ON HARDY'S INEQUALITY IN WEIGHTED REARRANGEMENT INVARIANT SPACES AND APPLICATIONS. II

LECH MALIGRANDA

ABSTRACT. This note is a sequel to the preceding one with the same title published in these Proceedings. The contents of the first paper are assumed to be known. References are in alphabetical order in each paper, but they, as well as the theorems, are numbered consecutively.

We shall now apply our results to imbedding theorems for weighted Sobolev spaces and interpolation theorems.

1. Under the hypotheses of Theorems 1 or 2, we have for every absolutely continuous function x defined in $I = (0, l)$ such that

$$x(0^+) = \lim_{t \rightarrow 0^+} x(t) = 0, \quad \kappa_1(t)x'(t) \in E,$$

the following inequality holds:

$$(24) \quad \|x\|_{E_{\kappa_0, t}} \leq C \|x'\|_{E_{\kappa_1, t}},$$

where $\kappa_0(t) = \kappa(t)[\psi(t)]^{-1}$ and $\kappa_1(t) = t\kappa(t)[\psi(t)]^{-1}$ or $\kappa(t)[\psi'(t)]^{-1}$. Hence

$$(25) \quad \|x\|_{E_{\kappa_0, t}} \leq C \left(\|x\|_{E_{\kappa_1, t}} + \|x'\|_{E_{\kappa_1, t}} \right) =: C \|x\|_{W_{E, \kappa_1}^1}.$$

2. A pair of quasi-Banach spaces (X_0, X_1) continuously embedded in some Hausdorff topological vector space $\mathfrak{X}$ is called a quasi-Banach couple. The sum of X_0 and X_1 , denoted by $X_0 + X_1$, is the set of all $x = x_0 + x_1$, with $x_i \in X_i$, $i = 0, 1$. For each $x \in X_0 + X_1$, we define the Peetre K -functional for x by

$$(26) \quad K(t, x) \equiv K(t, x; X_0, X_1) = \inf_{\substack{x = x_0 + x_1 \\ x_0 \in X_0, x_1 \in X_1}} (\|x_0\|_{X_0} + t\|x_1\|_{X_1}).$$

The K method generates interpolation spaces (see [22, Chapter 3]) by applying function quasi-norms Φ to $K(t, x)$. For example, if $0 < p \leq \infty$ and $f: \mathbf{R}_+ \rightarrow \mathbf{R}_+$ is a continuous nondecreasing function such that $0 < p^\infty(f) \leq q^\infty(f) < 1$, then let us denote by $(X_0, X_1)_{f, p; K}$ the set of all $x \in X_0 + X_1$ for which

$$(27) \quad \|x\|_{f, p; K} = \left\{ \int_0^\infty \left(\frac{K(t, x)}{f(t)} \right)^p dt/t \right\}^{1/p} < \infty$$

(see [24, p. 294]). In the sequel we write $(X_0, X_1)_{\theta, p; K}$ instead of $(X_0, X_1)_{f, p; K}$.

Received by the editors March 20, 1982.

1980 *Mathematics Subject Classification*. Primary 46E35, 46E30.

Key words and phrases. Interpolation spaces, rearrangement invariant spaces, indices.

©1983 American Mathematical Society
0002-9939/82/0000-0851/\$02.00

For $0 < \theta < \theta' < 1$ and $x \in (X_0, X_1)_{\theta,1;K} + (X_0, X_1)_{\theta',1;K}$, we define

$$(28) \quad \begin{aligned} K_{\theta}^{\theta'}(t, x) &\equiv K_{\theta}^{\theta'}(t, x; X_0, X_1) \\ &= K(t, a; (X_0, X_1)_{\theta,1;K}, (X_0, X_1)_{\theta',1;K}) \end{aligned}$$

and

$$(29) \quad \begin{aligned} (X_0, X_1)_{\Phi; K_{\theta}^{\theta'}} &= \{x \in (X_0, X_1)_{\theta,1;K} + (X_0, X_1)_{\theta',1;K} : \\ &\|x\|_{\Phi; K_{\theta}^{\theta'}} = \Phi(K_{\theta}^{\theta'}(t, x)) < \infty\}. \end{aligned}$$

2a. We shall show that the space $E_{f,p}$, which by definition consists of all (class of) measurable functions on $(0, 1)$ for which the norm

$$(30) \quad \|x\|_{f,p} = \int_0^1 x^*(t) dt + \left\{ \int_0^1 \left[\frac{t}{f(1/(1-\log t))} x^{**}(t) \right]^p \frac{dt}{t(1-\log t)} \right\}^{1/p}$$

is finite, coincides with the interpolation space $(L^1, L \log^+ L)_{f,p;K}$.

THEOREM 3 (BENNETT [2, THEOREM 8.1])—FOR $f(t) = t^{\theta}$, $0 < \theta < 1$. Let $1 \leq p \leq \infty$ and $0 < a \leq tf'(t)/f(t) \leq b < 1$. Then

$$(L^1, L \log^+ L)_{f,p;K} = E_{f,p}$$

holds with equivalent norms.

PROOF. As in [2] we have

$$\begin{aligned} \|x\|_{f,p;K} &\leq 2 \left\{ \frac{\int_0^{e^{-1}} [(1-\log t)f(1/(1-\log t))]^{-1} \int_t^1 x^{**}(s) ds]^p dt}{t(1-\log t)} \right\}^{1/p} \\ &\quad + f(2^{-1})^{-1} \left\{ \int_{e^{-1}}^1 [(-\log t)^{-1} \int_t^1 x^{**}(s) ds]^p e dt \right\}^{1/p} \\ &\quad + \left(\int_1^{\infty} f(t)^{-p} dt/t \right)^{1/p} \int_0^1 x^*(s) ds = I_1 + I_2 + I_3. \end{aligned}$$

We use the Hardy inequality (23) to obtain the estimate

$$I_1 \leq C \left\{ \frac{\int_0^1 [tf(1/(1-\log t))]^{-1} x^{**}(t)]^p dt}{[t(1-\log t)]} \right\}^{1/p}$$

The rest of the proof is the same as in [2, Theorem 8.1] and need not be repeated here.

2b. We consider $L(\varphi_i, q_i)$, $i = 0, 1$, spaces on $(0, \infty)$ (see Example 2). We assume that

$$(31) \quad \begin{cases} t^{-a}\varphi_1(t) \text{ increases for some } a > 0, \\ \eta(t) = \varphi_0(t)/\varphi_1(t) \text{ is increasing from 0 to } \infty, \\ t^{-b}\eta(t) \text{ increases for some } b > 0. \end{cases}$$

THEOREM 4. Suppose that r.i. space E on $(0, \infty)$ has Fatou norm or $\beta^\infty(E) < 1$. Let φ_0, φ_1 be as given in above and let $t\tau'(t) \approx \tau(t)$ (τ —function from Example 1). If

$$\Phi(h) = \|h(\eta(t)^{\theta'-\theta})/\varphi_\theta(t)\|_{E_{1,\tau^{-1}}}, \quad \varphi_\theta(t) = \varphi_0(t)^{1-\theta}\varphi_1(t)^\theta,$$

and

$$(32) \quad q^\infty(\varphi_{\theta'} \circ \tau^{-1}) < \alpha^\infty(E) \leq \beta^\infty(E) < p^\infty(\varphi_\theta \circ \tau^{-1}),$$

then

$$(33) \quad (L(\varphi_0, q_0), L(\varphi_1, q_1))_{\Phi, K_\theta^{\theta'}} = E_{1,\tau}^{(*),1}$$

with equivalent quasi-norms.

PROOF. From Theorem 2.1 of Holmstedt [26] (see also [22, p. 52]) and Theorem 4.2 of Torchinsky [32] it follows that

$$\begin{aligned} \varphi_\theta(t)x^*(t) &\leq C \int_0^{\eta(t)} u^{-\theta} \varphi_0 \circ \eta^{-1}(u) x^* \circ \eta^{-1}(u) du/u \\ &\leq C \int_0^{\eta(t)} u^{-\theta} \left[\sup_{0 < s \leq \eta^{-1}(u)} \varphi_0(s) x^*(s) \right] du/u \\ &\leq C \int_0^{\eta(t)} u^{-\theta} \|x^* \chi_{(0, \eta^{-1}(u))}\|_{L(\varphi_0, q_0)} du/u \\ &\leq C \int_0^{\eta(t)} u^{-\theta} K(u, x) du/u \leq CK_\theta^{\theta'}(\eta(t)^{\theta'-\theta}, x) \end{aligned}$$

and

$$\begin{aligned} K_\theta^{\theta'}(\eta(t)^{\theta'-\theta}, x) &\leq C \int_0^{\eta(t)} u^{-\theta} K(u, x) du/u + C \eta(t)^{\theta'-\theta} \int_{\eta(t)}^\infty u^{-\theta'} K(u, x) du/u \\ &\leq C \int_0^{\eta(t)} u^{-\theta} \|x^* \chi_{(0, \eta^{-1}(u))}\|_{L(\varphi_0, 1)} du/u \\ &\quad + C \int_0^{\eta(t)} u^{1-\theta} \|\varphi_1 x^* \chi_{[\eta^{-1}(u), \infty)}\|_{L_{1-1}^1} du/u \\ &\quad + C \eta(t)^{\theta'-\theta} \int_{\eta(t)}^\infty u^{-\theta'} \|x^* \chi_{(0, \eta^{-1}(u))}\|_{L(\varphi_0, 1)} du/u \\ &\quad + C \eta(t)^{\theta'-\theta} \int_{\eta(t)}^\infty u^{1-\theta'} \|\varphi_1 x^* \chi_{[\eta^{-1}(u), \infty)}\|_{L_{1-1}^1} du/u \\ &\leq C \varphi_\theta(t) \{ [P_{\varphi_\theta}(P_{\varphi_0} x^*)](t) + [P_{\varphi_\theta}(Q_{\varphi_1} x^*)](t) \\ &\quad + [Q_{\varphi_\theta}(P_{\varphi_0} x^*)](t) + [Q_{\varphi_\theta}(Q_{\varphi_1} x^*)](t) \}. \end{aligned}$$

Since (32) and $p(\varphi_\theta) \leq p(\varphi_0)$, $q(\varphi_1) \leq q(\varphi_{\theta'})$ so by Theorem 1 (see Remark 4) we have

$$\Phi(K_\theta^{\theta'}(\cdot, x)) \approx \|x\|_{E_{1,\tau}^{(*),1}}.$$

In the particular case, $\varphi_i(t) = t^{1/p_i}$, $1 \leq p_0 < p_1 < \infty$, and $\tau(t) = t^{r(1/p_0 - 1/p_1)}$, $0 < r < \infty$, Theorem 4 was obtained by Merucci [27, Theorem 1].

REMARK 7. If

$$\begin{aligned} K_\theta^1(t, x) &= K(t, x; (X_0, X_1)_{\theta, 1; K}, X_1), \\ K_0^{\theta'}(t, x) &= K(t, x; X_0, (X_0, X_1)_{\theta', 1; K}) \end{aligned}$$

and

$$K_0^1(t, x) = K(t, x; X_0, X_1)$$

then Theorem 4 holds for $0 \leq \theta < \theta' \leq 1$.

THEOREM 5. Suppose that *r.i.* space E on $(0, \infty)$ has Fatou norm or $\beta^\infty(E) < 1$. Let $\varphi_i, \psi_i, i = 0, 1, \eta(t) = \varphi_0(t)/\varphi_1(t), \xi(t) = \psi_0(t)/\psi_1(t)$ be as in (31), $1 \leq q_i < r_i \leq \infty$, and let $t\tau'(t) \approx \tau(t)$ (τ —function from Example 1).

If $A \in [L(\varphi_i, q_i), L(\psi_i, r_i)], i = 0, 1$, and

$$(34) \quad q^\infty(\varphi_1 \circ \tau^{-1}) < \alpha^\infty(E) \leq \beta^\infty(E) < p^\infty(\varphi_0 \circ \eta^{-1}),$$

then $A \in [E_{1, \tau^{-1}}^{(*)}, E_{\kappa, \delta}^{(*)}]$, where $\delta(t) = \xi^{-1} \circ \eta \circ \tau^{-1}(t)$ and $\kappa(t) = \psi_0 \circ \delta(t)/\varphi_0 \circ \tau^{-1}(t)$.

PROOF. Let $\Phi(h) = \|h \circ \eta(t)/\varphi_0(t)\|_{E_{1, \tau^{-1}}}$. Since

$$\begin{aligned} c\varphi_0(t)x^*(t) &\leq K(\eta(t), x; L(\varphi_0, q_0), L(\varphi_1, q_1)) \\ &\leq C\varphi_0(t)[P_{\varphi_0}x^*(t) + Q_{\varphi_1}x^*(t)] \end{aligned}$$

and (34), so by Theorem 1 or Theorem 2 (see Remark 4)

$$(L(\varphi_0, q_0), L(\varphi_1, q_1))_{\Phi, K} = E_{1, \tau^{-1}}^{(*)}.$$

On the other hand,

$$\begin{aligned} c\kappa(t)x^* \circ \delta(t) &\leq K(\eta \circ \tau^{-1}(t), x; L(\psi_0, r_0), L(\psi_1, r_1))/\varphi_0 \circ \tau^{-1}(t) \\ &\leq C\kappa(t)[(P_{\psi_0}x^*)(\delta(t)) + (Q_{\psi_1}x^*)(\delta(t))], \end{aligned}$$

and by Theorem 1,

$$(L(\psi_0, r_0), L(\psi_1, r_1))_{\Phi, K} = E_{\kappa, \delta}^{(*)}.$$

From the interpolation theorem (see [22 or 27, Lemma 1]) we have thus proved our theorem.

COROLLARY 5. Let $\varphi_i, \psi_i, q_i, r_i, i = 0, 1, \eta, \xi$ be as in Theorem 5. Assume that $\tau(t) = t$ and $E = L(\varphi, q)$, where $q^\infty(\varphi_1) < p^\infty(\varphi) \leq q^\infty(\varphi) < p^\infty(\varphi_0)$. If $A \in [L(\varphi_i, q_i), L(\psi_i, r_i)], i = 0, 1$, then $A \in [L(\varphi, q), L(\psi, q)]$, where $\psi(t)/\psi_0(t) = \varphi \circ \eta^{-1} \circ \xi(t)/\varphi_0 \circ \eta^{-1} \circ \xi(t)$.

REMARK 8. In the particular case, if $\varphi_i(t) = t^{1/p_i}, 1 \leq p_0 < p_1 < \infty, \psi_i(t) = t^{1/p'_i}, 1 \leq p'_0 < p'_1 < \infty$, then from Corollary 5 we have interpolation theorems of Sharpley [30], Bennett and Rudnick [21, Theorem B] and Persson [29, Theorem 2.1].

9. The reader will have no difficulty in supplying the proofs of Corollary 5 for the spaces $\Lambda(\varphi, q)$ because $\Lambda(\varphi, q) = L(\varphi^{1/q}, q)$.

10. The above theorems can be proved for Lorentz-Orlicz spaces $L(\varphi, F)$, $\Lambda(\varphi, F)$; some generalizations of Corollary 5 are in interesting papers of Torchinsky [31] and Heinig-Vaughan [25].

2c. In this part we extend the fundamental interpolation theorem of Krein and Semenov (see [10, Theorem 6.1, p. 175]) and the result of Pavlov [28, Theorem 3] to weighted r.i. spaces.

THEOREM 6. Suppose that r.i. space E on $\mathbf{R}_+$ has Fatou norm or $\beta^\infty(E) < 1$. Let φ_i , ψ_i , $i = 0, 1$, be positive nondecreasing concave functions on $\mathbf{R}_+$ such that

- (1) $\varphi_0(s)/\varphi_1(s)$ is nondecreasing, $\varphi_0(0^+) = 0$;
- (2) $\{\varphi_0(s)/\varphi_1(s): s \in \mathbf{R}_+\} \subset \{\psi_0(s)/\psi_1(s): s \in \mathbf{R}_+\}$. We assume that a linear operator $A: D_A \supset E_{\rho, t}^{(*)} \rightarrow L^0$, where ρ is a positive measurable function on $\mathbf{R}_+$, satisfies

$$(35) \quad \sup_{t>0} \psi_i(t)(Ax)^{**}(t) \leq C_i \int_0^\infty x^*(s) d\varphi_i(s), \quad i = 0, 1.$$

If

$$(36) \quad q^\infty(\varphi_1/\rho) < \alpha^\infty(E) \leq \beta^\infty(E) < p^\infty(\varphi_0/\rho)$$

then $A \in [E_{\rho, t}^{(*)}, E_{\kappa, \delta}^{(**)}]$, where δ is a positive measurable solution of equation

$$(37) \quad \psi_0(\delta(t))[\psi_1(\delta(t))]^{-1} = \varphi_0(t)[\varphi_1(t)]^{-1}$$

and

$$(38) \quad \kappa(t) = \psi_0(\delta(t))[\varphi_0(t)]^{-1}\rho(t) = \psi_1(\delta(t))[\varphi_1(t)]^{-1}\rho(t).$$

PROOF. We have (for $x \in \Lambda_{\varphi_0} + \Lambda_{\varphi_1}$)

$$(Ax)^{**}(\delta(t)) \leq C \left[\psi_0(\delta(t))^{-1} \int_0^t x^*(s) d\varphi_0(s) + \psi_1(\delta(t))^{-1} \int_t^\infty x^*(s) d\varphi_1(s) \right]$$

(see [10, p. 176]). Hence,

$$\begin{aligned} \kappa(t)(Ax)^{**}(\delta(t)) &\leq C\rho(t) [S_{\varphi_0}x^*(t) + T_{\varphi_1}x^*(t)] \\ &\leq C\rho(t) [P_{\varphi_0}x^*(t) + Q_{\varphi_1}x^*(t)]. \end{aligned}$$

By applying Theorem 1 we have

$$\|Ax\|_{E_{\kappa, \delta}^{(**)}} \leq C\|P_{\varphi_0}x^*\|_E + C\|Q_{\varphi_1}x^*\|_E \leq C\|\rho x^*\|_E = C\|x\|_{E_{\rho, t}^{(*)}}.$$

Theorem 6 can be applied to generalizations of interpolation theorems, Calderon-Mitjagin [10, Theorem 6.4] and Marcinkiewicz [9, Theorem 1; 25, Theorem 5] or Hausdorff-Young theorem for convolution operator [10, Theorem 6.16; 28, Theorem 4] which include weighted r.i. spaces as intermediate classes. The reader will have no difficulty in extending the results to the case of weighted r.i. spaces.

REFERENCES

21. C. Bennett and K. Rudnick, *On Lorentz-Zygmund spaces*, Dissertationes Math. (Rozprawy Mat.) **175** (1979), 67 pp. MR **81i** #42020.
22. J. Bergh and J. Löfström, *Interpolation spaces, an introduction*, Springer-Verlag, New York, 1976. MR **58** #2349.
23. A. P. Calderon, *Spaces between L^1 and L^∞ and the theorem of Marcinkiewicz*, Studia Math. **26** (1966), 273–299. MR **34** #3295.
24. J. Gustavsson, *A function parameter in connection with interpolation of Banach spaces*, Math. Scand. **42** (1978), 289–305. MR **80d** #46124.
25. H. P. Heinig and D. Vaughan, *Interpolation in Orlicz spaces involving weights*, J. Math. Anal. Appl. **64** (1978), 79–95. MR **80f** #46028b.
26. T. Holmstedt, *Interpolation of quasi-normed spaces*, Math. Scand. **26** (1970), 177–199.
27. C. Merucci, *Quelques resultats d'interpolation pour les espaces de Lorentz*, C. R. Acad. Sci. Paris Sér. A **280** (1975), 1731–1734. MR **51** #11085.
28. E. P. Pavlov, *An interpolation theorem*, Functional Analysis, No. 8: Operator Theory, Ul'janovsk. Gos. Ped. Inst. Ul'janovsk, 1977, pp. 113–119. (Russian) MR **58** #30149.
29. L. E. Persson, *On a weak-type theorem with applications*, Proc. London Math. Soc. **38** (1979), 295–308. MR **81b** #42062.
30. R. Sharpley, *Spaces $\Lambda_\alpha(X)$ and interpolation*, J. Funct. Anal. **11** (1972), 479–513. MR **49** #9611.
31. A. Torchinsky, *Interpolation of operations and Orlicz classes*, Studia Math. **59** (1976), 177–204. MR **55** #11024.
32. ———, *The K -functional for rearrangement invariant spaces*, Studia Math. **64** (1979), 175–190. MR **80k** #46032.

THE MATHEMATICAL INSTITUTE OF THE POLISH ACADEMY OF SCIENCES, POZNAŃ BRANCH,
MIELŻYŃSKIEGO 27/29, 61-725 POZNAŃ, POLAND

On Hardy's inequality in weighted rearrangement invariant spaces and applications. I, II.

Proc. Amer. Math. Soc. 88 (1983), no. 1, 67-74, 75-80.

Given a positive measurable function k on an interval $I=(0, l)$, a measurable $\delta: I \rightarrow I$ and a rearrangement invariant function space E on $(0, l)$, one defines a Banach space $E_{k, \delta}$ with norm $\|x\|_{E_{k, \delta}} = \|k(t)x(\delta(t))\|_E$. Given an increasing absolutely continuous function $\psi: I \rightarrow \mathbb{R}$ with $\psi(0)=0$, one can define various averaging operators, for example, $(P_\psi x)(t) = \psi(t)^{-1} \int_0^t x(s) \psi(s) s^{-1} ds$. The author gives some sufficient conditions on k, δ, E and ψ under which these operators are bounded on $E_{k, \delta}$. Some applications to embedding theorems for weighted Sobolev spaces and interpolation theorems are given in the second paper.

G. Schechtman (Rehovot)

Maligranda, Lech

46017

On Hardy's inequality in weighted rearrangement invariant spaces and applications. I.

Proc. Am. Math. Soc. 88, 67-74 (1983).

For a positive nondecreasing continuous function ψ on the interval $(0, 1)$ the average operators P_ψ, Q_ψ and S_ψ, T_ψ are considered. Main results are two valuable and very general theorems in which the author has given inequalities for the upper index $q^1(\psi)$ and upper index $\beta^1(E)$ in the sense of Boyd and estimates for the norms of average operators in some weighted rearrangement invariant spaces.

A. Waszak.

Maligranda, Lech

46018

On Hardy's inequality in weighted rearrangement invariant spaces and applications. II.

Proc. Am. Math. Soc. 88, 75-80 (1983).

The note contains theorems on the identity of the interpolation space $(L^1, L \log^+ L)_{\theta, p, K}$ and the space $E_{\theta, p}$ with equivalent norms, and of the spaces $(L(\varphi_0, q_0), L(\varphi_1, q_1))_{\theta, K, \theta'}$ and $E_{\theta, \theta'}$ with equivalent quasi-norms. Moreover, in the last part of the note the author has given a theorem on weighted rearrangement invariant spaces which states certain generalization of the fundamental interpolation theorem of Krein and Semenov and also of the theorem of Pavlov. [For part I see the review above].

A. Waszak.

11 Б1042. О неравенстве Харди в весовых перестановочно-инвариантных пространствах и приложения. I. On Hardy's inequality in weighted rearrangement invariant spaces and applications. I. Maligranda Lech. «Proc. Amer. Math. Soc.», 1983, 88, № 1, 67-74 (англ.)
Пусть $\psi(t)$ — положительная неубывающая непрерывная функция на $(0, l)$, где $0 < l \leq \infty$. Изучаются операторы

$$P_\psi x(t) = (\psi(t))^{-1} \int_0^t x(s) \psi(s) s^{-1} ds,$$

$$Q_\psi x(t) = (\psi(t))^{-1} \int_t^l x(s) \psi(s) s^{-1} ds,$$

$$S_\psi x(t) = (\psi(t))^{-1} \int_0^t x(s) \psi'(s) ds,$$

$$T_\psi x(t) = (\psi(t))^{-1} \int_t^l x(s) \psi'(s) ds.$$

В терминах индексов функции ψ и перестановочно-инвариантного пространства X получены достаточные, а иногда и необходимые условия ограниченности этих операторов в X . Библи. 20. Е. Семёнов

11 Б1043. О неравенстве Харди в весовых перестановочно-инвариантных пространствах и приложения. II. On Hardy's inequality in weighted rearrangement invariant spaces and applications. II. Maligranda Lech. «Proc. Amer. Math. Soc.», 1983, 88, № 1, 75-80 (англ.)

Продолжение статьи (реф. 11Б1042).

Изучается обобщенный K -метод, порожденный нестепенными функциями. Вычислены пространства $(L(\varphi_0, q_0), L(\varphi_1, q_1))_{\theta, K, \theta'}$, где $L(\varphi, q)$ — пространство Лоренца и

$$\|x\|_{\Phi, K, \theta'} = \Omega(K(t, x), (X_0, X_1)_{\theta, 1, K}, (X_0, X_1)_{\theta', 1, K}).$$

Доказана теорема об интерполяции операторов слабого типа в весовых перестановочно-инвариантных пространствах. Е. Семёнов