Fleet-Level Reliability Analysis of Repairable Units: A Non-Parametric Approach using the Mean Cumulative Function

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Abstract: This paper describes the use of the mean cumulative function (MCF) and linear estimates based on the recurrence rate to predict the expected number of failures in the future. Reliability data from two repairable units are used to verify the procedure and comparison. The empirical data used in the paper is based on field data gathered during the operational life of the Swedish military aircraft system FPL 37 Viggen from 1977 to 2006, which essentially is the whole life cycle of the system.

Keywords: Non-parametric analysis, repairable units, maintenance, aviation, reliability, mean cumulative function.

1. Introduction and Background

Complex technical systems are normally repaired rather than replaced when they fail. It is often desirable to analyse the reliability characteristics of these systems based on data generated in a customer use environment, in order to assess reliability, frequency of failure or other parameters which may be influenced by the systems' age and usage. Despite the advantages of continuously analysing reliability data to be able to improve the maintenance programme continuously, methods such as parametric and non-parametric analysis are often ignored due to a belief that the mean time between failures (MTBF) is sufficient to describe the reliability pattern of repairable units.

Complex technical systems such as military and civil aircraft are continuously maintained to assure a specific level of safety and availability at the lowest possible cost. When dealing with complex technical systems in a competitive environment, maintenance departments are required to ensure that their fleet will meet, or continue to meet, their established performance goals (*e.g.*, operational readiness, dispatch reliability, cost-effectiveness, *etc.*) and to make sure that demands for deliveries will be met. Moreover, during the development a maintenance programme, the quantification of the operational risk of aircraft system failure is a great challenge. The reason includes the inadequacy of in-service information, and a lack of understanding of the influence of failures [1].

Therefore, a formal reliability programme is needed which ensures the collection of important information about the aircraft system's reliability performance throughout the operational phase, and directs the use of this information in the implementation of analytical and management processes.

Aircraft systems can be classified into non-repairable and repairable systems. Non-repairable systems are those which are not repaired when they fail to perform one or more of their functions satisfactorily, and are instead discarded. The discard action does not necessarily mean that the unit cannot be repaired. In some cases repair actions are not economically effective since a repair would cost almost as much as acquiring a new unit. Repairable units are those which, after failing to perform one or more of their functions satisfactorily, can be restored to satisfactory performance by any method other than

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replacement of the entire system [2]. The reliability analysis of repairable units includes modelling the number of recurrent failure events over time rather than the time to the first failure, and the reliability of such units strongly depends on the effectiveness of the repair action. The quality or effectiveness of the repair action can be classified into three categories [2-3]:

- Perfect repair: *i.e.,* restoring the system to the original, a "like-new" condition.
- Minimal repair: *i.e.,* restoring the system to a functional but "like-old" condition.
- Normal repair: *i.e.,* restoring the system to any condition between 1 and 2.

Based on the quality and effectiveness of the repair action, a repairable system may end up in five different possible states after repair, *i.e.* an as-good-as-new, an as-bad-as-old, a better-than-old but worse-than-new, a better-than-new, and a worse-than-old condition [2-3]. If through a repair action a major modification takes place in the unit, it may end up in a condition better than new; and if a repair action causes some error or an incomplete repair is carried out, the unit may end up in a worse-than-old condition [3].

There are two major approaches to the reliability analysis of repairable units, namely parametric and non-parametric methods. The parametric approach includes the stochastic point process, and the analysis includes mainly the homogeneous Poisson process (HPP), the renewal process (RP), the non-homogeneous Poisson process (NHPP) and the generalized renewal process (GRP), introduced by Kijima [4]. A renewal process is a counting process where the inter-occurrence times are s-independent (*i.e.*, the inter-occurrence times are mutually stochastically independent) and identically distributed with an arbitrary life distribution [5]. The NHPP is often used to model repairable systems that are subject to a minimal repair. The HPP describes a sequence of s-independent and identically distributed (IID) exponential random variables. Conversely, an NHPP describes a sequence of random variables that are neither statistically independent nor identically distributed [5]. The GRP allows the goodness of repairs to be modelled from *as-good-as-new* repair (RP) to *same-as-old* repair (NHPP). The GRP is particularly useful in modelling the failure behaviour of a specific unit and understanding the effects of repair actions on the age of that system. An example of a system to which the GRP is especially applicable is a system which is repaired after a failure and whose repair does not bring the system to an *as-good-as-new* or an *as-bad-as-old* condition, but instead partially rejuvenates the system [6]. Different parametric methods are implemented to model the probability of failure for repairable units, *e.g.,* the power law process [4-9].

The application of these methods for a single system/unit is quite clear and straightforward. However, in practice the analyst is often dealing with multiple similar systems which are installed in different aircraft and which are running in different operating environments, with different other influencing factors. The aim of reliability analysis in this paper is to track field failures to provide information regarding failure rates and the expected number of failures at the fleet level, and not at the individual component level.

The application of parametric reliability analysis methods at the fleet level, even if it is very limited in scope, is quite complex and time-consuming for a variety of reasons. For instance, failure analysis is made more difficult by the highly multi-censored nature of the reliability data belonging to different failure modes. The analysis of time-censored and failure-censored data needs different treatment [8 and 10], with the application of different methods. Moreover, drawing conclusions at the fleet level from these individual analyses requires statistical assumptions which in practice entail a degree of uncertainty.

Moreover, when pooling data for units from an operational fleet, the associated failure data require that one should consider the statistical characteristics to assure the

applicability of the pooling. This means that the analyst should test the similarity of the distribution between failures, for the whole population, which in practice is quite difficult and time-consuming.

In general, using these parametric methods often requires some degree of statistical sophistication and sound statistical knowledge and experience on the part of the analyst. There is also often a problem communicating the results and ideas to customers and management within the industry [11]. Moreover, the management and the engineers and field service teams who maintain and support the aircraft systems can easily be daunted by such complex techniques. Hence it is very important to employ a failure analysis methodology which is statistically valid, yet communicates to the managers and engineers in a professional language with which they are familiar [12]. According to [12], monitoring a recurrent failure in a complex system such as an aircraft does not necessarily require complicated methods.

Non-parametric methods provide a non-parametric graphical estimate of the number of recurrences (repairs/failures) per unit and per the whole population, versus the utilization/age. The model used to describe a population of systems in this paper is based on the mean cumulative function (MCF) at the system age t. The MCF is non-parametric in the sense that it does not use a parametric model for the population. This estimation involves no assumptions about the form of the mean function or the process generating the system histories. Graphical methods based on the MCF [13-16] allow the monitoring of system failures and the maintenance of statistical rigour without resorting to complex stochastic techniques. The MCF is simple in that it is easy to understand, prepare, and present. It can be successfully used to track field failures and identify failure trends, anomalous systems, unusual behaviour, the effect of various parameters (*e.g.*, maintenance policies, environmental and operating conditions, *etc.*) on failures, *etc.* Thus it is a significant decision support tool which permits operators, maintenance providers, and manufacturers to make quantifiable and rational decisions in fields that have typically been the domain of guesswork and experience. This very useful and simple concept has been in existence for nearly two decades, but the literature remains highly theoretical and difficult for the average practitioners, and the reported applications have been very limited [12, 13 and 15].

The objective of this paper is to provide a relatively simple method for estimating the expected number of failures for large arrays of repairable units from an operational aircraft fleet. This task is motivated by the need to produce such estimates relatively quickly and simply, without having to use complex parametric methods. Since an aircraft, whether it is commercial or military, comprises a large number of repairable units with differing failure distributions, there is in practice neither the time nor the resources to study each type of unit in a detailed manner using parametric methods, and such methods are thus impractical. By using non-parametric methods, the mathematical manipulation of operational data is greatly simplified, albeit at the cost of losing some analytical rigour.

However, this is acceptable, since the objective is to find a procedure that is suitable for large-scale use, on a fleet-level basis, *i.e.*, to estimate the future maintenance requirements for an aircraft fleet typically comprising hundreds of separate types of units, and tens to hundreds of thousands of individual repairable units. Furthermore, the fact that operation profiles and operational environments frequently vary greatly between individual aircraft and over time means that the total number of relevant parameters becomes large, and that many of them are difficult to determine with any precision. This means that, by the time enough data (especially for military aircraft systems) have

accumulated to allow parametric methods to become useful, a large part of the life cycle of the studied system will already have passed.

Therefore, what is needed is a method which can be applied to a large number of populations of repairable units with a reasonable expenditure of resources, and which requires only relatively basic operational information about the studied unit. Furthermore, it should be simple to iterate the method and improve the estimates as more data becomes available.

The paper is organized as follows. In Section 2 the approach is described using the mean cumulative function for recurrence reliability data. The empirical reliability data and data collection process is described in Section 3. In Section 4 the results of the reliability data process is described. The paper ends with a summary and conclusions.

2.   Non-Parametric Model for Recurrent Event Reliability Data using MCF

In reliability theory and many other applications, *e.g.,* public health, engineering, sociology, economics and medicine, the focus of interest is directed on the study of processes which generate events repeatedly over time, *i.e.,* recurrent processes [17]. In the field of reliability analysis for repairable units, the focus of interest concerning recurrent events is on the number of repairs for a unit, the time to failure and the effectiveness of repair, for example.

Having n repairable units in the operational aircraft fleet, let $N_i(t)$ be the total number of cumulative recurrences for unit i, were $i = 1, \dots, n$ over a given time t. Then it follows that the counting process which counts the total observed cumulative number of failures for the whole fleet is given by equation (1).

$$N(t) = \sum_{i=1}^{n} N_i(t) \qquad (1)$$

The mean cumulative number of failures $\mu(t)$, also described as the mean cumulative function (MCF), is given by equation (2).

$$\mu(t) = E[N(t)] = \sum_{i=1}^{n} \mu_i(t) \qquad (2)$$

If $\mu(t)$ is differentiable, then the recurrence rate is given by equation (3).

$$\nu(t) = \frac{dE[N(t)]}{dt} = \frac{d\mu(t)}{dt} \qquad (3)$$

In the context of reliability $\nu(t)$ is also referred to as the failure intensity or the rate of occurrence of failures (ROCOF) [7]. During the late 1980's, Nelson [14] provided an appropriate point-wise estimator for the MCF, the so-called Nelson's estimate $\hat{\mu}(t_j)$.

Having the recurrence times t_{ij} of all the units n, let m be the unique recurrence times; *i.e.*, unit i has $j = 1, \dots, m_i$ recurrences. Order the recurrence times from the lowest to the highest, $t_1 < t_2, \dots, < t_m$. The MCF is then estimated according to equation (4).

$$\hat{\mu}(t_j) = \sum_{k=1}^{j} \left[\frac{\sum_{i=1}^{n} \delta_i(t_k) d_i(t_k)}{\sum_{i=1}^{n} \delta_i(t_k)} \right] \qquad (4)$$

where, $\delta_i(t_k) = \begin{cases} 1, \text{if unit } i \text{ is still functioning,} \\ 0, \text{otherwise,} \end{cases}$

and $d_i(t_k)$ is the number of recurrences for unit i at the time t_k. Plotting the point-wise estimated MCF, $\hat{\mu}(t_j)$, gives a step function with jumps at the recurrence times; see Section 4 for examples. When studying the point-wise estimated MCF, one is normally also interested in gaining an understanding of the variance in the studied data population, and the confidence level is therefore of great importance. Using a point-wise normal approximation, a confidence interval is estimated for the MCF [9] according to equation (5).

$$\hat{\mu}(t_j)/e^{\left[\left(z_{1-\frac{\alpha}{2}}\right)s\hat{e}_{\hat{\mu}(t)}/\hat{\mu}(t)\right]} < \hat{\mu}(t) < \hat{\mu}(t_j)e^{\left[\left(z_{1-\frac{\alpha}{2}}\right)s\hat{e}_{\hat{\mu}(t)}\hat{\mu}(t)\right]} \tag{5}$$

where $s\hat{e}_{\hat{\mu}(t)}$ is the standard error for the estimated MCF, and $z_{1-\alpha/2}$ is the value of a normal distribution at significance level α.

The process of estimating the MCF can be described by the following steps [9]:

i. Order the ages (*e.g.,* the operation times, calendar times, *etc*.) for the recurrences from the lowest to the highest times, *i.e.,* $t_1 < t_2, ..., < t_m$ for unit i according to equation (4) above, and note the censoring age. Of course, the censoring and recurrence age can be the same, and it may also happen that different units have the same censoring and recurrence time.

ii. Calculate the number of units at risk (*i.e.,* the number of units that have a recurrence probability greater than zero); at each unit age calculate the number of remaining units at risk at that age, *i.e.,* the number of units that risk a recurrence at a specific age, this number is given by $\delta_i(t_k)$ in equation (4) above.

iii. Calculate the mean numbers; for each recurrence, calculate the observed incremental mean number of recurrences per unit at that recurrence age as $1/n$; *i.e.,* one out of the n units passing through that age had a recurrence.

iv. Calculate the MCF according to $\hat{\mu}(t_j)$ given in equation (4) above; calculate the value of the sample MCF at each recurrence by adding the increment obtained in step iii and the preceding increments from the previous recurrences.

By using the MCF estimation described in equation (4) and a corresponding plot of the MCF-curve, one obtains very valuable reliability information about the studied population, *e.g.,* the number of repairs/unit, trends and anomalies, and information for estimating the future reliability behaviour.

By using the recurrence rate $v(t)$ to extrapolate the MCF with linear estimates, it is possible to estimate the future failure behaviour of the units. Based on the definition of the derivative, $v(t)$ can be approximated by equation (6).

$$v(t) \sim \frac{\hat{\mu}(t_{j+1})-\hat{\mu}(t_j)}{(t_{j+1})-t_j} \tag{6}$$

where $j = 1, ..., m - 1$, and m is the total number of recurrences for the studied population. Instead of taking the derivative between the two consecutive points ($\Delta = 1$) in the MCF estimation, we use overlapping derivatives, according to equation (7).

$$v(t) \sim \frac{\hat{\mu}(t_{j+\Delta})-\hat{\mu}(t_j)}{(t_{j+\Delta})-t_j} \tag{7}$$

where $j = 1, ..., m - \Delta$. By calculating the derivative over overlapping segments with a certain step length, Δ, greater than one, we obtain an estimate of the variability of the recurrence rate over the operational time. Calculating this "local recurrence rate" according to equation (7), and then dividing the calculated rates into different percentiles, we use the median of the derivatives as a nominal estimate and the 5th and 95th percentiles as the maximum and minimum estimates of the predicted recurrence rates. The 95th percentile is the value below which 95 percent of the derivatives are found, and the 5th percentile limit is the value below which 5 percent of the derivatives are found. These three measurements (the 95th percentile, the median and the 5th (percentile) define the slope of the linear estimates used to predict the future expected number of failures. See Section 4 for two empirical examples.

3.   Data Collection Process

The data analysis performed in this paper is based on two types of repairable components, see Table 1. The studied units were in operational service in the Swedish military aircraft system FPL 37 Viggen from 1974 to 2006, which is essentially the whole life cycle of the system. The data concern about 330 aircraft with a total of 615,000 flight hours, and include information about all the maintenance-significant events for the physical units, *i.e.,* the date of delivery, storage time, storage maintenance, installations, removals, failures, corrective maintenance, modifications, and discard. These events are associated with calendar dates and accumulated operating time in flight hours.

The first repairable unit studied was a radar transmitter (henceforth referred to as unit A). The radar transmitter was for the main surveillance radar (PS-37) of the AJ 37 strike aircraft. The fault modes for this unit were dominated by failures of a variety of electronic components, the most common being failures of thyratrons, magnetrons and pulse transformers. Since this unit was only subjected to corrective maintenance, there is only one failure category, *i.e.,* failures in service. The data were right-hand censored by two processes: failure and discard. The second unit studied was a cooling turbine (henceforth referred to as unit B), which was the principal unit of the environmental control system that delivered cooling air for the electronics and the cockpit air conditioning in all versions of the FPL 37 aircraft system. The cooling turbine was a heavily stressed mechanical unit with a fairly high failure rate in service, despite receiving preventive maintenance.

Table 1: Number of Units Included in the Study

Item	Hardware	#	Maintenance Strategy
A	Radar Transmitter	124	Corrective Maintenance
B	Cooling Turbine	372	Preventive/Corrective Maintenance

For units A and B, the failures and censoring are shown in event plots, see Figure 1 and Figure 2. Among the 124 units of type A, there are 18 units that were time-censored without any failures at all, and for unit B there are 257 units that were time-censored without failures among the 372 units, which can also be seen in the event plot.

Table 2 presents the number of units that have suffered a given number of failures, showing that for unit A there are 106 units which have encountered 1 failure, 96 units which have encountered 2 failures, *etc.* Based on Table 2, one can calculate the total number of failures for the whole population of unit A, *i.e.,* 413 failures. The corresponding data are presented for unit B, and the total amount of accumulated failures for unit B can be calculated to be 156 over the whole life cycle.

Table 2: The Number of Units that have Suffered a Given Number of Failures

Failures (#)	0	1	2	3	4	5	6	7	8	9
Unit A (#)	18	106	96	82	60	33	21	8	4	3
Unit B (#)	257	115	37	4	--	--	--	--	--	--

These two types of components were both relatively maintenance-intensive. The radar transmitter was only subjected to corrective (on-condition) maintenance, while the cooling turbine, as a heavily stressed mechanical unit, received preventive maintenance (periodic overhauls). The differences between unit A and B (A being an electronic unit and B being a mechanical unit, and B receiving both preventive and corrective maintenance and A receiving corrective maintenance only) resulted in the failure patterns for the two units being quite different; see Figure 3 and Figure 4 for the estimated MCFs.

Table 3 and Table 4 describe the number of failures and units in service for each 500-flight-hour interval for unit A and unit B, respectively. As mentioned above, the units were right-hand censored by two processes, either by discard or failure, and in the case of aircraft crashes, the units were, of course, time-censored at the operational time for the unit at the time of the crash.

Table 3: Failures and Units of Type A in Service for Each 500-Flight-Hour Interval

Operation Time [h]	Units in service [#]		Accumulated failures [#]	
0-500	124 – 104	(20 discards)	0 – 113	(113 failures in interval)
500-1000	104 – 98	(6 discards)	113 – 243	(130 failures in interval)
1000-1500	98 – 85	(13 discards)	243 – 340	(97 failures in interval)
1500-2000	85 – 50	(35 discards)	340 – 396	(56 failures in interval)
2000-2500	50 – 9	(41 discards)	396 – 411	(15 failures in interval)
2500-3000	9 – 1	(8 discards)	411– 412	(1 failure in interval)
3000-3500	1 – 0	(1 discard)	412 – 413	(1 failure in interval)

Table 4: Failures and Units of Type B in Service for Each 500-Flight-Hour Interval

Operation Time [h]	Units in service [#]		Accumulated failures [#]	
0-500	372 – 338	(34 discards)	0 – 39	(39 failures in interval)
500-1000	338 – 286	(52 discards)	39 – 74	(35 failures in interval)
1000-1500	286 – 217	(69 discards)	74 – 115	(41 failures in interval)
1500-2000	217 – 113	(104 discards)	115 – 143	(28 failures in interval)
2000-2500	113 – 33	(80 discards)	143 – 154	(11 failures in interval)
2500-3000	33 – 1	(32 discards)	154– 156	(2 failures in interval)

In Figure 1 and Figure 2 the decrease in the number of units over the operational time is plotted (for reasons of legibility only every third unit is plotted). The decrease in the number of units before 500 operating hours is largely due to losses through aircraft crashes during the early life of the aircraft system, while from about 1,500 hours the losses are mostly due to the discard of complete aircraft and surplus units. The difference between the slopes of the two event plots is largely due to the fact that Unit B was also installed in the JA 37 interceptor version of FPL 37, which had a slightly longer operational life than the AJ 37 strike version.
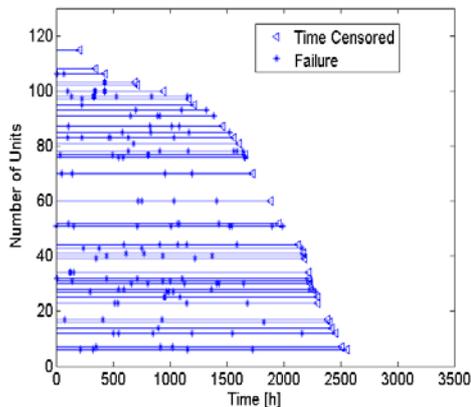

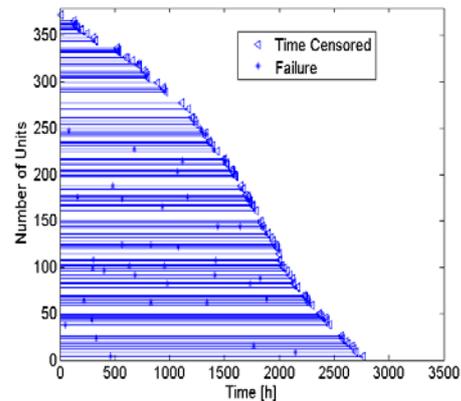
Figure 1: Event Plot for Units of Type A   **Figure 2:** Event Plot for Units of Type B

4. Reliability Data Analysis - MCF and Recurrence Rate

The first part of the analysis involved sorting and filtering the life data into an analysable format, *i.e.*, cleaning and censoring the data used (modifications, discards, suspensions,

and failures), and eliminating units that had been used for accelerated life testing in test rigs *etc*. Performing the MCF estimation as described in Section 2 according to equation (4), the estimation plots shown in Figure 3 and Figure 4 are obtained.

These figures present the MCF and its two-sided confidence levels at a significance level of 5% for our two studied types of units.
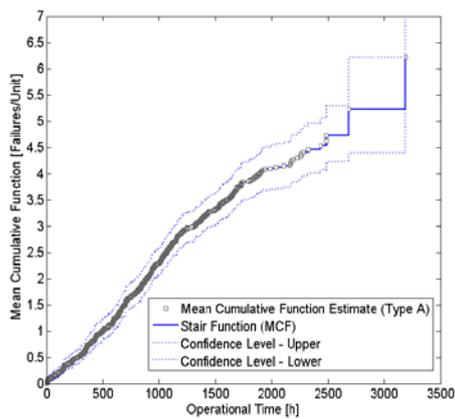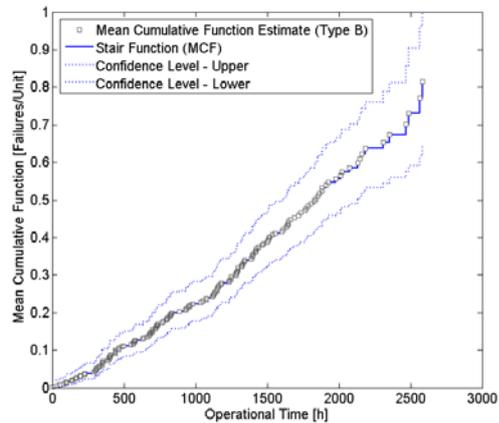

Figure 3: MCF for units of type A

Figure 4: MCF for units of type B

It is clear from Figure 3, that for the type A units the recurrence rate increased during the first ~700 flight hours, and then there seems to have been a reliability improvement around ~1,250 flight hours; the reason for this might be that the repair process became more efficient. In Figure 4 (showing the MCF for the units of type B), it is obvious that the trend indicates that the units deteriorated with operational time, which is not surprising since they are mechanical and wear out. In this particular case the repair process was clearly not perfect (*i.e.*, did not result in an as-good-as-new condition).

Calculating the "local recurrence rates" for Unit A, as described in Section 2 and according to equation (7), gives the results illustrated in Figure 5. This figure shows the variation of the "local recurrence rates" obtained with equation (7), divided into twenty equal bins, as well as the positions of the percentiles used (the 95th percentile, the median and the 5th percentile) on the x-axis. The "local recurrence rates" presented in Figure 5 are based on the first 500 hours of operation for unit A. This means that the three linear curves used to estimate the next 500 hours of operation are based on the first 500-hour segment, *i.e.*, on a measure of the variability of the MCF from the time when the units were new until the point where we start estimating the future MCF with linear approximation. The 500-hour interval was chosen as a realistic interval for estimates for this particular aircraft system, being equivalent to 3-5 years of operation.

In Figure 5 and all the other examples presented in the paper, the step length Δ is set to 15 recurrences (*i.e.,* points 1 to 15, 2 to 16, 3 to 17, *etc*.). Of course, varying the step length will affect the difference between the three estimated curves. Having a smaller step length will, of course, increase the "noisiness" of the data. The step length was selected as a compromise between not losing information about the reliability behaviour of the studied population and a reasonable spread of the estimates.

The illustrated example is based on estimates for 500-hour-long segments to evaluate the goodness of fit of the estimated linear curves against the actual MCF. This procedure is repeated at 500-hour intervals over the life cycle of the studied units to

evaluate how good the estimate of the failure intensity is over the entire life cycle of the units; see Figure 6 and Figure 7 for the estimates for the two different types of units. As shown in Figure 6 and Figure 7, the linear estimates of the expected number of failures for the succeeding 500-hour segments are in most cases well within the three estimates.
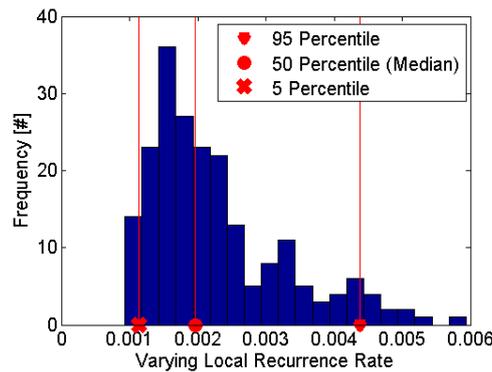

Figure 5: Variation of "Local Recurrence Rates" for the First 500 Hours for Unit A

The median-based estimate underestimates the expected number of failures at the beginning of the life cycle and overestimates it towards the end of life for unit A. A probable reason for this change is that the MCF curve in this case seems to have several inflection points which prevent constraint of the derivative within narrow limits. For unit B the estimation of the expected number of failures is quite accurate throughout the whole life cycle of the unit, perhaps with some underestimation in the middle of the life cycle. The reason for this good approximation for unit B is that, although the recurrence rate changes throughout the life cycle of the units, this change is consistent and unidirectional and is consequently well tracked by the estimation algorithm.
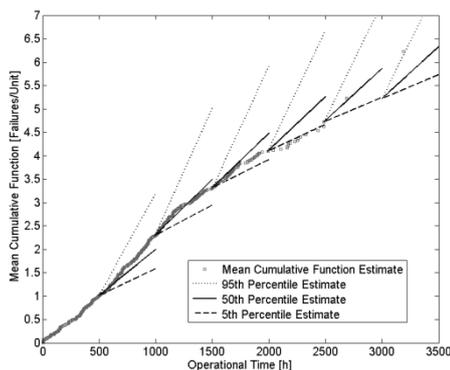

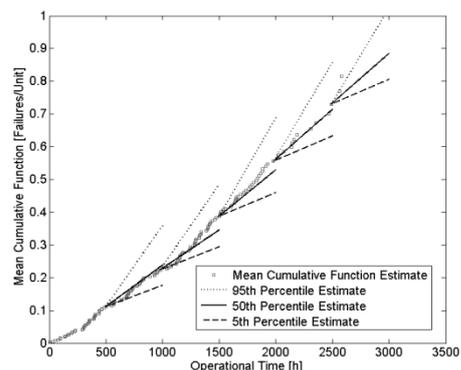**Figure 6:** Successive Estimates for Unit A   **Figure 7:** Successive Estimates for Unit B

The estimated numbers of failures for unit A and unit B are compared to the actual numbers of failures for all the 500-hour segments in Table 5 for unit A and Table 6 for unit B. The actual number of failures for each 500-hour interval is given in column two, the low (5%), nominal (median) and high (95%) estimates are given in column three, and the difference between the actual number of failures and the nominal estimates is given in column four. As seen in Table 6, the estimations for unit B are much better, with an

average error of 10% during the whole life cycle, which is rather better than one would expect. Even the estimates for unit A, with an average error of 32% over the life cycle, must be considered to be reasonably good results.

Table 5: A Comparison between Empirical and Modelled Number of Failures (Type A)

Operation Time [h]	Empirical Failures [#]		Estimated Failures [#] (5%, 50%, 95%)	Δ	Error [%]
0-500	0 – 113	(113)	N/A	N/A	N/A
500-1000	113 – 243	(130)	(59,100,223)	+30	23%
1000-1500	243 – 340	(97)	(61,112,251)	-15	13%
1500-2000	340 – 396	(56)	(41, 80,177)	-24	30%
2000-2500	396 – 411	(15)	(17,34,75)	-19	56%
2500-3000	411– 412	(1)	(1,3,6)	-2	67%
3000-3500	412 – 413	(1)	(1,1,1)	0	0%
					Average = 32%

Table 6: A Comparison between Empirical and Modelled Number of Failures (Type B)

Operation Time [h]	Empirical Failures [#]		Estimated Failures [#] (5%, 50%, 95%)	Δ	Error [%]
0-500	0 – 39	(39)	N/A	N/A	N/A
500-1000	39 – 74	(35)	(21,40,78)	-5	13%
1000-1500	74 – 115	(41)	(18,31,67)	+10	24%
1500-2000	115 – 143	(28)	(12,24,50)	+4	14%
2000-2500	143 – 154	(11)	(6,11,21)	0	0%
2500-3000	154 – 156	(2)	(1,2,3)	0	0%
					Average = 10%

5.   Conclusion

The objective of this paper has been to find a simple and easily understandable methodology for estimating the expected number of failures of repairable units, particularly during the latter part of the life cycle of the system concerned. This is a highly relevant and very important objective for military aircraft, whose planning horizon is longer than that of commercial aircraft and for which very long intervals between aircraft generations mean that spares and maintenance may become difficult and expensive to obtain towards the end of the system's life.

The method described above for estimating the expected number of failures seems to estimate the number of future failures with a reasonable precision due to its simplicity. The estimate for unit B is remarkably good, because of the consistent change in the failure rates over the life cycle of the unit. The fit for unit A is much worse, since the MCF for this unit has several inflection points and changes in a much more inconsistent way. For Unit A it would probably be very hard to find a good parametric estimate, and if this were at all possible, it would probably only be achieved quite close to the end of the units' life cycle.

In an actual operational situation, iterations will be performed at much shorter intervals. Furthermore, in a real situation the individual units in a population will have quite different numbers of operational hours at a given time. This means that for most units the actual MCF will be available as a more reliable estimate until they reach the number of operational hours of the current "lead unit", and that the linear estimate will only need to be invoked beyond this point. However, in this paper we have not been able to use this method of estimation, since we cannot obtain the exact number of operational hours for all the units of the studied types at a specific date from the available historical records.

In the analysis performed in this study, we have only considered failures in operation for unit B. Since this unit received preventive maintenance, a considerable number of incipient failures were actually found and repaired during overhauls. While these failures are significant for the total maintenance costs, spare parts requirements and decisions on the maintenance interval for preventive maintenance, they do not affect the rate of failure during operation, provided that there are no changes in the scope of the preventive maintenance, which is the reason why we have not included them in our study. The motivation for carrying out preventive maintenance is, of course, to minimize the number of operational failures. However, since this is never completely successful, estimating the number of operational failures that will occur for units receiving scheduled preventive maintenance and corrective maintenance is at least as important as doing so for units receiving corrective maintenance only. In an actual operational situation the number of overhauls (or other preventive maintenance actions) over an interval in the future can easily be calculated from the number of hours since the previous overhaul (the time since overhaul or TSO) for the individual units, and can be added to the estimated corrective maintenance actions

In short, the described method is probably sufficiently good to be useful for the operational planning of maintenance requirements, even though the estimates used are purely heuristic and have no explanatory power concerning the underlying causes of the failure process. In this connection we might well cite Sir Robert Watson-Watt, Head of British Radar Development during World War Two: "*Give them the third best to go on with, the second best comes too late, the best never comes.*"

Further studies will concern an application of the method to units in a currently operational aircraft fleet. In this phase the actual MCF will be calculated as far as the "lead unit" of each population. The failure of units trailing after the lead unit will be estimated by following this actual MCF until the operational time of the lead unit is reached, and the linear estimate described above will be used from that point onwards.

Furthermore, the changes over time in the distribution of the "local recurrence rates" (as shown in Figure 5) will be studied and applied to more types of components to search for recurring patterns in the distributions.

References

[1]   Ahmadi, A., Kumar, U. and Söderholm, P. *Operational Risk of Aircraft System Failure.* International Journal of Performability Engineering, 2010, vol. 6, No. 2, pp. 149-158.

[2]   Ascher, H., and H. Feingold. *Repairable Systems Reliability: Modeling, Inference, Misconceptions and Their Causes.* Dekker, New York, N.Y., 1984.

[3]   Modarres, M. *Risk Analysis in Engineering: Techniques, Tools, and Trends.* CRC/Taylor & Francis, Boca Raton, Fla., 2006.

[4]   Kijima, M., and U. Sumita. *A Useful Generalisation of Renewal Theory: Counting Processes Governed by Non-Negative Markovian Increments.* Journal of Applied Probability, 1986; 23: 71-88.

[5]   Rausand, M., and A. Høyland. *System Reliability Theory: Models, Statistical Methods, and Applications.* Wiley, Hoboken, N.J., 2004.

[6]   Kijima, M. *Some Results for Repairable Systems with General Repair.* Journal of Applied Probability, 1989; 26: 89-102.

[7]   Crow, L.R. *Reliability Analysis for Complex Repairable Systems*. Conference on Reliability and Biometry Held at the Florida State University, Tallahassee, July 9-27, 1973, SIAM, Philadelphia, Pa.; 379–410.

[8]   Rigdon, E.R., and A.P. Basu. *Statistical Methods for the Reliability of Repairable Systems.* Wiley, New York, N.Y., 2000.

[9]   Meeker, W.Q., and L.A. Escobar. *Statistical Methods for Reliability Data.* Wiley, New York, N.Y., 1998.

[10]   Caroni, C. *Failure Limited Data and TTT-based Trend Tests in Multiple Repairable Systems*. Reliability Engineering & System Safety, 2010; 95(6): 704–706.

[11]   Misra, K.B. *Handbook of Performability Engineering*. Springer Verlag, London, 2008.

[12]   Al-Garni, A.Z., M.Tozan, A.M. Al-Garni, and A. Jamal. *Failure Forecasting of Aircraft Air-conditioning/Cooling Pack with Field Data.* Journal of Aircraft, 2007; 44 (3): 996-1002.

[13]   O'Connor, P.D.T. *Practical Reliability Engineering*. Wiley, Chichester, 1991.

[14]   Nelson, W. *Graphical Analysis of System Repair Data*. Journal of Quality Technology, 1988; 20 (1): 24-35.

[15]   Nelson, W.B. *Recurrent Events Data Analysis for Product Repairs, Disease Recurrences, and Other Applications.* SIAM, Philidelphia, Pa., 2003.

[16]   P. Wang., and Coit, D.W. *Repairable Systems Reliability Trend Tests and Evaluation.* Annual Reliability and Maintainability Symposium, 24-27 January 2005; 416-421.

[17]   Zuo, J., W.Q. Meeker, and H. Wu. *Analysis of Window-Observation Recurrence Data.* Technometrics, 2008; 50(2): 128-143.

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