TECHNICAL NOTE

STRESS INTENSITY FACTORS FOR THE EXPANSION LOADED STAR CRACK

The star crack has been the subject of a number of papers, see for example [1-6]. References [1] and [4] treat the pressurized star crack with an arbitrary number of cracks of equal length, while [2] and [3] analyze the star crack with 4 arms of equal length subjected to a pressure of varying magnitude. The star crack with 4 arms of alternating lengths opened by a constant pressure of alternating magnitude was analyzed in [5] and [6] gives results for the pressurized star crack with an arbitrary number of cracks of alternating lengths as well as other loading cases.

This note is restricted to the star crack being defined as an arbitrary number n of equally spaced radial crack arms of equal length a with a common origin and situated in an infinite plane elastic medium, Fig. 1. We shall consider two loading cases here, the first being the pressurized star crack and the second being the star crack opened by centrally located expansion forces, see Fig. 1. Because of the symmetry of these, the stress intensity factor K_I will be the same for all crack tips in both configurations and further $K_{II} = 0$. The purpose of this note is to present two expressions for K_I which easily give accurate results for an arbitrary number of cracks.

For the pressurized star crack, opened by a constant pressure p , we define a normalized stress intensity factor $k_{Ip}(n)$ through

$$K_{Ip} = k_{Ip}(n) \cdot p \sqrt{(\pi a)}. \quad (1)$$

An approximate expression for $k_{Ip}(n)$ may be found if we replace the star crack by a hypocycloidal crack. This crack lends itself to the use of the complex mapping method for solving the boundary value equation, as was done in [6]. After comparatively little algebra we obtain

$$k_{Ip}(n) = 2\sqrt{(n-1)/n} \text{ when } n \geq 2. \quad (2)$$

When $n = 2$ the star crack reduces to the Griffith crack and eqn (2) yields the exact value $k_{Ip}(2) = 1$. References [2] and [3] give the result $k_{Ip}(4) = 0.8637$ and 0.8636 respectively where [4] gives the value of $k_{Ip}(n)$ within the interval $2 < n \leq 9$. Equation (2) yields values of $k_{Ip}(n)$ which are less than 0.5% larger than those reported in [2-4]. Moreover, the expression for $k_{Ip}(n)$ in eqn (2) has the proper asymptotic property $k_{Ip}(n) \sim 2/\sqrt{n}$ for large n , see [4]. It is thus believed that eqn (2) is quite satisfactory for engineering purposes.

We may also define a normalized stress intensity factor $k_{IF}(n)$ for the star crack opened by central expansion forces F . Here we have

$$K_{IF} = k_{IF}(n) \cdot F/\sqrt{(\pi a)}. \quad (3)$$

In deriving the exact expression for $k_{IF}(n)$ we shall make use of two results. First, according to [7], the radial displacement u_r of the crack junction of the pressurized star crack is given by

$$u_r = 2(1-\nu^2)/E \cdot \int_0^a K_{Ip} \left(\frac{\partial K_{IF}}{\partial F} \right) da, \quad (4)$$

for plane strain conditions. This expression may also be derived from [8]. Inserting eqns (1) and (3) into eqn (4) there results

$$u_r = 2(1-\nu^2)pa/E \cdot k_{Ip}(n)k_{IF}(n). \quad (5)$$

Secondly, from [4] we obtain the following expression for u_r ,

$$u_r = \frac{2(1-\nu^2)pa}{E} k_{Ip}(n) \sqrt{\left(\frac{n}{2}\right)} / \sqrt{\left(1 + \frac{n}{2\pi} \sin \frac{2\pi}{n}\right)}. \quad (6)$$

Fig. 1.

After eliminating u_r and $k_{1p}(n)$ we obtain

$$k_{1p}(n) = \sqrt{\left(\frac{n}{2}\right)} / \sqrt{\left(1 + \frac{n}{2\pi} \sin \frac{2\pi}{n}\right)} \text{ when } n \geq 2. \quad (7)$$

Apart from some numerical values given in [6], this result appears to be new.

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