence of flow pulsations is pointed out by H_1 . The method as a whole is detailed in [10]. The harmogram can detect a hidden periodicity in the flow and provide an estimation $\hat{T}_p$ of its period T_p . It is then possible to use the period estimation $\hat{T}_p$ as a value for the integration time T_i in order to reduce the error E_p (fig.D.2.3).

D.3 Simulations

Expression (D.6) is now used to simulate u_{mean} . The upper bound found in (D.17) is correct only if the power of the noise $a_n h(t) + b(t)$ is low compared to the power of the periodic signal. Otherwise, the flow velocity can no longer be

Figure D.6: The harmogram of the simulated mean flow velocity as it is modelled in (D.18). The maximum of the harmogram exceeds the threshold and therefore indicates the period $\hat{T}_p = 0.25$ s of the existing flow pulsations.

Figure D.7: A sequence of $\hat{u}_{mean}$. In the first part the integration time T_i is arbitrary chosen and equal to 0.3 s. In the second part, the integration time takes the value proposed by the harmogram $T_i = 1/f_p = 0.25$ s

considered as periodic. The following signal was tested:

$$u(t) = u_{mean} \left(1 + \sqrt{2} \left(\sum_{k=1}^3 a_k \sin(2k\pi f_{puls} t) \right) + w(t) \right), \quad (D.18)$$

where $u_{mean} = 1$, $\{a_1, a_2, a_3\} = \{1, 1/2, 1/4\}$, $f_{puls} = 4\text{Hz}$ and $w(t)$ is a normal distributed random variable with mean zero and variance 10^{-2} . $w(t)$ is used for modelling the total noise $a_n h(t) + b(t)$. The results are shown in fig.D.2.4 and fig.D.3. The harmogram detects the periodicity at 4Hz and reports it to the new integration time $T_i = 1/4 = 0.25\text{Hz}$. The mean flow velocity is integrated in fig.D.2.4. One can easily see that the precision error is considerably reduced by taking $1/f_d$ as the new value of the integration time. When the integration time is equal to 0.30s, the standard deviation is equal to 0.15 while it is equal to 0.02 when the integration is changed to 0.25s.

D.4 Conclusion

A pulsating flow can be modelled as the sum of components with constant amplitudes and instantaneous frequencies added to a background noise. This decomposition makes possible the detection of eventual flow pulsations by Hinich's harmogram. The use of the flowmeter data in district heating applications implies an integration of the output over a time interval. The length of the time interval influences the quality of the estimation of the mean flow velocity. When flow pulsations are involved, the integration time should be equal to a multiple of the period of the mean flow velocity. It allows then to considerably reduce the sampling error introduced in [3]. The harmogram can be used to estimate the period of the flow pulsations. The estimated period is then used as the new value of the integration time. The sampling error is then reduced all the better since the noise level is low.

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