Prediction of crack lengths in rock after cautious blasting with zero inter-hole delay

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ABSTRACT: Mechanics and dimensional analysis are used to derive new prediction equations for the lengths R_c of the radial cracks emanating from the half-casts in the contour after cautious blasting of a contour with a free toe and zero initiation delay between the charges. The influencing parameters are:

- density ρ_e , velocity of detonation D and adiabatic expansion exponent γ of the explosive;
- bore-hole diameter Φ_h and charge coupling ratio f ; and
- density ρ , sound velocity c and fracture toughness K_{Ic} of the rock.

A number of parameters in the prediction equations are determined by curve fitting to test data obtained from bench blasting in granite in the Vånga quarry in southern Sweden. An approximate design diagram for these equations is given and its use explained. This diagram gives a comprehensive view of how the different parameters influence the predicted crack lengths. The prediction equations are very good for low velocity of detonation (VOD) explosives but less so for high VOD ones. This is supported by independent Finnish data. The prediction equations can not be expressed as simple functions of the charge concentration q (kg/m).

KEYWORDS: cautious blasting, zero delay, blast damage, prediction equation, radial cracks, decoupling, velocity of detonation, VOD, fracture toughness.

LIST OF SYMBOLS

a	acceleration, m/s^2
a	exponent in curve fit Equation 34a
A	PPV prefactor in Equation 5a
B	burden, m
B_c	critical burden, m
c	sound velocity in rock, m/s
d	uncharged hole length, m
D	velocity of detonation, m/s
e	explosive energy, J/kg
e, e'	exponents in Equations 34b and A2
f	coupling ratio
$f_H (R/H)$	charge length correction
E	Young's modulus of rock, Pa
F	force, N
g_o	conversion constant, $kg\ m/s^2 / N$
H	height of explosive column, m

i	impedance function in Equation 34a
I	rock damping coefficient, 1/m
k	pressure prefactor, cm/MPa ^{0.68}
K_{Ia}	crack arrest toughness, Pa√m
K_{Ic}	fracture toughness, Pa√m
m	mass, kg
n	pressure prefactor
N	number of radial cracks
p	pressure, Pa
p_{CJ}	CJ-point pressure, Pa
p_e	constant volume explosion pressure, Pa
p_h	bore-hole pressure, Pa
$p_{h,min}$	critical bore-hole pressure, Pa
$p_{h,crack}$	experimental value of $p_{h,min}$
p_s	shock pressure amplitude, Pa
$p_{s,crit}$	critical shock amplitude, Pa
PPV	peak particle velocity, m/s
PPV _o	fully coupled PPV, m/s
PPV _c	critical PPV, m/s
q	charge concentration, kg/m
r	distance from hole center, m
r^2	coefficient of determination
R	dist. from charge to gauge, m
R_c	max. length of radial cracks, m
R_c^I	$R_c - \Phi_h/2$, m
s	Hugoniot slope
S	spacing, m
Δt	initiation delay, s
u_r	radial displacement, m
$\dot{u}_r$	radial particle velocity, m/s
$\dot{u}_s$	shock velocity, m/s
v	crack velocity, m/s

Greek

α	decoupling exponent
β	pressure influence exponent in Equations 17b and 31–34
β	PPV decay exponent in Equation 5a
δ	pressure decay exponent
σ_c	critical stress value, Pa
σ_t	uniaxial tensile strength, Pa
ε_θ	tangential or hoop strain
$\dot{\varepsilon}_\theta$	hoop strain rate, 1/s
γ	adiabatic exponent
λ	pressure prefactor
Φ_e	charge diameter, m

Φ_h	bore-hole diameter, m
ν	Poisson's ratio of rock
ρ	rock density, kg/m ³
ρ_e	explosive density, kg/m ³

1 INTRODUCTION

To be able to design blasting rounds so that the damage to the remaining rock doesn't create serious problems has always been of prime importance in rock excavation. In tunnels and drifts, the extra costs of cautious contour blasting can be recovered by less scaling and support work or less concrete backfill (Andersson 1994). In open pit mining, cautious blasting leads to improved safety from better functioning catchment berms and improved recovery of ore through steeper slopes (Siira 1994).

The term damage should be reserved for the degree of impaired function of the structure, e.g. its stability. It is, however, often used as synonymous with cracking; micro- or macrocracking, even movement in and extension of existing cracks. The design of cautious blasting is conceptually based on cracking but often expressed in terms of peak particle velocity, PPV (Holmberg & Persson 1980, Hustrulid et al. 1992). These designs mainly serve as a consistent way to keep down the charge concentration (Niklasson et al. 1994), i.e. the dynamic load level, and the resulting damage is seldom documented except by the superficial half-cast factor (Ouchterlony 1992).

Even in extensive blast damage monitoring studies (Ouchterlony et al. 1993, Olsson 1996), the main thrust in damage measurements lies in indirect methods, geo-physical, hydrological etc. Recently SveBeFo has developed a direct technique to measure the extent of cracking behind blast-holes and conducted an extensive series of tests under controlled conditions in a dimensional stone at Vånga in southern Sweden (Olsson & Bergqvist 1993a, b, 1996a, b). The purpose of this work has been to develop a basis for improved contour blasting of mainly tunnels.

At Vånga, 4–5 m long, $\Phi_h = 24$ –64 mm blast-holes in various burden-spacing patterns have been charged with different explosives and blasted in groups of three or four along benches. Over 300 holes have been fired since 1991, most of them with zero inter-hole delay, using electronic delay detonators. The remaining damaged contour has been carefully removed by a cord post-split and the resulting blocks wedge split and then sawn perpendicularly to the half-casts. The cracking has been visualized using dye penetrant techniques. Figure 1 shows a hole where the radial cracks clearly dominate.

The results obtained so far at Vånga may be summarized as follows. For otherwise constant conditions:

- The radial crack lengths decrease with a decreasing coupling ratio.
- Fully charged holes show a more complex crack pattern with interacting cracks.
- A high velocity of detonation (VOD) results in many short cracks around the hole.
- The crack lengths increase with an increasing charge concentration.
- For some explosives the crack lengths increase with an increasing burden.

Figure 1. Radial cracks behind a 51 mm blast-hole, Olsson & Bergqvist (1993a).

- Arc-shaped subsurface cracks may occur when the spacing is increased.
- The time delay between initiation of the different holes is important:
 - an instantaneous initiation of groups of holes gives the shortest cracks;
 - the crack lengths increase with an increasing delay time;
 - even delay times as low as 1 ms give crack lengths more like single hole blasts;
 - sequential microinterval (zipper) blasting gives longer cracks than instantaneous initiation;
 - traditional smooth blasting procedures give unnecessarily long cracks; and
 - alternating initiation sequences like 1, 3, 1, 3, ... ms give short cracks.

The Vånga work is expected to continue for at least the next 3 years.

In order to make the best use of the unique Vånga data base, it should be used to verify existing models for cracking or to develop new ones. Such models can then be used to design cautious blasting with increased confidence. Such work started within SveBeFo's 1993–96 research program and this paper gives the first results from a quasistatic modelling based on mechanics and dimensional analysis (Ouchterlony 1997).

2 DERIVATION OF MODEL

2.1 Description of process and general formula

A simplified picture of the Vånga bench blasts is shown in Figure 2. At Vånga there is a horizontal fracture plane which cuts off the toe and no subdrilling or stemming has been used. The primer has been much smaller than commonly used, 50 g in our case, so the blasting process is as nearly independent of hole length as one can hope to get in practice. In most cases, the charges have been radially decoupled, i.e. their diameter smaller than the blast-hole diameter and centrally located in the blast-holes.

A very brief description of the sequence of events of the blast and the main influencing parameters are given by the following list.

Event:	Influencing parameter:
1. Detonation of charge	ρ_e, D, γ
2. Gas pressure hits bore-hole wall	Φ_e, Φ_h, ρ, c
3. Crack growth is initiated	E, ν, σ_t, K_{Ic}
4. 'Symmetric' growth of cracks	N, p_h
5a. Shock wave from adjacent charge	$S, \Delta t, c$
5b. Stress wave reflection from free face	B, d, c
6. Preferred growth of cracks	S, B
7a. Cracks vent to free face	v, B
7b. Axial venting of gases	stemming, d, H
8a. Burden starts moving out	$B < B_c$
8b. Contour cracks stop moving	$K_{Ia}, R_c.$

Note that each event is not necessarily well defined. Nor is the time order necessarily always correct as indicated by 5a–5b etc. The list is meant to give a qualitatively correct idea of the parameters involved in the blasting process.

Figure 2. Side view of cautious blasting with a free bottom.

If we regroup the influencing parameters in a dimensional analysis scheme (Taylor 1974) we obtain

Explosive loading:	4	$\rho_e, D, \gamma, \Delta t$
Geometry:	6	$\Phi_e, \Phi_h, B, S, H, d$
Rock material:	7	$E, \rho, c, \nu, \sigma_t, K_{Ic}, K_{Ia}$
Result parameter:	1	R_c or N
Physical dimensions:	4	force $F(N)$, mass m (kg), length (m) and time (s)
Newton's second law:	1	$F = ma/g_0, g_0$ (kg m/s ² /N).

This means that the minimum number of dimensionless groups to describe the process is thus $4 + 6 + 7 + 1 - 4 + 1 = 15$. This number is by far too large to be practicable. $g_0 = 1$ in SI units and is therefore omitted in the sequel.

The first reduction occurs if we assume that c is the P-wave velocity, which for reasonable values of Poisson's ratio ν is relatively constant and not much different from the bar velocity, $c \approx \sqrt{E/\rho}$. This means that we can drop the parameters E and ν and the number of dimensionless groups is reduced by 2 to 13.

Further we shall assume that the process is two-dimensional, i.e. conditions along the blast-hole everywhere the same. This removes the parameters H and d . Thus the number of groups becomes 11. Then we further restrict the analysis to a constant blasting pattern, $S/B \approx \text{const}$, a single hole or instantaneous initiation of all holes, $\Delta t = 0$, and one strength parameter for the rock. We also assume that intact strength behavior is less important to our process than crack propagation and arrest which means that σ_t drops out. The arrest phenomenon is finally assumed to be governed by $K_{Ic} \approx K_{Ia}$.

Then only 7 groups remain and a general functional relationship can be stated as e.g.

$$\frac{R_c}{\Phi_h} = h\left(\frac{\rho_e \Phi_e^2}{\rho \Phi_h^2}, \frac{\Phi_e}{\Phi_h}, \frac{B}{\Phi_h}, \frac{c}{D}, \frac{\rho_e D^2 \sqrt{\Phi_h}}{K_{Ic}}, \gamma\right) \quad (1)$$

In this equation the groups may be rearranged to suit different purposes as long as all the remaining 10 parameters appear at least once. Other intermediate or derived parameters may replace the original ones on a one to one basis.

The charge concentration q (kg/m) and the coupling ratio f may, for example, be written

$$q = \frac{\pi}{4} \cdot \rho_e \Phi_e^2 \quad \text{and} \quad (2)$$

$$f = \frac{\Phi_e}{\Phi_h} \quad (3)$$

Thus, provided that the function $h(\dots)$ is known, Equation 1 answers the question: *Given q , f , D , and B etc., how long will the (maximum) crack length in a given rock be?* These are exactly the variables involved in the Vånga work (Olsson & Bergqvist 1993a, b, 1996a, b). Hence, their results could be used to determine h . We have no a priori knowledge of its functional form though.

2.2 Some simple formulas

In order to obtain some perspective on what the function h should look like, some simple formulas are investigated and their advantages and draw backs discussed. This discussion will set the stage for the derivation of h in Section 2.3.

As a Swedish rule of thumb, $R_c(m) = q$ (kg/m of Dynamex M equivalents) was often used before. In the work by Sjöberg (1980) this equation was replaced by

$$R_c = 1.9 \cdot q \text{ (kg/m of Dynamex B equivalents) when } q < 1.4 \quad (4)$$

For each explosive, the equivalent amount of Dynamex B must be calculated using the weight strength concept (Persson et al. 1993). This equation is the origin of several tables that have been in extensive use. Since it relates the depth of the damage zone only to the charge concentration it can not account for the sizeable effect of decoupling that we have seen in the Vånga tests. Thus it is insufficient for our modelling purposes.

In the engineering approach of Holmberg & Persson (1980), a critical PPV-value PPV_c is used to delimit the damage zone. Since the conditions are roughly the same along a large part of the charge, the PPV-value at a distance R in this part may be calculated from the simplified site scaling law (Ouchterlony et al. 1993)

$$PPV_o = A \cdot \left(\frac{\sqrt{f_H q}}{R} \right)^\beta \quad \text{with} \quad (5a)$$

$$f_H = \frac{\tan(H/2R)}{H/2R} \quad (5b)$$

Here A and β are site specific constants, determined by measurements. The function $f_H(R/H)$ is a correction for charge length H , based on the observation that close to the charge, only parts of the charge seem to contribute to the PPV value. When R/H becomes large, $f_H \rightarrow 1$.

The failure criterion $PPV_o \leq PPV_c$ when $R \geq R_c$ delimits the depth of the damage zone. Often the range $PPV_c = 700\text{--}900$ mm/s has been used. An inversion of Equation 5a yields R_c as a function of PPV_c (Ouchterlony et al. 1993).

Because the coupling ratio is missing, this equation is unsuitable in its present form. This effect may of course be added, see Atchison et al. (1964) or Atchison (1968) who have found that, relative to the fully coupled state PPV_o , the decoupled PPV value is given by

$$PPV = PPV_o \cdot \left(\frac{\Phi_c}{\Phi_h} \right)^{1.5} = PPV_o \cdot f^{1.5} \quad (6)$$

A commonly used value for the decay exponent in Equation 5a is $\beta \approx 1.5$. In cautious blasting shallow damage zones are the goal, $R_c \ll H$. Then Equations 5–6 reduce to

$$R_c = \pi \left(\frac{PPV_c}{A} \right)^{4/3} \cdot q \cdot f^2 \quad (7)$$

It gives a generalization of Equation 4 that incorporates the coupling ratio f . Since other explosive properties such as the velocity of detonation are not represented in Equations 5–7 they are, however, still insufficient for our modelling purposes.

It may be that an H -dependence like in Equations 5 should be incorporated into Equation 1 when $R_c \ll H$ no longer applies. In the Vånga tests $H = 4\text{--}5$ m and with few exceptions $R_c \leq 0.5$ m so the H -dependence is left out.

Recently Hustrulid et al. (1992) and Hustrulid (1994) have presented another PPV formula for blast damage zone predictions. It is based on a linearly elastic solution for the particle velocity from a spherical source and the stacking of equivalent explosive spheres of height $b = \sqrt{1.5} \cdot \Phi_h/2$. The first version of the formula may be written

$$\text{PPV} = \frac{0.61 p_h}{\rho c} \cdot \left(\frac{\Phi_h}{R} \right) \cdot \exp \left(- \frac{IR}{0.61 \Phi_h} \right) \quad (8)$$

Here p_h is the pressure on the bore-hole wall and I is a non-dimensional rock damping factor. In Hustrulid (1994), the factor $0.61 \Phi_h$ in the exponent has been omitted so there the damping factor has dimensions 1/m. Hustrulid et al. (1992) state that the PPV value is produced by the sphere closest to the gauge point at distance R and that multiple spheres need not, in general, be considered. Thus the charge length doesn't influence the PPV level in Equation 8, unlike in Equation 5.

The wall pressure in Equation 8 is related to the explosion pressure p_e by

$$p_h = p_e \cdot f^{2\gamma} \quad (9)$$

Ordinarily $p_e \propto \rho_e \cdot D^2$, see below. Thus Equation 8 contains more of the desired parameters that make up the dimensionsless groups in h in Equation 1.

Jiang et al. (1993) show however that a series of stacked spheres does not represent the radiation pattern produced by a column of explosives, either in full or half-space. This casts doubt on the adequacy of Equation 8. Furthermore, in both PPV approaches there is a tacit connection between PPV_c and a critical stress level $\sigma_c = \sigma_t$ e.g. which is based on a one-dimensional relationship between codirectional tensor and vector components,

$$\sigma = \rho c \text{PPV} \quad (10)$$

It holds only at the front of a three dimensional P-wave which impinges on a material at rest. Furthermore, this stress is compressive for an explosively generated wave. Behind the wave front, the radial expansion eventually sets up a tensile tangential stress field which can initiate radial cracks. See also the highly critical discussion by Blair & Minchinton (1996) who emphasize the lack of a simple PPV versus σ or ε relationship.

Thus these common blast damage approaches have serious drawbacks. In terms of dimensional analysis PPV is a result parameter, like R_c in the scheme above, but must be described by a different functional relationship than h in Equation 1. There is little point in deriving that relationship when there is no simple relationship between PPV and stress or strain, quantities that can govern material fracture and hence determine the depth of the damage zone.

The pressure aspects of the PPV approach will, however, be pursued a bit further. According to the simplest theory of ideal detonation; one dimensional flow, a thin shock in which the chemical reaction is instantaneous and the discontinuity is steady

(Fickett & Davis 1979, Persson et al. 1993), the CJ pressure of the detonation, p_{CJ} , is given by

$$p_{CJ} = \rho_e \frac{D^2}{\gamma + 1} \quad (11)$$

Since at high densities $\gamma \approx 3$ the following approximation is often used

$$p_{CJ} = \rho_e \frac{D^2}{4} \quad (12)$$

see Persson et al. (1993). At the CJ point, the explosive gases are compressed and, at a reexpansion to the original volume, the explosion pressure becomes

$$p_e = p_{CJ} \cdot \left(\frac{\gamma}{\gamma + 1} \right)^\gamma \approx \frac{p_{CJ}}{2} \quad (13)$$

see Persson et al. (1993), Equation 4.34. The postfactor actually lies within 0.42–0.45 when $1.5 < \gamma < 3$ though. Combining Equations 11 and 13 we obtain

$$p_e = \frac{\gamma^\gamma}{(\gamma + 1)^{(\gamma+1)}} \cdot \rho_e D^2 = n(\gamma) \cdot \rho_e D^2 \quad (14)$$

The prefactor in this equation lies in the interval 0.19–0.11 when $1.5 < \gamma < 3$.

We can estimate γ roughly from Fickett & Davis (1979) Equation 2A-21

$$\gamma = \sqrt{1 + \frac{D^2}{2e}} \quad (15)$$

Here e (J/kg) is the heat of complete reaction for a detonation in a polytropic gas. In turn, e may in an engineering sense be estimated by the available explosive energy given by the explosives manufacturer. Thus γ as an influencing parameter in the dimensional analysis scheme in Section 2.1 could be replaced by e but in this paper γ is retained.

Hustrulid et al. (1992) and Hustrulid (1994) state that p_e and γ can be supplied by the explosives manufacturers. The range of values that they use is $\gamma = 1.3$ –2.14. In order to calculate p_e and γ more exactly we would have to know the ingredients of the explosive and to use codes like Tiger (Cowperthwaite & Zwisler 1975) to compute ideal expansion isentropes. Along such curves γ varies from about 3 in the high pressure range to about 1.3 at full expansion. The matter of a constant γ versus one which varies is brought up again in the discussion in Section 4.

When the charge is fully coupled p_e becomes the bore-hole wall pressure p_h . For a decoupled charge further expansion of the gases is required before the pressure p_h is reached. Persson et al. (1993) use $2\gamma \approx 3$ in Equation 9 to express this. In the traditional design of presplit blasts, see Calder (1977) e.g. it is stated that the bore-hole pressure created by a decoupled charge is given by

$$p_h = n \cdot \rho_e D^2 \cdot f^{2.4} \quad (16)$$

Unlike in Equation 14, this prefactor n depends on explosive density. It is given in the form of a $n(\rho)$ diagram where n lies in the interval 0.19–0.08 when $0.8 \leq \rho \leq 2.0$.

The value 2.4 of the f -exponent in Equation 16 appears to go back to work in which decoupled spherical charges were fired in water filled metal drums and the

pressure calculated from the shock velocity (Bauer 1967). Note though that this experiment refers to a spherical decoupling and Equation 16 to a cylindrical one and yet that the exponent is the same.

In this review of pressure formulas we thus see decoupling dependence ranging from the simple f^2 , $f^{2.4}$ or f^3 to the more specific $f^{2\gamma}$ with a constant γ -value and to one which takes the actual gas expansion law for each explosive into account. The latter falls outside the simplistic description of events behind Equation 1 but as mentioned the matter is discussed again in Section 4.

To avoid the drawbacks of the PPV approach when accounting for the observed cracking in the Vånga tests, Olsson & Bergqvist (1996a) assumed simply that

$$p_h = 0.125 \rho_e D^2 \cdot f^{2\gamma} \quad \text{and} \quad (17a)$$

$$R'_c = k \cdot p_h^\beta \quad (17b)$$

Here

$$R'_c = R_c - \frac{\Phi_h}{2} \quad (18)$$

denotes that the crack length was measured from the bore-hole wall, not the center. Then k and the exponents $2\gamma = \alpha$ and β were constants determined by a least squares fit to average crack length data for 7 different sets of experiments, see Table 1 further below or Olsson & Bergqvist (1996a), Table 21. The result of the fitting was that $\alpha = 3.37$, $\beta = 0.68$ and $k = 1.52$ when p_h is given in MPa and R'_c in cm.

A plot of the results is shown in Figure 3. The agreement between the line and the measured values is expressed by the coefficient of determination $r^2 = 0.899$. It could be better for short cracks, but the exponent 3.37 is not far from the value 3 (Persson et al. 1993) and the exponent $0.68 < 1$ is consistent with Hustrulid's Equation 11 when $I \neq 0$. This is promising but the dimensional form of Equation 17b is awkward.

Figure 3. Curve fit diagram for results from Olsson & Bergqvist (1996a).

By the simple introduction of parameters Φ_h and K_{Ic} it can be made dimensionless however and thus serve as a first approximation of the function h in Equation 1, see Section 3 below.

In summary, the formulas in the PPV approach do not contain enough parameters to describe our Vånga data and the approach has a poor physical basis. This motivates the derivation of a new crack length equation to give the function h .

2.3 Derivation of new crack length equation

Consider a cylindrical hole in a plane elastic medium subjected to a dynamic pressure pulse. According to Grady & Kipp (1987), the number of cracks generated along the circumference is given by

$$N = \pi \Phi_h \cdot \left(\frac{\rho c \dot{\epsilon}}{6 K_{Ic}} \right)^{2/3} \quad (19)$$

where $\dot{\epsilon}$ is the strain rate associated with the loading. In their theory the driving force for the fragmentation is the kinetic energy with respect to the center of mass and our choice is to represent it by the circumferential strain rate, i.e. $\dot{\epsilon} = \dot{\epsilon}_\theta$, rather than by the radial one.

Further, according to Liu & Katsabanis (1993), a conservation of momentum across the shock wave requires that the pressure amplitude

$$p_s = \rho \dot{u}_r \dot{u}_s \quad (20)$$

where $\dot{u}_r$ is the radial particle velocity and $\dot{u}_s$ the shock velocity. The latter two are related through the shock Hugoniot, $\dot{u}_s = c + s \cdot \dot{u}_r$, where c is the elastic P-wave velocity in rock, see above. For rocks $c \approx 3000$ – 6000 m/s and the slope $s \approx 1$ – 2 . For reasonably low loadings, such as in cautious blasting, $\dot{u}_s \approx c$ and thus the radial particle velocity becomes

$$\dot{u}_r = \frac{p_s}{\rho c} \quad (21)$$

By definition the linearized circumferential strain $\epsilon_\theta = u_r/r$ where r is the radial coordinate, i.e. the distance from the center of the blast-hole. Hence,

$$\dot{\epsilon}_\theta = \frac{\dot{u}_r}{r} \quad (22)$$

Combining Equations 21 and 22 we obtain $\rho c \dot{\epsilon}_\theta = p_s/r$ or

$$\rho c \dot{\epsilon}_\theta = 2 \frac{p_h}{\Phi_h} \quad (23)$$

at the bore-hole wall where $p_s \approx p_h$.

Inserting this expression into Equation 19 yields

$$N = \frac{\pi}{3^{2/3}} \cdot \left(\frac{p_h \sqrt{\Phi_h}}{K_{Ic}} \right)^{2/3} \quad (24)$$

By requiring that $N \geq 2$ we obtain an expression for the minimum dynamic pressure required to split the bore-hole by initiation of cracks

$$p_h \geq p_{h,\min} = 1.52 \cdot \frac{K_{Ic}}{\sqrt{\Phi_h}} \quad (25)$$

For a $\Phi_h = 50$ mm bore-hole in granite with $K_{Ic} \approx 2 \text{ MPa}\sqrt{\text{m}}$ (Ouchterlony 1991) we obtain a reasonable $p_{h,\min} \approx 14 \text{ MPa}$. A combination of Equations 24 and 25 yields

$$N = 2 \left(\frac{p_h}{p_{h,\min}} \right)^{2/3} \quad (26)$$

To initiate 4 cracks dynamically would thus require $2\sqrt{2} \cdot p_{h,\min}$ or about 38 MPa.

An application of classic static stress intensity factor formulas to crack initiation leads to a result in which $p_{h,\min}$ is unrealistically independent of hole size. A similar application of star crack formulas, see Ouchterlony (1983) e.g. to obtain the arrest length R_c leads to an unrealistic $R_c \propto p_h^2$ dependence. Hence, it was decided to extend the arguments leading to Equations 24 and 25 instead. This means to obtain an expression for the number of radial cracks N in terms of pressure and to set $N \geq 2$ to obtain the critical pressure level at the border R_c between damaged and undamaged rock.

To do this we need an equation for the shock wave amplitude travelling outward from a cylindrical blast-hole. Liu & Katsabanis (1993) provide this. With some simplifications, their main results may be written

$$p_h = 0.167 \rho_e D^2 \cdot \left(\frac{\rho c}{\rho_e D} \right)^{0.25} \quad \text{and} \quad (27a)$$

$$\frac{p_s}{p_h} = \left(\frac{\Phi_h}{2r} \right)^\delta \quad \text{where } \delta \approx 1.5 \left(\frac{D}{c} \right)^{0.25} \quad (27b)$$

Here p_s is the shock amplitude at r . Actually, Liu & Katsabanis (1993) give a prefactor of 1.5 in Equation 27a instead of $0.167 \approx 1/6$, but that value is unnaturally high. A perusal of their data shows that a much better fit is obtained by the prefactor $1/6$.

Equation 27a introduces the concept that an 'impedance ratio' between explosive and rock influences the result, at least for a fully coupled charge. Assuming that there exists a critical shock amplitude $p_{s,\text{crit}}$, an inversion of Equation 27b yields at $r = R_c$,

$$\frac{R_c}{\Phi_h} = 0.5 \cdot \left(\frac{p_h}{p_{s,\text{crit}}} \right)^{1/\delta} \quad (28)$$

For many explosives $D \approx c \approx 5500 \text{ m/s}$, so for these the exponent $1/\delta = 2/3$. Further, the exponent 0.25 in $1/\delta$ makes the c/D factor relatively insensitive. For an explosive with low velocity of detonation however, $D = 2000 \text{ m/s}$, then $1/\delta = 2/3 \cdot (5.5/2)^{0.25} \approx 0.86$ which is not too far from $2/3$ e.g. Thus Liu & Katsabanis' (1993) equations are quite attractive.

If we let $\Phi_h \rightarrow 2R_c$ in Equations 19 and 23, use that $N \geq 2$, and Equation 25, we may write the critical dynamic pressure $p_{s,\text{crit}}$ at $r = R_c$ as

$$p_s \geq p_{s,\text{crit}} = 2p_{h,\min} \sqrt{\frac{\Phi_h}{2R_c}} = 1.08 \cdot \frac{K_{Ic}}{\sqrt{R_c}} \quad (29)$$

Insertion into Equation 27b gives

$$p_h \geq p_{h,crit} = 1.08 \cdot \frac{K_{Ic}}{\sqrt{R_c}} \cdot \left(\frac{2R_c}{\Phi_h} \right)^\delta \quad (30)$$

Combining Equation 14 and 27a, splitting the factors $3/2$ and $(D/c)^{0.25}$ in the exponent δ in Equation 27b and inserting Equation 29 into it, keeping however the values of the numerical constants as yet undetermined, we obtain in analogy with Equations 17a–b the following set of equations

$$p_h = n(\gamma)\rho_e D^2 \cdot \left(\frac{\rho c}{\rho_e D} \right)^{0.25} \cdot f^\alpha \quad \text{and} \quad (31a)$$

$$\left(\frac{2R_c}{\Phi_h} \right)^{(D/c)^{0.25}} = \lambda \left(p_h \frac{\sqrt{R_c}}{K_{Ic}} \right)^\beta \quad (31b)$$

Here α , β and λ are dimensionless constants which will be determined e.g. by a logarithmic least squares fit to the crack length data from Vånga. Possibly we should choose $\alpha = \alpha_o \cdot \gamma$, cf. Equations 9 and 17a.

Equations 31 represent a physically based estimate of the functional form of h in Equation 1. Unfortunately R_c/Φ_h isn't given directly in closed form but once α and β have been determined, the equations can be rearranged to do so, cf. Section 3. A comparison of Equations 31 and 1 shows that the parameter most clearly representing the charge concentration q , i.e. $\rho_e \Phi_e^2 / \rho \Phi_h^2$, is missing. It can, however, be reconstructed from the impedance ratio $\rho c / \rho_e D$, the coupling ratio f and the velocity ratio c/D . Further, the parameter B/Φ_h is also missing but this is acceptable for now because the Vånga results seem to be rather insensitive to this parameter, see below.

3 NUMERICAL RESULTS

Equations 31 are relatively complicated as compared to Equations 17 or any of the other simple formulas in Section 2.1. An important question is now whether or not the experimental data can support their degree of complexity regarding $n(\gamma)$, the impedance factor $(\rho c / \rho_e D)^{0.25}$, the exponent $\alpha = \alpha_o \cdot \gamma$, the exponent $(D/c)^{0.25}$ and the change from Φ_h to R_c as the length scale in the dimensionless pressure parameter in Equation 31b.

To answer this question we first write Equations 17 in dimensionless form as

$$p_h = 0.167 \rho_e D^2 \cdot f^\alpha \quad \text{and} \quad (32a)$$

$$\frac{2R_c}{\Phi_h} = \lambda \left[p_h \frac{\sqrt{\Phi_h/2}}{K_{Ic}} \right]^\beta \quad (32b)$$

to use as a reference. The crack length data obtained at Vånga are given by Table 1, see Olsson & Bergqvist (1996a). They represent, in slightly rounded numbers, the average crack lengths measured from the half-casts of the middle one or two blast-holes of groups of holes which have been fired simultaneously. This reduces possible

Table 1. Data for crack length calculation with Equations 17 and 31–35.

Explosive name	Charge diameter Φ_c (mm)	Hole diameter Φ_h (mm)	Charge concentration q (kg/m)	VOD D (m/s)	Coupling ratio f	Crack length R'_c (cm)
Gurit	17	51	0.21	2000	0.33	5
Gurit	17	38	0.21	2000	0.45	15
Gurit	22	64	0.40	2000	0.34	15
Gurit	22	24	0.40	2000	0.92	90
Emulet 20	51	51	0.45	1850	1.00	35
Kimulux 42	22	64	0.37	4800	0.34	25
Detonex 80	9	51	0.08	6500	0.21	15

edge effects and ensuing scatter in the data. Since each group consists of 3–4 holes this also reduces the number of available crack length data.

The data in Table 1 was obtained using different combinations of $B \cdot S$; 0.5·0.5 m, 0.8·0.5 m, 0.8·0.8 m and 1.0·0.8 m, so neither S/B nor B/Φ_h has been constant, cf. Equation 1. A large majority of the data represents the case 0.5·0.5 m, however, and the influence of S/B and B/Φ_h doesn't appear to be large. Thus, their effects are left out until the data base is sufficiently large for them to be more rigorously judged. The number of data behind Table 1 is nevertheless well over 50 if all combinations of $B \cdot S$ are used.

The necessary explosives data is given in Table 2. The material parameters chosen for the Vånga granite are $\rho = 2650 \text{ kg/m}^3$, $c = 5500 \text{ m/s}$ and $K_{Ic} = 2 \text{ MPa}\sqrt{\text{m}}$. Since the tests have only been made in this one rock, the correct values become important first when crack lengths in different rocks are compared.

The curve fitting algorithms have been implemented in Mathcad Plus 6.0 (Deng 1996). The parameters in Equations 32 become, $\alpha = 3.74$, $\beta = 0.72$ and $\lambda = 4.18$. These numbers differ a little from those in Equations 17, both because they now refer to a crack length which is measured from the center of the bore-hole and because Equation 32b is dimensionless. The quality of fit, as measured by the coefficient of determination r^2 is 0.857 which will be our reference, see Table 4.

First we change from Φ_h to R_c as the length scale in the dimensionless pressure parameter in Equation 32b to obtain

$$p_h = 0.167 \rho_e D^2 \cdot f^2 \quad \text{and} \quad (33a)$$

$$\frac{2R_c}{\Phi_h} = \lambda \left(p_h \frac{\sqrt{R_c}}{K_{Ic}} \right)^\beta \quad (33b)$$

The result of the curve fit is that $\alpha = 3.54$, $\beta = 0.58$, $\lambda = 2.32$ and $r^2 = 0.926$. Thus the improved curve fit supports the change of pressure length scale which the new crack length Equation 31b has introduced.

For the remaining curve fitting work Equations 31 are generalized as follows

$$p_h = n(\gamma) \rho_e D^2 \cdot i \left(\frac{\rho c}{\rho_e D} \right) \cdot f^{\alpha \cdot a(\gamma)} \quad \text{and} \quad (34a)$$

Table 2. Data for explosives used in Vånga tests.

Explosive name	VOD D (m/s)	Energy content e (MJ/kg)	Density ρ_e (kg/m ³)	Adiabatic exponent γ	Prefactor $n(\gamma)$
Gurit	2000	3.50	1000	1.254	0.213
Kimulux 42	4800	3.20	1100	2.145	0.140
Emulet 20	1850	2.35	250	1.315	0.205
Detonex	6500	5.95	1250	2.133	0.141

Table 3. Combinations of functions used in curve fitting procedure.

Parameter combination case no	Detonation prefactor n	Impedance prefactor i	Decoupling exponent a	Crack length exponent e
a	1/6	1	1	1
b	$\gamma^\gamma/(\gamma+1)^{(\gamma+1)}$	1	1	1
c	1/6	$(\rho c/\rho_e D)^{0.25}$	1	1
d	1/6	1	1	$(D/c)^{0.25}$
e	$\gamma^\gamma/(\gamma+1)^{(\gamma+1)}$	1	γ	1
f	1/6	$(\rho c/\rho_e D)^{0.25}$	1	$(D/c)^{0.25}$
g	$\gamma^\gamma/(\gamma+1)^{(\gamma+1)}$	1	1	$(D/c)^{0.25}$
h	$\gamma^\gamma/(\gamma+1)^{(\gamma+1)}$	$(\rho c/\rho_e D)^{0.25}$	1	$(D/c)^{0.25}$
i	$\gamma^\gamma/(\gamma+1)^{(\gamma+1)}$	$(\rho c/\rho_e D)^{0.25}$	γ	$(D/c)^{0.25}$

Table 4. Resulting parameter values form curve fitting procedure.

Parameter combination case no	Decoupling exponent α	Pressure exponent β	Pressure prefactor λ	Coefficient of determination r^2
ref	3.74	0.72	4.18	0.857
a	3.54	0.58	2.32	0.926
b	3.02	0.67	1.26	0.953
c	3.02	0.64	1.09	0.915
d	2.55	0.59	0.97	0.957
e	1.16	1.01	0.19	0.887
f	2.12	0.66	0.44	0.953
g	2.20	0.67	0.566	0.969
h	1.76	0.77	0.198	0.975
i	0.73	1.03	0.038	0.883

$$\left(\frac{2R_c}{\Phi_h}\right)^{e(D/c)} = \lambda \left(p_h \frac{\sqrt{R_c}}{K_{lc}}\right)^\beta \quad (34b)$$

where the functions a , e , i and n are either constants or the functions introduced in Section 2.3, see Equations 31, e.g. Equations 33 represent a special case of the parameter combinations in Equations 34, called case a. Starting with it the combinations a–i in Table 3 have been tried.

The results are shown in Table 4. The coefficient of determination increases relatively much as the degree of complexity of the curve fitting equations increases. Since the functions n , i , a and e were not arbitrary but motivated by a model, the outcome of the curve fitting supports the model. It can not be ruled out though that another choice of adding complexity, i.e. functions, would result in a still better fit, but to look for that is premature before the whole Vånga data set has been used.

The largest increase in r^2 has been obtained by changing the length scale in the dimensionless pressure parameter; ref case $\rightarrow$ case a. The second largest increase was obtained by then adding either a variable detonation prefactor $n(\gamma)$, case a $\rightarrow$ case b, or a variable crack length exponent $e(D/c)$, case a $\rightarrow$ case d.

Adding the bore-hole impedance factor $i(\rho c/\rho_e D)$ initially has a slightly negative influence on the curve fit since r^2 decreases somewhat; case a $\rightarrow$ case c and case d $\rightarrow$ case f. In the end the effect is slightly positive; case g $\rightarrow$ case h. Using a variable decoupling exponent $a(\gamma)$ finally reduces the quality of the curve fit considerably; case b $\rightarrow$ case e and case h $\rightarrow$ case i.

At this stage, the decoupling exponent should thus be kept constant in the final crack length prediction equation, i.e. we set $a = 1$. Table 4 also shows that the pressure exponent β is relatively constant, $\beta = 0.67 \pm 15\%$, if cases e and i are excluded. This β -value thus has a relatively high credibility.

Next, consider whether the impedance factor i should be included or not. Both i and the crack length exponent e stem from the work of Liu & Katsabanis (1993) so it could be argued that both or neither of them should be included.

There are several reasons why i should be excluded at this stage, i.e. set to $i = 1$. Firstly, it has a marginal effect on the curve fit; compare case g with $r^2 = 0.969$ with case h with $r^2 = 0.975$. Secondly, the number of data points on which our results are based is small. Thirdly, it relates to a fully coupled charge and fourthly the decoupling exponent $\alpha = 2.2$ for case g comes very close to the value 2.4 frequently used in the literature (Calder 1977). Thus the conclusion is that presently the 'best' crack length prediction equation is given by case g and

$$p_h = \frac{\gamma^\gamma}{(\gamma + 1)(\gamma + 1)} \cdot \rho_e D^2 \cdot f^{2.2} \quad \text{and} \quad (35a)$$

$$\left(\frac{2R_c}{\Phi_h} \right)^{(D/c)^{0.25}} = 0.566 \cdot \left(p_h \frac{\sqrt{R_c}}{K_{Ic}} \right)^{0.67} \quad (35b)$$

A comparison of Equation 35b with the data is shown in a log-log diagram in Figure 4 below. The vertical deviation of any average is about $\pm 20\%$. The Kimulux average has a higher pressure (or shorter crack length) than the fitted line, the Detonex average has a lower pressure (or longer crack length). The individual values naturally deviate more and the standard deviations for the measured data are discussed in Section 4 below.

The exclusion of the impedance factor i in Equation 34a, would seem to mean excluding q from our influence parameters. This conclusion is somewhat misleading, however, since an increase in charge size for a given explosive and at constant f means an increase in bore-hole size Φ_h and thus in crack length R_c , see also Section 4.

Figure 4. 'Best' crack length prediction equation, case g, and Vånga data.

Equations 35 form an implicit relationship between crack length R_c and the other blasting parameters which is easily rearranged to an explicit one. If we let $0.67 \approx 2/3$ and recognize that for cracking to occur at all, the bore-hole pressure must be larger than some critical value, $p_h \geq p_{h,crack}$, then Equation 35b may be rewritten

$$\frac{2R_c}{\Phi_h} = \left(\frac{p_h}{p_{h,crack}} \right)^{2/[3(D/c)^{0.25} - 1]} \quad \text{where} \quad (35b')$$

$$p_{h,crack} = 3.30 \cdot \frac{K_{Ic}}{\sqrt{\Phi_h}} \quad (35c)$$

The validity of Equation 35b' is restricted to $R_c \geq \Phi_h/2$, because cracks inside the bore-hole do not exist. The cracking pressure $p_{h,crack}$ is about twice as high as the theoretically predicted value $p_{h,min}$ from Equation 25 which has the prefactor 1.52. Thus theory and experiments are not quite consistent in this respect.

Equations 35 are easily implemented on a computer or a hand-held calculator. There is, however, also some merit in constructing a design diagram. This is done in the Appendix.

4 DISCUSSION

A pure dependence of R_c on charge concentration q is not to be generally expected since the combination $\rho_e \Phi_c^2$ is not the only one to describe the explosive in Equations 35 in this paper. Firstly, when $D = c$ then exponent of the p -factor in Equation 35b' becomes equal to 1 and when $D \approx 2000$ m/s and $c = 5500$ m/s

then it becomes 1.5. Next, using Equation 15 to replace D^2 in Equation 35a by the explosive energy e , there results for two cases above the following equations

$$R_c \approx \text{const} \cdot q \cdot \frac{e}{\Phi_e^{0.5}} \cdot f^{0.7} \quad \text{for } D = c \quad \text{and} \quad (36a)$$

$$R_c \approx \text{const} \cdot \frac{(q \cdot e)^{1.5}}{\Phi_e^{1.25}} \cdot f^{1.55} \quad \text{for } D = 0.36c \quad (36b)$$

For a given explosive for which $\rho_e = \text{const}$, $q \propto \Phi_e^2$ and the net effect of Equations 36 is that $R_c \approx \text{const} \cdot q^{0.75-0.88}$ when f is constant. This is slightly more forgiving for large charges than the linear relationship postulated by e.g. Equations 4 and 7. On the other hand, a slow burning explosive would seem to be relatively more sensitive to decoupling than a fast burning one. This may seem counterintuitive but it is a result of Liu & Katsabanis (1993) pressure decay relation, Equation 27b. The constant depends on γ so a comparison between the two equations for different explosives is not immediately apparent.

As mentioned in Section 3, two parameters in the crack length prediction equations have a certain degree of arbitrariness, c and K_{Ic} . Unless experiments are made in another type of rock, one can not be sure that these are the material parameters that will describe our test results best and that correct values for them have been chosen.

Because of Equations 25 and 35c, K_{Ic} is the easiest to treat. Instrumented dynamic pressure strength tests for bore-holes would validate or falsify the size effect implied by these equations. Further, another value for K_{Ic} than 2 MPa $\sqrt{\text{m}}$ would change the value of the prefactor λ in the right member in Equation 35b or 35b' but leave the prediction itself undisturbed. For fracture toughness measurements there are international standard testing methods valid for isotropic rock (ISRM 1988, 1995).

The matter of c is different because of the form chosen for the exponent in the left member of Equation 35b. Another value of c than 5500 m/s would probably influence all the curve fit constants α , β and λ and consequently the crack length prediction.

A comparison of the measured crack lengths and the values predicted by the design diagram or the formulas in Equations 35 are given in Table 5. An effective charge diameter for Detonex 80 of $\Phi_e \approx 9$ mm has been used, giving a coupling ratio of $f = 0.18$. The corresponding data and crack length intervals are also shown in Figure 5. Note that two of the points are represented by single data; $\Phi_e = 17$ mm

Table 5. Comparison of measured and calculated crack lengths.

Explosive name	Hole diameter Φ_h (mm)	Coupling ratio f	Measured cracks $R_{c,m}$ (cm)	Predicted cracks R_c (cm)	Difference %
Gurit	51	0.33	7.6	10.7	+41%
Gurit	38	0.45	16.9	16.9	$\pm 0\%$
Gurit	64	0.34	18.2	17.6	-3%
Gurit	24	0.92	91.2	81.3	-11%
Emulet 20	51	1.00	37.6	41.6	+11%
Kimulux 42	64	0.34	28.2	47.5	+69%
Detonex 80	51	0.18	17.6	13.4	-24%

Figure 5. Crack length prediction line, Equations 35, and ranges of Vånga results.

Gurit in $\Phi_h = 38$ mm holes and $\Phi_e = 22$ mm Gurit in $\Phi_h = 24$ mm holes. Their positions could thus contain relatively substantial errors. Note also, however, that the prediction line passes more or less straight through all the points representing the explosives with a low velocity of detonation (VOD), Gurit and Emulet 20.

In Table 5, three of the seven points deviate more than 20% from the prediction value; $\Phi_e = 17$ mm Gurit in $\Phi_h = 51$ mm holes, $\Phi_e = 22$ mm Kimulux 42 in $\Phi_h = 64$ mm holes, and Detonex 80 g/m in $\Phi_h = 51$ mm holes. In practice though, a 3.4 cm overestimate of the average crack lengths behind $\Phi_e = 17$ mm Gurit in $\Phi_h = 51$ mm holes is insignificant.

The two high VOD explosives Kimulux and Detonex are the outliers. Kimulux is the more severe case because the prediction line in Figure 5 above overestimates the measured average crack lengths by 69% and the whole crack length interval, mean $\pm$ std. deviation lies below the prediction line. On the other hand, a nonconservative estimate of the average crack lengths behind Detonex is not good either.

The Detonex and Kimulux points lie on each side of the prediction line so perhaps no common correction based on the D/c ratio alone can bring both back to the line? This may be the explanation why the attempt at using the decoupling exponent $\alpha = \alpha_o \cdot \gamma$ above failed to improve the curve fit and why the impedance factor $(\rho c / \rho_e D)^{0.25}$ was only marginally effective in the curve fitting procedure in Section 3.1.

The explosive behavior of a straight emulsion like Kimulux 42 is relatively well known, that of the PETN cord with wrapping seemingly less so. Sanchidrián et al. (1996) use a 'mock explosive' concept to describe the expansion properties of PETN cord, 3 g/m, 6 g/m and 12 g/m in $\Phi_h = 32$ mm bore-holes. As a first step they calculate bore-hole wall pressures which become 58.5, 29.5 and 14.5 MPa, respectively. With assumed properties of their PETN according to our Table 2, our Equation 35a gives the numbers 57.0, 27.2 and 12.4 MPa which are relatively close.

For 80 g/m PETN cord in $\Phi_h = 51$ mm holes, Sanchidrián's method gives a bore-hole pressure of 154 MPa, Equation 35a gives 163 MPa which is again close. Compared to this there seems to be no large error in our description, but the matter is raised again below.

Jokinen & Ylätaalo (1995, 1996) report so called one way or split blasting tests in Oulainen gabbro and Ristijärvi granite in Finnish dimensional stone quarries. Rows of $\Phi_h = 32$ mm holes behind a free surface were initiated simultaneously with 10 g/m detonating cord. The toe and the sides were free so the situation was very much like that at Vånga. They measured the crack lengths obtained with the same sawing and penetrant techniques that were used at Vånga. The difference is that they documented the cracks in the burden, not the ones behind the holes. The values given here are the longest radial cracks as measured from the bore hole wall and denoted $L_{\max}$ by Jokinen and Ylätaalo.

Data for their explosives are given in Table 6. K-17, F-17 and Formex are comparable to our low VOD explosives. Two or three pipe charges or cartridges tied to a 10 g/m cord downline were used per hole, giving a charge length of 0.8–1.4 m and 1.6–1.2 m uncharged without stemming, Jokinen (1997). No primer was used. Their PETN cord differs mainly in charge concentration. The density of the core has been assumed to be $\rho_e = 1250$ kg/m³, just like in Table 2 and the core diameter Φ_e calculated accordingly. The cord downline has not been included in the K-17, F-17 and Formex numbers.

The maximum crack lengths obtained are shown in Table 7 and compared with the predictions that Equations 35 give. Note that the values of Jokinen and Ylätaalo

Table 6. Data for explosives used by Jokinen & Ylätaalo (1995, 1996). Bore hole diameter $\Phi_h = 32$ mm.

Explosive name	VOD	Energy content	Density	Charge diameter	Coupling ratio
	D (m/s)	e (MJ/kg)	ρ_e (kg/m ³)	Φ_e (mm)	f
K-17	1900	1.4	950	17	0.53
F-17	2500	2.8	950	17	0.53
Formex	1800	4.3	600	22	0.69
PETN 40 g/m	6500	5.95	1250	6.4	0.20
PETN 20 g/m	6500	5.95	1250	4.5	0.14

Table 7. Comparison of crack lengths measured by Jokinen & Ylätaalo (1995, 1996) with the predictions of Equations 35.

Explosive name	Oulainen gabbro			Ristijärvi granite		
	Measured cracks	Predicted cracks	Difference	Measured cracks	Predicted cracks	Difference
	$R_{c,m}$ (cm)	R_c (cm)		$R_{c,m}$ (cm)	R_c (cm)	
K-17	13.6	13.9	+2%	15.6	16.6	+6%
F-17	27.6	25.9	−6%	2.1	29.1	—
Formex	4.6	20.5	—	15.6	24.0	+54%
PETN 40 g/m	10.6	8.0	−25%	19.6	8.9	−55%
PETN 20 g/m	—	3.7	—	7.6	4.2	−45%

have been increased by the hole radius 1.6 cm to make them comparable to ours. The rock data used was obtained from their report and by personal communication, Jokinen (1997).

- Oulainen gabbro: $\rho = 2900 \text{ kg/m}^3$, $c = 6900 \text{ m/s}$ and $K_{Ic} = 2.4 \text{ MPa}\sqrt{\text{m}}$;
- Ristijärvi granite: $\rho = 2660 \text{ kg/m}^3$, $c = 6100 \text{ m/s}$ and $K_{Ic} = 2.0 \text{ MPa}\sqrt{\text{m}}$.

Equations 35 predict that the crack lengths in Ristijärvi granite should be slightly longer than in Oulainen gabbro. This agrees with the experiments except for the F-17 explosive in the granite, a point which Jokinen & Ylätaalo (Part 2, 1996) discard in their analysis. The agreement between measured and predicted crack lengths is not bad for the K-17 and F-17 explosives, considering that the downline hasn't been included. The measured Formex value in Oulainen gabbro is too low, but it is based on only one hole.

Interestingly all three predicted crack lengths from the PETN cord holes in Table 7 are lower than the measured ones, just like our own results for the cord with a higher charge concentration, see Table 5. This strengthens the suspicion that our explosives description might be overly simple.

Recently, Brent (1995) has circumvented all the simplifications in the explosives description in this paper by using proprietary fundamental detonation codes which can model the non-ideal detonations in commercial explosives under the actual confinement and diameter. This amounts to saying that the value of γ depends on the pressure level, or density, in a way which is unique for each explosive. For ANFO he gives a range of $2.8 \rightarrow 1.3$ as the pressure approaches atmospheric conditions, for a doped emulsion $> 3.0 \rightarrow 1.3$ and presumably a higher decoupling exponent. Our decoupling exponent 2.2 corresponds to a γ -value of 1.1 which implies that Equation 35a might underestimate the effect of decoupling on the bore-hole pressure considerably.

Even if Brent doesn't give the explosive energy data for his two examples with decoupled charges a comparison can be attempted. His first example is the use of an ANFO-based explosive with $\rho_e = 800 \text{ kg/m}^3$ and a measured VOD of $D = 4360 \text{ m/s}$ in bore-holes with $\Phi_h = 270 \text{ mm}$. The designed bore-hole pressure is $p_h = 40 \text{ MPa}$ which requires a coupling factor of $f = \sqrt{0.12} \approx 0.35$. If we estimate the explosive energy as 3.5–4.0 MJ/kg (Persson et al. 1993) then Equation 35a yields $p_h = 230$ –240 MPa.

Brent's second example (1995) is the use of what could be an emulsion explosive with $\rho_e = 1230 \text{ kg/m}^3$ and a measured VOD of $D = 4790 \text{ m/s}$ in bore-holes with $\Phi_h = 89 \text{ mm}$. The designed bore-hole pressure is $p_h = 112 \text{ MPa}$ which this time requires a coupling factor $f = \sqrt{0.18} \approx 0.42$. If we estimate the explosive energy as 3.0–4.0 MJ/kg, then Equation 35a yields $p_h = 575$ –630 MPa.

In both cases Equation 35a overestimates Brent's pressures by a factor of 5–6. This amounts to underestimating the effect of decoupling on the bore-hole pressure considerably, at least for high-VOD explosives. The best way to ascertain whose pressure prediction is the most accurate would of course be to measure.

If Equation 35a overestimates the pressure in the holes charged with Kimulux, then the true position of the point would move leftward towards the prediction line in Figure 5. Since Brent (1995) doesn't give any data for PETN cord, one can unfortunately not say how the Detonex outlier would move.

Three other ways in which the Kimulux and Detonex points differ are, firstly, that for Kimulux one expects a diameter effect on D but that for Detonex one doesn't because the critical diameter is so small, less than 1 mm (Kaye 1978). Secondly, the VOD of the cord is superseismic with respect to the rock material, $D > c$. This might change the reflection properties at the bore-hole wall when the detonation wave impinges on it. Thirdly, the possible effect of the cord wrapping has already been mentioned.

However, an improvement of the explosives description used here is perhaps best guided by Brent's work (1995). Its two most salient features are an effective γ which decreases with decreasing pressure, or density, and a considerably lower pressure level. Using Abel's expansion law for the detonation products (Schmidt 1935) approximates such behavior by introducing one new material property, the covolume α , which has dimensions m^3/kg . An independent estimate of $\alpha(\rho)$ is given by Johansson & Persson (1970) so the degree of complication of the model itself is small. The prediction equations and the design diagram will, however, become more complicated.

If this explosives description doesn't improve the curve fit sufficiently, then a further relatively simple, engineering type, calibration is given by Davis (1995). His equation of state for the detonation products contains a smeared out step function that connects the high and low density region behaviors. The simple form presented has 3 new material parameters. Along the principal isentrope, $\gamma \approx 1.3$ after a large expansion and a pressure value are the two other parameters required. Davis' work is another way of expressing the isentropes referred to in Section 2.1 above.

The ultimate step might well be to use explosives descriptions like Brent (1995) that are based on the theories of non-ideal detonations and require advanced computer codes.

The discussion above points to certain deficiencies in the simplified description of the explosive properties which has been used in this paper. Each degree of complication distances the model presented here from common engineering use and the number of data behind some points in Figure 5 is quite limited. This suggests that the quality of the underlying data should be improved at the same time as a more complicated explosives description is attempted.

Note though that the results obtained here are far better than the first simple approach presented in Olsson & Bergqvist (1996a), see Figure 3, where the Gurit points do not fall on their prediction line, a line which in itself is dimensionally awkward.

Vånga represents a nearly ideal site to test contour blasting. Thus one could perhaps expect that continued testing would result in improved crack length predictions, Equations 35 and the design diagram in Figure A1. One way to do this would be to stepwise broaden the data base. From a research point of view, complementary tests should then be made, for example:

- Validate the decoupling influence by testing $\Phi_e = 17$ mm Gurit in $\Phi_h = 38$ mm holes and $\Phi_e = 22$ mm Gurit in $\Phi_h = 24$ and 38 mm holes.
- Scrutinize the VOD influence by testing Kimulux 42, $\Phi_e = 22$ mm in $\Phi_h = 51$ mm holes and $\Phi_e = 17$ mm (if available) in $\Phi_h = 51$ and 64 mm holes. Further, test detonating cord of some other strength than 80 g/m.

- Investigate why the high VOD explosives behave differently.
- Validate the VOD influence by testing an explosive with $D \approx 3000$ m/s. This opens the field to diluted explosives like Emulet, a styropore diluted emulsion with densities down to 250 kg/m^3 or heavily gassed emulsions deposited in a string so as not to fill the bore-hole.

These data can be used as input in a repeated curve fitting procedure, which is easy to do with existing programs (Deng 1996). As previously discussed, points 2 to 4 in the list above may require an improved explosives description. Thus some of these complementary tests should be instrumented with pressure gauges so that the resulting pressure part of the prediction equations is verified.

Secondly, the Vånga test program during 1997–99 will give data on the influence of factors like burden and spacing which can be included in an extended dimensional analysis and curve fitting.

Thirdly, there is a need to investigate the influence of the rock, both in another quarry with relatively fracture free rock with different values for c and K_{Ic} , and in rock masses which are more heavily fractured.

The end goal of this work and the Vånga tests is to help provide a basis for improved design of cautious blasting. In this light, the present equations and design diagram are only a first, limited step and the actual predictions that they yield must be used with great caution because of the limitations in the analysis.

It would, finally, be expected that the experimental work could be sped up by using dynamic computations (Song & Kim 1995, Minchinton & Lynch 1996) both to improve the understanding of the complex dynamic phenomena involved, to guide the testing and perhaps even to optimize certain aspects of cautious blasting.

5 CONCLUSIONS

In this paper, mechanics and dimensional analysis are used to derive new prediction equations for the lengths R_c of the radial cracks emanating from the half-casts in the contour after cautious blasting of a contour with a free toe and zero initiation delay between the charges. The influencing parameters are;

- density, VOD and adiabatic expansion exponent of the explosive;
- bore-hole diameter and charge coupling ratio; and
- density, sound velocity and fracture toughness of the rock.

The theory predicts, among other things, that the critical pressure for a dynamic pressurization of a bore-hole depends on hole size, $p_{h,min} = 1.52 \cdot K_{Ic} / \sqrt{\Phi_h}$. The experimental data is, however, best described by the prefactor 3.30, see below.

A number of parameters in the prediction equations are determined by curve fitting to test data obtained from bench blasting in granite in the Vånga quarry. These data do not motivate the use of the full form of the equations, however. A compromise between equation complexity and goodness of fit is obtained by

$$p_h = \frac{\gamma^\gamma}{(\gamma + 1)(\gamma + 1)} \cdot \rho_e D^2 \cdot f^{2.2} \quad (35a)$$

$$p_{h,crack} = 3.30 \cdot \frac{K_{Ic}}{\sqrt{\Phi_h}} \quad \text{and} \quad (35c)$$

$$\frac{2R_c}{\Phi_h} = \left(\frac{p_h}{p_{h,crack}} \right)^{2/[3(D/c)^{0.25} - 1]} \quad (35b')$$

The coefficient of determination for these equations is 0.97.

In an Appendix an approximate design diagram for these equations is given and its use explained. This diagram, see Figure A1, gives a comprehensive view of how the different parameters influence the predicted crack lengths. It is thus a step towards the end goal of this work which is to help provide a basis for improved design of cautious blasting. The prediction tools of this paper must however be used with great caution because of the limitations of both the analysis and of the underlying data.

A comparison with our Vånga data shows that the prediction equations are very good for the two low VOD explosives used, Gurit and Emulet 20 for which $D \approx 2000$ m/s. They are worse for the two high VOD explosives used, Kimulux 42 and Detonex 80. The crack lengths generated by Kimulux are overestimated by about 70% and those caused by Detonex underestimated by about 25%. A comparison with independent data confirms the prediction quality for two other low VOD explosives and that the crack lengths caused by Detonex are underestimated. This and other referenced work point to some deficiencies in the explosives description which should be considered.

When the charge concentration q (kg/m) is introduced in Equations 35, it can not replace all other explosives properties. For a range of VOD-values $0.36 \leq D/c \leq 1$, the net effect is however that $R_c \approx \text{const} \cdot q^{0.75-0.88}$ which is less dependent on q than previously used linear relationships. The other explosives properties are hidden in the constant and this makes a comparison between different explosives impossible.

At the end, tests are recommended and explosives descriptions suggested that could improve the predictive capacity of the equations presented in this paper. It should finally be repeated that the present equations and design diagram are based on an analysis with limitations such that any crack length predictions that can be made for a real, practical blast round must be used with great caution.

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REFERENCES

- Andersson, P. 1994. The damage zone at tunnel driving, seminar Oct. 1992. SveBeFo report 8. Stockholm: Swedish Rock Engineering Research (in Swedish).
- Atchison, T.C. 1968. Fragmentation principles. In E.P. Pfeider (ed.), *Surface Mining*, Ch. 7.2: 355–372. Seeley W Mudd Series. New York: Am. Inst. Min. Met. & Petr. Engrns.
- Atchison, T.C., Duvall, W.I. & Pugliese, J.M. 1964. Effect of decoupling on explosion-generated strain pulses in rock. Report RI 6333. Twin Cities, MN: US Bureau of Mines.
- Bauer, A. 1967. The status of practical rock mechanics in blasting. *Proc. 9th US Symp. on Rock Mechanics*: 249–262. New York: Am. Inst. Min. Met. & Petr. Engrns.
- Blair, D. & Minchinton, A. 1996. On the damage zone surrounding a single blasthole. In B. Mohanty (ed.), *Proc. 5th Intl Symp. Rock Fragmentation by Blasting*: 121–130. Rotterdam: Balkema.
- Brent, G.F. 1995. The design of pre-split blasts. *Proc. Explo '95 Conference*: 299–305, Publ. Series no. 6/95. Carlton, VIC: AusIMM.
- Calder, P. 1977. Pit slope manual, chapter 7 – Perimeter blasting. Report 77. Ottawa, ONT: CANMET, Ministry of Supply and Resources.
- Cowperthwaite, M. & Zwisler, W.H. 1975. Tiger computer program documentation. Publ. Z106. Menlo Park CA: Stanford Res. Inst.
- Davis, D.C. 1995. Equation of state for detonation products. *Proc. 10th Intl Detonation Symposium*: 369–376, Publ. ONR 33395-12. Arlington, VA: Office of Naval Research.
- Deng, J. 1996. Personal communication. Stockholm: SveBeFo.
- Fickett, W. & Davis, W.C. 1979. *Detonation*. Los Alamos Ser. In *Basic Appl. Scis.* Berkeley, CA: Univ. Calif. Press.
- Grady, D.E. & Kipp, M.E. 1987. Dynamic rock fragmentation. In B.A. Atkinson (ed.), *Fracture Mechanics of Rock*: 429–475. London: Academic Press.
- Holmberg, R. & Persson, P.-A. 1980. Design of tunnel perimeter blasthole patterns to prevent rock damage. *Trans Inst Mining and Metallurgy A*: 37–70.
- Hustrulid, W. 1994. The 'practical' blast damage zone in drift driving at the Kiruna mine. In Andersson (1994): 75–125.
- Hustrulid, W., Bennett, R., Ashland, F. & Lenjani, M. 1992. A new method for predicting the extent of the blast damaged zone. *Proc. Blasting Conference*: paper no 3. Gytterp: Nitro Nobel.
- ISRM. 1988. Suggested methods for determining the fracture toughness of rock. F. Ouchterlony coord. *Int. J. Rock Mechs Min. Sci.* 25: 71–96.
- ISRM. 1995. Suggested method for determining mode I fracture toughness using cracked chevron notched Brazilian disc specimen. R. Fowell coord. *Int. J. Rock Mechs Min. Sci.* 32: 57–64.
- Jiang, J., Baird, G.R. & Blair, D.P. 1993. Surface vibrations due to a buried source. In H.P. Rossmannith (ed.), *Proc. 4th Intl Symp. Rock Fragmentation by Blasting*: 89–96. Rotterdam: Balkema.
- Jokinen, J.U.S. & Ylätaalo, K. 1995, 1996. The effect of loosening techniques on the product quality in the dimensional stone quarrying industry, Part 1, Blasting tests, Gabbro of Oulainen, and Part 2, Blasting tests, Granite of Ristijärvi. Res. reports TK-KAL-A17 & A-18. Helsinki: Lab. of Rock Engng, Helsinki Univ. Techn.
- Jokinen, J.U.S. (ed.). 1996. Ibid part 4, Conclusion report. Res. report TK-KAL-A22. Helsinki: Lab. of Rock Engng, Helsinki Univ. Techn.
- Jokinen, J.U.S. 1997: Personal communication 1997-04-22.
- Johansson, C.H. & Persson, P.A. 1970. *Detonics of High Explosives*. London: Academic Press.
- Kaye, S.M. 1978. *Encyclopedia of Explosives and Related Items* 8: P95. Dover, NJ: US Army Arm. Res. & Dev. Command.
- Liu, Q. & Katsabanis, P.D. 1993. A theoretical approach to the stress waves around a borehole and their effect on rock crushing. In H.P. Rossmannith (ed.), *Proc. 4th Intl Symp. Rock Fragmentation by Blasting*: 9–16. Rotterdam: Balkema.
- Minchinton, A. & Lynch, P. 1996. Fragmentation and heave modelling using a coupled discrete element gas code. In B. Mohanty (ed.), *Proc. 5th Intl Symp. Rock Fragmentation by Blasting*: 71–80. Rotterdam: Balkema.

- Niklasson, B., Sjöberg, C., Tenlén, G. & Nilsson, L. 1994. Design instructions for cautious blasting in the tunnels of Ringen and Yttre Tvärleden projects. *Proc. Blasting Conference*: paper no. 14. Gytterp: Nitro Nobel (in Swedish).
- Olsson, M. & Bergqvist, I. 1993a. Crack growth in rock during cautious blasting. SveBeFo report 3. Stockholm: Swedish Rock Engineering Research (in Swedish).
- Olsson, M. & Bergqvist, I. 1993b. Crack lengths from explosives in small diameter boreholes. In H.P. Rossmanith (ed.), *Proc. 4th Intl Symp. Rock Fragmentation by Blasting*: 193–196. Rotterdam: Balkema.
- Olsson, M. & Bergqvist, I. 1996a. Crack growth during multiple hole blasting. SveBeFo report 18. Stockholm: Swedish Rock Engineering Research (in Swedish).
- Olsson, M. & Bergqvist, I. 1996b. Crack lengths from explosives in multiple hole blasting. In B. Mohanty (ed.), *Proc. 5th Intl Symp. Rock Fragmentation by Blasting*: 187–191. Rotterdam: Balkema.
- Olsson, O., Bäckblom, G., Ben Slimane, K., Cournut, A., Davies, N. & Mellor, D. 1996. Planning, organization, and execution of an EDZ experiment while excavating two test drifts by TBM boring and blasting, respectively. In G. Barla (ed.), *Proc. Euroc '96 2*: 1329–1336. Rotterdam: Balkema. See also companion articles on pp. 1337–1352.
- Ouchterlony, F. 1983. Analysis of cracks related to rock fragmentation. In H.P. Rossmanith (ed.), *Rock Fracture Mechanics*: 32–67. CISM Courses and Lect. no. 275. Wien: Springer.
- Ouchterlony, F. 1991. Fracture toughness testing of rock with core based specimens. *Engng Fract. Mechanics* 35: 351–366.
- Ouchterlony, F. 1992. Some recent research and development in Swedish tunnel blasting. SveDeFo report DS 1992:1. Stockholm: Swedish Detonic Research Foundation.
- Ouchterlony, F. 1997. Prediction of crack lengths in rock after cautious blasting with zero inter-hole delay. SveBeFo report 31. Stockholm: Swedish Rock Engineering Research.
- Ouchterlony, F., Sjöberg, C. & Jonsson, B.A. 1993. Blast damage predictions from vibration measurements at the SKB underground laboratories at Äspö in Sweden. *Proc. 9th Ann Symp on Explosives and Blasting Research*: 189–197. Cleveland, OH: ISEE.
- Persson, P.-A., Holmberg, R. & Lee, J. 1993. *Rock Blasting and Explosives Engineering*. Boca Raton, FL: CRC Press.
- Sanchidrián, J.A., Garcia-Bermudez, P., Tsoutrelis, C., Exadaktylos, G., Cojean, R., Fleurisson, J.A., Tsimidakis, J. & Rogakis, C. 1996. Improvement of productivity in quarrying dimension stone using new drilling and blasting techniques. *Workshop on Ornamental Stones*. Montpellier: Brite-Euram.
- Schmidt, A. 1935. Ueber die Detonation von Sprengstoffen und die Beziehung zwischen Dichte und Detonationsgeschwindigkeit. *Zeitschrift für das Gesamte Schieß- und Sprengstoffwesen mit der Sonderabteilung Gasschutz* 30: 364–369 (in German).
- Siira, O. 1994. The economic consequences of cautious blasting and stability in the Aitik mine. In Andersson (1994): 42–44 (in Swedish).
- Sjöberg, C. 1980. Crack propagation zones around small diameter charges. *Proc. Swedish Rock Blasting Committee*: 53–98. Stockholm: Bergsprängningskommittén.
- Song, J. & Kim, K. 1995. Numerical simulation of the blasting induced disturbed rock zone using the dynamic lattice network model. In H.P. Rossmanith (ed.), *Mechanics of Jointed and Faulted Rock*: 755–761. Rotterdam: Balkema.
- Taylor, E.S. 1974. *Dimensional Analysis for Engineers*. Oxford: Clarendon Press.

APPENDIX: CONSTRUCTION OF A DESIGN DIAGRAM FROM EQUATIONS 35

The final equations in Section 3 that best defined the depth of the damage zone, i.e. the length of the radial cracks behind a half-cast R_c were

$$p_h = \frac{\gamma^\gamma}{(\gamma + 1)(\gamma + 1)} \cdot \rho_e D^2 \cdot f^{2.2} \quad \text{and} \quad (35a)$$

Figure A1. Approximate design diagram for cautious blasting, based on Equations 35.

$$\frac{2R_c}{\Phi_h} = \left(\frac{p_h}{p_{h,crack}} \right)^{2/[3(D/c)^{0.25}-1]} \quad \text{where} \quad (35b')$$

$$p_{h,crack} = 3.30 \cdot \frac{K_{Ic}}{\sqrt{\Phi_h}} \quad (35c)$$

They are of course easily implemented on a computer or a hand-held calculator but a design diagram such as in Figure A1 gives an immediate feeling for the relative importance of the different parameters, see below.

The different steps in the going through the design diagram are as follows, see the encircled numbers in Figure A1:

– 1: Calculate, firstly, γ from the explosive properties and Equation 15 and, secondly, an effective density for the explosive,

$$\gamma = \sqrt{1 + \frac{D^2}{2e}} \quad \text{and} \quad (15)$$

$$\rho_{e,eff} = 6 \frac{\gamma^\gamma}{(\gamma+1)(\gamma+1)} \cdot \rho_e = 6n(\gamma)\rho_e \quad (A1)$$

see Table 2, e.g. for explosives tested at Vånga.

– 2: Start on the VOD-axis at the given D -value and draw a straight line leftward to the appropriate density line, $\rho_{e,eff} = 0.5, 1.0, 1.10$ and 1.25 . Continue vertically downward to the negative horizontal axis and read off the explosion pressure p_e .

– 3: Continue vertically downward to the appropriate decoupling line, $f = \text{const}$, and then draw a horizontal line rightward to the negative slanting pressure axis. Read off the bore-hole pressure p_h .

- 4: Calculate the critical bore-hole pressure for cracking from Equation 35c

$$p_{h,crack} = 3.30 \cdot \frac{K_{Ic}}{\sqrt{\Phi_h}} \quad (35c)$$

and divide p_h by this number to obtain the non-dimensional overpressure number $p_h/p_{h,crack}$.

- 5: Move from the negative pressure axis to the appropriate value of the overpressure number on the negative vertical axis.

- 6: Calculate the pressure exponent

$$e' = \frac{2}{3(D/c)^{0.25} - 1} \quad (A2)$$

- 7: Draw a straight line horizontally from the overpressure axis rightward to the appropriate relative damage line given by e' . Then draw a vertical line upward to the positive horizontal axis and read off the relative crack length $2R_c/\Phi_h$.

- 8: Continue vertically upward to the appropriate bore-hole diameter line. Then draw a horizontal line leftward to the vertical damage axis and read off the depth of the damage zone, R_c , in cm.

In Figure A1, two design examples are given, a $\Phi_e = 22$ mm charge of Kimulux 42 in a $\Phi_h = 64$ mm hole and a $\Phi_e = 17$ mm charge of Gurit in a $\Phi_h = 38$ mm hole. In this figure, the influence of D is given both by the starting point on the VOD-axis and the slope e' of the damage lines in the 3rd quadrant. For a given explosive, the value of $\rho_{e,eff}$ may be calculated once and for all. For the ones used here the value may be obtained directly from Equation A1 and Table 2.

The substantial influence of the decoupling is given by the lines in the 2nd quadrant. The influence of absolute hole size doesn't enter until the calculation of the critical bore-hole pressure $p_{h,crack}$ in Equation 35c. In Figure A1 the move from the bore-hole pressure axis to the overpressure axis will be horizontal for a $\Phi_h = 51$ mm hole in rock with a fracture toughness of $K_{Ic} = 2 \text{ MPa}\sqrt{\text{m}}$.

In the 3rd quadrant, the influence of the VOD is felt again through the pressure exponent e' from Equation A2. It is visible in the slope of the prediction line and can for given explosive-rock combination be calculated before hand. The influence of bore-hole size enters both into the value of $p_{h,crack}$, which decreases with size, and more directly into the lines in the 4th quadrant.