Cases of Coupled Vibrations and Parametric Instability in Rotating Machines

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Doctoral Thesis in Solid Mechanics

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PREFACE

A lot of thanks are addressed to my mother Louise, who raised me up and posed a stable foundation which has guaranteed my good intellectual development despite all difficult circumstances encountered in the Democratic Republic of Congo where I was born. My sisters Eunice and Rachel, thanks a lot for all that you did for me during my studies in high school.

Viktoria my precious wife and Leo my beloved son; you made all this possible. For about four years, I have been home in Stockholm during weekends only because I had to travel to Luleå for work every week. Thanks a lot dear Viktoria for your endurance and support all these years. You are my crown princess!

Prof Jan-Olov and Dr Rolf; thanks a lot for all scientific guidance you provided. I sincerely confess that I enjoyed the rich collaboration we had together. It was a great privilege for me being a PhD student within this interesting research field of rotordynamics.

To all my colleagues at the divisions of Solid mechanics and Machine elements, thanks a lot for the wonderful time we have had together at fika times, during the division trips and all daily scientific exchanges. I wish we meet again in our future professions.

I finally thank You God for keeping me mentally and physically healthy through the entire process. Thank You very much.
ABSTRACT

The principal task in this research project was to analyse the causes and consequences of coupled vibrations and parametric instability in hydropower rotors; where both horizontal and vertical machines are involved. Vibration is a well-known undesirable behavior of dynamical systems characterised by persistent periodic, quasi-periodic or chaotic motions. Vibrations generate noise and cause fatigue, which initiates cracks in mechanical structures. Motions coupling can in some cases augment the stability characteristics of a rotating machine, but it can also be a source of instability that causes self-excited vibrations. In this thesis, motions coupling due to a bearing’s design, gyroscopic effect and geometric misalignment in rotating components were studied. The performed studies include mathematical modelling and numerical simulation of the above named sources of motions coupling. Experiments were also performed in order to evaluate the derived analytical models.

Plain cylindrical hydrodynamic journal bearings cross couple the rotor translational motions. This cross coupling is the main source of oil induced instability. The inherent nonlinearity of plain cylindrical hydrodynamic journal bearings becomes strong for eccentricities greater than 60% of the bearing clearance, where most existing linear models are not able to predict accurately the rotor trajectory. Therefore, the journal bearing impedance descriptions method, a method that is valid for all bearing eccentricities and aspect ratios, was used to analyse the rotor steady-state imbalance response. Strong nonlinearities together with cross coupling are the source of complex dynamics in fluid-film journal bearings. The simulation results show that linear bearing models derived from the nonlinear impedance descriptions of the Moes-cavitated (π – \text{film}) finite-length bearing can predict the steady-state imbalance response of a rigid symmetric rotor that is supported by two identical journal bearings at high eccentricities. This is, however, only the case when operating conditions are below the threshold speed of instability and when the system has period one solutions. The speed at which oil induced instability occurs, also called the instability threshold speed, will depend upon the low stability characteristics of the less loaded bearing for an offset rigid rotor. However, for a flexible rotor the gyroscopic coupling effect will increase the instability threshold. The gyroscopic coupling effect not only increases the instability threshold, but the journal trajectories' magnitude also significantly increases. This is normally not a preferable condition since high vibrations will induce heat and stress in babbited bearings. Adding rotor imbalance would enable the system to be operated beyond its threshold speed of instability with reduced vibration amplitudes.
A tilting-pad combi-bearing is a bearing designed as a combination of both tilting-pad journal and thrust bearings. Thrust bearing is a component used in vertical rotating machines and shafts designed to transmit thrust, e.g. hydropower rotors and aircraft engines. The total axial load is normally carried by one thrust bearing. In hydropower applications, the influence of the combi-bearing is strongly simplified in the rotor dynamic modelling. The derived linear model shows that the combi-bearing couples the rotor’s lateral and angular motions at the contact point between the combi-bearing and the rotor. However, if the thrust bearing’s pads arrangement is not symmetrical or if all the pads are not angularly equidistant, the rotor vertical (axial) and angular motions are also coupled. This last case of coupling will also occur if the axial equivalent stiffness is not evenly distributed over the thrust bearing. A defected pad or unequal hydrodynamic pressure distribution on the pads’ surfaces may be the cause. The Porjus U9’s simulation results show that the combi-bearing influences the dynamic behavior of the machine. The rotor motions’ coupling due to combi-bearing changes the system’s natural frequencies and vibration modes. Introducing an angular misalignment in the combi-bearing’s rotating collar will generate an asymmetry in the rotor system at the combi-bearing’s location. The rotor system’s stiffness in its two translational directions differ at the combi-bearing’s location. Constant parameters and/or coefficients in rotating asymmetric structures appear to change with time when observed in the stationary frame. These time dependent parameters (coefficients) are the source of parametric instability in rotating systems. If the collar angular misalignment is located in one plane, all rotor motions in this plane at the contact point between the combi-bearing and the rotor will be coupled. A parametric instability is observed within certain ranges of the rotor speeds, depending on the magnitude of the angular misalignment.

The studied cases of motions coupling due to plain cylindrical hydrodynamic journal bearings, motions coupling due gyroscopic effect, motions coupling due to tilting-pad combi-bearing and motions coupling due to manufacture or assembling errors in rotating components are cases that can be encountered in hydropower rotors. The results have revealed in particular that if the combi-bearing is manufactured or assembled with a certain angular misalignment, this may cause a parametric instability in the hydropower rotor. The parametric instability can even occur below the rotor critical speed, which would cause problems for undercritical machines as hydropower plants. The outcomes of these studies will contribute in further understanding of vibration problems and particularly in helping to improve and sustain the functionality of new and existing hydropower plants in Sweden.
LIST OF APPENDED PAPERS

This doctoral thesis comprises a survey of the following six papers:


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1. INTRODUCTION

The principal task in this research project was to analyse the causes and consequences of coupled vibrations and parametric instability in rotating machines; where both horizontal and vertical machines are involved. The outcomes of these studies will particularly help to improve and sustain the functionality of new and existing hydropower plants in Sweden.

1.1. SWEDISH HYDROPOWER TECHNOLOGY

Today, hydroelectric power accounts for about one fifth of the world’s power supply and is by far the most important renewable energy source on the planet. Up until the 1960s, hydroelectric power plants accounted for almost 100% of Swedish electrical power production. Today, the hydropower share of the total national annual production of electrical energy is 45–50%, depending on the precipitation conditions of the previous year. The major large-scale hydropower development in Sweden took place during the 1940s, 1950s and 1960s. With new operating conditions on ageing machines and the replacement of old components by new designs, new analyses are necessary in order to predict and/or explain the dynamic behaviour of the machine.

1.2. DYNAMICS OF ROTATING MACHINES

The rotor constitutes the main component in rotating machines. The rotor is often an assembly of many other components, such as shaft, turbine, generator, flying wheel, geared wheels and many others. The non-rotating components, including bearings, stator, casing and vanes, will certainly influence the dynamic behaviour of the rotating components by means of the physical properties of the interfaces. Fluid structure interaction is one of the main encountered physical phenomena at the interface between stationary and rotating components in rotating machines. Fluid-film lubrication between bearings and rotor, magnetic pull forces between generator and stator, aerodynamic forces between a rotor blade’s tip and stator are a few examples of the encountered interactions. The dynamic analysis of the overall assembly of rotating and non-rotating components together with their interaction properties constitutes the engineering field of rotor dynamics. A more detailed introduction to rotor dynamics is given in a number of books [1, 2, 3].
1.2.1. Linear dynamics of symmetric rotors

The assumption of small motions or weak nonlinear characteristics in a rotor system results in the linearised equations of motion. Consider a Jeffcott rotor, which consists of a rigid disc mounted offset (case when $a \neq b$) or mid span (case when $a = b$) on a massless flexible shaft according to Fig. 1. The linearised equations of motion of the rotor disc, Eq. 1, are second order ordinary differential equations and suitable for eigenanalysis. The equilibrium equations, Eq. (2) and Eq. (3), between the elastic forces of the bending rotor shaft and the bearings' reaction forces should be solved simultaneously with the rotor disc's equations of motion, Eq. (1). This is because the journals (the portion of rotor shaft located inside the bearing) displacements $x_{11}, y_{11}, x_{22}, y_{22}$ are needed in the rotor disc's equations of motion, and, at the same time, the rotor disc's displacements $x_r, y_r$ are also needed in the equilibrium equations. The solution of the eigen value problem can be presented in the form of a Campbell diagram as shown in Fig. 2(a, b). The Campbell diagram is a diagram where the system's natural frequencies are plotted as functions of the rotor speed. A line of synchronous excitation (commonly called a 1X vibration) can be drawn into the Campbell diagram. The intersections between this line and the curves of the system's natural frequencies give the rotor critical speeds. A more general and useful definition of the rotor critical speed is that it is a rotational speed of a machine at which one of the frequency components of the excitation (forcing) coincides with a natural frequency of the system [4]. The other way to detect the critical speed for linear rotor systems is to generate the system's response to a certain known excitation. The rotor speed at which the system's response goes into resonance (large vibration amplitude) is the critical speed. For an offset rotor supported by rigid or isotropic bearings, the forward natural frequencies increase with the rotor speed, while the backward frequencies decrease with rotor speed, as can be seen in Fig. 2(a). The increase in forward natural frequencies is due to the gyroscopic coupling effect which stiffens the rotor system. When the rotor disc is located at mid-span, the rotor disc angular and lateral motions are decoupled, ($k_{14} = k_{41} = -k_{21} = -k_{22} = 0$) in Eq. (4). This will decrease the system's natural frequencies. The system's natural fundamental mode becomes independent of the rotor speed, as can be observed in Fig. 2(b), as a straight horizontal line. The decoupling of the rotor motions has resulted in a certain decrease of the system's natural frequencies.
Fig. 1: a) Jeffcott rotor on two identical linear bearings, b) Graphical illustration of the rotor rigid body motion

\[
\begin{align*}
\begin{bmatrix}
    m & 0 & 0 & 0 \\
    0 & m & 0 & 0 \\
    0 & 0 & I_d & 0 \\
    0 & 0 & 0 & I_d \\
\end{bmatrix}
\begin{bmatrix}
    \dot{x}_r \\
    \dot{y}_r \\
    \dot{\theta}_r \\
    \dot{\theta}_r \\
\end{bmatrix}
+ \begin{bmatrix}
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    0 & 0 & I_p & 0 \\
    0 & 0 & 0 & -I_p \\
\end{bmatrix}
\begin{bmatrix}
    \dot{x}_r \\
    \dot{y}_r \\
    \dot{\theta}_r \\
    \dot{\theta}_r \\
\end{bmatrix}
+ \begin{bmatrix}
    k_r & 0 & 0 & k_{14} \\
    k_r & k_{23} & 0 & 0 \\
    k_{32} & k_{14} & 0 & 0 \\
    k_{41} & 0 & 0 & k_{\phi_r} \\
\end{bmatrix}
\begin{bmatrix}
    x_r - \Delta_x \\
    y_r - \Delta_y \\
    \theta_r + \alpha_r \\
    \theta_r - \alpha_r \\
\end{bmatrix}
= \overline{\Phi}
\end{align*}
\]

(1)

\[
\begin{align*}
\begin{bmatrix}
    c_{bx} & 0 \\
    0 & c_{by} \\
\end{bmatrix}
\begin{bmatrix}
    \dot{x}_{b1} \\
    \dot{y}_{b2} \\
\end{bmatrix}
+ \begin{bmatrix}
    k_{bxx} & 0 \\
    0 & k_{bxy} \\
\end{bmatrix}
\begin{bmatrix}
    x_{b1} \\
    y_{b2} \\
\end{bmatrix}
= \begin{bmatrix}
    k_1 & 0 & 0 & k_{14} \\
    0 & k_1 & k_{123} & 0 \\
\end{bmatrix}
\begin{bmatrix}
    x_r - \Delta_x \\
    y_r - \Delta_y \\
    \theta_r + \alpha_r \\
    \theta_r - \alpha_r \\
\end{bmatrix}
\end{align*}
\]

(2)

\[
\begin{align*}
\begin{bmatrix}
    c_{bx} & 0 \\
    0 & c_{by} \\
\end{bmatrix}
\begin{bmatrix}
    \dot{x}_{b2} \\
    \dot{y}_{b2} \\
\end{bmatrix}
+ \begin{bmatrix}
    k_{bxx} & 0 \\
    0 & k_{bxy} \\
\end{bmatrix}
\begin{bmatrix}
    x_{b2} \\
    y_{b2} \\
\end{bmatrix}
= \begin{bmatrix}
    k_2 & 0 & 0 & k_{24} \\
    0 & k_2 & k_{223} & 0 \\
\end{bmatrix}
\begin{bmatrix}
    x_r - \Delta_x \\
    y_r - \Delta_y \\
    \theta_r + \alpha_r \\
    \theta_r - \alpha_r \\
\end{bmatrix}
\end{align*}
\]

(3)

\[
\Delta_z = x_{b1} + \frac{a}{a+b}(x_{b2} - x_{b1}), \quad \Delta_y = y_{b1} + \frac{a}{a+b}(y_{b2} - y_{b1})
\]
\[
\alpha_x = \frac{(y_{2x} - y_{1x})}{a + b}, \quad \alpha_y = \frac{(x_{2x} - x_{1x})}{a + b}
\]

\[
k_1 = k_y = 3EI \frac{a^3 + b^3}{a^3 b^3}, \quad k_{b1} = k_{b2} = 3EI \frac{a + b}{ab}, \quad k_{14} = k_{41} = -k_{23} = -k_{32} = 3EI \frac{a^2 - b^2}{ab^3}
\]

\[
k_1 = k_{a1} = 3L_1 \frac{b}{a + b}, \quad k_{14} = k_{a1} = 3L_1 \frac{a}{a + b}, \quad k_{12} = k_{21} = 3L_1 \frac{a}{a + b}
\]

If the rotor disc is located at mid-span, \(a = b\), the equations (1) to (3) reduced to the following equations (4) and (5).

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & I_x & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & I_{xy} & 0 & 0 & 0 & 0 & 0 & 0 \\
[u] & \beta & \beta & \beta & \beta & \beta & \beta & \beta & \beta & \beta \\
\end{bmatrix}
\begin{bmatrix}
\dot{x}_x \\
\dot{y}_x \\
\dot{\beta}_x \\
\dot{\beta}_y \\
\end{bmatrix}
+ \omega
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\dot{x}_x \\
\dot{y}_x \\
\dot{\beta}_x \\
\dot{\beta}_y \\
\end{bmatrix}
= \begin{bmatrix}
k_x & 0 & 0 & 0 & x_{x} - x_{b} \\
k_y & 0 & 0 & 0 & y_{x} - y_{b} \\
0 & 0 & k_{b1} & 0 & \theta_x \\
0 & 0 & 0 & k_{b2} & \theta_y \\
\end{bmatrix}
\]

\[
(4)
\]

\[
2 \begin{bmatrix}
c_{bxx} & 0 \\
0 & c_{byy} \\
\end{bmatrix}
\begin{bmatrix}
\dot{x}_x \\
\dot{y}_b \\
\end{bmatrix}
+ 2 \begin{bmatrix}
k_{bxx} & 0 \\
0 & k_{byy} \\
\end{bmatrix}
\begin{bmatrix}
x_b \\
y_b \\
\end{bmatrix}
= \begin{bmatrix}
k_x & 0 & x_{x} - x_{b} \\
k_y & 0 & y_{x} - y_{b} \\
\end{bmatrix}
\]

\[
(5)
\]

where \(E, m, I_x, I_y, \theta_x, \theta_y, x_x, y_x, y_{b}, k_x, k_y, k_{b1}, k_{b2}, \omega\) are rotor Young’s modulus, rotor disc weight, shaft area moment of inertia, rotor disc diametric moment of inertia, rotor disc polar moment of inertia, rotor disc angular displacement about the \(X\) axis, rotor disc angular displacement about the \(Y\) axis, rotor disc lateral displacement in the \(X\) direction, rotor disc lateral displacement in the \(Y\) direction, journal displacement in the \(X\) direction, journal displacement in the \(Y\) direction, shaft translational stiffness in the \(X\) direction, shaft translational stiffness in the \(Y\) direction, rotor disc angular stiffness about the \(X\) axis, rotor disc angular stiffness about the \(Y\) axis, and rotor angular speed, respectively. The vector \(\bar{Q}\) can contain the external applied forces, such as the rotor imbalance. The parameters \(c_{bxx}, c_{byy}\) are the bearing’s damping coefficients, and \(k_{bxx}, k_{byy}\) are the bearing’s stiffness coefficients. Factor 2 in Eq. (5) stands for two identical bearings for a mid-span rotor system. For a symmetric rotor, the rotor shaft translational stiffness is equal \((k_x = k_y)\) and de facto the rotor shaft angular stiffness is also equal \((k_{\theta x} = k_{\theta y})\).
In addition to the evaluation of the rotor natural frequencies, linear analysis can also be used to obtain the rotor response to different kinds of excitation. Knowing the rotor response, stresses and strains can be computed to evaluate the life-length of the system's components. Another practical example for the use of linear analysis is the rotor balancing technique.

### 1.2.2. Nonlinear dynamics of symmetric rotors

When the assumption of small motions is not valid, the nonlinear analysis is the only valid tool to solve such problems. There are different approximation methods to analyse problems with weak nonlinearities, such as the averaging technique and the perturbation methods [5]. The nonlinearities in the rotor system studied in the appended papers are principally caused by plain cylindrical hydrodynamic journal bearings operating at high eccentricities. The nonlinear characteristics of plain cylindrical hydrodynamic journal bearings become strong at eccentricities larger than 0.6. Therefore, the bifurcation technique is suitable for the analysis of the rotor-bearing systems operated at high eccentricity. The bifurcation technique is a graphical method, via numerical simulation, which consists on showing the qualitative nature of the solution for a range of parameter values of the system. The rotor speed was the system's parameter varied when the bifurcation diagrams were computed. A more detailed description of the bifurcation diagrams and other graphical methods can be found in [6].

### 1.2.3. Asymmetric rotor systems

The most illustrated case of asymmetry in rotating structures is stiffness orthotropy [7]. This means that the rotor stiffness is not the same in the two symmetry planes. A rotor shaft with a rectangular cross section can be
given as an example of a rotor with orthotropic stiffness. If \( k_x \neq k_y \) in Eq. (4), the mid-span rotor system becomes asymmetric and de facto will also \( k_y \neq k_y \). In an asymmetric rotor system, all parameters and coefficients appear to change with time when observed in the stationary frame. Therefore, the equations of motion need to be derived in the rotating frame, because in the rotating frame all coefficients and parameters remain constant and the eigenanalysis can be carried out as usual. However, the results and their interpretation are treated in the context of the rotating frame [8]. The time dependent parameters or/and coefficients may be a source of parametric instability in rotating systems [7, 17, 18]. The rotor mass inertia asymmetry, the intermittent rotor/stator rub, a pulsating torque, and an excessive ball bearing clearance are some other sources of parametric instability in rotating machines [12]. The transformation of the Eq. (4) to the rotating frame gives [16]:

\[
\begin{bmatrix}
\ddot{q}_d + \omega [\ddot{\mathbf{M}}_d] \dot{q}_d + (\ddot{\mathbf{K}} + \omega^2 [\ddot{\mathbf{M}}_{sy}] + [\ddot{\mathbf{G}}_{sd}])(\ddot{q}_d - \ddot{q}_b) = \ddot{\mathbf{Q}}_d
\end{bmatrix}
\]

(6)

The analysis of asymmetric rotors in the rotating frame is only possible when the bearings are isotropic. In cases where asymmetric rotors are supported by anisotropic bearings and if their steady-state response is periodic, the Floquet theory can be used to study the system’s stability [16]. The Floquet theory provides the stability of the system for a given set of parameters. If any of the eigenvalues of the Floquet transformation matrix
(monodromy matrix) have magnitude greater than 1, then at least one of the solutions of the equations of motions grows and the system is unstable.

1.2.4. Vertical hydropower rotors

Most hydropower plants are vertical machines. A turbine (runner), a generator and an exciter are the principal rotating components attached to the rotor shaft as shown in Fig. 3(a, b). The total weight of all rotating components and the rotor shaft is carried by a single machine component called a thrust bearing, and the radial loads are carried by radial bearings. A combi-bearing is a machine component designed to combine a tilting-pad journal and thrust bearings in one machine element. In the hydropower industry, it is common practice to simplify strongly the influence of the combi-bearing in the rotor dynamic models, as shown in Fig. 3(a). Therefore, a more detailed (complete) analysis of the combi-bearing’s dynamic influence on vertical hydropower rotors has been carried out in this research project according to Fig. 3(b). Neglecting damping, Figures 3(c, d) show how the combi-bearing was modelled in this research project.

Fig. 3 (a, b) Schematic view of a vertical hydropower unit together with a discretised rotor: (a) shows how the thrust bearing is omitted in the rotor model, and (b) shows the integration of the combi-bearing in the rotor system. (c, d) show how the combi-bearing was modelled.
1.3. RESEARCH QUESTION

Motions coupling can, in some cases, be a source of self-excited vibrations in rotating machines. Vibrations generate noise and cause fatigue, which initiates cracks in mechanical structures. It is therefore important that rotating machines are designed against self-excited motions. Manufacturing or assembling error in a machine’s rotating components can generate asymmetry in the rotor system. An asymmetry in rotating structure can be a source of parametric instability. The research question can be summarised as follows:

*What are the causes and consequences of coupled vibrations and parametric excitation in horizontal and vertical rotating machines?*

2. COUPLED VIBRATION IN ROTATING MACHINES

Vibration is a well-known undesirable behaviour of dynamical systems characterised by persistent periodic, quasiperiodic, or chaotic motions. Vibrations generate noise and cause fatigue, which initiates cracks in mechanical structures. For systems with several degrees of freedom, motion in one direction can induce motion in the other and/or vice versa. That means that there is a certain coupling between the two motions. Coupling is in some cases a source of instability that induces self-excited vibrations in rotating machinery. The bearings’ characteristics, the rotor gyroscopic effect, the rotor imbalance and the magnetic-pull forces are some examples of the reasons for motions coupling in rotating machines. Ales Tondl [9] derived the equations of motions for a vertical rigid rotor and found that the rotor imbalance couples the rotor lateral and torsional motions. L. Lundström, R. Gustavsson, J.-O. Aidanpää, N. Dahlbäck and M. Leijon [10] showed how the magnetic-pull force has two components, a radial and a tangential. The generator (rotor) radial displacement generates a pulling force in both the lateral X and Y directions. The rotor motions’ coupling due to the bearings’ characteristics, the rotor gyroscopic effect and coupling due to the components’ manufacture or assembling errors have been studied in this research project.

2.1. Coupling due to fluid-film journal bearings

Plain cylindrical hydrodynamic journal bearings provide high damping to the rotor system, but they also cross couple the rotor’s translational motions. This cross coupling is the main source of oil-induced instability; therefore, the rotor speed should not exceed the speed at which oil-induced instability occurs. A careful analysis of the dynamic characteristics of rotors supported by plain cylindrical hydrodynamic journal bearings is primordial in the early design process. For a mid-span Jeffcott rotor on plain cylindrical hydrodynamic journal bearings, the Eq. (5) become:
The cross-coupling terms $c_{b_{xy}}, c_{b_{yx}}, k_{b_{xy}}, k_{b_{yx}}$ are not zero. It is important to mention that the bearing coefficients are functions of the rotor speed when fluid-film lubricated bearings are used.

2.2. Gyroscopic coupling effect on rotor forward and backward critical speeds

Childs [7], Chen and Gunter [11] have demonstrated the gyroscopic stiffening effect on forward critical speeds for rotors supported by rigid/or isotropic bearings. E. Swanson, C. D. Powell and S. Weissman [13] mention that it is possible for fluid-film bearings to change the neat generalisation that forward critical speeds increase with the rotor speed, while backward critical speeds decrease. Depending on rotor and bearing characteristics, and how the latter change with speed, it is possible that a forward mode might actually decrease with speed, and/or a backward mode increase with speed. Therefore, a thorough analysis of the rotor gyroscopic coupling effects on the instability threshold and critical speeds of rotors supported by fluid-film bearings is important in the early design process of the machine. The gyroscopic matrix $[G]$ in Eq. (1) clearly shows the coupling between the rotor disc’s angular motions $\theta_x$ and $\theta_y$.

2.3. Coupling due to tilting-pad combi-bearing in vertical rotors

A tilting-pad combi-bearing is a machine component used in vertical hydropower rotors, as shown in Fig. 3(b). This component is designed to combine tilting-pad journal and thrust bearings in one machine element. The thrust bearing is principally used in vertical machines and shafts designed to transmit thrust. Shiau, Hsu and Chang [14] show that there is a coupling between the rotor translational and rotational displacements at the thrust bearing’s location. White, Torbergsen and Lumpkin [15] modelled and simulated a vertical pump with tilting-pad journal bearings. They assumed that the encountered disagreement between measurement and simulation result of the response at the lower no-drive end (NDE) tilting-pad journal bearing was due to the neglected stiffness and damping coefficients of the thrust bearing. In this thesis, the analysed combi-bearing is a fluid-film lubricated tilting-pad thrust and journal bearings combined together. Only linear fluid-film and support structure stiffness were taken into account in the model while fluid-film damping and pads inertia effects were neglected.
3. PARAMETRIC INSTABILITY IN VERTICAL ROTOR SYSTEMS

When possible manufacturing or assembling errors in the combi-bearing’s rotating collar are introduced as angular misalignments $\alpha$ and $\beta$ according to Fig. 4, the rotor stiffness in its translation directions at the combi-bearing’s location differ. As a consequence, the equations of motion of the rotor system in the stationary frame will have coefficients that vary sinusoidally with time. This will result in a system that may be parametrically excited. The angular misalignment in the combi-bearing’s rotating collar couples the rotor lateral, vertical and angular motions at the contact point $P$. Static forces and moments are also generated, and parametric instability can be observed within certain ranges of the rotor speed. Paper F gives a detailed analysis of the rotor-bearings system shown in Fig. 4.

![Fig. 4](image)

**Fig. 4** (a) Symmetry plane of a vertical rotor with a misaligned collar, (b) Collar translation in the $X$, $Z$ directions and rotation about the $Y$ axis, (b) Insertion of angular misalignments $\alpha$ and $\beta$ in the $X$-$Z$ plane, (c) Free body diagram.
4. EXPERIMENTAL VERIFICATION OF THE ANALYTIC MODELS

The main purpose of the performed experiments was to verify the analytic models of the plain cylindrical hydrodynamic journal bearing and the combi-bearing by comparing the numerical simulation results with the experiments.

4.1 Bently Nevada Rotor Kit RK4 with oil whirl/whip options

The Bently Nevada Rotor Kit RK4 with oil whirl/whip options, Fig. 3, was used to verify the analytic models of the plain cylindrical hydrodynamic journal bearing.

(a)                                                       (b)

Fig. 3 Bently Nevada Rotor Kit RK4 with Oil whirl/whip option: (a) Rotor-bearing system, (b) Oil supplier and instruments monitor display
4.1 Vertical rotor rig

A small scale vertical rotor rig equipped with a combi-bearing, Fig. 4, was designed and constructed to verify the analytic model of the combi-bearing.

Fig. 4: (a) Schematic view of the vertical rig’s symmetry plane, (b) Digital picture of the actual vertical rig.

Fig. 5: (a) Digital picture of the combi-bearing, (b) Digital picture of the upper view of the radial bearing.

The rollers and ball bearings, the springs and the flexible coupling were manufactured by SKF, LESJÖFORS SPRINGS & PRESSINGS AB, and BOMEX, respectively. Some of the equipment used in the experiments is
from the Bently Nevada Rotor Kit Model RK 4: the electric motor, the motor speed controller device, the proximitor assembly, the proximity probes for motor speed control and rotor motion measurement. The rotor motion is measured at 25 mm from the bottom ball bearing due to the large vibration’s amplitudes when the rotor runs near its critical speed. The proximity probe for rotor motion measurement is mounted on a support structure which is attached to the fundament, while the probe for motor speed control is attached to the bottom ball bearing’s support structure, as can be seen in Fig. 4. (b). All the other components, which have not been mentioned above, were designed by the author, and the manufacture was handled by the technicians at the workshop of Luleå University of Technology. In Fig. 5, springs are mounted between the pin-roller assembly and a screw. A schematic illustration of the pin-roller assembly is shown in Fig. 6. The rollers are used to reduce friction between the rotating collar and the bearings.

Fig. 6 Screw-spring-pin-roller assembly.
5. RESULTS AND SUMMARY OF THE APPENDED PAPERS

5.1. Dynamic analysis of journal bearing at high eccentricity

The results in Paper A showed that linear bearing models derived from the nonlinear impedance descriptions of the Moes cavitated ($\pi - \text{film}$) finite-length hydrodynamic bearing can predict the steady-state imbalance response of a rigid symmetric rotor supported by two identical journal bearings at high eccentricities. This is, however, only the case when operating conditions are below the threshold speed of instability and when the system has period one solutions. The error will increase in the vicinity of the resonance speed. The result shown in Fig. 7 is the bifurcation diagram of the $X$ displacements for the rotor system with a nonlinear bearing model. In this bifurcation diagram (not maximum or minimum displacements), the rotational speed is increased in steps of 100 rpm (revolutions per minute) and 100 Poincaré sections are plotted after the transients have decayed, 200 periods from the start. Figures 8 and 9 display the steady-state unbalance response of the rotor system with a nonlinear bearing model (blue), and the steady-state unbalance response of the rotor system with a linear bearing model (red). The solutions are presented as rotor trajectories plots for the selected rotational speeds of 6000 and 6300 rpm.

![Bifurcation Diagram](image)

**Fig. 7** $X$ displacements bifurcation diagram for a case example studied.
Fig. 8 Steady-state unbalance response of the rotor system with a nonlinear bearing model (blue) and the steady-state unbalance response of the rotor system with a linear bearing model (red) at rotor speed of 6000 rpm. The rotor non-dimensional coordinates at static equilibrium position are: $X_0 = 0.4182, Y_0 = -0.6755, \varepsilon_0 = 0.7945$ (static eccentricity ratio)

Fig. 9 Steady-state unbalance response of the rotor system with a nonlinear bearing model (blue) and the steady-state unbalance response of the rotor system with a linear bearing model (red) at rotor speed of 6300 rpm. The rotor non-dimensional coordinates at static equilibrium position are: $X_0 = 0.4205, Y_0 = -0.6682, \varepsilon_0 = 0.7895$ (static eccentricity ratio)
5.2. Gyroscopic coupling effect on oil whirl instability and journal trajectories

The results in Paper B show that the instability threshold of a rigid offset rotor-bearing system will depend on the low stability characteristics of the less loaded bearing. However, taking into account the shaft flexibility and gyroscopic coupling effect will increase the instability threshold. The gyroscopic coupling effect not only increases the instability threshold, but the journal trajectories’ magnitude also significantly increases. This is normally not a preferable condition since high vibrations will induce heat and stress in babbited bearings. The result displayed in Fig. 10 shows the “Journal 1” steady state dimensionless orbit at the threshold speed of instability, which is about 7850 rpm for the rigid offset rotor with nonlinear bearings. At rotor speeds lower than 7850 rpm, the journal converges to a static equilibrium position. Journal 1 is the journal of the less loaded bearing 1. The orbits in Fig. 11 are the steady state dimensionless orbits of journals 1 and 2 and rotor mass centre at threshold speed of instability for the flexible offset rotor with nonlinear bearings. Instability occurs at about 11934 rpm.

![Graph showing journal 1 dimensionless orbit](image)

**Fig. 10** “Journal 1” dimensionless orbit for the rigid offset rotor with nonlinear bearings, N=7850rpm. At rotor speeds lower than 7850 rpm, the journal converges to a static equilibrium position. Journal 1 is the journal of the less loaded bearing 1.
Fig. 11 Dimensionless orbits for the flexible offset rotor with nonlinear bearings, N=11934.01rpm. At rotor speeds lower than 11934 rpm, the journal converges to a static equilibrium position.

5.3. Coupling due to tilting-pad combi-bearing

In Paper C, the derived linear model shows that the combi-bearing couples the rotor’s lateral and angular motions at the contact point between the combi-bearing and the rotor. However, if the thrust bearing’s pads arrangement is not symmetrical or if all the pads are not angularly equidistant, the rotor vertical (axial) and angular motions are also coupled. This last case of coupling will also occur if the axial equivalent stiffness is not evenly distributed over the thrust bearing. A defected pad or unequal hydrodynamic pressure distribution on the pads’ surfaces may be the cause. The Porjus U9’s simulation results show that the combi-bearing influences the dynamic behaviour of the machine. The rotor motions’ coupling due to combi-bearing changes the system’s natural frequencies and vibration modes. Fig. 12(a, b) contain the Campbell diagrams showing the undamped natural frequencies of the rotor system with combi-bearing and the rotor system without thrust bearing. The plotted undamped natural frequencies are expressed in cycles per minute [cpm], while the rotor driving frequencies are given in revolutions per minute [rpm]. Fig. 13 displays the rotor first lateral vibration modes of the system with combi-bearing and system without thrust bearing. For the case of the rotor system without thrust bearing, parameters $L$ and $r$ were set to $L=r=0$; meaning that the journal-bearing is exactly located at the contact point $(P)$ and the thrust bearing is removed.
Fig. 12 Campbell diagrams of the rotor system with combi-bearing (dotted line), and the rotor system without thrust bearing (solid line): (a) first and second natural frequencies, (b) third natural frequencies.

Fig. 13 Rotor first lateral vibration modes: (a) rotor system with combi-bearing, (b) rotor system without thrust bearing.

5.4. Experimental verification of oil-induced instability

The experimental results in Paper D confirmed the accuracy of the analytical bearing impedance descriptions method used to model both linear and nonlinear hydrodynamic journal bearings. Compared with the experimental results, the threshold speed of instability of the rotor-bearing system studied was predicted by both linear and nonlinear analyses with an error of 5.69% and 5.36%, respectively. The analytically predicted journal trajectory was also in agreement with the experiments. Fig. 14 shows the experimental and theoretical journal unbalance responses when oil whirl has started. The Fast Fourier Transform (FFT) of both theoretical and experimental unbalance responses has revealed that the frequencies of the responses are about half the rotor speed. When oil whirl becomes unstable, the journal motion is basically bounded by the bearing clearance geometry, as can be experimentally observed in Fig. 15. Theoretically
speaking, the journal orbit increases to infinity when oil whirl becomes unstable. The experimental bifurcation diagram in Fig. 16 shows clearly the period doubling of journal motions when oil whirl has occurred.

Fig. 14 Theoretical (N=6340rpm) and experimental (N=6000rpm) journal trajectories (orbits) when the oil whirl phenomenon has started. The theoretical results are obtained from the unbalance response of the rotor system with nonlinear journal bearing.

Fig. 15 Experimental journal trajectory (orbit) when the oil whirl becomes unstable (N=6200rpm). The journal motion is basically bounded by the bearing clearance geometry.
5.5. Experimental verification of the combi-bearing model

The simulated vertical rotor-bearings system is a small-scale vertical machine constructed to verify the analytically derived combi-bearing’s model. The results in Paper E show that there was good agreement between the simulation and experimental results. Simulation and experimental results showed that the journal (radial) bearing’s position relative to the contact point between the combi-bearing’s collar and the rotor influences the rotor system’s fundamental natural frequencies. The combi-bearing model needs to be included into the rotor dynamic models. Neglecting the effect of this component (combi-bearing) may cause significant errors in the predicted results. Fig. 17 contains the theoretical Campbell diagram showing the undamped fundamental natural frequencies of the rotor system for different configurations of the combi-bearing. The parameter $L$ was set to $L=25\text{mm}$ (solid blue lines) and $L=55\text{mm}$ (dotted blue lines). The result for $L=r=0$ (green line) corresponds to a model where the axial springs are removed and the radial bearing is located at the contact point $(P)$ as they frequently do in the hydropower industry. The plotted undamped fundamental natural frequencies are expressed in cycles per minute [cpm], while the rotor driving frequencies are given in revolutions per minute [rpm]. Figs. 18(a, b) display the theoretical and experimental unbalance responses of the rotor system. The rotor response was computed and measured at a distance of $25\text{mm}$ (node 6) from the bottom ball bearing.

Fig. 16 Experimental bifurcation diagram of $X$ displacements collected from 5000 rpm.
5.6. Consequences of geometric misalignment in combi-bearing

The results in Paper F show that an angular misalignment in the combi-bearing’s rotating collar generates an asymmetry in the rotor system. The rotor system’s stiffness in its two translational \(X\) and \(Y\) directions differ at the combi-bearing’s location. Constant parameters and/or coefficients in rotating asymmetric structures appear to change with time when observed in the stationary frame. These time dependent parameters (coefficients) may cause a parametric instability in rotating systems. If the collar angular misalignment is located in the \(X-Z\) plane, all rotor motions in this plane at the contact point between the combi-bearing and the rotor will be coupled. A parametric instability is observed within certain ranges of the rotor
speeds, depending on the magnitude of the angular misalignment. In Figs. 19(a, b), the real parts of the eigenvalues of the rotor system in rotating frame are plotted as functions of the rotor speed for \( \alpha = \beta \). Fig. 20(a) displays the imaginary parts of the rotor system’s eigenvalues in the rotating frame as functions of the rotor speed for \( \alpha = \beta \). A value of 5 degrees angular misalignment for both \( \alpha \) and \( \beta \) was used. The rotor critical speeds and the regions of parametric instability are also shown in Fig. 20(a). Fig. 20(b) shows the rotor steady-state response as a function of rotor speed for \( \alpha = \beta = 5^\circ \). Resonances can be observed at rotor critical speeds where the vibrations’ amplitudes theoretically grow to infinity. Figs. 21(a, b) show the rotor response in the vertical \( Z \) direction as a function of time for \( \alpha = \beta, N = 1355 \text{rpm} \) and \( \alpha = \beta, N = 2600 \text{rpm} \). The rotor response was computed at node 2, which is the contact point between the rotor and the combi-bearing.

![Fig. 19](image1.png)

**Fig. 19** Real part of the rotor system’s eigenvalues in the rotating frame. The blue colour is for \( \alpha = \beta = 1^\circ \), and the red colour is for \( \alpha = \beta = 5^\circ \).

![Fig. 20](image2.png)

**Fig. 20** (a) Imaginary part of the eigenvalues in the rotating frame, (b) Rotor steady-state response at combi-bearing’s location for \( \alpha = \beta \).
Fig. 21 Rotor time-response in the vertical Z direction at node 2. (a): $\alpha = \beta$ and $N = 1355$ rpm, (b): $\alpha = -\beta$ and $N = 2600$ rpm.
6. CONCLUSIONS

The results obtained in these studies are useful in the design process and maintenance of rotating machines in general, and hydropower plants in particular. The derived models could serve as a simulation tool during design process/modifications or in analysis of failures. Due to the large scale of real hydropower units, simulations are useful because they are more time and cost efficient than running full scale experiments. They also facilitate the analysis of a large number of operating conditions and design modifications. In cases where plain cylindrical hydrodynamic journal bearings operate at high eccentricities, nonlinear dynamic analysis is required to locate the operating conditions where linear analysis is valid. Gyroscopic coupling has a stiffening effect on rotors supported by plain cylindrical hydrodynamic bearings, but it also increases the vibrations' amplitude, which is an undesirable consequence. Neglecting the effect of a thrust bearing in rotor dynamic modelling of vertical machines or shafts designed to transmit thrust can cause errors in the predicted results. The analysis of the combined thrust-journal bearing design, commonly called combi-bearing, shows that there is coupling in rotor motions at the contact point between the combi-bearing and the rotor. The combi-bearing influences the rotor natural frequencies and the vibration modes. Errors in manufacturing or assembling rotating components can cause parametric instability in rotating systems. It is therefore important to predict the system’s operating conditions susceptible to parametric instability. This can be achieved by careful modelling of the assumed possible errors that can occur during the manufacture or assembling processes; for example, angular misalignments or errors in parallelism.

7. DISCUSSIONS

A profound analysis of the combi-bearing’s influence on the dynamic behaviour of vertical hydropower rotors has been carried out in this research project. The results presented in this research work show that it is an advantage for the hydropower industry to strive after a development of better rotor dynamic models. Good models of the dynamic characteristics of the machine’s components should be included in the rotor dynamic model. The calculations of the critical speeds or the unbalance response are not the only important dynamic characteristics dictating the design or troubleshooting of rotating machines. There are many other dynamic characteristics the designer or the investigator should consider, such as the risk of parametric instability due to manufacture or assembling errors in rotating components.
8. FURTHER WORK

The author realises that there is a need for further studies in analysing the consequences which would result in simultaneous misalignments in the two symmetry planes of the combi-bearing’s rotating collar. Experimental verification of the modelled defected combi-bearing is important in order to verify the accuracy of the analytic models. There is also a need to evaluate the influence of the combi-bearing on real hydropower rotors.

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REFERENCES

Paper A
Use of nonlinear journal-bearing impedance descriptions to evaluate linear analysis of the steady-state imbalance response for a rigid symmetric rotor supported by two identical finite-length hydrodynamic journal bearings at high eccentricities

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Abstract This paper concerns the investigation of validity limits of linear models in predicting rotor trajectory inside the bearing clearance for a rigid symmetric rotor supported by two identical journal bearings operating at high eccentricities.

The inherent nonlinearity of hydrodynamic journal bearings becomes strong for eccentricities greater than 60% of the bearing clearance where most existing linear models are not able to accurately predict the rotor trajectory.

The usefulness of nonlinear journal-bearing impedance description method in this investigation is due to the analytical formulations of the linearised bearing coefficients, and the analytical nonlinear bearing models. These analytically derived bearing coefficients do not require any numerical differentiation (or integration) and are therefore more accurate for large eccentricities. The analytically derived nonlinear bearing models markedly decrease the simulation time while valid for all \( L/D \) (length to diameter ratios) and all eccentricities.

The results contained in this paper show that linear models derived from the nonlinear impedance descriptions of the Moes-cavitated (\( \pi \)-film) finite-length bearing can predict the steady-state imbalance response of a symmetric rigid rotor supported by two identical journal bearings at high eccentricities. This is, however, only the case when operating conditions are below the threshold speed of instability and when the system has period-one solutions. The error will become larger closer to the resonance speed.

Keywords Nonlinear model · Linear model · Journal bearing · Impedance descriptions · Dynamics · Imbalance response · Rotor

Abbreviations

- \( C = C_r \) Bearing radial clearance [m]
- \( C_{ij}, i, j = X, Y \) Dimensional bearing damping coefficients [Ns/m]
- \( c_{ij}, i, j = X, Y \) Non-dimensional bearing damping coefficients [–]
- \( D \) Bearing inner diameter [m]
- \( E \)Rotor mass eccentricity [m]
- \( e \) Journal (rotor) dimensional eccentricity [m]
- \( F_0 \) Half-static load (for a symmetrically loaded rotor) [N]
- \( F_x, F_y \) Bearing reaction force \( \hat{F} \) [N] components in the \( x \) and \( y \) directions
- \( F_{X}, F_{Y} \) Bearing reaction force \( \hat{F} \) [N] components in the \( X \) and \( Y \) directions
- \( g \) Gravity acceleration [m/s²]
- \( K_{ij}, i, j = X, Y \) Dimensional bearing stiffness coefficients [N/m]
1 Introduction

Undercritical industrial heavy rotating machinery sometimes operates at high eccentricities, for example hydropower or steam turbine with horizontal rotors. Therefore, high eccentricity dynamic analysis of rotor-bearing systems is of industrial interest.

Linear models are widely used in most industrial design processes due to their simplicity and the ease with which it is possible to interpret the results. In the area of rotor-bearing system dynamics, most developed linear bearing models derived from the numerical differentiation approach are proved valid enough at low eccentricities.

In predicting the critical mass for a rigid symmetric rotor, Ram Turaga, A.S. Sekhar and B.C. Majumdar [1] used the finite element method to calculate the bearing coefficients and found that linear models were valid for eccentricities less than 0.6.

A.K. Tieu and Z.L. Qiu [2] used Lund’s infinitesimal perturbation method [3] and found that the rotor trajectory by the linear analysis assumes a significant error already at eccentricity 0.6 when the whirl amplitude is greater than 20 per cent of the bearing clearance.

D. Childs [4] and Lund [5] have demonstrated that the rotor is completely stable for eccentricities greater than 75 per cent of the bearing clearance and for all $L/D$ ratios.

Bearings with low $L/D$ ratios in particular are more stable than others [6], therefore they will be the object of the present work, using $L/D = 0.25$.

R.D. Brown, G. Drummond and P.S. Addison [7] showed that a rigid rotor supported on a hydrodynamic bearing film at high eccentricity satisfied the conditions for chaos.

Linear models are not valid for motions other than period-one, therefore a bifurcation diagram is needed to locate such operating conditions and validate linear models outside these regions.

The purpose of this paper is to use the journal-bearing impedance description method for both linear and nonlinear models of bearing reaction forces.
to investigate how well linear models can approximate nonlinear models at high eccentricities. The high nonlinear characteristics of bearing reaction forces at large eccentricities make both numerical differentiation and integration approaches less accurate to linearise the bearing reaction forces [6].

2 Method

D. Childs, H. Moes, H. van Leeuwen [6], D. Childs [4] and H. Moes and R. Bosma [8] derived the journal impedance descriptions for rotordynamic applications, which consist of defining the bearing reaction force components as analytical nonlinear functions of the journal motion (displacement and velocity). They also derived the analytical bearing stiffness and damping coefficients by linearisation of the bearing reaction forces using Taylor series expansion, where second and higher order differential terms have been dropped. These analytical expressions for bearing coefficients are particularly suitable in rotordynamics because they yield more accurate results and less computational time than the existing numerical differentiation and pressure-integration approaches. They also provide very accurate models for all eccentricities and \(L/D\) ratios.

The Moes-cavitated (\(\pi\)-film) finite-length bearing model [8] will be used in present studies, with the following impedance (dimensionless bearing-load-force vector due to pure squeezing) components:

\[
W_x \approx 6(1-\epsilon^2)^{-1} \left( \left(1 - y^2 \right) T + x \right)^{-1} + 3^{-1}(1 \\
+ 2x^2 - y^2)T + x \right)^{-1}(1 - \epsilon^2) \left( \frac{L}{D} \right)^{-2} \right)^{-1},
\]

\[
W_y \approx 6y(1-\epsilon^2)^{-1} \left( \left(1 - y^2 \right) T + x \right)^{-1} + 3^{-1} \left(1 - x^2 \right) \left( \frac{L}{D} \right)^{-2} \right)^{-1},
\]

\[
T = 2(1-\epsilon^2)^{\frac{1}{2}} \times \tan^{-1} \left( \left(1 - y^2 \right)^{\frac{1}{2}} \left(1 - y^2 \right)^{\frac{1}{2}} - x \right)^{-1},
\]

\[
E = \frac{1 - \epsilon^2}{1 - y^2}^{-1},
\]

\[
\epsilon = \frac{e}{C} = \sqrt{x^2 + y^2} < 1.
\]

The dimensional bearing reaction forces in \(x\) and \(y\) directions are:

\[
F_x = -V_x 2 \mu L \left( \frac{R}{C} \right)^3 W_x,
\]

\[
F_y = -V_y 2 \mu L \left( \frac{R}{C} \right)^3 W_y.
\]

Transformation to the rotor fixed reference coordinates \(X, Y, Z\) is carried out as follows:

\[
x = \frac{X \cos \zeta + Y \sin \zeta}{C},
\]

\[
y = \frac{Y \cos \zeta - X \sin \zeta}{C},
\]

\[
\zeta = \tan^{-1} \left( \frac{\hat{Y} - \hat{\omega}X}{X + \hat{\omega}Y} \right).
\]

\[
V_x = \|\hat{V}_x\| = |\hat{V}_x - \hat{\omega} \hat{k} \times \hat{C} | = \left\{ (\hat{X} + \hat{\omega}Y)^2 + (\hat{X} - \hat{\omega}X)^2 \right\}^\frac{1}{2},
\]

\[
\hat{V}_y = \left[ \hat{X} \quad \hat{Y} \quad 0 \right]^T, \quad \hat{\omega} = \left[ 0 \quad 0 \quad \hat{\omega} \right]^T,
\]

\[
\hat{C} \hat{e} = \left[ X \quad Y \quad 0 \right]^T.
\]

Vectors \(\hat{V}_x, \hat{V}_y, \hat{C} \hat{e}, \hat{W}, \hat{F}\) and angles \(\gamma, \zeta\) are illustrated in Fig. 15 in Appendix A.

The dimensional bearing reaction forces in \(X\) and \(Y\) directions are:

\[
F_X = F_x \cos(\zeta) - F_y \sin(\zeta),
\]

\[
F_Y = F_x \sin(\zeta) + F_y \cos(\zeta).
\]

The above bearing reaction forces are nonlinear functions of rotor displacement and velocity in the bearing clearance.

3 Modelling

A rigid symmetric rotor is supported by two identical plain cylindrical journal bearings with coordinates and parameters as shown in Fig. 1.
3.1 Nonlinear bearing model

3.1.1 Equations of motion

For an unbalanced rigid symmetric rotor supported by two identical journal bearings, the equations of motion of the rotor mass centre are:

\[
\ddot{X} = \frac{2}{M} F_X + E \omega^2 \cos(\omega t),
\]

\[
\ddot{Y} = \frac{2}{M} F_Y + E \omega^2 \sin(\omega t) - g,
\]

where \(M, F_X, F_Y, E, \omega, t, g\) are rotor mass (kg), bearing reaction force (N) in the \(X\) direction, bearing reaction force (N) in the \(Y\) direction, rotor mass eccentricity (m), rotor rotational speed (rad/s), time (s) and gravity acceleration (m/s²), respectively.

3.2 Linear bearing model

The bearing reaction forces \(F_X, F_Y\) in (2) are linearised about the equilibrium position and are expressed as linear functions of rotor displacement and velocity.

3.2.1 Equations of motion about the equilibrium position

Introducing the dimensionless variables:

\[
\tilde{X} = \frac{X}{C}, \quad \tilde{X}' = \frac{\dot{X}}{\omega C}, \quad \tilde{X}'' = \frac{\ddot{X}}{\omega^2 C},
\]

\[
\tilde{Y} = \frac{Y}{C}, \quad \tilde{Y}' = \frac{\dot{Y}}{\omega C}, \quad \tilde{Y}'' = \frac{\ddot{Y}}{\omega^2 C},
\]

\[
\tau = \omega t,
\]

(3) can be written in non-dimensional form as follows:

\[
p^2 \begin{bmatrix}
\tilde{X}'' \\
\tilde{Y}''
\end{bmatrix} = - \begin{bmatrix}
c_{XX} & c_{XY} \\
c_{YX} & c_{YY}
\end{bmatrix} \begin{bmatrix}
\tilde{X}' \\
\tilde{Y}'
\end{bmatrix}
+ \begin{bmatrix}
k_{XX} & k_{XY} \\
k_{YX} & k_{YY}
\end{bmatrix} \begin{bmatrix}
\tilde{X} \\
\tilde{Y}
\end{bmatrix}
+ \frac{E}{C} \begin{bmatrix}
\cos \tau \\
\sin \tau
\end{bmatrix},
\]

where \(p^2 = \frac{C \omega^2}{g}\), and the 8 non-dimensional bearing coefficients are obtained according to [6] as follows:

\[
k_{XX} = \frac{\cos \gamma_0}{\varepsilon_0} - \sin \gamma_0 \left( \frac{\partial \gamma}{\partial \varepsilon} \right)_a,
\]

\[
k_{XY} = \frac{\sin \gamma_0}{\varepsilon_0} + \cos \gamma_0 \left( \frac{\partial \gamma}{\partial \varepsilon} \right)_a,
\]

\[
k_{YX} = - \frac{\sin \gamma_0}{\varepsilon_0} - \sin \gamma_0 \left( \frac{\partial W}{\partial \varepsilon} \right)_a,
\]

\[
k_{YY} = \frac{\cos \gamma_0}{\varepsilon_0} + \cos \gamma_0 \left( \frac{\partial W}{\partial \varepsilon} \right)_a,
\]

(5)
Use of nonlinear journal-bearing impedance descriptions to evaluate linear analysis

The partial derivatives required to evaluate the above coefficients (see (5)) are found in Appendix B and the complete expressions for \( \gamma_0, W_0 \) are found in Appendix A.

At static equilibrium, the Sommerfeld number is:

\[
S = \mu \left( \frac{\omega}{2\pi} \right) \left( \frac{R}{ar{C}} \right)^2 \frac{DL}{F_0} = \frac{1}{\pi \epsilon_0 W_0} > 0, \tag{6}
\]

or

\[
f(\epsilon_0) = \mu \left( \frac{\omega}{2\pi} \right) \left( \frac{R}{ar{C}} \right)^2 \frac{DL}{F_0} - \frac{1}{\pi \epsilon_0 W_0} = 0. \tag{7}
\]

Equation (7) is a nonlinear function of \( \epsilon_0 \) to be solved in the interval \([0, 1]\).

4 A case example

Some of the following data used for simulations are taken from [9], where a short-bearing solution was assumed. Results in [9] for eccentricities \((\epsilon < 0.7)\) were compared with results obtained using the nonlinear finite-length bearing impedance description method.

For investigation of high eccentricities \((\epsilon > 0.7)\) the bearing radial clearance \( C \) was doubled, and the short bearing solution was no longer valid:

\[
M = 22.6796 \text{ [Kg]}, \quad R = 2.54 \times 10^{-2} \text{ [m]}, \quad D = 2 \times R, \quad L = 0.5 \times R, \quad \frac{L}{D} = 0.25,
\]

\[
C = 101.6 \times 10^{-6} \text{ [m]}, \quad \mu = 0.0069 \text{ [Pa s]}, \quad E = 8.128 \times 10^{-6} \text{ [m]}.\]

5 Results

The results shown in Figs. 2 and 3 are the bifurcation diagrams for the system with a nonlinear bearing model. In these bifurcation diagrams (not maximum or minimum displacements) the rotational speed is increased in steps of 100 rpm and 100 Poincaré sections are plotted after the transients have decayed, 200 periods from start.

Figures 4 to 10 display the steady-state imbalance response solutions of (2) for the system with a nonlinear bearing model (blue), and the steady-state imbalance response solutions of (4) for the system with a linear bearing model (red). The solutions are presented as plots of rotor trajectories for the selected rotational speeds of interest: 1000, 2000, 3000, 6000, 6300, 7700, 11000 rpm.

Figures 11 and 12 show the differences between the maximum absolute values of \( \bar{X} \) and \( \bar{Y} \) displacements of steady-state solutions of (2) and (4) for a range of rotational speeds from 5000 to 10000 rpm.

Figures 13 and 14 are the bifurcation diagrams and steady-state imbalance response, respectively. These results were obtained by using data from [9] where the bearing radial clearance \( C = 50.8 \times 10^{-6} \text{ [m]} \) is half of that used in the case example studied. The bifurcation diagrams in Fig. 13 differ markedly from those in Figs. 2 and 3. The bearing radial clearance is the only parameter changed to obtain the resulting differences.

Equations (2) and (4) were numerically integrated using the 5th order Runge–Kutta numerical integration method with adaptive integration time steps.

Equations (3) were transformed into non-dimensional form (see (4)) just for numerical convenience, using the derived bearing coefficients in their non-dimensional form. The dimensional solutions \((X, Y)\) displacements of (2) are converted to their non-dimensional quantities by dividing by the bearing clearance \( C \).

The initial conditions for \( X \) and \( Y \) variables were to be chosen in the interval \([0, C]\) for the numerical integration of (2), and for the numerical integration of (4) the initial conditions were \((\bar{X}_0, \bar{Y}_0)\). The remaining initial conditions for velocities are \( \dot{X}_0 = \dot{Y}_0 = \dot{X}'_0 = \dot{Y}'_0 = 0 \).
Fig. 2  $X$ displacements bifurcation diagram for the case example studied

Fig. 3  $Y$ displacements bifurcation diagram for the case example studied
Fig. 4  (a) Steady-state unbalance response solutions of (2) for the system with a nonlinear bearing model (blue) and the steady-state unbalance response solution of (4) for the system with a linear bearing model (red) at rotor speed of 1000 rpm. The rotor non-dimensional coordinates at static equilibrium position are: \( \bar{X}_0 = 0.2928 \), \( \bar{Y}_0 = -0.8669 \), \( \varepsilon_0 = 0.91241 \).

(b) In-zoomed Fig. 4a
Steady-state unbalance response solutions of (2) for the system with a nonlinear bearing model (blue) and the steady-state unbalance response solution of (4) for the system with a linear bearing model (red) at rotor speed of 2000 rpm. The rotor non-dimensional coordinates at static equilibrium position are: $\bar{X}_0 = 0.3366$, $\bar{Y}_0 = -0.8101$, $\epsilon_0 = 0.880644$

Steady-state unbalance response solutions of (2) for the system with a nonlinear bearing model (blue) and the steady-state unbalance response solution of (4) for the system with a linear bearing model (red) at rotor speed of 3000 rpm. The rotor non-dimensional coordinates at static equilibrium position are: $\bar{X}_0 = 0.3616$, $\bar{Y}_0 = -0.7679$, $\epsilon_0 = 0.855044$
Use of nonlinear journal-bearing impedance descriptions to evaluate linear analysis

**Fig. 7** Steady-state unbalance response solutions of (2) for the system with a nonlinear bearing model (blue) and the steady-state unbalance response solution of (4) for the system with a linear bearing model (red) at rotor speed of 6000 rpm. The rotor non-dimensional coordinates at static equilibrium position are: $\bar{X}_0 = 0.4182$, $\bar{Y}_0 = -0.6755$, $\varepsilon_0 = 0.7945$

**Fig. 8** Steady-state unbalance response solutions of (2) for the system with a nonlinear bearing model (blue) and the steady-state unbalance response solution of (4) for the system with a linear bearing model (red) at rotor speed of 6300 rpm. The rotor non-dimensional coordinates at static equilibrium position are: $X_0 = 0.4205$, $Y_0 = -0.6682$, $\varepsilon_0 = 0.7895$
Fig. 9 Steady-state unbalance response solutions of (2) for the system with a nonlinear bearing model (blue) and the steady-state unbalance response solution of (4) for the system with a linear bearing model (red) at rotor speed of 7700 rpm. The rotor non-dimensional coordinates at static equilibrium position are: $\bar{X}_0 = 0.4290$, $\bar{Y}_0 = -0.6366$, $\varepsilon_0 = 0.7677$

Fig. 10 Steady-state unbalance response solutions of (2) for the system with a nonlinear bearing model (blue) and the steady-state unbalance response solution of (4) for the system with a linear bearing model (red) at rotor speed of 11000 rpm. The rotor non-dimensional coordinates at static equilibrium position are: $\bar{X}_0 = 0.4404$, $\bar{Y}_0 = -0.5750$, $\varepsilon_0 = 0.7243$
Use of nonlinear journal-bearing impedance descriptions to evaluate linear analysis

**Fig. 11** Maximum $\bar{X}$ displacements (linear versus nonlinear bearing model)

![Graph showing maximum $\bar{X}$ displacements](image1)

**Fig. 12** Maximum $\bar{Y}$ displacements (linear versus nonlinear bearing model)

![Graph showing maximum $\bar{Y}$ displacements](image2)
Fig. 13  Bifurcation diagrams (data taken from [9]), where the bearing radial clearance $C = 50.8 \times 10^{-6} \text{[m]}$ is half that used in the case example studied.

$\text{Bifurcation Diagrams}$

$\bullet$ Bifurcation of X-displacements
$\bullet$ Bifurcation of Y-displacements

![Bifurcation Diagrams](image)

Fig. 14  Rotor steady-state imbalance response (data taken from [9]), where the bearing radial clearance $C = 50.8 \times 10^{-6} \text{[m]}$ is half that used in the case example studied.

$\text{Rotor Mass Center Orbits}$

![Rotor Mass Center Orbits](image)
6 Discussion

The results of this investigation show that linear bearing models derived from the nonlinear impedance descriptions of the Moes-cavitated (π-film) finite-length bearing can predict the steady-state imbalance response of a rigid symmetric rotor supported by two identical journal bearings at high eccentricities. However, this is only the case when operating conditions are below the threshold speed of instability (around 11400 rpm) and when the system has period-one solutions. The error will become larger closer to the resonance speed (3000 rpm), see Fig. 6.

When the system has period-one solutions, the trajectories predicted by linear models are in acceptable agreement with the nonlinear models. However, deviations are to be expected because the linear models will always predict elliptical trajectories, while the nonlinear models display trajectories of a banana shape and are generally asymmetrical with respect to the static equilibrium position, as illustrated in [9].

As can be seen in Figs. 11 and 12, the deviations in the maximum magnitudes of the $\bar{X}$ and $\bar{Y}$ displacements around the static equilibrium position are less than 30 per cent in regions where the system has period-one solutions (not at the resonance speed). The greatest difference in the $\text{max}(|X|, |Y|)$ is around 0.035 and the corresponding largest $\text{max}(|X|, |Y|)$ value is about 0.135. The smallest difference in the $\text{max}(|X|, |Y|)$ values is around 0.015 and the corresponding largest $\text{max}(|X|, |Y|)$ value is about 0.08. In per cent, this makes the deviations into the interval 19–26%. The deviations are expected, especially for hydrodynamic bearings operating at large eccentricities, which exhibit strong nonlinear characteristics. All the other existing numerical methods for linearising the hydrodynamic bearing forces by computing the so-called bearing stiffness and damping coefficients have been proven to be inaccurate for predicting the journal trajectory at large eccentricities (see references [4, 9]), and they are therefore limited at (moderate) low eccentricities (less than 0.6 according to several authors). Linear models derived from the impedance description method are valid at large eccentricities. This is because they do not require any numerical differentiation or integration. They are obtained in a closed analytical form. This paper deals exclusively with journal bearings at large eccentricity; therefore, the other existing linear numerically derived models (methods) are not valid for comparison. With differences in the range of 19–26%, the journal trajectories from a linear model could be considered as an acceptable prediction in the present studied case where the original physical model is strongly nonlinear.

A certain minimum speed is required to generate the hydrodynamic pressure inside the bearing clearance and therefore the rotor speed cannot be zero. The case studied in this paper has a resonance at around 3000 rpm. Due to the large error expected, the objective is not to investigate the linear model in the region around this speed (2200–4000 rpm). At low speeds, less than 2200 rpm, the journal oscillations are small due to the low imbalance force. At 1000 rpm, the journal oscillations are less than 1% of the bearing clearance and can be neglected. Therefore, due to the low imbalance excitation force, the low speed cases can just be considered as static. The threshold speed of instability of the journal bearing is around 11400 rpm; therefore, the system becomes unstable when running near or above this speed. The hydrodynamic lubrication will fail and metal-to-metal contact will be established if this speed is reached. The highest speed, which could be reached, with imbalance force excitation in the system, is 11300 rpm. The simulations were carried out with rotor speeds of up to 11000 rpm.

Between 6300–7500 rpm, the rotor whirls with half of its spinning frequency. Between 9000–10000 rpm, the motions are quasiperiodic with regions of phase-lock.

Increasing the rotor speed further, close to the threshold speed of instability (11400 rpm), the rotor system reaches instability and the hydrodynamic lubrication fails.

Figures 13 and 14 show the case with data from [9] where the system operates at low eccentricities, having only period-one solutions at the displayed frequencies under the threshold speed of instability. In this case, a linear model will predict all displayed rotor trajectories (Fig. 14) obtained from the nonlinear bearing model.
7 Conclusion

Linear models derived from the nonlinear impedance descriptions of the Moes-cavitated (π-film) finite-length bearing can predict the steady-state imbalance response of a symmetric rigid rotor supported by two identical journal bearings at high eccentricities. However, this is only the case where operating conditions are below the threshold speed of instability and when the system has period-one solutions. The error will also increase closer to the resonance speed.

The deviations in the maximum magnitudes of the $\bar{X}$ and $\bar{Y}$ displacements about the static equilibrium position are in the interval between approximately 19 and 26% when the system has period-one solutions, except at the resonance speed. These deviations depend on the net difference in shape between the two respective rotor trajectories and also on the fact that the nonlinear models generate trajectories which are generally asymmetrical with respect to the static equilibrium position.

This paper concerns the dynamics of a bearing model with $L/D = 0.25$ at high eccentricities. However, the described procedure is general and may be used in investigations of systems with bearings with other $L/D$ ratios.

Appendix A

$$W_0 = \left[0.15(E_0^2 + G_0^2)\right]^{1/2} \left(1 - \xi_0\right)^{3/2}.$$

$$E_0 = 1 + 2.12Q_0.$$

$$G_0 = 3\eta_0 \left(1 + 3.6Q_0\right) / 4(1 - \xi_0).$$

$$Q_0 = (1 - \xi_0) \left(\frac{L}{D}\right)^{-2}.$$

$$\xi_0 = \dot{\xi}_0 \cos \gamma_0,$$

$$\eta_0 = \dot{\eta}_0 \sin \gamma_0,$$

$$\gamma_0 \equiv \left[1 - \xi' \left(1 - \eta'^2\right)^{1/2}\right] \tan^{-1} \left(\frac{4(1 + 2.12B_0)(1 - \eta'^2)^{1/2}}{3(1 + 3.6B_0)\eta'}\right) - \pi \eta' \left(\frac{\eta'}{2}\right) + \sin^{-1} \eta' + \alpha_0 - \sin^{-1} \eta'.

$$\begin{align*}
B_0 &= \left(1 - \xi_0^2\right) \left(\frac{L}{D}\right)^{-2}, \\
\xi' &= \dot{\xi}_0 \cos \alpha_0, \\
\eta' &= \dot{\eta}_0 \sin \alpha_0, \\
\alpha_0 &= \frac{\pi}{2}.
\end{align*}

Appendix B

$$\begin{align*}
\left(\frac{\partial W}{\partial \nu}\right)_a &= \frac{4}{3} \left[2(b - a)b^{-2} - a \frac{\dot{\xi}_0^2}{b}\right] \\
&\times \cos^2 \gamma_0 (1 - \xi_0^2)^{-1/2},
\end{align*}$$

$$\begin{align*}
a &= 1 + 2.12B_0, \\
b &= 1 + 3.6B_0, \\
\left(\frac{\partial W}{\partial \xi}\right)_a &= \left(\frac{\partial W}{\partial \xi}\right)_\eta \left(\frac{\partial \xi}{\partial \eta}\right)_a + \left(\frac{\partial W}{\partial \eta}\right)_\xi \left(\frac{\partial \xi}{\partial \xi}\right)_a, \\
\left(\frac{\partial W}{\partial \nu}\right)_\eta &= -W_0 \left(E_0^2 + G_0^2\right)^{-1/2} \left[\frac{3}{4} G_0 \eta_0 \dot{d}^{-2}\right] \\
&\times \cos^2 \gamma_0 (1 - \xi_0^2)^{-1/2}.
\end{align*}$$
Use of nonlinear journal-bearing impedance descriptions to evaluate linear analysis

\[ d = 1 - \xi_0, \]

\[ \frac{\partial W}{\partial \eta} \bigg|_\xi = -W_0 \left( E_0^2 + G_0^2 \right)^{-1} \frac{G_0^2}{\eta_0}, \]

\[ \frac{\partial \xi}{\partial \alpha} |_\varepsilon = \cos \gamma_0 - \varepsilon_0 \sin \gamma_0 \left( \frac{\partial y}{\partial \varepsilon} \right)_a, \]

\[ \frac{\partial \eta}{\partial \varepsilon} |_a = \sin \gamma_0 + \varepsilon_0 \cos \gamma_0 \left( \frac{\partial y}{\partial \varepsilon} \right)_a, \]

\[ \frac{\partial y}{\partial \alpha} |_\varepsilon = \left[ y_0 - \frac{\pi}{2} + \sin^{-1} \varepsilon_0 \right] \varepsilon_0 \left( 1 - \varepsilon_0^2 \right)^{-\frac{1}{2}}, \]

\[ \frac{\partial W}{\partial \alpha} |_\varepsilon = \left( \frac{\partial W}{\partial y} \right)_{\eta} \left( \frac{\partial \xi}{\partial \alpha} \right)_\eta, \]

\[ \frac{\partial W}{\partial \gamma} |_\varepsilon = \left( \frac{\partial W}{\partial \gamma} \right)_{\eta} \left( \frac{\partial \xi}{\partial \gamma} \right)_\eta + \left( \frac{\partial W}{\partial \eta} \right)_{\xi} \left( \frac{\partial \eta}{\partial \gamma} \right)_\xi, \]

\[ \frac{\partial \xi}{\partial \gamma} |_\varepsilon = -\varepsilon_0 \sin \gamma_0, \]

\[ \frac{\partial \eta}{\partial \gamma} |_\varepsilon = \varepsilon_0 \cos \gamma_0. \]

References

Paper B
Effects of Shaft Flexibility and Gyroscopic Coupling on Instability Threshold Speeds of Rotor-Bearing Systems.

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ABSTRACT
The driving speeds at which self-excited motions occur in rotor-bearing systems are commonly referred to as “instability threshold”. These speeds and the magnitude of rotor (journal) trajectories are two important variables characterising the limits and states of a rotating machinery.

The hydrodynamic lubrication in journal-bearing provides damping and reduces friction on rotor systems; therefore the journal amplitude should not exceed the bearing radial clearance. Linear bearing models are not able to accurately predict the journal trajectories for rotor-bearing system operating in conditions where the system does not have period one solutions, or when the journal motion is larger than 20-30% of the bearing radial clearance.

Therefore the nonlinear bearing impedance descriptions method was used to model the hydrodynamic reaction forces.

Two cases were analysed: 1) a rigid non-symmetric rotor and 2) a flexible non-symmetric rotor. The two models consist of a rotor supported by two identical finite-length hydrodynamic journal bearings of length to diameter ratio L/D=1, with same lubricant properties. The flexible non-symmetric rotor was modelled by the finite element method (FEM).

Simulation results show that the instability threshold of the rigid non-symmetric rotor-bearing system (case1) depends on the low stability characteristics of the less loaded bearing. But when the shaft flexibility and the gyroscopic coupling effect are taken into account, the instability threshold increases for the flexible non-symmetric rotor-bearing system (case2).

The gyroscopic coupling effect does not only increase the instability threshold, but the journal trajectories magnitude has also significantly increased. This is normally not a preferable condition since high vibrations will induce heat and stress in babbitted bearing.

NOMENCLATURE

- $K_1$, Dimensional stiffness coefficients for bearing 1
- $K_2$, Dimensional stiffness coefficients for bearing 2
- $k_1$, Non-dimensional stiffness coefficients for bearings
- $k$, Unit vector in z and Z directions
- $L$, Bearing length [m]
- $m$, Rotor disc mass [kg]
- $O$, Bearing centre
- $O_t$, Journal centre
- $R$, Bearing inner radius
- $\omega$, Revolutions per minute
- $\Omega$, Sommerfeld number
- $V_r$, Journal velocity vector [m/s]
- $\hat{V}_r$, Journal’s pure-squeeze-velocity vector [m/s]
- $V_x$, Journal’s pure-squeeze-velocity magnitude [m/s]
- $\hat{W}$, Bearing impedance $\hat{W}$’s magnitude at the static position
- $W$, Bearing impedance $W$ magnitude in x, y axes
- $x,y,z$, Coordinate system with abcisse $(x,y,z)$ fixed to the vector $\hat{V}_r$
- $X,Y$, Rotor (journal) displacements in X and Y directions
- $\gamma$, Journal attitude angle
- $\gamma_t$, Journal attitude angle at static equilibrium position
- $\hat{e}$, Journal non-dimensional eccentricity
- $\hat{e}$, Journal non-dimensional eccentricity vector
- $\varepsilon$, Journal non-dimensional static eccentricity
- $\zeta$, Angle between x and X
- $\mu$, Oil dynamic viscosity [Pa*s]
- $\omega$, Rotor angular speed [rad/s]
- $\Omega = \frac{\omega}{2}$, Angular speed of x,y,z coordinates relative to X,Y,Z

INTRODUCTION
Stability is a desirable characteristic of all dynamical systems. Two principal definitions of stability are commonly used for mechanical systems: static and dynamic stability. Static stability is the initial tendency of the system to return to its equilibrium state after a disturbance, while dynamic stability concerns the time history of the system motion after being disturbed from its equilibrium point [1]. A system is dynamically stable if the disturbances from the equilibrium point are decaying with time.

For rotor systems supported by hydrodynamic journal-bearings, the oil induced instability is a frequently encountered phenomenon causing the system instability. Oil induced instability occurs as oil whirl or oil whip. Oil whirl happens when the rotor loses its static equilibrium position and starts to whirl, the static equilibrium position becomes an equilibrium orbit in which the rotor performs periodic motion and forms a $\gamma$ closed orbit [2]. But when the rotor whirl frequency equals to the rotor first natural bending frequency, then the oil whip phenomenon has occurred [3].
The results presented in this paper concern the oil whirl instability characteristics, while considering the influence of rotor flexibility and gyroscopic coupling effects. The rotor critical speed, speed at which the system resonance peak frequency is excited by the rotor unbalance, has been shown to be about half the system threshold speed of instability for rotors supported by oil film journal-bearings with coordinates and parameters as shown in the figure below:

Fig. 1 Plain cylindrical journal-bearing coordinates system

\[ W^* = 6 \left( \frac{1 - \epsilon^2}{\epsilon^2} \right)^2 \left[ \left( \frac{\epsilon^2}{\epsilon^2} \right)^2 + \left( \frac{2\epsilon^2 - 1}{\epsilon^2} \right)^2 \right] \left( \frac{L}{D} \right)^2 \]

\[ W^* = 6 \left( \frac{1 - \epsilon^2}{\epsilon^2} \right)^2 \left[ \left( \frac{\epsilon^2}{\epsilon^2} \right)^2 + \left( \frac{2\epsilon^2 - 1}{\epsilon^2} \right)^2 \right] \left( \frac{L}{D} \right)^2 \]

However, the threshold speed of instability for rotors supported by rigid or isotropic bearings. The bearing coefficients are almost constants. At large eccentricities and/or low rotor speeds, the bearing coefficients are strong nonlinear functions of journal displacement and velocity in the bearing clearance. The stiffness and damping characteristics of fluid-film bearings depend strongly on the rotor speed. They also have cross-coupling between the vertical and horizontal axes. W. J. Chen and E. J. Gunter [5] have demonstrated the gyroscopic stiffening effect on forward critical speeds for rotors supported by rigid or isotropic bearings. The rotor critical speed, instability characteristics, while considering the influence of rotor flexibility and gyroscopic coupling effects. The rotor critical speed, speed at which the system resonance peak frequency is excited by the rotor unbalance, has been shown to be about half the system threshold speed of instability for rotors supported by oil film journal-bearings with coordinates and parameters as shown in the figure below:

Fig. 1 Plain cylindrical journal-bearing coordinates system

Nonlinear model. The Moes cavitated \( \epsilon = \text{film} \) Finite-length bearing model [7,8] will be used, with the following impedance \( W \) (dimensionless bearing-load-force vector due to pure squeezing) components:

\[ V, V, \hat{C}, \hat{W}, \hat{F} \] and angles \( \gamma, \zeta \) are illustrated in Fig16, in Appendix A.

The dimensional bearing reaction forces in \( x \) and \( y \) directions are:

\[ F_x = -V_x, \quad F_y = -V_y \]

Transformation to the rotor fixed reference coordinates \( X, Y, Z \) is done as following:

\[ x = \frac{X \cos \zeta + Y \sin \zeta}{C}, \quad y = \frac{Y \cos \zeta - X \sin \zeta}{C} \]

\[ \zeta = \tan \left( \frac{\beta - \beta_0}{\lambda + \beta_0} \right) \]

Vectors \( \hat{V}, \hat{V}, \hat{C}, \hat{W}, \hat{F} \) and angles \( \gamma, \zeta \) are illustrated in Fig16, in Appendix A.

The dimensional bearing reaction forces in \( X \) and \( Y \) directions are:

\[ F_x = F_x \cos \zeta - F_y \sin \zeta \]

\[ F_y = F_x \sin \zeta + F_y \cos \zeta \]

The above bearing reaction forces are nonlinear functions of journal displacement and velocity in the bearing clearance.

**Linear model.** The bearing reaction forces are modelled as linear functions of journal displacement and velocity in the bearing clearance.

\[ \begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \]

\[ K_{xx} = \frac{k_x}{F_x}, \quad C_{xx} = \frac{c_x}{F_x}, \quad \beta = \frac{r (m_{\text{air}} + m_{\text{water}})}{m_{\text{air}}} \]
Where the 8 non-dimensional bearing coefficients \((k_i, c_i)\) are obtained according to [7] as following:

\[
k_{ij} = \frac{\cos \gamma y_i - \sin \gamma y_i}{\varepsilon_i}
\]

\[
k_{ij} = \frac{\sin \gamma y_i + \cos \gamma y_i}{\varepsilon_i}
\]

\[
k_{ij} = \frac{-\sin \gamma y_i \cos \varepsilon_i}{\varepsilon_i}
\]

\[
k_{ij} = \frac{-\cos \gamma y_i \sin \varepsilon_i}{\varepsilon_i}
\]

\[
c_{ij} = \frac{\sin \gamma y_i}{\varepsilon_i}
\]

\[
c_{ij} = \frac{-\cos \gamma y_i}{\varepsilon_i}
\]

\[
c_{ij} = \frac{\sin \gamma y_i}{\varepsilon_i}
\]

\[
c_{ij} = \frac{-\cos \gamma y_i}{\varepsilon_i}
\]

The partial derivatives required to evaluate the above coefficients (equations (3)) together with the complete expressions for \(p, \omega\), are obtained according to [7] and reproduced in Appendices A and B.

At static equilibrium, the Sommerfeld number is:

\[S = \frac{\mu \omega}{2 \pi} \sqrt{\frac{DL}{\varepsilon_i}} > 0\]

Or

\[f(\varepsilon_i) = \frac{\mu \omega}{2 \pi} \sqrt{\frac{DL}{\varepsilon_i}} \frac{j}{\varepsilon_i} > 0\] (4)

Equation (4) is a nonlinear function of \(\varepsilon_i\) to be solved in the interval \(\varepsilon_i\). \(F_e\) is the bearing static load.

2. Rotor

The rotor is modelled in two different ways; first as a rigid non-symmetric rotor and secondly as a flexible non-symmetric rotor.

Rigid Non-symmetric Rotor with nonlinear bearings. The journal motion for bearing 1 and 2 can be derived by means of Newton’s second law as following:

\[
\begin{align*}
\overrightarrow{p}_1 &= \frac{1}{M_{x1}}(F_{x1}) \\
\overrightarrow{p}_1 &= \frac{1}{M_{x2}}(F_{x2}) - g \\
\overrightarrow{p}_2 &= \frac{1}{M_{x1}}(F_{x1}) \\
\overrightarrow{p}_2 &= \frac{1}{M_{x2}}(F_{x2}) - g
\end{align*}
\] (5a, 5b)

\(M_{x1}\) and \(M_{x2}\) being the rotor weight carried by bearing 1 and 2 respectively, \(F_{x1} (i=1, 2; j = X, Y)\) are the bearings reaction force components, \(g\) is the gravity acceleration.

Discrete multi-degrees-of-freedom Rotor model with nonlinear bearings

The flexible rotor can be modelled as a discrete multi-degrees-of-freedom system using the finite element method (FEM):

\[\begin{align*}
&[M] \overrightarrow{\ddot{q}} + [C] \overrightarrow{\dot{q}} + [K] \overrightarrow{q} = \overrightarrow{F} \\
&[M] \overrightarrow{\ddot{q}} + [C] \overrightarrow{\dot{q}} + [K] \overrightarrow{q} = \overrightarrow{F}
\end{align*}\] (6)

\[\overrightarrow{F}_e = \overrightarrow{F}_{e1} + \overrightarrow{F}_{e2} + \overrightarrow{F}_{e3} + \overrightarrow{F}_{e4} \]

Where \(n=1, 2...\) total number of elements; \([M]_i, [C]_i, [K]_i\) are the element translational mass matrix, rotatory mass matrix, gyroscopic matrix, stiffness matrix and external applied load vector respectively; these matrices can be found in [9]. \(\overrightarrow{F} = \overrightarrow{F}_{e1} + \overrightarrow{F}_{e2}\), where the vector \(\overrightarrow{F}_{e1}\) is the element gravitational force vector and can be found in [10]. \(\overrightarrow{F}_e\) is the bearing reaction force vector. For the present case the number of elements is 4 and the number of Nodes = number of elements + 1 = 5. The rotor material inner damping has been neglected in equation (6).

\[\overrightarrow{F}_e = 0\] for the 1st and 4th elements.

\[\overrightarrow{F}_e = \overrightarrow{F}_{e1} + \overrightarrow{F}_{e2} + \overrightarrow{F}_{e3} + \overrightarrow{F}_{e4}\]

Global system (elements assembling). The entire system becomes:

\[\begin{align*}
&{\left[\begin{array}{c}
[M]_1 & [C]_1 & [K]_1 \\
[M]_2 & [C]_2 & [K]_2 \\
&M & K & M & K
\end{array}\right]} \overrightarrow{\ddot{q}} + \overrightarrow{F}_{e1} + \overrightarrow{F}_{e2} + \overrightarrow{F}_{e3} + \overrightarrow{F}_{e4} = \overrightarrow{0} \\
&{\left[\begin{array}{c}
[M]_1 & [C]_1 & [K]_1 \\
[M]_2 & [C]_2 & [K]_2 \\
&M & K & M & K
\end{array}\right]} \overrightarrow{\ddot{q}} + \overrightarrow{F} = \overrightarrow{0}
\end{align*}\] (7)

\[\overrightarrow{F} = \overrightarrow{F}_{e1} + \overrightarrow{F}_{e2} + \overrightarrow{F}_{e3} + \overrightarrow{F}_{e4}\]

And \([M]_i [G]_i [K]_i\) are the assembled mass matrix (translational + rotatory), assembled gyroscopic matrix, assembled stiffness matrix and assembled external applied loads respectively.
SIMULATION STRATEGY

The system of equations (7) can be reformulated into its state space form which is suitable for simulation of any initial value problem.

Let $\mathbf{X}$ be the new state variable vector defined as:

$$
\mathbf{X} = \begin{bmatrix}
\mathbf{q} \\
\dot{\mathbf{q}}
\end{bmatrix}, \quad \mathbf{X}_{new} = \begin{bmatrix}
\mathbf{q} \\
\dot{\mathbf{q}}
\end{bmatrix}, \quad \mathbf{X}_{new} = \begin{bmatrix}
\mathbf{q} \\
\dot{\mathbf{q}}
\end{bmatrix}
$$

(8)

and $\mathbf{X}(t=0) = \mathbf{X}_0$ is a known initial condition.

The above variable substitution decreases the order of the previous ordinary differential equations (7), and the new system becomes:

$$
\mathbf{X} = [\mathbf{X}] \mathbf{X} + [\mathbf{Z}] \mathbf{Z}_{new}
$$

Where

$$
\frac{d}{dt} = \begin{bmatrix}
\mathbf{q} \\
\dot{\mathbf{q}}
\end{bmatrix} - [\mathbf{M}] [\mathbf{K}] - a[M] [\mathbf{G}]
$$

and

$$
\frac{d}{dt} = \begin{bmatrix}
\mathbf{q} \\
\dot{\mathbf{q}}
\end{bmatrix} - [\mathbf{M}] [\mathbf{K}]
$$

Eigen value problem and the Campbell diagram

The rotor system eigen value problem is solved by linear analysis. Linear bearing model is used, defining the bearing reaction force as linear function of the journal motion using the bearing coefficients.

Fig. 3 Non-symmetric rotor with linear bearings

Where $x, Lx = (a + t + b), ml, m2, m$ are rotor disc thickness, rotor total length, shaft weight on bearing 1 side, shaft weight on bearing 2 side and rotor disc weight respectively. The bearings static reaction forces $R1$ and $R2$ are calculated as following:

$$
\begin{align*}
R1 & = ml \frac{Lx}{2} + m(a + t + b) \frac{Lx}{2} - g \\
R2 & = m2(a + t + b) \frac{Lx}{2} - g
\end{align*}
$$

R1 and R2 are going to be used to solve the equation (4), by setting $F_s = R1$ for bearing 1 and $F_s = R2$ for bearing 2.

Equations (7) can be rewritten in homogenous form as:

$$
[M] \ddot{\mathbf{q}} + a[M] [\mathbf{G}] \dot{\mathbf{q}} + [\mathbf{K}] \mathbf{q} = \mathbf{0}
$$

(9)

Bearings reaction forces $\mathbf{P}$ are linear functions of journal motions.

$$
\mathbf{P} = [K_1] \mathbf{q} + [K_2] \mathbf{q}
$$

Using the substitutions in equations (8), the homogenous equations (10) can be rewritten as following:

$$
[z] [\mathbf{X}] + [\mathbf{Z}] \mathbf{X}_{new} = \mathbf{0}
$$

(11)

Assuming a solution to the equations system (11):

$$
\mathbf{X} = \mathbf{T} e^{\lambda t}
$$

Equations (11) become:

$$
\begin{align*}
[z]' [\mathbf{X}] + [\mathbf{Z}] \mathbf{X}_{new} & = \mathbf{0} \\
\mathbf{X} & = \mathbf{0} \\
n(s) & = -[z]' \\
[n(s)] - [\mathbf{K}] [\mathbf{X}] & = \mathbf{0} \\
\dot{\mathbf{X}} & = [\mathbf{K}] [\mathbf{X}] + [\mathbf{Z}] \mathbf{X}_{new} = \mathbf{0}
\end{align*}
$$

$\lambda = \sigma + j\omega_i$, $\omega_i, \sigma, T$ are system damped natural frequencies, system damping exponents and time respectively.

Static equilibrium

Due to the asymmetry of the rotor system, the bearings stiffness and damping coefficients will be 16, 8 coefficients for each bearing. The coefficients for bearing 1 will be computed by solving the equations (4) and (5) using $F_s = R1$ and the coefficients for bearing 2 by using $F_s = R2$. 

4
RESULTS

Given data:
\[ \begin{align*}
L_x &= 1.5m \\
\rho &= 7850 \text{ kg/m}^3 \\
r &= \frac{L_x}{2} \\
R &= 2.54 \times 10^{-3} \text{ m} \\
a &= \frac{2}{3} (L_x - r) \\
D &= 2R \\
b &= \frac{1}{3} (L_x - r) \\
d &= 0.3m \\
C &= 140 \times 10^6 \text{ m} \\
Dr &= 3d \\
\mu &= 0.03 \text{ Pa s} \\
E &= 210 \times 10^6 \text{ Pa}
\end{align*} \]

Without unbalance excitation

1) Rigid non-symmetric rotor. The result displayed on Fig4 is the solution of equations (5a), showing the “Journal 1” steady state dimensionless orbit at threshold speed of instability, which is about 7850rpm (revolutions per minute) for the rigid non-symmetric rotor with nonlinear bearings. At rotor speeds lower than 7850rpm, the journal converges to a static equilibrium point. Journal 1 is the journal of the less loaded bearing 1.

2) Flexible non-symmetric rotor. The result displayed on Fig5 is the solution of equations (7), and shows the steady state dimensionless orbits of journals 1 & 2 and rotor mass center at threshold speed of instability for the flexible non-symmetric rotor with nonlinear bearings. Instability occurs at about 11934rpm (revolutions per minute). At rotor speeds lower than 11934rpm, the journal converges to a static equilibrium point.

Forward synchronous unbalance excitation

1) Rigid non-symmetric rotor. The result on Fig6 shows the differences between max (X,Y) and min (X,Y) displacements of the unbalance response for the rigid non-symmetric rotor with nonlinear bearings. The rotational speed at which resonance occurs in bearing 1 is found to be about 2800rpm. The system becomes unstable at rotational speeds larger than 8500rpm.

The result on Fig7a shows the steady state unbalance response of “Journal 1” for the rigid non-symmetric rotor with nonlinear bearings. While Fig7b is the Fast Fourier Transform (fft) of “Journal 1” X-displacements to check out the frequency content of the journal whirl motion. The subsynchronous whirl can be revealed, having a frequency of about 3131rpm.

Fig. 4 “Journal 1” dimensionless orbit for rigid non-symmetric rotor with nonlinear bearings, N=7850rpm

Fig. 5 dimensionless orbits for flexible non-symmetric rotor with nonlinear bearings, N=11934.01rpm

Fig. 6 “Journal 1” dimensionless \([\text{max}(X,Y)-\text{min}(X,Y)]\) of a synchronous unbalance response for rigid non-symmetric rotor with nonlinear bearings

Fig. 7a “Journal 1” dimensionless rotor response to a synchronous unbalance excitation for rigid non-symmetric rotor with nonlinear bearings, N=8050rpm

Fig. 7b Fast Fourier Transform (fft) of “Journal 1” X-displacements on Fig. 7a to check the frequency content of the journal whirl motion
2) Flexible non-symmetric rotor. The result on Fig8 shows the differences between max (X,Y) and min (X,Y) displacements of the unbalance response for the flexible non-symmetric rotor with nonlinear bearings. The rotational speeds at which resonance occurs in bearing 1 are found to be about 2600rpm and 5400rpm.

The result on Fig9a shows the steady state unbalance responses of journals 1 & 2 and rotor mass center for the flexible non-symmetric rotor with nonlinear bearings. While Fig9b is the Fast Fourier Transform (fft) of "Journal 1" X-displacements to check out the frequency content of the journal whirl motion. The subsynchronous whirl can be revealed, having a frequency of about 2999rpm.

A Campbell diagram (whirl speed map) for the flexible non-symmetric rotor with linear bearings is shown on Fig10. For a rotational speed range from 45 to 30000rpm (scaled with steps of 100rpm), two critical speeds (2845, 5445rpm) are obtained by the intersection between the synchronous excitation line and the back or forward damped natural modes of the rotor-bearings system. These two obtained critical speeds are close to the two resonance speeds (2600 and 5400rpm) found by using nonlinear bearing model on Fig8.

DISCUSSIONS AND CONCLUSION

In this paper the instability threshold is analysed for two cases: 1) a rigid non-symmetric rotor and 2) a flexible non-symmetric rotor. The two models consist of a rotor supported by two identical finite-length hydrodynamic journal bearings of length to diameter ratio L/D=1, with same lubricant properties.

The simulations show that the gyroscopic coupling effect on the flexible rotor has increased the threshold speed of instability, from 7850rpm (rigid non-symmetric rotor) to 11934rpm (flexible non-symmetric rotor). But it has also markedly increased the magnitude of both journals orbits.

By linearising the bearing reaction forces at each driving speed, a Campbell diagram can be constructed. The Campbell diagram has predicted the existence of two (synchronous) critical speeds for the flexible non-symmetric rotor system, at: 2845 and 5445rpm. These two predicted critical speeds are close to the two resonance speeds (2600 and 5400rpm) obtained by using nonlinear bearing model. Observe that the lowest (first) computed resonance speed of the flexible rotor system depends mainly on bearing 1 characteristics, as it is in case of the rigid rotor. The second and higher resonance speeds depend on both rotor modes and bearing characteristics.

The whirl-frequency ratio $\frac{N_{\text{whirl}}}{N_{\text{synch}}} \ll 0.5$ due to the fact that the bearings are operated at large eccentricities, see [Lund (1966)] and [3].

It is not advisable to operate the rotor system near or beyond the instability threshold speed, because the journal orbits will be large and basically bounded by the bearing clearance geometry. A babbited bearing will not survive under this high vibration induced heat and stress. The rotor unbalance has shown a positive effect on flexible non-symmetric rotor, a possibility to operate the rotor system beyond its threshold speed of instability with reduced vibration amplitudes.
REFERENCES


APPENDIX A

\[ W_i = \left[ \begin{array}{ccc} \cos \alpha & \cos \beta \sin \alpha & \sin \beta \sin \alpha \\ -\sin \alpha & \cos \beta \cos \alpha & \sin \beta \cos \alpha \\ 0 & -\sin \beta & \cos \beta \\ \end{array} \right] \]

\[ \theta_i = 1 + 2.12\beta \theta_i \]

\[ \phi_i = 2 \beta \alpha \theta_i \]

\[ \gamma_i = \cos \gamma \theta_i \]

\[ \delta_i = \sin \gamma \theta_i \]

\[ \kappa = 2 \beta (\theta_i - \delta_i) \]

\[ \zeta = \cos \gamma \]
Paper C
OIL INDUCED INSTABILITY: ANALYTIC STUDY AND EXPERIMENTAL VERIFICATION ON FLEXIBLE ROTOR SUPPORTED BY A JOURNAL-BEARING AT ONE END

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KEYWORDS: Instability; oil whirl; nonlinear bearing; linear bearing; flexible rotor.

ABSTRACT
Oil induced instability, is a frequently encountered phenomenon causing system instability for rotors supported by hydrodynamic journal-bearings. In this paper a flexible rotor, simply supported at one end and with oil lubricated journal-bearing at the other, is analytically modelled. The rotor system is modelled in two ways namely as a discrete system by finite element method (FEM) with nonlinear journal-bearing and as a lumped inertia system with linear journal-bearing.

The analysed rotor-bearing system is a Bently Nevada Rotor Kit Model RK4 with Oil whirl/whip option. Results obtained from the simulation of the discrete rotor model with a nonlinear journal-bearing indicate at which rotational speed the oil induced instability (oil whirl) will occur. Campbell diagrams are shown for the lumped inertia rotor model with linear journal-bearing and the critical speeds are predicted. From the results the accuracy of the analytical speed-dependent bearing coefficients are evaluated. These coefficients were derived from the nonlinear bearing impedance descriptions by D. Childs. The bearing impedance descriptions method is a method valid for all L/D (length to diameter) ratios, and all journal eccentricities. The simulation time is significantly reduced by using a lumped inertia rotor model with linear journal-bearing. Critical speed obtained from Campbell diagram predicts a threshold speed of instability which is about 0.35% higher than that predicted by the discrete rotor model with a nonlinear journal-bearing. Compared with results collected from experiment, the simulation results predict a threshold speed of instability which is about 5.69% higher (linear analysis), or 5.36% higher (nonlinear analysis).

INTRODUCTION
For rotor systems supported by hydrodynamic journal-bearings, the oil induced instability is the frequently encountered phenomenon causing the system instability. Oil induced instability occurs as oil whirl or oil whip. Oil whirl happens when the rotor looses its static equilibrium position and starts to whirl, the static equilibrium position becomes an equilibrium orbit in which the rotor performs periodic motion and forms a closed orbit [1]. But when the rotor whirl frequency becomes equal to the rotor first natural bending frequency, then the oil whirl phenomenon has occurred [2].
MODELS

1. Journal-Bearing

The rotor is supported at one end by a plain cylindrical journal-bearing with coordinates system and parameters as shown in the figure below:

![Fig.1 Plain cylindrical journal-bearing coordinates system](image)

**Nonlinear model** The Moes cavitated (\(s \rightarrow \infty\)) finite-length bearing model \([4, 5]\) will be used, with the following impedance \(\hat{w}\) (dimensionless bearing-load-force vector due to pure squeezing) components:

\[
\begin{align*}
W_x &= \omega \left( [x - 1] \left( \frac{y}{l} \right) \frac{y}{l} \frac{x}{l} \frac{y}{l} \right) \\
W_y &= \omega \left( [x - 1] \left( \frac{y}{l} \right) \frac{y}{l} \frac{x}{l} \frac{y}{l} \right)
\end{align*}
\]

The dimensional bearing reaction forces in \(x\) and \(y\) directions are:

\[
\begin{align*}
F_y &= \omega \left( [x - 1] \left( \frac{y}{l} \right) \frac{y}{l} \frac{x}{l} \frac{y}{l} \right) \\
F_x &= \omega \left( [x - 1] \left( \frac{y}{l} \right) \frac{y}{l} \frac{x}{l} \frac{y}{l} \right)
\end{align*}
\]

Transformation to the rotor fixed reference coordinates \(X,Y,Z\) is done as following:

\[
\begin{align*}
x &= \frac{X \cos \gamma + Y \sin \gamma}{C} \\
y &= \frac{X \sin \gamma \cos \gamma \left( \frac{Y}{X} \right) \frac{x}{y}}{C} \\
z &= \omega \left( \frac{Y - \omega x}{x} \right)
\end{align*}
\]

\[
\begin{align*}
V_x &= \left[ \dot{x} + \frac{x}{x} \right] \frac{y}{y} \frac{z}{z} \\
V_y &= \left[ \dot{y} + \frac{x}{x} \right] \frac{y}{y} \frac{z}{z} \\
V_z &= \left[ \dot{z} + \frac{x}{x} \right] \frac{y}{y} \frac{z}{z}
\end{align*}
\]
Vectors \( \tilde{V}, \tilde{W}, \tilde{f}, \tilde{F} \) and angles \( \varphi \) are illustrated in Fig. 12, in Appendix B. The dimensional bearing reaction forces in \( X \) and \( Y \) directions are:

\[
F_x = F_{x, \text{load}} + F_{x, \text{unbalance}}
\]

\[
F_y = F_{y, \text{load}} + F_{y, \text{unbalance}}
\]

The above bearing reaction forces are nonlinear functions of journal displacement and velocity in the bearing clearance.

**Linear model** The bearing reaction forces are modelled as linear functions of journal displacement and velocity in the bearing clearance.

\[
\begin{bmatrix}
F_x \\
F_y
\end{bmatrix} =
\begin{bmatrix}
C_{xx} & C_{xy} & X \\
C_{yx} & C_{yy} & Y
\end{bmatrix}
\begin{bmatrix}
X \\
Y
\end{bmatrix}
\]

The above bearing reaction forces are nonlinear functions of journal displacement and velocity in the bearing clearance.

\[
F_x = F_{x, \text{load}} + F_{x, \text{unbalance}}
\]

\[
F_y = F_{y, \text{load}} + F_{y, \text{unbalance}}
\]

Where the 8 non-dimensional bearing coefficients \( (k, c) \) are obtained according to [4] as following:

\[
\begin{align*}
k_{xx} &= \frac{1}{2} \frac{2m_{ij}J_{ij}}{f_{ij}} \\
k_{yy} &= \frac{1}{2} \frac{2m_{ij}J_{ij}}{f_{ij}} \\
k_{xy} &= \frac{1}{2} \frac{2m_{ij}J_{ij}}{f_{ij}} \\
k_{yx} &= \frac{1}{2} \frac{2m_{ij}J_{ij}}{f_{ij}}
\end{align*}
\]

The partial derivatives required to evaluate the above coefficients (equations 3) together with the complete expressions for \( f_{ij}, W_{ij} \), are found in [4] and reproduced in Appendices A and B.

At static equilibrium, the Sommerfeld number is:

\[
S = \mu \frac{(k/2) \frac{dL}{dx}}{2F} = \frac{1}{2m_{ij}f_{ij}} > 0
\]

Or

\[
f_{ij} = \mu \frac{1}{2F} \frac{dL}{dx}
\]

Equation (4) is a nonlinear function of \( c \), to be solved in the interval \([b, b] \), where \( b \) is the journal-bearing static load.

**2. Rotor**

The rotor system is modelled in two ways namely as a discrete system by finite element method (FEM) with nonlinear journal-bearing and as a lumped inertia system with linear journal-bearing.

**Discrete multi-degree-of-freedom Rotor model with nonlinear journal-bearing** The flexible rotor can be modelled as a discrete multi-degree of freedom system using the finite element method (FEM):

\[
\begin{bmatrix}
\mathbf{M} \quad \mathbf{0} \\
\mathbf{0} \quad \mathbf{M}
\end{bmatrix}
\begin{bmatrix}
\mathbf{d} \\
\mathbf{d}
\end{bmatrix}
\begin{bmatrix}
\mathbf{x} \\
\mathbf{x}
\end{bmatrix}
\]

**Subsystem (element formulation)** The equations of motion for a typical eight degrees-of-freedom rotating element are [7]:

\[
\begin{bmatrix}
\mathbf{w}_1 \\
\mathbf{w}_2 \\
\vdots \\
\mathbf{w}_8
\end{bmatrix} =
\begin{bmatrix}
\mathbf{M} \cdot \mathbf{d} \cdot \mathbf{d} \cdot \mathbf{d} \\
\mathbf{R} \cdot \mathbf{M} \cdot \mathbf{d} \cdot \mathbf{d} \cdot \mathbf{d}
\end{bmatrix}
\]

where \( n \) is the number of elements; \( \mathbf{M} \) is the element translational mass matrix, \( \mathbf{R} \) is the unbalance force vector, \( \mathbf{C} \) is the bearing reaction force vector. For the present case the rotor inner damping is neglected \( [C] = 0 \). The number of elements is 4 and the number of Nodes = number of elements + 1 = 5.

**Global system (elements assembling)** The entire system becomes:

\[
\begin{bmatrix}
\mathbf{M} \cdot \mathbf{d} \cdot \mathbf{d} \cdot \mathbf{d} \\
\mathbf{R} \cdot \mathbf{M} \cdot \mathbf{d} \cdot \mathbf{d} \cdot \mathbf{d}
\end{bmatrix}
\begin{bmatrix}
\mathbf{x} \\
\mathbf{x}
\end{bmatrix}
\]

And \( [\mathbf{M}] \cdot [\mathbf{K}] \cdot [\mathbf{d}] \) are the assembled mass matrix (translational + rotatory), assembled gyroscopic matrix, assembled stiffness matrix and assembled external applied loads respectively.
SIMULATION STRATEGY

Journal whirl trajectories (orbits)

The system of equations (6) can be reformulated into its state space form which is suitable for simulation of any initial value problem. Let \( \mathbf{\tau} \) be the new state variable vector defined as:

\[
\mathbf{\tau} = \begin{bmatrix} \mathbf{\tau}_1 \\ \mathbf{\tau}_2 \\ \mathbf{\tau}_3 \end{bmatrix}, \quad \mathbf{\tau} = \begin{bmatrix} \mathbf{\tau}_1 \\ \mathbf{\tau}_2 \end{bmatrix}, \quad \frac{d}{dt} \mathbf{\tau} = \begin{bmatrix} \mathbf{\tau}_1 \\ \mathbf{\tau}_2 \end{bmatrix}
\]

and \( \mathbf{\tau}(t=0) = \mathbf{\tau}_0 \) is a known initial condition.

The above variable substitution decreases the order of the previous ordinary differential equations (6), and the new system becomes:

\[
\mathbf{\tau} = [\mathbf{\tau}, \mathbf{\tau}] \mathbf{\tau}_0,
\]

\[
[\mathbf{\tau}] = \begin{bmatrix} \mathbf{\tau}_1 \\ \mathbf{\tau}_2 \\ \mathbf{\tau}_3 \\ \mathbf{\tau}_4 \end{bmatrix} \quad \text{and} \quad [\mathbf{\tau}] = \begin{bmatrix} \mathbf{\tau}_1 \\ \mathbf{\tau}_2 \end{bmatrix}
\]

Eigen value problem and the Campbell diagram

The rotor system eigen value problem is solved by linear analysis. Linear bearing model is used, defining the bearing reaction force as linear functions of the journal motion using the bearing coefficients. In present case, the shaft diameter is much smaller (about seven times smaller) than the rotor disc diameter; therefore the rotor system can be modelled as a lumped inertia system.

![Fig.3 Lumped inertia rotor system with linear journal-bearing at static equilibrium position](image)

The bearings static reaction forces \( R1 \) and \( R2 \) are calculated as following:

\[
R2 = (M_r \times x) \frac{\partial}{\partial x}, \quad R1 = M_r \times x - R2
\]

where \( x, L_r = (a+b) \), \( M_r \) are the rotor disc thickness, shaft total length and rotor disc weight respectively. The equations of motion of the lumped inertia rotor system with linear journal-bearing (Fig.3) can be derived as follows:

\[
\begin{bmatrix} M_r & 0 & 0 & 0 \\ 0 & M_r & 0 & 0 \\ 0 & 0 & M_r & 0 \\ 0 & 0 & 0 & M_r \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \begin{bmatrix} k_{1s} & k_{1a} & k_{1b} & k_{1c} \\ k_{2s} & k_{2a} & k_{2b} & k_{2c} \\ k_{3s} & k_{3a} & k_{3b} & k_{3c} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} I_0 \dot{q}_1^2 \\ \frac{1}{2} I_0 \dot{q}_2^2 \\ \frac{1}{2} I_0 \dot{q}_3^2 \end{bmatrix}
\]

where \( I_0, I_1, I_2, I_3, L_r, \theta, a, b, \gamma, X, Z \) are rotor disc diametral moment of inertia, rotor disc polar moment of inertia, shaft area moment of inertia, rotor disc diameter, shaft diameter, rotor disc rotation with respect to \( X \) axis, rotor disc rotation with respect to \( Y \) axis, rotor disc geometric centre displacement in \( X \) direction, rotor disc geometric centre displacement in \( Y \) direction, journal displacement in \( X \) direction, shaft elasticity modulus, unbalance mass and unbalance distance respectively.

Static equilibrium

The two last equations of equations (9) are the equilibrium forces at the journal-bearing. The bearing stiffness and damping coefficients are calculated at static equilibrium position, by solving the equations (3) and (4) using \( \dot{\mathbf{\tau}} = 0 \). The homogenous form of equations (9) is obtained by putting the right hand sides of rows one to four equal to zero; giving the following equations system:

\[
[\mathbf{\tau}] \mathbf{\tau}^+ = \mathbf{\tau}
\]

\[
\begin{bmatrix} \mathbf{\tau} \\ \mathbf{\tau} \end{bmatrix} = \mathbf{I}
\]

\[
[\mathbf{\tau}] \mathbf{\tau}^+ = \mathbf{I}
\]

\[
[\mathbf{\tau}] \mathbf{\tau}^+ = \mathbf{I}
\]

\[
[\mathbf{\tau}] \mathbf{\tau}^+ = \mathbf{I}
\]

\[
[\mathbf{\tau}] \mathbf{\tau}^+ = \mathbf{I}
\]

\[
[\mathbf{\tau}] \mathbf{\tau}^+ = \mathbf{I}
\]

\[
[\mathbf{\tau}] \mathbf{\tau}^+ = \mathbf{I}
\]

where \( I_1, I_2, I_3, D_r, d, \theta, \gamma, X, Z \) are found in Appendix C. Assume a solution \( \mathbf{\tau} = \mathbf{\tau}^+ \) to the homogenous equations system (10); then equations (10) can be rewritten as:
\[
\begin{align*}
[x(t), y(t)] &= 0, \\
[x(t) - \hat{x}_n(t)], [y(t) - \hat{y}_n(t)] &= 0
\end{align*}
\]

\[\lambda = \sigma + jo_{\lambda}\]

\(\omega_0\), \(\sigma\) are the system damped natural frequencies and damping exponents respectively.

SIMULATION RESULTS (UNBALANCE RESPONSE)

System given data:

- \(\omega = 0.035\) m
- \(h = 0.0762\) m
- \(t = 0.0252\) m
- \(L_a = \sqrt{a^2 + b^2} = 0.137\) m
- \(d = 0.0105\) m
- \(D = 0.0072\) m
- \(\varphi = 7850\) kg/m
- \(E = 30 \times 10^8\) N/m
- \(B = 0.0127\) m
- \(R = 2\) m
- \(L = 2\) m
- \(\lambda = \frac{I}{R^2}\)
- \(C = 200 \times 10^{-6}\) m
- \(\mu = 0.02 Pa \times m\)

Fig.4 Theoretical (N=6340rpm) and experimental (N=6000rpm) journal trajectories (orbits) when Oil whirl phenomenon has started. The theoretical results are obtained from the unbalance response of the rotor system (FEM) with nonlinear journal-bearing

Fig.5 Experimental journal trajectory (orbit) when Oil whirl becomes instable (N=6200rpm). The journal motion is basically bounded by the bearing clearance geometry.

Fig.6 Experimental X, Y displacements at 6000, 6100 and 6200rpm

Fig.7 Experimental bifurcation diagram of X displacements collected from 5000rpm

Fig.8 FFT, for the theoretical oil whirl frequency search

Fig.9 FFT, for the experimental oil whirl frequency search (N=6000rpm)
the theoretical and experimental (for N=6000rpm) transforms (FFT) in Fig.8 and Fig.9 reveal the frequency of doubling of the system unbalance responses at more. The bifurcation diagram in Fig.6 show a period keep increasing with time and do not form a closed orbit any displacements become larger than the bearing clearance; they showed that at rotor speeds larger than hydrodynamic lubrication. The theoretical analysis has geometry; which is simply a sign of instability or a failure in the journal is basically bounded by the bearing clearance has started. The experiments in Fig.5 show that at rotor unbalance response when the oil whirl phenomenon Fig.4 show both theoretical and experimental journal orbits of bearing, and the bush bearing is assumed rigid. The results in presence of the rubber seal at the hydrodynamic journal-bearing, assumed rigid in theoretical analysis, is a self lubricated bearing to avoid high friction and wear. The rubber seal at the hydrodynamic journal-bearing together with the self lubricated bush bearing will add a certain amount of damping to the system. According to the instructions, the inlet pressure to the hydrodynamic journal-bearing should be kept at 8psi during experiment.

The theoretical models do not take into account the presence of the rubber seal at the hydrodynamic journal-bearing, and the bush bearing is assumed rigid. The results in Fig.4 show both theoretical and experimental journal orbits of the rotor unbalance response when the oil whirl phenomenon has started. The experiments in Fig.5 show that at 6200rpm the journal is basically bounded by the bearing clearance geometry; which is simply a sign of instability or a failure in hydrodynamic lubrication. The theoretical analysis has showed that at rotor speeds larger than 6340rpm, the journal displacements become larger than the bearing clearance; they keep increasing with time and do not form a closed orbit any more. The bifurcation diagram in Fig.6 show a period doubling of the system unbalance responses at 6000, 6100 and 62000rpm as displayed in Fig.5. The Fast Fourier Transforms (FFT) in Fig.8 and Fig.9 reveal the frequency of the theoretical and experimental (for N=6000rpm) X displacements: The theoretical whirl frequency is found to be $N_{\text{whirl-theory}} = 2997\text{rpm}$, and the experimental whirl frequency $N_{\text{whirl-experiment}} = 3000\text{rpm}$. The whirl ratio is $\frac{N_{\text{whirl-theory}}}{N} = 0.5$ for both theoretical and experimental results (2997/6340= 0.4727, 3000/6000=0.5), because the hydrodynamic journal-bearing operates at low eccentricities as stated in [1, 2].

The rule of thumbs [1, 2] says that the system critical speed (or resonance speed), is about half the threshold speed of instability $N_c = \frac{N}{2} + N_{\text{instability}}$ for rotor-bearing system operating at low eccentricities. Theoretical Campbell diagram, Fig.10 (zoomed in Fig.11) obtained from the lumped inertia rotor system model with linear journal-bearing, display good results which predict the system critical speed $N_c = 6340\text{rpm}$. The system threshold speed of instability predicted by the Campbell diagram can be found to be: $N = 2N_c = 2 \times 6340\text{rpm} = 12680\text{rpm}$, which is about 0.35% higher than the result (N=6340rpm) predicted by the theoretical rotor system model with a nonlinear journal-bearing. Compared to the experimental results, the system threshold speed of instability predicted by the Campbell diagram is about 5.69% higher than the result (N=6000rpm) predicted by the experiments. The theoretical model of the rotor system with a nonlinear journal-bearing has also predicted a high threshold speed of instability (N=6340rpm), about 5.36% higher than the experimental results (N=6000rpm). A possible reason is in the assumption of a rigid bush bearing in both linear and nonlinear analyses. The results presented in this paper evaluate the accuracy of the analytical bearing impedence descriptions method derived by D. Childs. The threshold speed of instability of the rotor-bearing system studied has been predicted by both linear and nonlinear analyses with an error of 5.69% and 5.36%. The analytically predicted journal trajectory is in agreement with the experiments as shown in Fig.4.

DISCUSSIONS AND CONCLUSION

The hydrodynamic journal-bearing on the Bently Nevada Rotor Kit Model RK4 (with Oil whirl/whip option), has a seal in rubber to prevent oil leakage. The other bearing (bush bearing), assumed rigid in theoretical analysis, is a self lubricated bearing to avoid high friction and wear. The theoretical and experimental results (2997/6340= 0.4727, 3000/6000=0.5), because the hydrodynamic journal-bearing operates at low eccentricities as stated in [1, 2].

The rule of thumbs [1, 2] says that the system critical speed (or resonance speed), is about half the threshold speed of instability $N_c = \frac{N}{2} + N_{\text{instability}}$ for rotor-bearing system operating at low eccentricities. Theoretical Campbell diagram, Fig.10 (zoomed in Fig.11) obtained from the lumped inertia rotor system model with linear journal-bearing, display good results which predict the system critical speed $N_c = 6340\text{rpm}$. The system threshold speed of instability predicted by the Campbell diagram can be found to be: $N = 2N_c = 2 \times 6340\text{rpm} = 12680\text{rpm}$, which is about 0.35% higher than the result (N=6340rpm) predicted by the theoretical rotor system model with a nonlinear journal-bearing. Compared to the experimental results, the system threshold speed of instability predicted by the Campbell diagram is about 5.69% higher than the result (N=6000rpm) predicted by the experiments. The theoretical model of the rotor system with a nonlinear journal-bearing has also predicted a high threshold speed of instability (N=6340rpm), about 5.36% higher than the experimental results (N=6000rpm). A possible reason is in the assumption of a rigid bush bearing in both linear and nonlinear analyses. The results presented in this paper evaluate the accuracy of the analytical bearing impedence descriptions method derived by D. Childs. The threshold speed of instability of the rotor-bearing system studied has been predicted by both linear and nonlinear analyses with an error of 5.69% and 5.36%. The analytically predicted journal trajectory is in agreement with the experiments as shown in Fig.4.

REFERENCES

APPENDIX A

\[
\begin{align*}
E_q &= 1 - \sin^2 \phi - \cos^2 \phi \\
E_r &= \cos \phi - \sin \phi \\
E_p &= 1 - \cos \phi - \sin \phi \\
\gamma &= \frac{1}{2} (\phi - \phi') \\
\gamma' &= \frac{1}{2} (\phi + \phi') \\
\beta &= \frac{1}{2} (\phi - \phi') \\
\beta' &= \frac{1}{2} (\phi + \phi')
\end{align*}
\]

APPENDIX B

\[
\begin{align*}
\left( \frac{\partial^2}{\partial x^2} \right) &= \frac{1}{f(x)} \left[ \phi - \phi' \right] \\
\left( \frac{\partial^2}{\partial y^2} \right) &= \frac{1}{f(x)} \left[ \phi + \phi' \right] \\
\left( \frac{\partial^2}{\partial z^2} \right) &= \frac{1}{f(x)} \left[ \phi - \phi' \right]
\end{align*}
\]

Fig 12. Kinematic variables for impedances and mobilities (from [2, 4])
Paper D
1 Introduction
Thrust bearing is a component used in vertical rotating machinery and shafts designed to transmit thrust. The total axial load is normally carried by one thrust bearing. In hydropower applications the influence of the thrust bearing is strongly simplified in the rotor dynamic modeling. Iliev [1] analyzed the failure of hydrogenerator thrust bearing with eight spring-supported pads, and found that the possible cause of bearing failure was a misalignment of bearing with respect to the axis of rotation. White, Torbergsen, and Lumpedkin [2] modeled and simulated a vertical pump with tilting-pad journal bearings. They assumed that the encountered disagreement between measurement and simulation result of the response at the thrust bearing collar was due to the neglected stiffness and damping coefficients of the thrust bearing. Shiau, Hsu, and Chang [3] showed that there is a coupling between the rotor translation and rotation displacements at the thrust bearing location. The thrust bearing cross coupling stiffness is directly proportional to the axial load [4]. An increase in axial load or a reduction of the total axial clearance in hydrodynamic thrust bearing leads to a significant increase in the first critical speed, in the whirl amplitude at the first critical speed, and the threshold speed of instability of the system [4]. Berger, Bonneau, and Prétre [5] studied the influence of axial thrust bearing on the dynamic behavior of an elastic shaft. They showed that a lateral dynamic force (imbalance force) can influence the axial dynamic behavior of the shaft and an axial dynamic load can excite the first critical bending frequency of a flexible shaft. The bearings characteristics, the rotor gyroscopic effect, the rotor imbalance, and the magnetic-pull forces are some examples of the reasons for motions coupling in rotating machinery. Tondl [6] derived the equations of motions for a vertical rigid rotor and found that the rotor imbalance couples the rotor lateral and torsional motions. Lundström et al. [7] showed how the magnetic-pull force has two components: a radial component and a tangential component. The generator (rotor) radial displacement generates a pulling force in both lateral x and y directions.

The studies contained in this paper concern particularly the analysis of a combined tilting-pad thrust-journal bearing dynamic influence on the vertical Porjus U9 hydropower turbogenerator at Porjus hydropower center. The Porjus hydropower center is a center for research, training, and development in hydropower technology situated in northern Sweden. The Porjus U9 hydropower unit is a full-scale unit, which is experimentally used for checking the effects of changes in scale and testing new design features and new materials to meet future demands for efficient and environmentally friendly electricity production (Fig. 1). Porjus U9’s combi-bearing is a fluid-film lubricated tilting-pad thrust and journal bearings combined together. Only linear fluid-film stiffness was taken into account in the model while fluid-film damping and pads inertia effects were not taken into account. The linearized model shows that the combi-bearing couples the rotor’s lateral and angular motions. However, if the thrust bearing’s pads arrangement is not symmetrical or if all the pads are not angularly equidistant the rotor axial and angular motions are also coupled. This last case of coupling will also occur if the thrust bearing equivalent total stiffness is not evenly distributed over the thrust bearing. A defective pad or unequal hydrodynamic pressure distribution on the pads’ surfaces may be the cause. The Porjus U9’s simulation results show that the combi-bearing influences the dynamic behavior of the machine. The rotor motions’ coupling due to combi-bearing changes the system’s natural frequencies and vibration modes. [DOI: 10.1115/1.4005002]

Keywords: combi-bearing, vertical rotor, natural frequency, natural mode, coupled motions

2 Modeling
The combi-bearing to be modeled is the upper bearing (bearing 3) in a vertical hydropower turbogenerator as illustrated in Fig. 2.
This upper bearing is a combination of both journal (radial) and thrust bearings, commonly called combi-bearing.

2.1 Tilting-Pad Combi-Bearing. If the thrust bearing collar is assumed to be rigid and massless, the dynamic characteristics of the combi-bearing free of manufacture and assembly errors can be modeled as follows.

The present analysis will consider the pads numbering order as shown in Fig. 3. A free body diagram of the thrust bearing collar (rotating part) together with the journal is an important tool to derive the forces and moments applied to the rotor at contact point (P) as shown in Fig. 4(c).

All reaction force vectors are drawn orthogonal to the collar and journal [see Fig. 4(c)] because the tilting-pad surface always lays parallel to the collar or the journal. It is enough to make a careful parametric model of the thrust bearing characteristics at a single pad position. Only the geometric parameter(s) will change due to the position of the pad. The thrust bearing reaction force in Z direction for a single pad (i) is calculated as follows:

$$F_{iz} = -k_i (l_i - l_0)$$

where $l_i = l_0 - z + (r \sin \phi_i + y) \tan \theta_i + (r \cos \phi_i - x) \tan \theta_i$.

In direction perpendicular to the thrust bearing rotating collar:

$$F_{iz, \text{top}} = \frac{F_{iz}}{\cos \theta_i}$$

Having the journal-bearing stiffness coefficients, the reaction forces in X and Y directions are assumed to be linear functions of the journal displacements and can therefore be obtained as follows:

$$F_{R_x} = -k_i [x + (L - z) \tan \theta_i]$$

$$F_{R_y} = -k_i [y - (L - z) \tan \theta_i]$$

In directions perpendicular to the journal:

$$F_{R_{x, \text{top}}} = \frac{F_{R_x}}{\cos \theta_j}$$

$$F_{R_{y, \text{top}}} = \frac{F_{R_y}}{\cos \theta_j}$$
The total reaction force will be the summation of all forces computed at each pad location together with the journal-bearing reaction forces:

\[ F_{\text{total}} = \left( \sum_{i=1}^{n} F_{i} \right) - \left( F_{B,x,y,z} \right) \sin \theta_i \]

\[ - \frac{F_{B,x,y,z}}{f_{\text{pad}}} \sin \theta_i \]

\[ F_{\text{total}} = F_{B,x} + \left( \sum_{i=1}^{n} F_{i} \right) \sin \theta_i = F_{B,y} + \left( \sum_{i=1}^{n} F_{i} \right) \tan \theta_i \]

With help from the free body diagram in Fig. 4(c), the thrust bearing reaction moments applied to the rotor at contact point \( P \) are derived for a single pad as follows:

\[ M_{i} = -F_{B,\text{xy},\text{yz},\text{zx}} \left( \sum_{i=1}^{n} \left( F_{i} \sin \phi_i + y \right) \cos \theta_i \right) \]

\[ M_{i} = F_{i} \left( \sum_{i=1}^{n} \left( F_{i} \sin \phi_i + y \right) \sin \theta_i \right) \]

The total reaction moments will be the summation of all moments computed for a single pad together with the moments generated by the journal-bearing reaction forces:

\[ M_{\text{total}} = \left( \sum_{i=1}^{n} M_{i} \right) - \left( F_{B,\text{xy},\text{yz},\text{zx}} \right) \left( \frac{L-z}{\cos \theta_i} - R \tan \theta_i \right) \]

Considering small motions, the derived total reaction forces and moments become

\[ F_{\text{total}} = -k_{L} \left[ z + \left( L - z \right) \tan \theta_i \right] \]

\[ F_{\text{total}} = -k_{L} \left[ z + \left( L - z \right) \tan \theta_i \right] \]

\[ F_{\text{total}} = -k_{L} \left[ z + \left( L - z \right) \tan \theta_i \right] \]

\[ M_{\text{total}} = \left( \sum_{i=1}^{n} M_{i} \right) + \left( F_{B,\text{xy},\text{yz},\text{zx}} \right) \left( \frac{L-z}{\cos \theta_i} - R \tan \theta_i \right) \]
Thus, Eqs. (11) to (15) become

\[
\sum_{i=1}^{n} (\sin \phi_i) = \sum_{i=1}^{n} (\cos \phi_i \sin \omega_i) = \left( \sum_{i=1}^{n} \cos \phi_i \right) = \left( \sum_{i=1}^{n} \sin \phi_i \right) = 0
\]

Thus, Eqs. (11) to (15) become

\[
F_{x_{\text{new}}} = -k_x z
\]

(16)

\[
F_{y_{\text{new}}} = -k_y x - k_y L \theta_x
\]

(17)

\[
M_{x_{\text{new}}} = \sum_{i=1}^{n} k_i (\sin \phi_i \sin \omega_i) \theta_x + k_i L_y - k_i L \theta_x
\]

(18)

\[
M_{y_{\text{new}}} = \sum_{i=1}^{n} k_i (\sin \phi_i \cos \omega_i) \theta_x - k_i L_x - k_i L \theta_x
\]

(19)

(20)

Where \( k_x, k_y \) are the equivalent total stiffness coefficients of the tilting-pad journal bearing in \( X \) and \( Y \) directions, \( k_i \) is the equivalent total stiffness coefficient of the thrust bearing in \( Z \) direction. The computed equivalent stiffness has taken into account both oil film and support structure stiffness while neglecting the pads inertia effect. \( k_i \) is the equivalent stiffness coefficient of the thrust bearing at a single pad \( (i = 1, \ldots, n) \) position in \( Z \) direction. In case of a perfect thrust bearing \( k_i = k_{i,n} \), \( n \) being the total number of the pads. \( \theta_x \) and \( \theta_y \) are the rotations about \( X \) and \( Y \) axes passing through the point \( P \) shown in Fig. 4a, \( r \) is the effective radius of the thrust bearing, \( L \) is the length from the contact point \( (P) \) to the journal-bearing’s position, \( \phi_i \) is the pad \( (i) \) angular position measured from a fixed reference.

2.2 Vertical Flexible Rotor (Finite Element Method). The vertical flexible rotor can be modeled as a discrete multidegree of freedom system using the finite element method (FEM) according to Fig. 5b. The derived combi-bearing (bearing 3) model is inserted into the U9 rotor finite element model at node 8.

The equations of motion for a typical eight degrees-of-freedom rotating element [8] are

\[
\begin{align*}
\left[ M_e \right] \ddot{q} + \left[ C \right] \dot{q} + \left[ K \right] q &= \dot{Q} \\
\end{align*}
\]

(21)

Where \( n = 1, 2, \ldots \) total number of elements; \( [M_e], [K], [G] \) are the element translational mass matrix, rotatory mass matrix, gyroscopic matrix, structural stiffness matrix, and external applied load vector, respectively; these matrices can be found in [8]. For the present case, the number of elements is eight and the number of nodes = number of elements + 1 = 9. The rotor material inner damping has been neglected in the model. The entire assembled system becomes

\[
\begin{align*}
\left[ M_{\text{new}} \right] \ddot{q} + \left[ C_{\text{new}} \right] \dot{q} + \left[ K_{\text{new}} \right] q &= \left[ Q_{\text{new}} \right] \\
\end{align*}
\]

\[
\ddot{q} = [X_1, Y_1, \theta_1, \theta_1, \ldots, X_9, Y_9, \theta_9, \theta_9]^T
\]

\[
\left[ M_{\text{new}} \right] \ddot{q} + \left[ C_{\text{new}} \right] \dot{q} + \left[ K_{\text{new}} \right] q = \left[ Q_{\text{new}} \right]
\]

(21)

Fig. 5 (a) Porjus U9 configuration. (b) Porjus U9 rotor finite element model.
\[ q_{\text{new}} = \begin{bmatrix} X_1 & Y_1 & \theta_{x1} & \theta_{y1} & \cdots & X_6 & Y_6 & \theta_{x6} & \theta_{y6} & z_1 \end{bmatrix}^T \]

\[ [M_{\text{new}}] = \begin{bmatrix} \frac{1}{36} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad [Q_{\text{new}}] = \begin{bmatrix} \frac{1}{36} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ K_{\text{new}} = \begin{bmatrix} 0 \end{bmatrix} \]

\[ F_{\text{new}}[F_{\text{new}}, m_{\text{rot}} \times g] \]

\[ q_{\text{rot}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ F_{\text{new}}[F_{\text{new}}, m_{\text{rot}} \times g] \]

\[ F_{x,x} = \begin{bmatrix} \begin{array}{c} [-41, \times X_1] \noalign{\medskip} (-k_1, \times Y_1) \noalign{\medskip} 0 \noalign{\medskip} 0 \end{array} \end{bmatrix} \]

\[ F_{y,y} = \begin{bmatrix} \begin{array}{c} [-42, \times X_1] 
oalign{\medskip} (-k_2, \times Y_1) \noalign{\medskip} 0 \noalign{\medskip} 0 \end{array} \end{bmatrix} \]

\[ F_{z,z} = \begin{bmatrix} \begin{array}{c} [-k_{\text{combi}}, \times X_1] 
oalign{\medskip} (-k_{\text{combi}}, \times Y_1) \noalign{\medskip} 0 \noalign{\medskip} 0 \end{array} \end{bmatrix} \]

\[ F_{\text{exc}} = \begin{bmatrix} \begin{array}{c} [-k_{\text{exc}}, \times X_1] 
oalign{\medskip} (-k_{\text{exc}}, \times Y_1) \noalign{\medskip} 0 \noalign{\medskip} 0 \end{array} \end{bmatrix} \]

\[ r = 0.4725 \text{ m} \]

\[ H = 0.841 \text{ m} \]

\[ L = 0.1927 \text{ m} \]

\[ k_{\text{new}} = k_{\text{combi}} = -64 \times 10^5 \text{ N m}^{-1} \]

\[ k_{\text{exc}} = -12.8 \times 10^5 \text{ N m}^{-1} \]

\[ k_1 = k_1 = 0.69 \times 10^5 \text{ N m}^{-1} \]

\[ k_2 = k_2 = 0.53 \times 10^5 \text{ N m}^{-1} \]

\[ k_3 = k_3 = 0.49 \times 10^5 \text{ N m}^{-1} \]

\[ l_{\text{pij}} = 9121 \text{ kg m}^2 \]

\[ l_{\text{pij}} = 74 \text{ kg m}^2 \]

\[ m_{\text{nem}} = 2600 \text{ kg} \]

\[ m_{\text{nem}} = 1900 \text{ kg} \]

\[ m_{\text{nem}} = 60 \text{ kg} \]

\[ m_{\text{nem}} = 49416 \times 10^5 \text{ kg} \]

\[ d_1 = 0.35 \text{ m} \]

\[ d_1 = 0.4 \text{ m} \]

\[ d_1 = 0.65 \text{ m} \]

\[ d_1 = 0.5 \text{ m} \]

\[ d_1 = 0.45 \text{ m} \]

\[ d_1 = 0.3 \text{ m} \]

\[ N = 600 \text{ rpm} \]

\[ E = 210 \times 10^6 \text{ N m}^{-2} \]

Figures 6(a) and 6(b) contain the Campbell diagrams showing the undamped natural frequencies of the rotor system with combi-bearing and the rotor system without thrust bearing. The results in Fig. 6(a) were obtained by using the combi-bearing’s model according to Eqs. (16) to (20). For the case of the rotor system without thrust bearing, parameter \( L \) was set to \( L = 0 \); meaning that the journal bearing is exactly located at the contact point (P). The plotted undamped natural frequencies are expressed in cycles per minute (rpm), while the rotor driving frequencies are given in revolutions per minute (rpm).

Figures 7, to 9 display the rotor’s first, second, and third lateral vibration modes of the system with combi-bearing and system without thrust bearing.

4 Discussions and Conclusions

The linear analysis shows that the combi-bearing couples the rotor lateral and angular motions. This coupling is clearly observed in the forces and moments equations (17)–(20). But if the pads arrangement is not symmetrical or if all pads are not angularly equivalent the rotor axial and angular motions are coupled as can be seen in Eqs. (11), (14), and (15). This last case of coupling will also occur if the thrust bearing equivalent total stiffness \( k_i \) is not evenly distributed around the thrust bearing. A defected pad or unequal hydrodynamic pressure distribution on the pads’ surfaces may be the cause.

The Campbell diagrams of the studied case (Fig. 6) show that the combi-bearing reduces the system’s first and second natural frequencies, but on the other hand, it increases the system’s third natural frequencies. It can be observed that at a rotor operational spin speed of 600 rpm, the system’s first forward natural frequency has decreased by about 2.6% from 1464 cpm (system without thrust bearing, solid line) to 1445 cpm (system with combi-bearing, dotted line), while the system’s second forward natural frequency has decreased by about 1.4% from 1574 cpm (system without thrust bearing, solid line) to 1552 cpm (system with combi-bearing, dotted line). However, the system’s third natural frequency has increased with about 4.8% from 2096 cpm (system without thrust bearing, solid line) to 2615 cpm (system with combi-bearing, dotted line). Figures 7 and 9 show that the rotor’s
The derived model shows that a possible way to eradicate (or to weaken) the rotor motions’ coupling at the combi-bearing’s location is to position the journal-bearing at (or as near as possible) the contact point (P) by decreasing the value of parameter L. In the studied case on Porjas U9, small values of L elevate the system’s first, second, and third natural frequencies to values larger than those for a system without thrust bearing. The rotor motions’ coupling due to combi-bearing has affected the dynamic behavior of the machine, because it has changed the system’s natural frequencies and vibration modes. Most of the rotor dynamics software used in the hydropower industry today does not include a detailed model of the thrust bearing. The results presented in this paper highlight the influence of the combi-bearing on the dynamics of a vertical rotating machine.
Acknowledgment

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Nomenclature

\[ E = \text{Young’s modulus (Nm}^{-2}\text{)} \]
\[ g = \text{gravity acceleration (ms}^{-2}\text{)} \]
\[ H = \text{length (m) from the thrust bearing pads surface to the contact point (P)} \]
\[ k_{x} = \text{total equivalent journal-bearing stiffness coefficient (Nm}^{-1}\text{) in X direction} \]
\[ k_{y} = \text{equivalent journal-bearing stiffness coefficient (Nm}^{-1}\text{) in Y direction} \]
\[ k_{z} = \text{equivalent thrust bearing stiffness coefficient (Nm}^{-1}\text{) in Z direction} \]
\[ L' = \text{element length (m)} \]
\[ L = \text{length (m) from the contact point (P) to the journal-bearing’s position} \]
\[ l_{0} = \text{spring initial length (m)} \]
\[ m_{\text{rot}} = \text{rotor total weight (kg)} \]
\[ N = \text{rotor spin speed in revolutions per minute (rpm)} \]
\[ r = \text{effective radius (m) of the thrust bearing} \]
\[ \theta = \text{angular rotation (rad) about the X axis} \]
\[ \phi = \text{pad (i) angular position (rad) measured from a fixed reference} \]
\[ \rho = \text{rotor density (kg m}^{-3}\text{)} \]
\[ \omega = \text{rotor angular spin speed (rad/s)} \]

Fig. 9 Rotor third lateral vibration mode: (a) rotor system with combi-bearing and (b) rotor system without thrust bearing

References

Paper E
Experimental Verification of a Combi-Bearing Model for Vertical Rotor Systems

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ABSTRACT

Combi-bearing is a combined thrust-journal bearing design used in vertical hydropower rotors. The dynamic characteristics of this component (combi-bearing) were analytically modelled in [2]. This analytic model was inserted into a finite element model of a vertical rotor rig and numerically simulated. In this paper, the simulated vertical rotor-bearings system is a small-scale vertical machine constructed to validate the analytically derived combi-bearing model. Good agreement was found between the simulation and experimental results. The simulation and experimental results showed that the journal (radial) bearing’s position relative to the contact point between the combi-bearing’s collar and the rotor influences the rotor system’s fundamental natural frequencies. Therefore the combi-bearing model needs to be included into rotor dynamic models. Neglecting the effect of this component may cause significant errors in the predicted results.

KEYWORDS: Combi-bearing, vertical rotor, natural frequency, unbalance response

NOMENCLATURE

d: Rotor shaft diameter [m]
E: Young’s modulus [N m⁻²]
g: Gravity acceleration [m s⁻²]
\([G^*]\): Element gyroscopic matrix
h: Length [m] from the contact point (P) to the collar’s centre of mass (O)
H: Length [m] from the collar’s bottom to the contact point (P)
I₉: Collar’s diametric moment of inertia [kg m²]
I₀: Collar’s polar moment of inertia [kg m²]
\([K^*]\): Element stiffness matrix
kₒ: Ball bearing radial stiffness coefficient [N m⁻¹]
k₂: Stiffness coefficient [N m⁻¹] of axial spring (i) in the Z direction
kₙ₋ₖₙ: Combi-bearing’s radial stiffness coefficients [N m⁻¹] in the X and Y directions
L: Length [m] from the contact point (P) to the journal-bearing’s position
l₁...₆: lengths [m] of elements 1 to 6
L₀: Initial length [m] of axial springs
L₉: Rotor total length [m]
\([M^*_g]\): Element translational mass matrix
\([M^*_r]\): Element rotatory mass matrix
m: Weight of the collar [kg]
mₑ: Imbalance in the collar [kg m]
mₘₑ: Rotor total weight [kg]
N: Rotor spin speed in revolutions per minute [rpm]
r: Effective radius [m] of the thrust bearing
θₓ: Angular rotation [radians] about the X axis
θᵧ: Angular rotation [radians] about the Y axis
φᵢ: Spring (i) angular position [radians] measured from a fixed reference
ρ: Rotor density [kg m⁻³] of structural steel material
ω: Rotor angular spin speed [rad/s]
INTRODUCTION

The influence of a thrust bearing is normally simplified or just ignored in rotor dynamic modelling of horizontal rotating machines. The main reason is that the axial load is considered small for most horizontal machines, except those machines designed to transmit thrust, such as aircraft engines. In vertical rotating machines, such as hydropower rotors, or in the rotating machines designed to transmit thrust, the thrust bearing becomes an important machine component that can influence the dynamic of the entire machine. White, Torbergsen and Lumpkin [1] modelled and simulated a vertical pump with tilting-pad journal bearings. They assumed that the encountered disagreement between the measurement and simulation result of the response at the lower no-drive-end (NDE) tilting-pad journal bearing was due to the neglected stiffness and damping coefficients of the thrust bearing. T. N. Shiau, W. C. Hsu, J. R. Chang [4] showed that there is a coupling between the rotor translational and rotational displacements at the thrust bearing location.

A combined thrust-journal bearing design, commonly called combi-bearing, is widely used in vertical hydropower plants. Most of the rotor dynamics software used in the hydropower industry today do not include a sufficiently accurate detailed model of this component (combi-bearing). The results presented in [2] highlighted the influence of the combi-bearing on the dynamics of a vertical rotating machine. The main purpose of the performed experiments is to validate the analytically derived model of the combi-bearing derived in [2] by comparing the numerical simulation results with the experimental ones. The simulation and experimental results of the rotor rig showed that the journal (radial) bearing’s position relative to the contact point between the combi-bearing’s collar and the rotor influences the rotor system’s fundamental natural frequencies.

1. VERTICAL ROTOR WITH A COMBI-BEARING

1.1 Experimental set-up

The experiments were performed on a vertical rotor rig, which is schematically represented in Fig. 1(a). Fig. 1(b) is a digital picture of the rig set-up. The radial and thrust bearings together constitute a machine component commonly called a combi-bearing. The rollers and ball bearings, the springs and the flexible coupling were

![Schematic and Digital Picture of Vertical Rotor Rig](image)

Fig. 1. (a): Schematic view of the vertical rig’s symmetry plane, (b): Picture of the actual vertical rig
manufactured by SKF, LESJÖFORS SPRINGS & PRESSINGS AB and BOMEX, respectively. Some of the equipment used in the experiments is from the Bently Nevada Rotor Kit Model RK 4: the electric motor, the motor speed controller device, the proximitor assembly, the proximity probes for motor speed control and rotor motion measurement. The rotor motion is measured at 25 mm from the bottom ball bearing due to the large vibration’s amplitudes when the rotor runs near its critical speed. The proximity probe for rotor motion measurement is mounted on a support structure which, is attached to the fundament, while the probe for motor speed control is attached on the bottom ball bearing’s support structure as it can be seen in Fig. 1. (b). All the other components that have not been mentioned above were designed by the authors of this paper, and the manufacture was handled by the technicians at the workshop of Luleå University of Technology. Figures 2a and 2b show a symmetry plane and the upper view of the combi-bearing’s collar. The collar’s geometric parameters and inertia axes coordinates are also show in the figures. In Fig. 3, springs are mounted between the pin-roller assembly and a screw. A schematic illustration of the pin-roller assembly is shown in Fig. 4. The rollers are used to reduce friction between the rotating collar and the bearings.

1.2 Combi-bearing

Assuming a rigid and massless collar, the linearized total reaction forces and moments applied to the rotor at contact point (P) were derived in [2]. However, when the weight of the collar is taken into account, and if the springs in both axial and radial directions are angularly equidistant the linearized total reaction forces and moments applied to the rotor at contact point (P) become:

\[ F_{x \text{ axial}} = -n k_y z \]  
\[ F_{y \text{ axial}} = -k_y x - k_x L \theta_x \]  
\[ F_y = -k_y y + k_x L \theta_x \]  
\[ M_{x \text{ axial}} = \sum_{i=1}^{n} k_y \left[ r^2 \sin \phi_i \sin \phi_i \right] + k_y L_y - k_x L^2 \theta_x - (m_c g h) \theta_x \]  
\[ M_{y \text{ axial}} = \sum_{i=1}^{n} k_y \left[ r^2 \cos \phi_i \cos \phi_i \right] - k_x L_x - k_x L^2 \theta_x - (m_c g h) \theta_y \]

Where \( k_x, k_y \) are the stiffness coefficients of the radial bearing in the \( X \) and \( Y \) directions, \( k_i \) (\( i = 1, \ldots, n \)) is the stiffness coefficient of a single vertical spring, and \( n \) being the total number of the vertical springs. The angles \( \theta_x \) and \( \theta_y \) are the rotations about the \( X \) and \( Y \) axes passing through the point (P) shown in Fig. 2(a, b). \( r \) is the effective radius of the thrust bearing, \( L \) is the length from the contact point (P) to the radial bearing’s position, \( \phi_i \) is the angular position of the vertical spring \( i \) measured from the fixed \( X \) axis of reference as shown in Fig. 2b. The thrust bearing has \( n = 4 \) identical vertical springs, which are angularly equidistant with \( \Delta \phi = \frac{360^\circ}{n} = 90^\circ \), thus \( \phi_{i} = i \times \Delta \phi \).
1.3 Vertical flexible rotor (finite element method)

The vertical flexible rotor was modelled as a discrete multi-degree of freedom system using the finite element method (FEM). The entire rotor is divided into six elements with seven nodes numbered in ascending order from the top to the bottom of the vertical rotor. Two ball bearings are located at the end nodes, node 1 (top bearing) and 7 (bottom bearing), and the combi-bearing is located at node 2. The derived model of the combi-bearing will be inserted into the rotor finite element model at the appropriate node 2. The damping of the rotor material has been neglected in the model.

The equations of motion for a typical eight degrees-of-freedom rotating element [3] are:

\[
[M_f] + [M_s] \ddot{\Theta} + \omega \dot{\Theta} + [K_g] \ddot{\Theta} - [G_g] \dot{\Theta} - [M_c] \ddot{Q} = \dot{Q}
\]

\[
\ddot{q} = [X_n \ Y_n \ \dot{\Theta}_n \ \dot{\Theta}_n \ X_{n+1} \ Y_{n+1} \ \dot{\Theta}_{n+1} \ \dot{\Theta}_{n+1}]
\]

Where \( n=1, 2 \ldots \) total number of elements; \([M_f], [M_s], [K_g], [G_g], [M_c]\) are the element translational mass matrix, rotatory mass matrix, gyroscopic matrix, stiffness matrix and external applied load vector, respectively; these matrices can be found in [3]. For the present case, the number of elements is 6 and the number of nodes = number of elements + 1 = 7.

The contact point \( P \) between the collar and the rotor is not at the collar centre of mass, therefore the collar inertia properties need to be added at the corresponding node 2 (which is the contact point \( P \)). The added collar inertia properties at point \( P \) are:

\[
[M_s] = \begin{bmatrix}
 m_c & 0 & 0 & (m_c h) \\
 0 & m_c & -(m_c h) & 0 \\
 0 & -(m_c h) & (I_{cc} + m_c h^2) & 0 \\
 (m_c h) & 0 & 0 & (I_{cc} + m_c h^2)
\end{bmatrix}, \quad [G_s] = \begin{bmatrix}
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & I_{dc} \\
 0 & 0 & -I_{dc} & 0
\end{bmatrix}
\]

(6)
\[ \begin{bmatrix} M_c \end{bmatrix} \text{ and } \begin{bmatrix} G_c \end{bmatrix} \text{ will be added to the assembled mass and gyroscopic matrices, respectively, at the corresponding node 2. } I_{dc} \text{ is the collar diametric moment of inertia about the axis passing through its centre of mass, and } I_{pc} \text{ is the collar polar moment of inertia about the Z axis.} \]

One more degree of freedom (the Z dimension) should be added to the global system. The rotor is assumed rigid in its axial dimension, meaning that the Z dimension defines a rigid body motion. The entire assembled vertical rotor-bearings system becomes:

\[
\begin{bmatrix}
\ddot{q}
\end{bmatrix} + \omega \begin{bmatrix} G \end{bmatrix} \begin{bmatrix}
\ddot{q}
\end{bmatrix} + \begin{bmatrix} K \end{bmatrix} \begin{bmatrix}
\ddot{q}
\end{bmatrix} = \begin{bmatrix} F \end{bmatrix}
\]

(7)

\[
\begin{bmatrix}
X_1, Y_1, \theta_1, \ldots, X_7, Y_7, \theta_7, z
\end{bmatrix}
\]

\[
\begin{bmatrix}
M_{tot}
\end{bmatrix} \begin{bmatrix}
q
\end{bmatrix} \begin{bmatrix}
K
\end{bmatrix} \begin{bmatrix}
G
\end{bmatrix} \begin{bmatrix}
M
\end{bmatrix} \begin{bmatrix}
B
\end{bmatrix} \begin{bmatrix}
F
\end{bmatrix} = \begin{bmatrix}
0
\end{bmatrix}
\]

(11)

where \( \begin{bmatrix} K_{gb} \end{bmatrix} \) contains the bearings coefficients from equations (8), (9) and (10). The linear homogenous equations (11) can be reformulated into its state space form for eigenvalue analysis.

An imbalance in the collar will generate a force and a moment on the rotor at the contact point (P) which is node 2 in the rotor finite element model. Assuming a harmonic excitation \( \begin{bmatrix} F_e \end{bmatrix} \), the system response can be obtained as follows:

\[
\begin{bmatrix}
M \end{bmatrix} \begin{bmatrix}
q
\end{bmatrix} + \omega \begin{bmatrix} G \end{bmatrix} \begin{bmatrix}
q
\end{bmatrix} + \begin{bmatrix} K \end{bmatrix} \begin{bmatrix} \ddot{q} \end{bmatrix} = \begin{bmatrix} F_e \end{bmatrix}
\]

(12)

\[
\begin{bmatrix}
F_e
\end{bmatrix} = \begin{bmatrix}
F_x, F_y, M_x, M_y
\end{bmatrix}
\]

(13)

With use of this harmonic excitation, the amplitude of the system response is computed at node 6. Node 6 is located at a distance of 25 mm from the bottom ball bearing at node 1. The rotor response was measured far from the collar due to the large vibration’s amplitudes when the rotor runs close to its critical speed.
1.5 Results

The rotor rig’s data and parameters are:

\[ r = 61.3 \times 10^{-3} \text{ m} \]
\[ H = 77 \times 10^{-3} \text{ m} \]
\[ h = 0.0663 \text{ m} \]
\[ k_b = 34.71 \times 10^7 \text{ Nm}^{-1} \]
\[ k_i = k_y = 14.26 \times 10^3 \text{ Nm}^{-1} \]
\[ k_i = 3.43 \times 10^3 \text{ Nm}^{-1} \]
\[ m_c = 0.5031 \text{ kg} \]
\[ \rho = 7810 \text{ kg m}^{-3} \]
\[ E = 210 \times 10^9 \text{ Nm}^{-2} \]
\[ I_{dc} = 0.0027 \text{ kg m}^2 \]
\[ I_{pc} = 0.00091 \text{ kg m}^2 \]
\[ \rho = 1077 \text{ Nm} \]
\[ \rho = 1026.14 \text{ Nm} \]
\[ \rho = 1071.34 \text{ Nm} \]
\[ L_v = 0.785 \text{ m} \]
\[ d = 0.00721 \text{ m} \]
\[ l_0 = 0.025 \text{ m} \]
\[ l_2 = l_5 = l_4 = l_3 = 0.155 \text{ m} \]
\[ l_1 = 0.14 \text{ m} \]

Where \( l_1, l_2, l_3, l_4, l_5 \) and \( l_6 \) are the lengths of rotor elements one to six, respectively.

---

Fig. 5. Theoretical Campbell diagram:
Green line (\( L=r=0 \)), Solid blue line (\( L=25 \text{ mm} \)), Dotted blue line (\( L=55 \text{ mm} \))

Fig. 6. Theoretical rotor unbalance responses computed at node 6

Fig. 7. Rotor unbalance response computed and measured at node 6: (a) \( L=25 \text{ mm} \), (b) \( L=55 \text{ mm} \)

Fig. 5 contains the theoretical Campbell diagram showing the undamped fundamental natural frequencies of the rotor system for different configurations of the combi-bearing. These results were obtained by using the combi-bearing’s model according to equations (1) to (5) and the rotor finite element model according to [3]. The parameter \( L \) was set to \( L=25 \text{ mm} \) (solid blue lines) and \( L=55 \text{ mm} \) (dotted blue lines). The result for \( L = r = 0 \) (green line) corresponds to a model where the axial springs are removed and the radial bearing is located at the contact point (\( P \)) as they frequently do in industry. The plotted undamped fundamental natural
frequencies are expressed in cycles per minute [cpm], while the rotor driving frequencies are given in revolutions per minute [rpm].

Fig. 6 and Figs. 7(a, b) display the theoretical and experimental unbalance responses of the rotor system. The rotor response was computed and measured at a distance of 25 mm (node 6) from the bottom ball bearing.

2. DISCUSSIONS AND CONCLUSIONS

The experimental results in Figs. 7(a, b) verify the accuracy of the combi-bearing’s model derived in [2]. The theoretical Campbell diagram in Fig. 5 predicts the first critical speeds of rotor rig to be 1377 rpm (revolutions per minute) when \( L=25 \text{ mm} \) and 1506 rpm when \( L=55 \text{ mm} \). These theoretically predicted critical speeds are confirmed by the experimental results in Figs. 7(a, b). The experimental rotor unbalance response in Fig. 7(a) goes into resonance between \( N = 1300 \text{ rpm} \) and \( N = 1525 \text{ rpm} \) when the radial bearing is set to a distance of \( L = 25 \text{ mm} \) from the contact point. When the radial bearing is set to a distance of \( L = 55 \text{ mm} \); the experimental rotor unbalance response in Fig. 7(b) goes into resonance between \( N = 1450 \text{ rpm} \) and \( N = 1670 \text{ rpm} \). For safety reasons, it is not advised to run a machine at or near the critical speed due to large vibration amplitudes. The proximity sensors used in the experiments also have a limited sensing range of \( \pm 1 \text{ mm} \) which is exceeded at rotor speeds close to the theoretically predicted values: 1377 rpm \( (L = 25 \text{ mm}) \) and 1506 rpm \( (L = 55 \text{ mm}) \). Therefore, all experimental data were collected far down near the bottom ball bearing where the vibration amplitudes are expected to be small. The results in Fig. 5, Fig. 6 and Fig. 7 clearly reveal that for the present studied case the combi-bearing stiffens the rotor system by increasing the natural frequencies with about 13% when \( L=25 \text{ mm} \) and with about 23% when \( L=55 \text{ mm} \). In the hydropower industry, it is common practice to omit or neglect the influence of the combi-bearing when modelling rotor dynamics. The results presented in this paper show that the combi-bearing model might be needed in rotor dynamic models. Neglecting the effect of this component (combi-bearing) can cause errors in the predicted results.

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4. REFERENCES

Paper F
Misalignment in Combi-Bearing: A Cause of Parametric Instability in Vertical Rotor Systems

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ABSTRACT
The dynamic characteristics of the combi-bearing (combined thrust-journal bearing) in vertical rotor systems were analytically modelled in [1] and experimentally validated in [2]. An angular misalignment, which may be caused by a possible manufacture or assembling error, is introduced in the combi-bearing’s rotating collar. A new model of the defected combi-bearing has been derived. The derived model shows that the angular misalignment in the combi-bearing’s rotating collar generates an asymmetry in the rotor system at the combi-bearing’s location. The rotor system’s stiffness in its two translational $X$ and $Y$ directions differ at the combi-bearing’s location. Constant parameters and/or coefficients in rotating asymmetric structures appear to change with time when observed in the stationary frame. These time dependent parameters (coefficients) are the source of the so-called parametric instability in rotating systems. If the collar angular misalignment is located in $X$-$Z$ plane all rotor motions in this plane at the contact point between the combi-bearing and the rotor will be coupled. A parametric instability is observed within certain ranges of the rotor speed depending on the magnitude of the angular misalignment.

KEYWORDS: Combi-bearing, vertical rotor, angular misalignment, asymmetry, parametric instability

NOMENCLATURE

d: Rotor shaft diameter [m]
$D_r$: Rotor disc diameter
$E$: Young’s modulus [N m$^{-2}$]
$g$: Gravity acceleration [m s$^{-2}$]
$h$: Length [m] from the contact point ($P$) to the collar’s centre of mass ($O$)
$H$: Length [m] from the collar’s bottom to the contact point ($P$)
$I_d$: Rotor disc diametric moment of inertia [kg m$^2$]
$I_p$: Rotor disc polar moment of inertia [kg m$^2$]
$I_{dc}$: Collar’s diametric moment of inertia [kg m$^2$]
$I_{pc}$: Collar’s polar moment of inertia [kg m$^2$]
$k_r$: Radial stiffness coefficient [N m$^{-1}$] for the bearings at rotor ends
$k_i$: Stiffness coefficient [N m$^{-1}$] of axial spring ($i$) in the $Z$ direction
$k_x, k_y$: Combi-bearing’s radial stiffness coefficients [N m$^{-1}$] in the $X$ and $Y$ directions
$L_r$: Rotor total length [m]
$L$: Length [m] from the contact point ($P$) to the journal-bearing’s position
$L_0$: Initial length [m] of axial springs
$m_c$: Weight of the collar [kg]
$m_r$: Rotor disc weight [kg]
$N$: Rotor spin speed in revolutions per minute [rpm]
r: Effective radius [m] of the thrust bearing
t: Rotor disc thickness
$\alpha, \beta$: Angular misalignments in the rotating collar
$\theta_x$: Angular rotation [radians] about the $X$ axis
$\theta_y$: Angular rotation [radians] about the $Y$ axis
$\phi$: Spring ($f$) angular position [radians] measured from a fixed reference
$\rho$: Rotor density [kg m$^{-3}$] of structural steel material
$\omega$: Rotor angular spin speed [rad/s]
INTRODUCTION

A combined thrust-journal bearing design, commonly called a combi-bearing, is widely used in vertical hydropower plants as shown in Fig. 1(a). Most of the rotor dynamics software used in the hydropower industry today does not include a sufficiently detailed model of this component (combi-bearing). The results presented in [1, 2] highlighted the influence of the combi-bearing on the dynamics of a vertical rotating machine.

When an angular misalignment, due to manufacture or assembling error in the combi-bearing’s rotating collar, is introduced into the model according to Fig. 1(b) the transverse stiffness of the rotor in one plane differs from that in the other plane at the combi-bearing’s location. Then the resultant equations of motion of the rotor system in the stationary frame will have coefficients that vary sinusoidally with time. This will result in a system that may be parametrically excited. Ecker and Pumhössel [3] studied a case of parametric excitation due to a periodic axial force on a vertical Laval-Jeffcott-rotor and they showed that if the frequency of the axial force is chosen to be a combination of certain natural bending frequencies of the rotor, an increase of stability is noticed. A detailed analysis of parametrically excited asymmetric rotors can be found in [4] and many other textbooks as [5, 6, 7]. By introducing an angular misalignment in the combi-bearing’s rotating collar, sinusoidally time-dependent coefficients appear in the derived equations of motion. The eigenanalysis is not feasible for systems having time-dependent coefficients; therefore the system’s equations of motion should be transformed into the rotating frame of reference. In the rotating frame of the reference all coefficients and parameters are constant. A simplified rotor model (Fig. 1(b)) similar to that studied in [2] was used for the analysis, but with a misaligned collar instead. A free-body diagram was used to derive all forces and moments applied to the rotor at the contact point with the combi-bearing’s rotating collar. The discretization of the rotor shaft was done by means of the finite element method. The rotor disc and the combi-bearing’s collar were assumed rigid; therefore their inertia properties were applied at respective nodes in the rotor finite element model. The results are presented in form of plots of the real and imaginary parts of the the system’s eigenvalues in the rotating frame. The rotor response is also plotted to show how the motion becomes unstable for a rotor speed within the range of parametric instability.

1. A MISALIGNMENT IN THE COMBI-BEARING’S ROTATING COLLAR

When angular misalignments $\alpha$ and $\beta$ are introduced in the rotating collar according to Fig. 1, 3(b, c), the combi-bearing’s dynamic influence on the entire system needs to be reanalysed.

![Fig. 1. (a) Shematic view (right) of a vertical hydropower unit together with a discretized rotor (left), (b) Symmetry plane of a vertical rotor model with a misaligned collar](image)

The radial and thrust bearings together constitute a machine component commonly called a combi-bearing. The rotating collar is connected to the rotor at point $P$. Misalignment may occur due to assembly or manufacturing errors. This is shown in Fig. 1, where $\alpha$ is an angular misalignment due to a possible assembly error, and $\beta$ is
an angular misalignment due to a manufacturing error. The combi-bearing’s radial stiffness in the two symmetry planes are $k_x$ and $k_y$ and the axial stiffness for a single spring is $k_i$. The rotor ends are supported by two radial bearings, each with stiffness $k_b$. The balls between the springs and collar/rotor are to represent frictionless contacts.

1.1 Combi-bearing with a misaligned rotating collar

The dynamic characteristics of the combi-bearing with angular misalignments in the rotating collar can be modelled as follows:

\[ F_{ii} = k_i (L_0 - L_i) \]

Where $L_i = L_0 - z + (r \sin \phi_i + y) \tan \theta_i + (r \cos \phi_i - x) \tan (\theta_i - \alpha)$. 

\[ F_{ij} = -k_i (L_0 - L_j) = -k_i [z - (r \sin \phi_j + y) \tan \theta_j - (r \cos \phi_j - x) \tan (\theta_j - \alpha)] \]

The radial bearing’s reaction forces in the $X$ and $Y$ directions are linear functions of the collar displacements and can therefore be obtained as follows:
\[
F_{JBx} = -k_y \left[ x + (L - z) \tan \theta_y + \beta \right] \\
F_{JBy} = -k_y \left[ y - (L - z) \tan \theta_y \right]
\]  

(2)

In directions perpendicular to the collar:
\[
F_{Jx} = \frac{F_{r}}{\cos(\theta_y - \alpha)}; \quad F_{Jz} = \frac{F_{r}}{\cos \theta_y}; \quad F_{JBx} = \frac{F_{r}}{\cos(\theta_y + \beta)}; \quad F_{JBy} = \frac{F_{r}}{\cos \theta_y}
\]

The total reaction force will be the summation of all forces’ contributions from both thrust and radial bearings’ computed reaction forces as follows:
\[
F_{x_{r,\text{total}}} = \left( \sum_{i=1}^{n} F_{Jx_i} \right) \cos \theta_y - \left( \sum_{i=1}^{n} F_{Jz_i} \right) \sin \theta_y
\]  

(4)

\[
F_{y_{r,\text{total}}} = F_{h,\text{rad}} + \left[ \sum_{i=1}^{n} F_{Jx_i} \right] \sin(\theta_y - \alpha) = F_{h,\text{rad}} + \left[ \sum_{i=1}^{n} F_{Jx_i} \right] \tan(\theta_y - \alpha)
\]

(5)

\[
F_{z_{r,\text{total}}} = -k_y \left[ x + (L - z) \tan(\theta_y + \beta) \right] + \left[ \sum_{i=1}^{n} \left\{ -k_y \left[ - (r \sin \phi_i + y) \tan(\theta_y + \beta) - (r \cos \phi_i - x) \tan(\theta_y - \alpha) \right] \right\} \right] \tan \theta_y
\]

(6)

The total reaction moments will be the summation of all moments’ contributions from both thrust and radial bearings’ computed reaction moments as follows:
\[
M_{x_{r,\text{total}}} = \left( \sum_{i=1}^{n} M_{x_i} \right) - \left[ \sum_{i=1}^{n} F_{Jx_i} \right] \left[ \frac{L - z}{\cos \theta_y} \right] \cos \theta_y - \left[ \sum_{i=1}^{n} F_{Jz_i} \right] \left[ \frac{L - z}{\cos \theta_y + \beta} \right] \cos(\theta_y + \beta)
\]  

(7)

\[
M_{y_{r,\text{total}}} = \left( \sum_{i=1}^{n} M_{y_i} \right) + \left[ \sum_{i=1}^{n} F_{Jx_i} \right] \left[ \frac{L - z}{\cos(\theta_y + \beta)} \right] \cos(\theta_y + \beta) - \left[ \sum_{i=1}^{n} F_{Jz_i} \right] \left[ \frac{L - z}{\cos \theta_y} \right] \cos \theta_y
\]

(8)

The thrust bearing reaction moment applied to the rotor at contact point \((P)\) is derived for a single vertical spring as follows:
\[
M_{x_{t,\text{rad}}} = F_{t,\text{rad}} \cos \theta_y \left[ r \sin \phi_i + y \right] - \left[ F_{t,\text{rad}} \right] \sin \theta_y \left[ H \cos \theta_y - (r \sin \phi_i + y - H \sin \theta_y) \tan \theta_y \right]
\]

(9)

\[
M_{y_{t,\text{rad}}} = \left[ F_{t,\text{rad}} \right] \left[ r \sin \phi_i + y \right] - \left[ F_{t,\text{rad}} \right] \tan \theta_y \left[ H \cos \theta_y - (r \sin \phi_i + y - H \sin \theta_y) \tan \theta_y \right]
\]

(10)

The vertical springs are angularly equidistant:
\[
\sum_{i=1}^{n} \left( \sin \phi_i \right) = \sum_{i=1}^{n} \left( \cos \phi_i \sin \phi_i \right) = \sum_{i=1}^{n} \left( \cos \phi_i \right) = \sum_{i=1}^{n} \left( \sin \phi_i \right) = 0
\]

Considering small motions \(x, y, z, \theta_x, \theta_y\) and \(\theta_y \times \tan \alpha \ll 1, \theta_y \times \tan \alpha \ll 1\) gives:
\[
\tan(\theta_y - \alpha) = \frac{\tan \theta_y - \tan \alpha}{1 + (\tan \theta_y \times \tan \alpha)} = \theta_y - \tan \alpha, \quad \text{and} \quad \tan(\theta_y + \beta) = \frac{\tan \theta_y + \tan \beta}{1 - (\tan \theta_y \times \tan \beta)} = \theta_y + \tan \beta
\]
The linearized total reaction forces and moments become:

\[ F_{\text{total}} = -k_z (\tan \beta)^2 + k_z \mathbf{z} + (k_x \tan \alpha + k_z \tan \beta) x + 2(k_x L \tan \beta) \theta_x + k_x L (\tan \beta)^2 \]  \hspace{1cm} (11)

\[ F_{\mathbf{r}} = -k_z \mathbf{r} + k_x L \theta_x + (k_x \tan \alpha + k_z \tan \beta) \mathbf{z} - k_x L \tan \beta \] \hspace{1cm} (12)

\[ F_{\mathbf{z}} = -k_z \mathbf{z} + k_x L \theta_x \] \hspace{1cm} (13)

\[ M_{\text{total}} = -\sum_{i=1}^{n} k_x r^2 (\sin \phi_i)^2 \theta_i + k_x L \mathbf{z} - \mathbf{m} g \mathbf{h} - \mathbf{m} \mathbf{g} \] \hspace{1cm} (14)

\[ M_{\mathbf{r}} = -\sum_{i=1}^{n} k_x r^2 (\cos \phi_i)^2 \left[ \mathbf{l} + (\tan \alpha)^2 \right] \theta_i - \mathbf{m} g \mathbf{h} \mathbf{r} + k_x L H \tan \alpha - \mathbf{M}_{\mathbf{r}} \] \hspace{1cm} (15)

where

\[ M_{\mathbf{r}} = -k_x \mathbf{r} \left( L - R \sin \beta \right) \mathbf{x} - k_x L \left( 2L \sin \beta - 2R \sin \beta \right) \left( \cos \beta \right)^2 + \mathbf{M}_{\mathbf{r}} \]

\[ + k_x \tan \beta \left( 2L - R \sin \beta \right) \mathbf{z} - k_x L \tan \beta \left( L - R \sin \beta \right) \mathbf{y} \]

and \( k_x = \sum_{i=1}^{n} k_i \).

The parameters \( k_x, k_z \) are the combi-bearing’s radial stiffness coefficients in the \( X \) and \( Y \) directions, \( k_i, i = 1 \ldots n \) is the stiffness coefficient of a single vertical spring, and \( n \) being the total number of the vertical springs. The angles \( \theta_x \) and \( \theta_y \) are the rotations about the \( X \) and \( Y \) axes passing through the point \( P \) in Fig. 2(a, b), \( r \) is the effective radius of the thrust bearing, \( L \) is the length from the contact point \( P \) to the radial bearing’s position, \( \theta_i \) is the angular position of the vertical spring \( i \) measured from the fixed \( X \) axis of reference as shown in Fig. 2b, \( h \) is the distance from the contact point \( P \) to the collar’s centre of mass and \( R \) is the collar radius according to Fig. 2a. The angular misalignments \( \alpha \) and \( \beta \) couple the rotor motions \( x, z, \theta_x \), and generate static forces and moment at the contact point \( P \) as it can be seen in equations (11), (12) and (15).

In the force equation (12), it can be observed that the term \( (\tan \alpha)^2 \mathbf{z} \) is due to the angular misalignment \( \alpha \) of the lower part of the rotating collar. In the present analyses, the radial bearing is isotropic, meaning that \( k_x = k_z \), therefore the term \( (\tan \alpha)^2 \mathbf{z} \) generates a difference between the stiffness in the \( X \) and \( Y \) directions. Due to the rotating motion of the collar, the coefficient \( k_x (\tan \alpha)^2 \) will periodically oscillate when observed in the stationary frame. This time dependent coefficient \( k_x (\tan \alpha)^2 \) will generate a parametric excitation in the rotor system. For the sake of simplicity, let the thrust bearing have four \( n = 4 \) identical vertical springs which are angularly equidistant with \( \Delta \phi = \frac{360^\circ}{n} = 90^\circ \), thus \( \phi_i = i \times \Delta \phi, \ i = 1,2,\ldots,4 \), and \( k_x = 4 \times k_i \).

2. ASYMMETRIC FLEXIBLE ROTOR SYSTEM

Due to the asymmetry of the rotor system, the equations of motion are going to be derived in the rotating frame according to [4]. In the rotating frame, all coefficients (parameters) remain constant. The vertical flexible rotor is modelled as a discrete multi-degree of freedom system using the finite element method (FEM). The entire rotor is divided into six elements with seven nodes numbered in ascending order from the top to the bottom of the vertical rotor. Two ball bearings are located at the ends’ nodes, node 1 (top bearing) and 7 (bottom bearing); and the combi-bearing is located at node 2. The derived model of the combi-bearing will be inserted into the rotor finite element model at the appropriate node 2. The damping of the rotor material has been neglected in the model.
2.1 Shaft element

The equations of motion for a typical eight degrees-of-freedom rotating element in the rotating frame of reference are [4]:

\[
\begin{bmatrix}
\tilde{\mathbf{M}}_s + \mathbf{K}_s
\end{bmatrix}\ddot{\mathbf{q}}_s + \omega \left(\begin{bmatrix}
\tilde{\mathbf{M}}_s + \mathbf{K}_s
\end{bmatrix} + \mathbf{G}_s\right)\dot{\mathbf{q}}_s + \omega^2 \left(\begin{bmatrix}
\tilde{\mathbf{M}}_s + \mathbf{G}_s
\end{bmatrix} + \mathbf{K}_s\right)\mathbf{q}_s = \tilde{\mathbf{Q}}_s
\]

(16)

All matrices in Eq. (16) are found in [4].

2.2 Discs

The equations of motion for the disc (collar) in the rotating frame of reference are [4]:

\[
\begin{bmatrix}
\tilde{\mathbf{M}}_d + \mathbf{K}_d
\end{bmatrix}\ddot{\mathbf{q}}_d + \omega \left(\begin{bmatrix}
\tilde{\mathbf{M}}_d + \mathbf{K}_d
\end{bmatrix} + \mathbf{G}_d\right)\dot{\mathbf{q}}_d + \omega^2 \left(\begin{bmatrix}
\tilde{\mathbf{M}}_d + \mathbf{G}_d
\end{bmatrix} + \mathbf{K}_d\right)\mathbf{q}_d = \tilde{\mathbf{Q}}_d
\]

(17)

The contact point \((P)\) between the collar and the rotor is not at the collar centre of mass, therefore the collar inertia properties need to be evaluated and added at node 2 (which is the contact point \((P)\)). The collar inertia properties evaluated at point \((P)\) are:

\[
\begin{bmatrix}
m_c & 0 & 0 & (m_c h) \\
0 & m_c & -(m_c h) & 0 \\
0 & -(m_c h) & (I_{dc} + m_c h^2) & 0 \\
(m_c h) & 0 & 0 & (I_{dc} + m_c h^2)
\end{bmatrix}, \begin{bmatrix}
G_c \\
I_{cd} \\
I_{dc} \\
I_{cc}
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & I_p & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

(18)

\(m_c, I_{dc}, I_p\) are the collar weight, the collar diametric moment of inertia about the axis passing through its centre of mass, the collar polar moment of inertia about the Z axis, respectively. The rotor disc inertia properties \(\tilde{\mathbf{M}}_{rd}\) and \(\mathbf{G}_{rd}\) will be added at corresponding node 5 where the rotor disc is located.

\[
\begin{bmatrix}
m_r & 0 & 0 & 0 \\
0 & m_r & 0 & 0 \\
0 & 0 & I_r & 0 \\
0 & 0 & 0 & I_r
\end{bmatrix}, \begin{bmatrix}
G_{rd} \\
I_{rd} \\
I_{rd}
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

(19)

\(m_r, I_r\) and \(I_{rd}\) are the rotor disc weight, the rotor disc polar moment of inertia, and the rotor disc diametric moment of inertia, respectively. All matrices in Eq. (17) are found in [4], and the matrices \(\tilde{\mathbf{M}}_{rd}, \mathbf{G}_{rd}\) are the principal input for Eq. (17).

2.3 Assembling the matrices

The process of assembling the matrices is exactly the same as that used for models in the stationary frame of reference when the rotor system is symmetric. However, there are now more matrices to assemble. Assembling all the elements and discs (collar) matrices gives the following equations of motions [4]:

\[
\begin{bmatrix}
\mathbf{M}_s + \omega \left(\begin{bmatrix}
\mathbf{M}_s + \mathbf{G}_s\end{bmatrix} + \mathbf{K}_s\right) + \omega^2 \left(\begin{bmatrix}
\mathbf{M}_s + \mathbf{G}_s\end{bmatrix} + \mathbf{K}_s\right)\end{bmatrix}\ddot{\mathbf{q}} = \tilde{\mathbf{Q}}
\]

(20)

\[
\tilde{\mathbf{q}} = \begin{bmatrix}
X_1 & Y_1 & \theta_1 & \ldots & \ldots & X_7 & Y_7 & \theta_7 & \ldots & \theta_7 & z
\end{bmatrix}^T
\]

\(\tilde{\mathbf{Q}}\) is a vector containing the bearings’ total reaction forces and moments together with the external applied forces and moments.
\[
\mathbf{Q} = \left[ \begin{array}{cccc}
\mathbf{F}_{34} & \mathbf{F}_{32} & \mathbf{\bar{Q}}_1 & \mathbf{\bar{Q}}_2 \\
\mathbf{F}_{34} & \mathbf{F}_{32} & \mathbf{\bar{Q}}_1 & \mathbf{\bar{Q}}_2 \\
\mathbf{F}_{34} & \mathbf{F}_{32} & \mathbf{\bar{Q}}_1 & \mathbf{\bar{Q}}_2 \\
\mathbf{F}_{34} & \mathbf{F}_{32} & \mathbf{\bar{Q}}_1 & \mathbf{\bar{Q}}_2 \\
\end{array} \right] + \left[ \begin{array}{c}
m_{rot} \\
m_{rot} \\
m_{rot} \\
m_{rot} \\
\end{array} \right], \quad m_{rot} \text{ being the rotor total weight.}
\]

\[
\mathbf{F}_{34} = \begin{bmatrix} -k_{b1}X_1 & -k_{b2}Y_1 & 0 & 0 \end{bmatrix};
\mathbf{F}_{32} = \begin{bmatrix} F_{x_{rot}} & F_{y_{rot}} & M_{x_{rot}} & M_{y_{rot}} \end{bmatrix};
\]

\[
\mathbf{\bar{Q}}_{34} = \begin{bmatrix} -k_{b1}X_1 & -k_{b2}Y_1 & 0 & 0 \end{bmatrix}
\]

\[
\mathbf{F}_{34}, \mathbf{F}_{32} \text{ and } \mathbf{\bar{Q}}_{34} \text{ contain the bearings' reaction forces and moments applied at nodes 1, 2 and 7, respectively.}
\]

\[
k_{b1} \text{ is the stiffness coefficient for the radial bearings at rotor ends, } k_{x}, k_{y} \text{ are the radial stiffness coefficients in } X \text{ and } Y \text{ directions for the combi-bearing.}
\]

### 2.4 Eigenvalue problem and the system response

To obtain the system’s natural frequencies in the rotating frame, the eigenvalue problem has to be formulated for the homogenous system. By omitting gravitational and imbalance forces, the equations (20) can be rewritten in homogenous form as follows:

\[
\begin{bmatrix} \mathbf{\bar{M}}_0 \mathbf{\bar{q}} + \omega^2 \left[ \mathbf{\bar{K}}_0 + \mathbf{\bar{G}}_0 \right] \mathbf{\bar{q}} + \mathbf{\bar{K}}_0 \mathbf{\bar{q}} \end{bmatrix} = \mathbf{\bar{Q}}_{\text{new}}
\]

\[
\mathbf{\bar{Q}}_{\text{new}} = \begin{bmatrix} \mathbf{F}_{34} & \mathbf{F}_{32} & \mathbf{\bar{Q}}_1 & \mathbf{\bar{Q}}_2 \end{bmatrix}^{T}
\]

The linear bearings’ reaction forces and moments \( \mathbf{\bar{Q}}_{\text{new}} \) can be written as: \( \mathbf{\bar{Q}}_{\text{new}} = \mathbf{K}_b \mathbf{y} \), where \( \mathbf{K}_b \) contains the bearings coefficients. The homogenous equations (21) become:

\[
\begin{bmatrix} \mathbf{\bar{M}}_0 \mathbf{\bar{q}} + \omega^2 \left[ \mathbf{\bar{K}}_0 + \mathbf{\bar{G}}_0 \right] \mathbf{\bar{q}} + \mathbf{\bar{K}}_0 \mathbf{\bar{q}} \end{bmatrix} = \mathbf{\bar{Q}} = 0
\]

which can be reformulated into its state space form for eigen analysis. Integrating the equations (20) gives the rotor response \( \mathbf{\bar{q}} \) due to the angular misalignments in the collar.

### 3 RESULTS

The following data and parameters were used in the numerical simulation of the rotor system:

\[
\begin{align*}
 r &= 61.3 \times 10^{-3} \text{m} \\
 R &= 15 \times 10^{-3} \text{m} \\
 H &= 77 \times 10^{-3} \text{m} \\
 L &= 25 \times 10^{-3} \text{m} \\
 b &= 0.0663 \text{m} \\
 k_b &= 34.71 \times 10^7 \text{N/m} \\
 k_x &= k_y &= 14.26 \times 10^3 \text{Nm} \\
 l_1 &= l_2 &= l_3 &= l_4 &= l_5 &= l_6 \\
 I_{pc} &= 0.00091 \text{kg m}^2 \\
 I_p &= 0.0066 \text{kg m}^2 \\
 I_{dc} &= 0.0027 \text{kg m}^2 \\
 m_c &= 0.5031 \text{kg} \\
 m_l &= 0.0225 \text{m} \\
 m_r &= 0.785 \text{m} \\
 m_d &= 0.00721 \text{m} \\
 l_1 &= l_2 = l_3 = l_4 = l_5 = l_6 &= 0.129 \text{m} \\
 l_7 &= 0.14 \text{m}
\end{align*}
\]

where \( l_1,l_2,l_3,l_4,l_5 \) and \( l_6 \) are the lengths of rotor finite elements 1 to 6, respectively.
Fig. 4. Real part of the rotor system’s eigenvalues in the rotating frame. The blue color is for $\alpha = \beta = 1^\circ$, and the red color is for $\alpha = \beta = 5^\circ$. The figures (b) and (c) are the magnifications of the regions of interest in figure (a).
Fig. 5. Real part of the rotor system’s eigenvalues in the rotating frame. The blue color is for $\beta = 1^\circ, \alpha = -1^\circ$, and the red color is for $\beta = 5^\circ, \alpha = -5^\circ$. The figures (b) and (c) are the magnifications of the regions of interest in figure (a).
Fig. 6. Imaginary part of the eigenvalues in the rotating frame.

Fig. 7. Rotor time-response in the vertical Z direction at node 2. (a): $\alpha = \beta$ and $N = 1355$ rpm, (b): $\alpha = -\beta$ and $N = 2600$ rpm.

Fig. 8. Rotor steady-state response at node 2. (a): $\alpha = \beta$, (b): $(\alpha = -\beta)$

In Figs. 4(a, b, c) and Figs. 5(a, b, c) the real parts of the eigenvalues of the rotor system in the rotating frame are plotted as functions of the rotor speed for two different combinations of the angular misalignments $\alpha$ and $\beta$: $(\alpha = \beta), (\alpha = -\beta)$. Figs. 4(b, c) and Figs. 5(b, c) are zoomed figures of Fig. 4(a) and Fig. 5(a) respectively, where the regions of instability are magnified. These results were obtained by using the combi-bearing’s model according to equations (11) to (15) and the rotor finite element model in Eq. (22).
Figs. 6(a, b) display the imaginary parts of the rotor system’s eigenvalues in the rotating frame as functions of the rotor speed for the two different combinations of the angular misalignments $\alpha$ and $\beta$. A value of 5 degrees angular misalignment for both $\alpha$ and $\beta$ was used. The rotor critical speeds and the regions of parametric instability are shown in these figures.

Figs. 7(a, b) show the rotor response in the Z direction as a function of time for two different combinations of the angular misalignments $\alpha$ and $\beta$. A value of 5 degrees angular misalignment for both $\alpha$ and $\beta$ was used, and the rotor speeds of interest were $N = 1355$ rpm and $N = 2600$ rpm.

Figs. 8(a, b) show the rotor steady-state response as a function of rotor speed for two different combinations of the angular misalignments $\alpha$ and $\beta$. Resonances can be observed at rotor critical speeds where vibrations’ amplitudes theoretically grow to infinity. A value of 5 degrees angular misalignment for both $\alpha$ and $\beta$ was used.

4. DISCUSSIONS AND CONCLUSIONS

When the angular misalignments are present in the rotating collar, the system becomes unstable within a certain range of rotor speeds, depending on the size (magnitude) and the direction of the angular misalignment as shown in Figs. 4(b, c) and Figs. 5(b, c). These figures show clearly that the region of instability increases with the magnitude of the angular misalignment. Increasing the angular misalignment will also increase the magnitude of the real parts of the system’s eigenvalues which implies an increase in the exponential growth of the rotor motion. When comparing the results of the two angular misalignment combinations $\{\alpha = \beta\}$ and $\{\alpha = -\beta\}$, one can observe the significant differences between the results in Figs. 4(a, b, c) and Figs. 5(a, b, c). For the present case, the combination $\{\alpha = -\beta\}$ in Fig. 5 is the most critical because the number and the range of the unstable regions increase; and the magnitude of the real parts of the system’s eigenvalues also increases. The angular misalignment combination $\{\alpha = \beta\}$ will cause instability at a low rotor speed about $N = 152$ rpm as shown in Figs. 4(a, b). This should therefore be avoided when the working conditions of the machine are undercritical. At rotor speed $N = 2600$ rpm, the rotor response for the combination $\{\alpha = -\beta\}$ grows more rapidly than for the combination $\{\alpha = \beta = 5^\circ\}$ at rotor speed $N = 1355$ rpm as shown in Figs. 7(a, b). The collar angular misalignments do not influence the system’s critical speeds. But, on the other hand, they generate both lateral and axial vibration in the rotor system as shown in Figs 8(a, b). It is important to mention that due to the asymmetry in the rotating collar, the diametric moments of inertia of the collar about the two perpendicular principal axes (axes passing through its centre of mass) will not be equal and should therefore be calculated separately. However, for the present studied case, with angular misalignments of $1^\circ$ and $5^\circ$, the differences in the collar diametric moments of inertia are insignificant.

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