Electromagnetic Characterization of Power Electronic Systems

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To Mike, Jennie, and Rose
Propelled by increased global awareness and demand for clean energy systems, there is a growing trend in transportation, utility, industrial, and residential applications towards the utilisation of power electronic systems with enhanced power flow controllability and efficiency. Examples of power electronics applications include terminal converters in high-voltage direct Current (HVDC) transmission; flexible AC transmission systems (FACTS); and converters to interface alternative energy systems such as wind turbines to the grid, variable-speed motor drives in pump systems, vehicular propulsion systems, air-conditioners, and refrigerators.

The basic functionality of power electronic components is achieved by switching high voltages and currents. Recent advancements in semiconductor technology have significantly improved the current and voltage handling capabilities and the switching frequencies of power electronic devices. However, this rapid switching of high currents and voltages in turn generates electromagnetic disturbances that could distort the functionality of the power electronic equipment and other devices in the vicinity. Electromagnetic compatibility (EMC) regulations and functionality requirements impose restrictions on the design of power electronic systems. To design robust power electronic systems, a thorough understanding of the related electromagnetic issues is required.

This thesis focuses on the EMC characterisation of power electronic systems and contains two major phases.

In the first phase, the high frequency characterisation of air-core reactors was considered. Air-core reactors are typically used in power systems for current limiting, filtering, shunting, and neutral grounding applications. It is of interest to understand the behaviour of air-core reactors in the presence of high frequency signals, especially from switching operations in the power electronic components. Using the partial element equivalent circuit (PEEC) approach, air-core reactor models, helpful in design and electromagnetic analysis, were created. The PEEC models were able to predict the current and voltage distributions and the eventual electromagnetic emissions at different frequencies.

The second phase involved the characterisation of electromagnetic emissions from PWM drives using both modeling and measurement. A case study was performed on a prototype hybrid electric vehicle (HEV). Typically, emissions from PWM drives are expected at harmonics of the PWM switching frequency \( f_s \) and harmonics of the fundamental frequency \( f_0 \) of the phase voltages. In this study, it was established that space vector PWM drives generate low-frequency pulsating (LFP) emissions at a frequency of \( 6f_0 \). The switching of voltage vectors generates common mode current \( i_{cm} \) spikes be-
cause of the presence of stray capacitances and inductances. The $i_{cm}$ spikes superpose across sector boundaries, forming spikes of double or triple amplitude that constitute the LFP emissions. The amplitudes of these pulsations were shown to be dependent on the drive parameters, such as the load, the speed, and the voltage slew rates. These common mode emissions enhance the emissions at harmonics of the switching frequency, create low-frequency emissions, and when injected into an electric motor, could cause torque pulsations and speed fluctuations that may degrade drive functionality. Measurements from an HEV prototype show the LFP emissions, and theoretical models were developed to characterise them.
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This work has been carried out at EISLAB, Department of Computer Science and Electrical Engineering, Luleå University of Technology, Sweden, between 2005 and 2010, supervised by Professor Jerker Delsing and Associate Professor Jonas Ekman. Professor Kalevi Hyyppä joined later in the second phase of the work. I am very grateful for all the support and directions provided throughout this work. I also want to thank the staff at the department for their hospitality.

The work was performed in two phases. The first phase presents an application of the Partial Element Equivalent Circuit (PEEC) approach in the creation of high frequency electromagnetic models for high power components, with emphasis on Air-Core Reactors. There has been a periodic follow-up by a reference group consisting of Professor Rajeev Thottappillil (Uppsala University), Roger Byström (Banverket), Professor Math Bollen (STRI AB) and Gunnar Russberg (ABB Corporate Research) who was later replaced by Dierk Bormann (ABB Corporate Research, Västerås).

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Part I: Thesis Introduction


carbon emissions and other pollutants from millions of vehicles and the numerous fossil fuel power plants around the world are posing serious problems to modern civilization, including global warming, deterioration of air quality and depletion of fossil fuel resources [1]. Increasing public environmental awareness, the influence of environmental activists and politicians, and increasing energy costs are driving research and technology development towards more efficient alternative, environmentally friendly, clean energy systems [1, 2]. In this context, power electronic systems offer enhanced power flow controllability and efficiency, as well as energy-saving options [3]. This efficient performance accounts for the increased utilization of power electronic systems, especially in residential, industrial, utility, and transportation applications. In particular, the energy-saving options offered by power electronic-controlled, variable-speed drives, discussed in Chapter 2, seem quite attractive.

1.1 Electric motor drives

Electric motor drives are used in a wide range of applications from high-precision, low-power (a few watts) servo-drive applications in robotics to applications in the kilowatt range, for example, variable-speed drives adjusting the flow rates of pumps. Conventional electric motor drives are typically constant or single-speed drives. With a power electronic converter interface, the drive speed can be varied with the capacity of the load, thus saving energy during light load conditions [3]. A schematic of an electric motor drive system is shown in Fig. 1.1. The power output to the motor is controlled by the power converter. A feedback control loop is necessary to keep the output power within a specified reference range. Different types of motor drives include direct current (DC) motor drives, induction motor drives, and synchronous motor drives. A more elaborate discussion on motor drives and their related electromagnetic compatibility (EMC) issues is presented in Chapter 6.
1.1.2 Power Electronics and EMC

Power electronic systems enhance power flow controllability and energy efficiency, as discussed in Chapter 2. However, the basic functionality of power electronic components, in power converters for example, is obtained by fast switching of high currents and voltages. This rapid switching in turn generates current and voltage transients, electromagnetic emissions, and heat losses. If undamped, the generated transients are injected into the system and might degrade the system’s functionality. Functionality issues related to current and voltage harmonics from power converters include torque pulsations in electric generators and motors [3, 4, 5, 6], generation of shaft voltages [7], and audible noise. The electromagnetic field emissions might also affect sensitive equipment in the vicinity.

In vehicular applications, the hybridization of vehicles involves the introduction of an electric drive system alongside the conventional internal combustion engine (ICE) for propulsion [1, 2]. This addition entails the introduction of power electronic components, such as electric machines and power converters, with voltage ratings of about 700 V and peak current ratings of about 750 A. The current and voltage transients from power converter switching are over 1 kV and 1 kA, respectively. These transients might generate significant electromagnetic emissions and distort sensitive systems, such as CAN communication. This increases the challenges in designing low electromagnetic emission vehicles with robust functionality, satisfying EMC regulations.

In utilities, communication lines running parallel to high-voltage direct current (HVDC) lines are disturbed by harmonics generated by HVDC terminal converters. In other utility applications, such as variable-speed drives in pump systems, static variable compensation, and converters interfacing power from variable-speed renewable energy sources to the grid, the switching transients and the consequent electromagnetic disturbances present large challenges. Aside from functionality-related issues, EMC regulations impose further constraints on the design of power electronic systems [8]. Electromagnetic characterization is thus essential in designing robust power electronic systems [9].

1.1.3 Modeling power electronic components

PSpice and the Electromagnetic Transient Program (EMTP) [10] are widely used to model the basic functionality of power electronic systems. These programs both use a
lumped circuit modelling approach in which only the electrical parameters of the various components are considered. In this approach, the component geometry and any stray capacitive and inductive couplings to other objects in close proximity are not modelled. This approach is more suitable for low frequency modeling where these coupling effects are negligible.

In electromagnetic modelling, the component geometry and any couplings to nearby objects should be modelled. The finite element method (FEM) [11], the method of moments (MOM)[12], and the partial element equivalent circuit (PEEC) method [13, 14] are all suitable electromagnetic modelling techniques. The FEM method is based on the differential form of Maxwell’s Equations, and entails detailed modelling of the component geometry and the space around the object. The model output contains the current and voltage distributions and the fields around the object. The MOM and the PEEC method are both based on the integral form of Maxwell’s equations. These methods require detailed modelling of the component geometry and solve for the current and voltage distributions in the component. The currents and voltages can then be post-processed to obtain interesting quantities, such as the emitted fields and the Joule heating. A major advantage of the integral methods is that they do not require modeling of the space around the object, significantly reducing the size of the problem. Integral approaches are suitable for modelling components, such as cables, air-core reactors, car chassis, as well as isolated components, such as power electronic switches. A more detailed discussion of electromagnetic modelling is presented in Chapter 3.

1.2 Scope

The integration of more power electronics components in power systems, with converters switching in the hundred kilohertz range or higher, calls for an investigation of the behaviour of the existing structures, in the presence of transients and high frequency harmonics from the power electronic components. This includes major components like transformers, large inductors or reactors, capacitor banks, and the cable network.

The utilization of variable-speed drive systems in residential, transportation and power systems applications is expanding. Power converters play a central role in electric drive systems and seem to constitute a major source of electromagnetic disturbances.

To design and integrate cost-effective, efficient power electronic systems, any inherent functionality issues must be addressed, including electromagnetic interference (EMI) issues, harmonics, and losses. This thesis focuses on the electromagnetic characterization of power electronic systems. The study can be summarized by the following research questions:

1. Can full wave 3D electromagnetic modeling feasibly be applied to the design and analysis of power electronic devices?

2. What are the major sources of electromagnetic emissions from an electric drive system? How can these sources be characterized?
1.2.1 Approach

This thesis is based on two major studies.

The first study was focused on the design of high frequency models of power electronic components; in particular, air-core reactors were considered. Air-core reactors are typically used in HVDC lines as filters or smoothing reactors to damp transients from HVDC line terminal converters. Air-core reactors are also used in current-limiting applications, for example, to limit the inrush currents when large capacitor banks are turned on. Another application is reactive power compensation when large inductive loads, such as generators, are disconnected from the network. Existing models of air-core reactors are mainly lumped circuit models [15, 16, 17, 10], which do not account for the high frequency response. In this study, the PEEC method was used to create high frequency electromagnetic models of air-core reactors, helpful in design and analysis. The modelling results were compared against both measurements and lumped circuit modelling results.

The second major study was focused on the source characterization of electromagnetic emissions from drive systems, with a case study performed on a prototype hybrid electric vehicle (HEV). The study was limited to electromagnetic issues resulting from the hybridization of conventional vehicles. Power converters play a central role in drive systems and constitute a major source of electromagnetic disturbances because of the transients generated during switching operations. Some of the energy of these transients is radiated as electromagnetic emissions, while a large part is injected into the systems being controlled, posing possible functionality issues. For example, large pulsating torque components at frequencies of $6f_0$ resulting from imperfections in the motor drive system, where $f_0$ is the fundamental frequency of the phase voltages, have been reported [4, 5, 6, 18, 19, 20, 21, 22, 23]. Among PWM schemes, the space vector PWM scheme is preferred for its flexible speed control capabilities [1, 2, 24, 25]. However, some EMI and functionality issues related to the space vector scheme have been reported. In [26], it was shown that the amplitude of current ripples at the carrier or switching frequency ($f_c$) are influenced by the placement of active vectors within each half carrier or PWM period. Issues related to the crossing of sector boundaries in the space vector hexagon have also been reported [4, 19, 27]. For example, in [19], the formation of common mode current spikes due to sector boundary crossing was mentioned. In [4], the generation of large torque pulsations due to sector boundary crossing was reported. The issues related to sector boundaries have not been formally characterized. In this phase of the study, the sources of transients from PWM drives were investigated. The formation of large-amplitude current spikes during sector boundary crossings, mentioned in [19], was characterized. The dependence of the emissions on different drive train parameters, such as speed, load, and voltage slew rates, were investigated using theoretical models.

1.3 Thesis Outline

This thesis consists of two parts: Part I and Part II. Part I is the thesis introduction and consists of 8 chapters. Chapter 1 presents the background and scope. Chapter
2 presents an overview of power electronic applications, with an emphasis on energy-saving alternatives. Chapter 3 discusses different electromagnetic modeling approaches. Chapter 4 presents a more detailed discussion of the PEEC modeling approach. Chapter 5 presents high frequency electromagnetic models of air-core reactors created using the PEEC approach. Chapter 6 presents drive systems and discusses source characterization of electromagnetic emissions from drive systems using measurements and theoretical modeling. A summary of scientific contributions is presented in Chapter 7, while Chapter 8 summaries the thesis with some discussion and conclusions. Part II consists of appended scientific contributions.
Chapter 2

Energy Saving Potential of Power Electronic Applications

This chapter presents an overview of power electronics in residential, industrial, utility, and transportation applications. It emphasizes the role of power electronics in the enhancement of power flow controllability and energy efficiency. The sustainability and energy-saving options discussed are based on reports from the European Environmental Agency on the total energy consumption by sector [28] and reports from the European Union Directorate-General for Energy and Transport (DG TREN) on the total carbon (CO\textsubscript{2}) emissions by sector [29], presented in Fig. 2.2 and Fig. 2.1, respectively.

2.1 Residential Applications

Approximately 26 percent of the European Union’s (EU’s) total energy consumption and 19 percent of the carbon emissions are from residential applications, as shown in Fig. 2.1 and Fig. 2.2. Residential applications include space heating, air conditioning, refrigeration, water heating, cooking, lighting, television and other applications. Energy savings in residential applications could be enhanced using power electronics in the following ways:

1. Space heating and air conditioning: About 25 percent of residential energy consumption is used for space heating and air conditioning. The energy efficiency of heat pumps could be boosted by approximately 30 percent if the conventional constant speed drives were replaced by variable-speed drives, allowing for load-proportional capacity modulation [3].

2. Lighting: Lighting accounts for about 15 percent of residential energy consumption. The efficiency of conventional 50 Hz fluorescent lamps could be increased by 20–30 percent by operating at frequencies greater than 25 kHz [3] through the use of power electronic components. Moreover, a daylight energy-saving option could be achieved by incorporating a dimming control.
3. Inductive cooking: During cooking, a significant amount of heat is lost to the surroundings when the traditional 50 Hz electric cookers are used. This could be improved by using inductive cooking, where the 50 Hz AC is converted to 25 kHz AC that supplies an inductive coil. The coil heats up a metal pan resting on top of
2.2 Industrial Applications

In the EU, the industrial sector accounts for about 27.5 percent of the total energy consumption and for more than 50 percent of the total carbon emissions, as shown in Fig. 2.2 and Fig. 2.1, respectively. Driven by rising energy costs and the energy-saving potential offered by variable-speed drives, there is a growing trend toward the integration of variable-speed drives in medium-voltage applications. These applications include drives for rolling mills, gas compressors, and extraction pumps [30, 31].

Other applications of power electronics in the industry include induction heating and welding. In induction heating, the heat in the work piece is produced by eddy currents generated by electromagnetic induction. Induction heating allows defined sections of a work piece to be heated with high precision and minimizes heat loss to the environment. [3, 30]. The heating depth is easily chosen by selecting the induction frequency. Low-frequency induction heating applications include melting large work pieces. High-frequency applications include forging, soldering, hardening, and annealing.

In welding applications, the typical current and voltage ratings of electric welders are 50 V and 500 A DC, respectively. Low current ripple and isolation of the welder output from the utility supply is usually required. Line frequency transformers are usually used to step down the utility voltage, while the conversion to the control DC is achieved using thyristor rectifiers. Series inductors are used to damp current ripples. The limitations of this approach include the large size, the weight and the low efficiency of the line-frequency transformers. In addition, large inductors are required to damp the low-frequency output current ripples. A more efficient approach is to first convert the utility voltage to uncontrolled DC using diode rectifiers and then convert to a higher frequency using switch-mode inverters. A high frequency transformer is then used to step the voltage down to the required welder voltage and provide isolation from the utility voltage. Major gains include the relatively small size and weight of the high-frequency transformer, and much smaller series inductors are required to damp the output current ripples, which will then be at higher frequencies [3].

2.3 Utility Applications

The applications of power electronics in utilities include converters interfacing HVDC lines to AC systems (to the grid), converters interfacing alternative energy sources such as wind turbines to the grid, static and dynamic variable compensation, and variable-speed pumps replacing the conventional single-speed pumps in energy conversion plants. Power electronic systems can enhance functionality in utilities in the following ways:
1. HVDC transmission interface: Power electronic converters facilitate the interconnection of HVDC transmission lines to AC systems (to the grid). The transmission of electrical energy from generating plants to distribution centres is handled mostly by AC transmission lines. Over long distances, power losses due to the inductance of the line become large, and series compensation is required for power factor correction. In HVDC transmission, the AC losses related to the inductance and capacitance of the line are minimized, but the cost of installing the terminal converters is high. HVDC transmission lines become more economical for distances longer than 300 miles. Moreover, HVDC lines facilitate the interconnection of unsynchronised AC systems. Usually, AC filters are installed on the AC side of the converters to prevent the harmonics generated by the converters from entering the AC system. On the DC side, large series inductors or smoothing reactors of about a hundred millihenries are placed to minimized ripples in the DC voltage [3, 32].

2. Interconnection of renewable energy sources: Power electronic converters can interface variable-speed renewable energy sources such as wind turbines to the 60 Hz grid. A synchronous generator connected directly to the grid does not allow for speed variations. However, with a converter interface, the generator speed can vary with the wind speed while supplying a steady 60 Hz output to the grid [3].

3. Static var compensation: In utilities, it is necessary to keep the voltage within a small range around a nominal value (about ±5 percent). Given a load with complex power $S = P + jQ$, the active power is given by $P = I_L V_L \cos \theta$, and the reactive power $Q = I_L V_L \sin \theta$, where $\cos \theta$ is the power factor and $I_L$ and $V_L$ are the load current and terminal voltage, respectively. Voltage instabilities usually result from perturbations in the reactive power when large inductive loads are connected to the AC system. Traditionally, these loads are compensated for by manually connecting large capacitor banks and inductors [33]. With the increasing size and dynamic nature of today’s grids, this approach has limited utility because of the inherent delay in the responses of the capacitors and inductors. Dynamic voltage stability can be attained using dedicated power electronic converters that inject currents and voltages into the grid at appropriate amplitudes and phases for reactive power compensation [3].

4. Active filters: Currents drawn by nonlinear loads usually contain a distortion component. Dedicated power converters can actively cancel out the distortion components by injecting negative distortion current components.

5. Variable-speed drives: Generators and pumps connected directly to the grid are constrained to run at a single speed with minimal variation. Using a power electronic converter interface, the generator or pump speed could be varied continuously, depending on the load capacity, thus saving energy under light load conditions.
2.3. Utility Applications

2.3.1 Transportation Applications

In the EU, the transportation sector consumes over 31 percent of the energy produced and accounts for over 23.1 percent of the carbon emissions, as presented in Fig. 2.2 and Fig. 2.1, respectively. Conventional vehicles use internal combustion engines (ICE), which depend on oil. The initiatives to minimize fuel consumption and carbon emissions are forcing automobile manufacturers to turn to hybrid electric vehicles (HEV) and electric vehicles (EV). A typical hybrid vehicle incorporates an electric propulsion system or electric drive train alongside a conventional ICE. The electric drive train can be powered by a number of alternative energy sources. The efficiency of a HEV is enhanced by about 25 percent relative to conventional vehicles because either the electric machine or the ICE engine can be operated at their optimal operating points, and energy can be regained from the vehicle’s inertia by regenerative braking [1, 2]. For example, both the ICE and electric drive train can be used to provide transient power during peak acceleration, while only the ICE provides power during cruising. The pure electric mode can be used in urban areas that require fast start-stop cycles or in emission-free zones. Zero-emission vehicles can be constructed by using an electric drive train as the sole propulsion system, as in electric vehicles. This technology is limited today by the low energy density of batteries, causing electric vehicles to require frequent recharging, as in plug-in electric vehicles. Aircraft could also have zero carbon emissions if they were to use all-electric propulsion systems. Power electronics play major roles in the development of hybrid vehicles, including the following:

1. Variable-speed drives: Variable-speed drives are used in the electric drive train. 
   Energy is regained from the drives by regenerative breaking, with the drive acting in generator mode. The drives are controlled by power electronic converters.

2. Interfacing alternative energy sources: Power electronic converters are used to interface electrical energy from alternative energy sources.

Power electronic applications seem to provide attractive options in the design of energy-efficient and sustainable systems. However, inherent issues resulting from the high slew rates in power electronic devices need to be thoroughly characterized. These issues include heat dissipation and EMC. EMC issues are discussed in more detail in Chapters 3 to 6, using both modelling and measurement approaches.
The increasing complexity of power electronic systems poses increasing challenges in design and analysis. The traditional rules of thumb must be replaced by efficient modelling tools. This chapter focuses on electromagnetic modelling of power electronic systems.

3.1 Electromagnetic modeling approaches

Computational electromagnetics is becoming increasingly popular in both research and industry. Modeling of power electronics exploits techniques developed in computational electromagnetics. Electromagnetic modelling in general involves solving Maxwell’s equations (3.1) - (3.4) either directly or indirectly. Maxwell’s equations are a set of coupled partial differential equations relating electromagnetic fields \((E, H)\) to current and charge distributions \((J, \rho)\) and material characteristics \((\varepsilon, \mu)\) in a system.

**Maxwell’s equations**

\[
\begin{align*}
\nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} & \oint_{L} \mathbf{H} \cdot d\mathbf{l} &= \int_{S} \left( \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{S} \quad (3.1) \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} & \oint_{L} \mathbf{E} \cdot d\mathbf{l} &= -\int_{S} \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} \quad (3.2) \\
\nabla \cdot \mathbf{D} &= \rho_{v} & \int_{S} \mathbf{D} \cdot d\mathbf{S} &= \int_{v} \rho_{v} \, dv \quad (3.3) \\
\nabla \cdot \mathbf{B} &= 0 & \oint_{S} \mathbf{B} \cdot d\mathbf{S} &= 0 \quad (3.4)
\end{align*}
\]

Electromagnetic modeling approaches can be classified into differential equation–based methods and integral equation–based methods. A brief discussion of both approaches is presented in this section. For a more comprehensive discussion, see [34] and [35].
3.2 Differential Equation based methods (DE)

Differential equation (DE)–based methods involve directly solving Maxwell’s equations for the fields that permeate all space. First, a numerical grid (mesh) of the problem space is constructed, and the fields propagate between any two points in the grid. Because of memory limitations, it is generally impossible to mesh the entire space. Usually only a finite problem domain (box) is meshed, and appropriate boundary conditions such as the absorbing boundary condition (ABC) or the perfect matching layer (PML) [35] are applied to simulate fields propagating to infinity. The solution involves solving large, sparse matrices with a large number of unknowns. Because the field propagates from grid point to grid point, small errors that occur at the grid points accumulate, leading to large grid dispersion errors for larger simulations. This problem is usually minimized by using a finer grid, but this creates a larger number of unknowns and hence a larger problem size. Examples of DE-based techniques include the following:

3.2.1 Finite Difference Time Domain (FDTD)

The finite-difference time-domain (FDTD) method is one of the earliest modeling techniques developed in electromagnetics, dating back to the 1960s, and introduced staggered grids for electric and magnetic field quantities [36]. It involves meshing the entire problem space and solving Maxwell’s differential equations directly. Similar to other DE-based approaches, it uses appropriate boundary conditions to terminate the problem space. Several techniques have been developed to enhance the accuracy and stability of the method; e.g., the staggered grid system for propagation of electric and magnetic fields in space [35]. Instabilities, in the form of spurious resonances, may arise from improper time stepping. Time stepping is usually constrained by the Courant–Friedrichs–Lewy (CFL) criterion [37]. A major advantage of this approach is its ability to treat nonlinear phenomena in inhomogeneous media. Problems may arise when trying to model over different time scales.

3.2.2 Finite Element Method (FEM)

The finite element method (FEM) [38] involves meshing the whole problem domain into discrete cells called finite elements. Suppose the given problem is characterized by the differential equation

\[
L \phi = f, \tag{3.5}
\]

where \( L \) is a differential operator, \( f \) is the excitation function, and \( \phi \) is the unknown quantity. An approximation to \( \phi \) that minimizes the expression (3.5) is proposed. Simple local functions \( \phi^e \) (or basis functions) are used to approximate the variation of \( \phi \) within each element. The approximating function for \( \phi \) can be expressed as a linear combination of the basis functions with unknown coefficients.
A major advantage of the FEM is its great flexibility in describing the problem geometry. On the other hand, it is very challenging to treat radiation and scattering problems with the FEM, as these problems entail discretization of the entire problem domain. Though this is usually sorted out by using boundary conditions, such as the ABC or the PML, the required amount of computer resources is relatively high compared to integral techniques, such as the MOM or the PEEC method (treated in the next section). Spurious resonances may be observed when the tangential continuity of the fields across the boundaries between adjacent elements is not properly treated.

3.3 Integral Equation based methods (IE)

Integral equation (IE) techniques are based on the integral forms of Maxwell’s equations. IE-based methods aim to solve for field sources (currents and voltages) on surfaces or boundaries, thus reducing the dimensionality of the problem. Similar to DE-based methods, IE-based methods also involve solving matrix equations, but the matrices of IE-based techniques are denser. With the development of recent fast solvers [39], new horizons for integral methods have opened up. As opposed to DE-based methods, IE-based methods do not require discretization of the entire problem domain; instead, only the problem geometry is discretized. Thus, IE-based methods have far fewer unknowns than DE-based methods given the same problem. As opposed to DE-based methods, the field propagates from points A to B using exact, closed-form solutions, thus minimizing grid dispersion errors. Spurious resonances may occur with IE-based methods [34], sometimes resulting from the meshing schemes. Usually, appropriate measures are taken to suppress them. Examples of IE-based methods include the following:

3.3.1 Method of Moments (MOM)

The Method of Moments (MOM) [12] is an IE-based approach. It aims to reduce an integral operation to a set of linear equations. For example, given the integral operator equation

\[ L(I) = f, \]  

(3.6)

where \( L \) is a linear operator, \( f \) the excitation function and \( I \) the unknown current function. \( I \) can be expressed as a linear combination of basis functions \( I^j \) with unknown coefficients. Using a suitable inner product, (3.6) can be converted into matrix equations, which can be solved to obtain the unknown coefficients. The MOM approach gives a solution in the form of the current distribution within the structure being analyzed.

The MOM is suitable for analyzing thin wires (antennas), homogenous dielectrics and unbounded radiation problems.
3.3.2 Partial Element Equivalent Circuit (PEEC) approach

The partial element equivalent circuit (PEEC) approach [13, 40] is also an IE-based technique. It is based on the electric field integral equation (EFIE), from which equivalent circuits are extracted. The main difference between the PEEC approach and the MOM is the PEEC’s ability to extract equivalent circuits from the integral equations. The PEEC method creates fewer unknowns than differential equation–based methods because it does not require discretization of the space around the geometry being analyzed. Though the resulting matrices are dense, recent fast solvers have greatly improved the solution time of PEEC simulations. Similar to other IE-based techniques, spurious resonances may occur [34], most likely resulting from poor geometric meshing. A more detailed description of PEEC theory is presented in Chapter 4.

3.4 Equivalent Circuit Lumped Models

Equivalent circuit lumped models, or simply lumped models, are a traditional way to model power components [15, 16]. They involve partitioning the component to be analyzed into several lumped sections. The equivalent circuit parameters (lumped parameters), such as the resistance, inductance, capacitance and conductance, are calculated for each lumped section using analytical or numerical routines. The electromagnetic couplings between the lumped sections are considered through mutual inductions and mutual capacitances. The lumped parameters are later assembled using Kirchhoff’s voltage and current laws. An attempt to model air-core reactors in a lumped modelling approach is considered in Chapter 5. However, this approach presents a frequency limitation, in that the models are limited to cases in which the dimensions of the lumped sections are smaller than the minimum wavelengths of interest. Equivalently, it is difficult for the lumped models to characterize the propagation of very fast pulses that have significant electromagnetic delays within each lumped section. In these cases, distributed models similar to the PEEC models are more suitable.

Based on the discussion in this section, the PEEC modelling approach was selected to model air-core reactors in Chapter 5.
As mentioned in Chapter 3, the PEEC method, like other integral methods, does not require the free space around the object to be discretized, significantly reducing the problem size. It is thus well-suited for analyzing antenna-like structures, such as cable harnesses, for example. A particular advantage of the PEEC approach is the possibility to extract equivalent circuits from a given geometric problem, which can then be analyzed using circuit solvers such as PSpice. This chapter presents a detailed discussion of the PEEC modeling approach.

4.1 The PEEC approach

In the PEEC method, the electric field integral equation (EFIE) is interpreted as Kirchhoff’s voltage law applied to a basic PEEC cell, resulting in a complete circuit solution for 3D geometries. A more extensive discussion of PEEC theory is given in [13, 40]. The PEEC approach to creating electromagnetic models involves the following phases:

- Meshing.
- Equivalent circuit interpretation of the EFIE.
- Matrix formulation: Obtaining circuit equations for the meshed structure.
- Matrix solution: Solving the circuit equations to obtain the currents and potentials in the meshed structure.
- (Optional) Post-processing of the current and potentials to obtain field variables.

4.2 Meshing of structure

Two meshing schemes are required for PEEC analysis: first, a volume cell mesh to model the current distribution and second, a surface mesh to model the charge distribution, as
explained in the previous section. The partial inductances given in (4.8) and the DC resistances given in (4.13) are calculated from the volume cells, and the coefficients of the potential given in (4.12) are calculated from the surface cell mesh.

The largest cell in the mesh must be smaller than $\lambda_{\text{min}}/20$, where $\lambda_{\text{min}}$ is the minimum wavelength of interest (corresponding to the highest frequency of excitation).

### 4.3 Equivalent circuit interpretation of EFIE

Consider the electric field on a conductor given by

$$E_i(r, t) = \frac{J(r, t)}{\sigma} + \frac{\partial A(r, t)}{\partial t} + \nabla \phi(r, t),$$  \hspace{1cm} (4.1)

where $E_i$ is an incident (externally) applied electric field, $J$ is the current density in the conductor, $A$ is the magnetic vector potential, $\phi$ is the scalar electric potential, and $\sigma$ is the electrical conductivity. By using the basic definitions of the electromagnetic potentials as in (4.2) and (4.3),

$$A(r, t) = \mu \int_{v'} G(r, r') J(r', t_d) dv'$$  \hspace{1cm} (4.2)

$$\phi(r, t) = \nabla \epsilon_0 \int_{v'} G(r, r') q(r', t_d) dv'.$$

where the Green’s function $G(r, r') = \frac{1}{|r-r'|}$. Substituting (4.2) in (4.1), the electric field integral equation (4.3), at the point $r$ in the conductor is obtained.

$$E_i(r, t) = \frac{J(r, t)}{\sigma}$$  \hspace{1cm} (4.3)

$$+ \mu \int_{v'} G(r, r') \frac{\partial J(r', t_d)}{\partial t} dv'$$

$$+ \nabla \epsilon_0 \int_{v'} G(r, r') q(r', t_d) dv'.$$

Expanding the current density [14] as $J = J^C + J^P$, where the free current density $J^C = \sigma E$, and the polarization current density $J^P = \epsilon_0 (\epsilon_r - 1) \frac{\partial E}{\partial t}$, the EFIE can be re-written as

$$E_i(r, t) = \frac{J(r, t)}{\sigma}$$  \hspace{1cm} (4.4)

$$+ \mu \int_{v'} G(r, r') \frac{\partial J(r', t_d)}{\partial t} dv'$$

$$+ \epsilon_0 (\epsilon_r - 1) \mu \int_{v'} G(r, r') \frac{\partial^2 E(r', t_d)}{\partial t^2} dv'$$

$$+ \nabla \epsilon_0 \int_{v'} G(r, r') q(r', t_d) dv'.$$
4.3. Equivalent circuit interpretation of EFIE

The third term in the righthand side of (4.4) vanishes for ideal conductors \( \epsilon_r = 1 \), thus permitting a separation of the ideal conductor and ideal dielectric properties.

Assume an ideal conductor consisting of \( k \) subconductors, and further partition each subconductor into \( n_\gamma \) volume cells, each of constant current density \( J_{\gamma nk} \), where \( n_\gamma = n_x, n_y, n_z \) for partitions in the x-, y-, or z-direction. Further defining pulse functions as in (4.5),

\[
P_{\gamma nk} = \begin{cases} 
1, & \text{inside the } nk: \text{th volume cell} \\
0, & \text{elsewhere} 
\end{cases} \quad (4.5)
\]

and taking a weighted volume integral over each \( v_{\gamma nk} \) volume cell, the second term on the righthand side of (4.4) represent the inductive voltage drop \( v_L \) over the conductor as

\[
v_L = \sum_{k=1}^{K} \sum_{n=1}^{N_\gamma} \mu \frac{1}{4\pi a_n a_{\gamma nk}} \int_{v_{\gamma nk}} \int_{v_{\gamma nk}} \frac{\partial I_{\gamma nk}(r_{\gamma nk}^\prime t_{\gamma nk})}{|r - r'|} dv_{\gamma nk} dv_{\gamma nk} \quad (4.6)
\]

where \( J_{\gamma nk} = \frac{I_{\gamma nk}}{a_{\gamma nk}} \). The inductive voltage drop can also be expressed as

\[
v_L = \sum_{k=1}^{K} \sum_{n=1}^{N_\gamma} L_{p_{\gamma nk}} \frac{\partial}{\partial t} I_{\gamma nk}(t - \tau_{\gamma nk}) \quad (4.7)
\]

where \( \tau_{\gamma nk} \) is the center-to-center delay between the volume cells \( v^\prime \) and \( v_{\gamma nk} \) and \( L_{p_{\gamma nk}} \) are partial inductances which are generally defined for volume cells \( v_a \) and \( v_b \) as

\[
L_{p_{ab}} = \mu \frac{1}{4\pi a_a a_b} \int_{v_a} \int_{v_b} \frac{1}{|r_{\alpha} - r_{\beta}|} dv_{\alpha} dv_{\beta}. \quad (4.8)
\]

The \( L_{p_{ab}} \) terms are referred to as the self partial inductances while the \( L_{p_{ai}} \) terms are the mutual partial inductances representing the inductive couplings between the volume cells.

The capacitive voltage over the \( m \)th volume cell is obtained from the fourth term of the right-hand side of (4.4). Extracting \( S_{mk} \) surface cells from the \( m \)th volume cell to obtain a surface representation of the charge distribution over the volume cell and using pulse functions defined as

\[
P_{mk} = \begin{cases} 
1, & \text{inside the } mk: \text{th surface cell} \\
0, & \text{elsewhere} 
\end{cases} \quad (4.9)
\]

and the following finite difference approximation

\[
\int_{v} \frac{\partial}{\partial \gamma} F(\gamma) dv \approx a \left[ F \left( \gamma + \frac{l_m}{2} \right) - F \left( \gamma - \frac{l_m}{2} \right) \right] \quad (4.10)
\]
the capacitive voltage over the $m^{th}$ volume cell is obtained as

$$v_C = \sum_{k=1}^{K} \sum_{m=1}^{M_k} q_{mk}(t_{mk}) \frac{1}{4\pi \varepsilon_0} \int_{S_{mk}} \frac{1}{|r^+ - r'|} ds'$$

where the vectors $r^+$ and $r^-$ are associated with the positive and negative end of the cell respectively [14]. From (4.11) the coefficient of potential is defined as

$$p_{ij} = \frac{1}{S_i S_j 4\pi \varepsilon_0} \int_{S_i} \int_{S_j} \frac{1}{|r_i - r_j|} dS_j dS_i$$

The $p_{ii}$ terms are referred to as the self coefficients of potential while the $p_{ij}$ terms are the mutual coefficients of potential representing the capacitive couplings between the surface cells.

The resistive voltage drop over the $m$th volume cell is obtained from the first term on the right-hand side of (4.4), from which the resistances can be defined as

$$R_\gamma = \frac{l_\gamma}{a_\gamma \sigma_\gamma}$$

where $l_\gamma$ is the length of the volume cell in the $\gamma$ direction, $a_\gamma$ is the cross-section of the volume cell normal to the $\gamma$ direction, and $\sigma_\gamma$ is the conductivity.

This interpretation of the EFIE allows for a systematic approach to constructing equivalent circuit representations of electromagnetic problems for mixed conductor-dielectric structures. Furthermore, the PEEC model allows active and passive circuit elements to be added to the analysis of the electromagnetic problem. Figure 4.1 shows the PEEC model of a conducting bar. The magnetic field couplings are considered through the mutual partial inductances represented by a voltage source $V_{mm}$, while the electric field couplings are accounted for by the mutual coefficients of the potentials represented by the current sources $I_{ip}$ and $I_{pj}$. Each node is connected to infinity by the corresponding self coefficients of potential $\frac{1}{p_{ii}}$ and $\frac{1}{p_{jj}}$, as shown in Figure 4.1.

### 4.4 Matrix formulation

This phase involves the formulation of circuit equations from the equivalent circuit representation of the meshed structure. If the complete equivalent circuit is expressed in a SPICE-compatible .cir-file, the circuit equations can be formulated and solved directly in freeware SPICE-like solvers. However, in the full-wave case, when time retardation is included, special solvers have to be used [41]. The circuit equations are formulated from the equivalent circuit representation of the conducting bar shown in Fig. 4.1 by applying
4.5 Matrix solution

Kirchhoff’s voltage law on the inductive loop and enforcing Kirchhoff’s current law at each node. This results in the following circuit equations

\[-A \Phi(t) - RI_L(t) - L_p \frac{\partial i_L(t)}{\partial t} = v_s(t)\]  \hspace{1cm} (4.14)

\[P^{-1} \frac{\partial \Phi(t)}{\partial t} - A^T i_L(t) = i_s(t)\]

where \( P \) is the coefficients of potential matrix, \( A \) is a sparse matrix containing the connectivity information, \( L_p \) is a dense matrix containing the partial inductances, \( R \) is a matrix containing the volume cell resistances, \( \Phi \) is a vector containing the node potentials (solution), \( i_L \) is a vector containing the branch currents (solution), \( i_s \) is a vector containing the current source excitations, and \( v_s \) is a vector containing the voltage source excitations [42]. The first row in the equation system in (4.14) is Kirchhoff’s voltage law for each inductive loop, while the second row satisfies Kirchhoff’s current law at each node.

4.5 Matrix solution

This phase involves solving the equation system in (4.14) for the potential and current distributions in the meshed structure. As shown in the previous section, the modified nodal analysis (MNA) method [43] was adopted. In this approach, the nodal potentials and volume cell currents are solved simultaneously, and the system coefficient matrix has two dense blocks (upper right and lower left). The MNA method also allows additional active and passive circuit elements to be simply included within the electromagnetic model.
In the solution of (4.14), the time derivatives can be calculated by, for example, a backward Euler scheme [44], as shown here for the $j$th node potential:

$$\frac{\partial \Phi_j(t)}{\partial t} = \frac{\Phi_j^n - \Phi_j^{n-1}}{\Delta t}$$

(4.15)

where $\Delta t$ is the time step separating the two discrete time instances $n$ and $n-1$. Discretizing (4.14) in time gives

$$
\begin{bmatrix}
-A & -(R + L_p \frac{1}{\Delta t}) \\
-P^{-1} \frac{1}{\Delta t} & A^T
\end{bmatrix}
\begin{bmatrix}
\Phi^n \\
\Phi_{n-1}^n
\end{bmatrix}
=
\begin{bmatrix}
-v_{s,n} - L_p \frac{1}{\Delta t} i_L^{n-1} \\
i_s + P^{-1} \frac{1}{\Delta t} \Phi^{n-1}
\end{bmatrix}
$$

(4.16)

when written in a matrix fashion with the sub-matrices as described in the previous section.

In a quasi-static (QS) solution of (4.14), only the potentials and currents at the $n$th and the $n-1$th time steps are used in the evaluation of the derivatives. For a full-wave (FW) solution accounting for time retardation in the electromagnetic couplings, a history of currents and node potentials is needed. The time step ($\Delta t$) should be carefully chosen because an extremely small $\Delta t$ can lead to numerical problems.

The time-domain PEEC simulation is verified against experimental measurements in paper E.

### 4.6 Postprocessing

The node potentials and volume cell currents can be post-processed to obtain electromagnetic field variables. This post-processing is performed in [45] for antenna problems, in [46] for printed circuit board problems, and in [47] for the computation of fields from and air-core reactors.
Chapter 5

Air-Core Reactor Modeling

Faced with the trend towards increasing power converter switching frequencies, there is a strong need for high-frequency characterization of other network components, including reactors. The high-frequency harmonics generated from the converter switching are injected into these components, triggering resonances. Switching devices such as insulated gate bipolar transistors (IGBT’s) have been reported to operate at 120 kHz and higher [48]. Such devices generate harmonics in the megahertz range. There is thus a need for models that can analyze component behaviour in the megahertz range. The case of air-core reactors will be considered here. The traditional lumped modelling approach, which involves partitioning the reactor into electromagnetically coupled lumped sections [15, 16, 17], presents a frequency limitation. Thus, distributed modelling approaches, such as the PEEC method, that account for detailed electromagnetic couplings are required. This chapter begins with a brief discussion about the applications of air-core reactors in power systems. This is followed by a detailed discussion of the modelling of air-core reactors using both lumped circuit modelling and the PEEC approach.

5.1 Applications of Air-Core Reactors in Power Systems

This section presents the applications of air-core reactors in power systems. A more detailed discussion of reactors is given in the ABB transformer handbook [48]. Reactors are usually used in power systems for current-limiting, filtering, and reactive power compensation. The reactors can be either air-cored (dry-type) or oil-immersed. Air-core reactors are large coils in free space, without a magnetic core and enclosure. The coils are cooled by air convection. The coil inductance is usually on the order of a few millihenry and does not saturate during operation because of the absence of a core. The reactor is subjected to large forces resulting from the flow of short-circuit currents (~ 1 kiloamperes). The magnitude of the short-circuit currents determines the choice and size of the supporting material. The magnetic field from the reactor spreads in its vicinity and may
heat nearby metallic structures. For this reason, air-core reactors are usually installed in a large outdoor space. Because of the risk of dielectric breakdown, oil-immersed reactors are more suitable than air-core reactors for high-voltage distribution systems in heavily polluted areas. An example oil-immersed reactor is shown in Fig. 5.1. Oil-immersed reactors are usually enclosed in a metal tank, cooled by the oil, and have a magnetic core, sometimes with air gaps. The field is well confined to within the metal tank. They are relatively small and require less space for installation than air-core reactors. The motivation for using air-core reactors in medium high-voltage systems is their relative cheapness. In the following subsections, the different applications of air-core reactors are summarized.

5.1.1 Shunting application

Shunt air-core reactors are commonly used to absorb reactive power in medium-voltage distribution systems for voltage stabilization during light load conditions. Under normal load conditions, there is a voltage drop through the series inductance and resistance of the line. Under light load conditions or immediately after the disconnection of a heavy load, the capacitance to earth at the receiving end (load end) draws a capacitive current through the line inductance. This causes voltage accumulation at the receiving end, a phenomenon referred to as the Ferranti effect [48]. To obtain voltage stabilization, the line inductance is compensated by series capacitors at different intervals along the line, while the line capacitance to ground is compensated by shunt reactors placed at the end of the line. The shunt reactors absorb the reactive power produced by the capacitive voltage rise. Figures 5.1 and 5.2 (d) are examples of shunt reactors in service.

5.1.2 Current limiting reactors

Current-limiting reactors are series reactors. They increase the series impedance of the line to limit short-circuit currents to lower levels for circuit breakers. Current-limiting reactors could be used to limit inrush currents when a heavy motor is turned on, to limit capacitor discharge current and to protect other devices against high fault currents. Other examples include smoothing reactors in high-voltage DC (HVDC) transmission lines, which damp transients from terminal converters to reduce current ripples on the DC side. Figure 5.2 (b) is an example of a current-limiting reactor.

5.1.3 Filter reactors

Filter reactors are used in series or parallel configurations with capacitors to tune resonance circuits in the audio frequency range or to block communication frequencies. An example of a filter reactor is shown in Fig. 5.2 (c).
5.2 Lumped circuit modeling of air-core reactors

Lumped modelling basically involves partitioning the reactor into sections that are electromagnetically coupled, neglecting couplings within sections. Each section (partition) is assumed to be electrically small and represented by the lumped circuit parameters $L, C$ and $R$. The inductance and capacitance of each partition are obtained using closed formulas. The lumped parameters can be implemented in SPICE-like simulators, such as EMTDC [10], for a quasi-static analysis. The couplings between the inductive partitions (magnetic field couplings) are represented by the mutual inductances, while the mutual capacitances represent the capacitive couplings (electric field couplings).
Figure 5.2: Examples of air-core reactors from www.nokiancapacitors.com. (a) Earthing reactor. (b) Current limiting reactor. (c) Filter reactor. (d) Shunt reactor.
Consider a reactor of $N$ turns discretized into $i$ inductive partitions and $i+1$ capacitive partitions. For the lumped models $N \gg i$, because several turns are represented in one discrete circuit element. The lumped representation of this reactor is shown in Fig. 5.3.

The lumped inductances $L_{km}$ are calculated using the expressions for coaxial circular filaments [49] as

$$L_{km} = \frac{\mu_0 N_k N_m 2R}{\kappa_L} \left[ \left( 1 - \frac{\kappa_L^2}{2} \right) K(\kappa_L) - E(\kappa_L) \right]$$

(5.1)

where $N_k$ and $N_m$ are the number of turns in the $k$th and $m$th inductive partitions respectively, $R$ the radius of the inductive partitions, and $K(\kappa)$ and $E(\kappa)$ are complete elliptic integrals of the first and second kinds respectively. The term $\kappa_L$ in (5.1) is defined as

$$\kappa_L = \frac{2R}{\sqrt{z_{km}^2 + 4R^2}}$$

(5.2)

where $z_{km}$ is the spacing between the $k$th and $m$th inductive partitions.

The coefficients of the potential $P_{ij}$ between the capacitive partitions $i$ and $j$ are calculated using the expression for the coefficients of the potential of two coaxial cylindrical cells [50] given by

$$P_{ij} = \frac{1}{2\pi^2 \varepsilon_0 R} \int_{-\ell_i}^{\ell_i} \int_{-\ell_j}^{\ell_j} \frac{K(\kappa_C)}{A} dZ_j dZ_i$$

(5.3)

where $\varepsilon_0$ is the permittivity of free space and

$$A = \sqrt{z_{ij}^2 + 4R^2},$$

(5.4)
\[ \kappa_C = \frac{2R}{l_i} \text{, and } l_i, l_j, z_{ij} \text{ and } R \text{ are defined in Fig. 5.3 (b) and } K \text{ is the complete elliptical integral of the first kind.} \]

This modeling approach has a frequency limitation, as mentioned in section 3.4. A comparison of lumped model results against measurements, shown in Fig. 5.9, clearly shows the low frequency limitation of the approach. Details of this analysis is presented in paper A.

5.3 PEEC air-core reactor modeling

This section deals with the creation of electromagnetic models for air-core reactors using the PEEC modeling approach.

5.3.1 Geometry description

In the presented model, each turn of the reactor is made up of a finite number of bars with a rectangular cross-section in one plane (i.e., pitch angle neglected). The end of one turn is connected to the beginning of the next turn by a short circuit, until the complete reactor winding is created. Figure 5.4 shows a sample 4-turn reactor model. In this case, each turn is formed by only 6 bars, for simplicity. The PEEC model for one turn with 6 bars per turn is shown in Fig. 5.5 (corresponding to one turn of the geometry in Fig. 5.4). The effects of ignoring the pitch angle of the turns have been studied in [17] and are not expected to be major sources of error.

![Figure 5.4: Schematic description for 4 turn reactor model formed by 6 bars (volume cells in the PEEC model) per turn.](image)

5.3.2 Partial Element Evaluation

As mentioned previously, magnetic field couplings between the bars are represented by partial mutual inductances, and the electric field couplings are represented by the mutual coefficients of the potential. These so-called partial elements are the foundation of the PEEC model and have to be calculated with great accuracy and efficiency. Analytical formulas only exist for calculating the partial inductances and the coefficients of
the potential for orthogonal structures in parallel or perpendicular orientations [51, 52]. Therefore, these routines can be used when modelling reactors with a rectangular cross-section (when 4 perpendicular bars represent one turn) [32].

To represent circular turns, the bars need to be inclined at arbitrary angles, as shown in Fig. 5.4 for 6 interconnected bars with an inclination of 60 degrees. For these types of problems—orthogonal geometries with arbitrary orientations—numerical integration routines are used to evaluate the partial elements [40]. Numerical integration routines are usually time-consuming and susceptible to numerical errors. For orthogonal geometries in arbitrary relative orientations, numerical routines can be avoided by estimating the partial elements, as is done in the following subsections.

### 5.3.2.1 Partial inductances

The partial inductance between two volume cells was given in (4.8) and can be expanded to show the current directions as

$$L_{pij} = \frac{\mu}{4\pi} \frac{1}{a_i a_j} \int_{a_i} \int_{a_j} \int_{l_i} \int_{l_j} \frac{dl_i \cdot dl_j}{|r_i - r_j|} da_i da_j$$  \hspace{1cm} (5.5)

where $l_i$ and $l_j$ are the lengths of the cells, $a_i$ and $a_j$ are the cross-sectional areas, and $r_i$ and $r_j$ are position vectors of arbitrary points in the $v_i$ and $v_j$ volume cells, respectively. $dl_i$ and $dl_j$ are vectors along the lengths ($l_i$, $l_j$) or the assumed current direction of the $i$th and $j$th volume cell, respectively. The volume cells are illustrated in Fig. 5.6. Analytical formulas for $L_{pij}$ are given, for example, in [13, 52, 53, 54], for the case of cells in parallel or perpendicular orientations. For the case of cells inclined at an arbitrary
angle $\alpha$, the inclination is handled by the dot product $(dl_i \cdot dl_j)$. This approximation gives fairly accurate solutions and is much faster than numerical integration routines.

![Diagram](image)

**Figure 5.6: Basic geometry for evaluating mutual partial inductances.** $v_i$ and $v_j$ are volume cells of lengths $l_i$ and $l_j$, respectively and $\alpha$ is the inclination of $v_j$ relative to $v_i$.

### 5.3.2.2 Coefficients of potential

The partial coefficients of the potential are obtained from the corresponding surface cells. The expression for the coefficients of the potential is given in (4.12), in which orthogonal surfaces lying on parallel planes are considered. The coefficient of the potential for the two orthogonal surfaces $S_i$ and $S_j$ shown in Fig. 5.7 will have a maximum value $P_{ij\text{max}}$ when $\alpha = n\pi$ and a minimum value $P_{ij\text{min}}$ when $\alpha = (n + 1/2)\pi$, where $n$ is an integer, given that $l_j > w_j$. For all $\alpha$, $P_{ij}$ is approximated as

$$P_{ij} = \cos^2 \alpha P_{ij\text{max}} + \sin^2 \alpha P_{ij\text{min}}. \quad (5.6)$$

Analytical expressions for $P_{ij\text{max}}$ and $P_{ij\text{min}}$ are given in [51, 52]. The expression (5.6) is fairly accurate and much faster to compute than numerical integration routines. Therefore, this approach is used for the circular air-core models.

The partial element evaluation for inclined cells presented in this section is used in paper B in the creation of high-frequency models of air-core reactors with circular cross-sections.

### 5.3.3 Skin and proximity effects

Skin and proximity effects give rise to nonuniformity in the current distribution along a cross-section of a conductor. The increase in current density near the conductor surface
and around the edges (due only to changing fields within the conductor itself) is termed the skin effect \[55\]. This phenomenon is noticeable for conductors/bars in which the cross-section (width or thickness) is larger than the skin depth, \(\delta\), defined as

\[
\delta = \frac{1}{\sqrt{4\pi\mu\sigma f_m}},
\]

where \(f_m\) is the maximum frequency of interest.

In this study, the volume filament (VFI) technique was applied \[13, 56\]. The idea is to make a 3D discretization of the interiors of the bars composing the reactor windings with maximum width and thickness of \(\delta/2\). However, the VFI approach is expensive in terms of cell count, and an optimal method would be to use a nonuniform meshing scheme in which a coarser mesh is used in the center of the cell (with a more uniform current distribution) and a finer mesh is used closer to the edges, respecting the \(\delta/2\) rule.

The current distribution in one volume filament can be influenced by changing fields in adjacent filaments. This is termed the proximity effect \[55\] and can be pronounced in reactor-type structures. Figure 5.8 is an illustration of the current distribution in bars due to skin and proximity effects. The current distribution in a multi-conductor system is a combination of skin and proximity effects.

Paper C presents air-core reactor models capturing skin and proximity effects.

5.3.4 Model Validation

In order to validate the PEEC reactor modelling approach, laboratory air-core reactor models were constructed by winding copper wire or tape around a sparse wooden or plastic support. PEEC simulation results were compared against measurements done on
Figure 5.8: Current distribution due to skin and proximity effects. Volume cell currents in same direction (a). Currents in opposite directions (b). Only skin-effect, no proximity effect (c).

Figure 5.9: Input impedance (magnitude) for 90 turn reactor. A comparison of PEEC model results against lumped model results and measurements.

the constructed reactors, and good agreement was found. A comparison of the PEEC model results with the measurements and the results from the lumped modelling approach of Section 5.2 is shown in Fig. 5.9. These results illustrate the limitations of the lumped models and the robustness of the PEEC approach. Details of these comparisons are included in the appended papers.
5.4 Electromagnetic Field computation from reactor
-Infinitesimal dipole approach.

The length of the current segments, \( l \), in the PEEC model is always chosen such that \( l < \frac{\lambda_{\text{min}}}{20} \), where \( \lambda_{\text{min}} \) is the minimum wavelength of interest. This allows us to approximate the current segments as infinitesimal dipoles. The field at any point in the vicinity of the reactor is considered to be a vector sum of the fields from all the infinitesimal dipoles.

The magnetic vector potential at a point \( r \) in space due to an infinitesimal dipole on the z-axis is given by

\[
A(r, t) = \hat{z} \frac{1}{4\pi\epsilon_0} I_0 e^{-jkr} r,
\]

assuming constant current \( I_0 \) in the segment, letting \( k \) be the wave number, and omitting the \( e^{j\omega t} \) term. The magnetic field is thus obtained as \( H = \frac{1}{2\pi\epsilon_0} (\nabla \times A(r, t)) \), while the electric field derived from the magnetic field as \( E = \frac{1}{2\pi\epsilon_0} \nabla \times H \). According to [57] and [58], the magnetic fields created by the dipoles can be written in a spherical coordinate system as

\[
H_\phi = jk I_0 l \frac{\sin(\theta)}{4\pi r} \left( 1 + \frac{1}{jkr} \right) e^{-jkr}
\]

while the electric field is given by

\[
E_r = \eta I_0 l \frac{\cos(\theta)}{2\pi r^2} \left( 1 + \frac{1}{jkr} \right) e^{-jkr}
\]

\[
E_\theta = j\eta I_0 l \frac{\sin(\theta)}{4\pi r} \left( 1 + \frac{1}{jkr} - \frac{1}{(kr)^2} \right) e^{-jkr}.
\]

The magnetic field components \( H_r = H_\theta = 0 \), while the electric field component \( E_\phi = 0 \).

The magnetic field in the DC case, \( H_\phi^{DC} \) given in (5.11), is obtained by taking the limiting value of \( H_\phi \) when \( k \) tends to zero and is in agreement with the Biot-Savart law.

\[
H_\phi^{DC} = \frac{I_0 l \sin(\theta)}{4\pi r^2}.
\]

Because the PEEC methods gives the solution down to DC, the field calculations are valid from DC to the upper frequency limit imposed by the meshing. This field computation approach was used in paper D to estimate fields from an air-core reactor.

5.5 Synthesizing reduced circuits by vector fitting

Though the PEEC models produce a broadband characterization of air-core reactors, the model is very computationally demanding to include in power system simulation software, such as ATP-EMTP [59] and EMTDC [10]. The PEEC frequency domain admittance
response can be approximated using a rational function, and reduced equivalent circuits can be extracted later from the rational function. This has been detailed in [60, 61, 62] but is briefly repeated here for completeness.

For a given admittance response \( y(s) \), where \( s = j\omega \), a rational function of the form (5.12) is fitted.

\[
y(s) = \sum_{m=1}^{N} \frac{c_m}{s - a_m} + d + se
\]

The parameters \( a, c, d \) and \( e \) define a state equation for \( Y(s) \) for a given input \( u(s) \) such that

\[
y(s) = Y(s)u(s) = (C(sI - A)^{-1}B + D + se)u(s). \tag{5.13}
\]

\( C \) is a matrix of residues \( c_m \), scaled such that the matrix \( B \) takes just values ones and zeros. \( A \) is a diagonal matrix of poles \( a_m \) of \( y(s) \) which could be real or conjugate pairs. While \( D \) and \( E \) contain parameters \( d \) and \( e \) which could be set to zero. The parameters \( a, b, c, d \) and \( e \) are obtained from a least square solution of (5.12). Equivalent circuits are synthesized from (5.12) by defining equivalent circuit parameters according to (5.14) - (5.17). The equivalent circuit parameters are used to construct a stub as in Fig. 5.10. The number of branches in the stub depends on the number of poles in the rational approximation (5.12).

The parameters \( C_0 \) and \( R_0 \) are defined by

\[
C_0 = e, \quad R_0 = 1/d. \tag{5.14}
\]
5.5. Synthesizing reduced circuits by vector fitting

Each real pole gives an RL–branch with parameters
\[ R_1 = -a/c, \quad L_1 = -1/c, \] (5.15)
while each complex conjugate pair
\[ \frac{c' + j\omega}{s - (a' + ja'' \omega)} + \frac{c' - j\omega}{s - (a' - ja'' \omega)} \] (5.16)
gives an RLC–branch with the following parameters
\[ L = \frac{1}{2c'}, \] (5.17a)
\[ R = (-2a' + 2(c'a' + a''a'')L), \] (5.17b)
\[ \frac{1}{C} = (a'^2 + a''^2 + 2(c'a' + c''a'')R)L, \] (5.17c)
\[ G = -2(c'a' + c''a'')CL. \] (5.17d)

Thus for a given PEEC model, reduced order equivalent electrical network includable in an ATP-EMTP type program can be extracted [62, 63]. Major issues with the vector fitting approach include convergence and passivity.

The conventional vector fitting (VF) approach [60, 61] has been shown to require a number of iterations for convergence, which could be computationally demanding for broadband problems [64]. The orthonormal vector fitting approach (OVF) [65], enhances the conventional vector fitting (VF) approach by using orthonormal basis functions as opposed to monomial basis functions used in VF. With the orthonormal vector fitting, convergence is achieved within a few iterations, making it more suitable for broadband applications [64]. The accuracy and convergence of the vector fitting procedure has been further enhancement in [66] by considering the derivatives.

Passivity issues arise when negative eigenvalues are obtained in the least square fit procedure, leading to unstable time domain simulations. This results in non-passive network components or components that do not absorb active power which is non-physical. Passivity is enforced by perturbing the residues, and constraining the real part of the admittance matrix \( Y(s) \) to be positive definite [67, 68, 65, 66].
Chapter 6

EMC characterization of drive systems

In this chapter, emphasis is placed on source characterization of current transients and electromagnetic emissions from drive systems through an analysis of the currents and voltages exiting the converters. The chapter starts with a brief discussion of drive systems and related EMI issues. The simple case of a DC motor drive is then considered, followed by the more complex case of an AC drive system used in a prototype hybrid electric vehicle (HEV). The generation of low-frequency pulsating (LFP) emissions from space vector PWM drives as well as potential functionality issues are discussed.

6.1 Drive systems

The utilization of drive systems in utilities, residential, industrial and vehicular applications has been discussed in Chapter 2. Power converters play a central role in drive systems and constitute a major source of electromagnetic disturbances because of their inherent switching operations. The switching operations generate current and voltage transients because of the presence of stray capacitances and inductances. These transients are injected into the systems being controlled and could pose some functionality issues. For example, large pulsating torque components at frequencies 6 times the fundamental frequency of the phase voltages, resulting from imperfections in the motor drive system, have been reported [4, 5, 6, 18, 19, 20, 21, 22, 23].

Emissions from PWM drives are normally expected at harmonics of the switching frequency ($kf_c$) and harmonics of the fundamental frequency $mf_0$, where $k$ and $m$ are integers [27, 69, 70, 71, 72, 73, 74, 75, 76, 26]. Several different PWM schemes exist: the sinusoidal PWM scheme, the random PWM scheme, and the space vector PWM scheme, among others [1, 24, 25]. Amongst these, the space vector PWM scheme is generally preferred for its flexible speed control capabilities [1, 24, 25, 2]. However, some EMI and functionality issues related to the space vector scheme have been reported. For example,
the harmonics generated by the space vector PWM scheme have been shown to have some
dependence on the placement of the zero vectors. In [26], it was shown that the amplitude
of current ripples at the carrier or switching frequency \( f_c \) are reduced by centering the
active vectors within each half carrier or PWM period. Issues related to the crossing
of sector boundaries in the space vector hexagon have been reported [4, 19, 27]. For
example, the formation of common mode current spikes due to sector boundary crossing
was mentioned in [19]. The generation of large torque pulsations due to sector boundary
crossing was reported in [4]. These issues related to sector boundaries have not been
formally characterized. In this chapter, the generation of low-frequency pulsating (LFP)
emissions from space vector PWM drives because of sector boundary crossing is discussed.
The formation of current spikes during sector boundary crossings, mentioned in [19], are
characterized. The dependence of the emissions on different drive train parameters, such
as speed, load, and voltage slew rates, are investigated using theoretical models.

6.2 Common Mode Currents \( (i_{cm}) \)

Stray capacitances exist because of the close proximity of objects, such as the metal
casing, car chassis, or vehicle frame to the converter. There are also stray inductances
that provide magnetic field couplings to objects in close proximity to the converter. Any
voltage transient would charge-discharge these stray capacitances, generating common
mode current \( (i_{cm}) \) spikes [69, 77]. The presence of stray inductances would cause damped
oscillations because of the oscillation of energy between the electric and magnetic fields.
The circulation of common mode currents in an AC motor drive system is illustrated in
Fig. 6.1. Part of the common mode currents are radiated as electromagnetic emissions,
6.3 Differential Mode Currents ($i_{dm}$)

while some are injected into the motor and have a return path through the stray or common mode capacitances, as shown in Fig. 6.1. The electromagnetic emissions from the common mode currents add up and account for more than 70 percent of the emissions from the drive system [69]. Common mode currents can be measured using a current probe (a Rogowski coil, for example) around all three conductors on the AC link, as shown in Fig. 6.4. Neglecting the stray inductances, the magnitude of $i_{cm}$ depends on the capacitance to ground or the common mode capacitance ($C_m$) and the voltage slew rate $dV/dt$, as given below:

$$i_{cm} = C_m \frac{dV}{dt}.$$  \hspace{1cm} (6.1)

The common mode capacitance ($C_m$) depends on the proximity to metal objects in the vicinity and the cable configuration. For EMI analysis, $i_{cm}$ is thus a suitable quantity because it is sensitive to changes in the vicinity through $C_m$, it is nonzero only when $dV/dt$ is nonzero, and it accounts for a large part of the emissions from the drive [69]. In this study, the common mode currents were measured and modelled to account for the electromagnetic emissions from the drive system.

6.3 Differential Mode Currents ($i_{dm}$)

Differential mode currents ($i_{dm}$), in the case of the DC motor drive, are the currents that flow to and from the load to ensure basic functionality. In a three-phase motor, the differential mode currents are the three-phase sinusoidal currents necessary for basic functionality of the motor. If the phase currents are perfectly synchronous, the electromagnetic emissions caused by them would cancel out, provided that the cables are also collocated in space. However, in reality, there are always emissions from differential mode currents due to residual phase errors in the currents and the non-collocation of the phase cables. These emissions are usually much weaker than the contribution from common mode currents [69]. A current probe around the cables could also measure the residual differential mode currents, if they were significant.

6.4 DC Motor Drives

To better understand the source of emissions from drive systems, a DC motor drive is considered first. This section deals with the characterization of electromagnetic emissions from DC motor drives. PSpice simulation of the drive circuit was done first, and a prototype was later constructed on which measurements were performed.

6.4.1 Drive circuit modeling

The drive circuit was an H-bridge constructed with power MOSFETs as switching devices. The drive circuit was designed to switch a 24 V DC input controlling a DC motor. The
H-bridge was switched diagonally using a trapezoidal pulse train \( v_P(t) \) of 0–5 V. The PSpice model of the drive circuit is shown in Fig. 6.2. The average output voltage \( V_0 \) and motor current \( I_0 \) were controlled by controlling the duty cycle of \( v_P(t) \).

The stray line-to-ground capacitances of the motor cables are represented as \( C_{m1} \) and \( C_{m2} \). The total common mode current \( i_{cm} \) was modelled as the sum of the currents through \( C_{m1} \) and \( C_{m2} \), meaning \( i_{cm}(t) = i_{cm1}(t) + i_{cm2}(t) \). This estimate of \( i_{cm} \) is equivalent to having a Rogowski coil or current probe around both cables to the motor [78].

The modeling results are shown in Fig. 6.3, where \( M_1, M_2, M_3 \) and \( M_4 \) are MOSFETs and \( v_{DS} \) are the respective drain source voltages. The voltage across the motor \( v_0 \) and the control pulse train \( v_p \) are shown. The common mode currents \( i_{cm} \) on the motor cables are also shown. It was observed that voltage transitions generated common mode current spikes, as shown in Fig. 6.3. The magnitude of the \( i_{cm}(t) \) spikes depended on the voltage slew rates \( \frac{\Delta V}{\Delta t} \) and the stray capacitances to ground. These conclusions were supported by measurements performed on a constructed prototype. The details of this analysis are presented in paper G.
Figure 6.3: PSpice model results showing the generation of $i_{cm}$ spikes during voltage switchings. $M_1, M_2, M_3$ and $M_4$ are MOSFETs, while $v_{DS}$ are the respective drain source voltages.

6.5 AC Motor Drives

This section deals with the emissions from an AC motor drive system. The case of drives used in a prototype hybrid electric vehicle (HEV) is considered.

The EM emissions resulting from PWM switching are normally expected at the harmonics of the PWM switching frequency and the harmonics of the fundamental frequency $m f_0$ [27, 69, 70, 71, 72, 73, 74, 75, 76]. This study illustrates the generation of low-frequency pulsating (LFP) common mode emissions at a frequency of $6 f_0$ from space vector PWM drives originating from the superposition of $i_{cm}$ spikes formed during the switching of either active or zero voltage vectors close to sector boundaries. These common mode pulsations enhanced the emissions at harmonics of the switching frequency ($f_s$) by a factor of three, creating low-frequency emissions; when injected into an electric motor, these emissions could cause torque pulsations and speed fluctuations that could deteriorate the functionality of the drive. The study focused on the characterization of
these pulsating emissions by analyzing the common mode currents. Measurements from an HEV demonstrating the existence of LFP emissions are presented, and the source is characterized using simple theoretical models. The potential consequences of the LFP emissions are discussed.

6.5.1 Measurements on Hybrid Electric Vehicle (HEV)

This section presents common mode current measurements performed on an HEV and discusses potential sources of errors.

6.5.2 Measurement setup

The common mode current measurements were performed on an HEV prototype. Fig. 6.4 shows a schematic of the hybrid drive system on which the measurements were performed. The electric machine was a three-phase synchronous machine. The phase current was measured at location A with a Tektronix current probe (model TCP404XL) of bandwidth DC to 2 MHz and a peak current rating of 750 A DC. The common mode current was
measured at location B with a Powertek Rogowski coil (model CWT 6B) of bandwidth
0.1 Hz to 16 MHz and a peak current rating of 1.2 kA and 8 kA/µs. The measurements
were recorded on a Tektronix digital oscilloscope (TDS7254) with a bandwidth of 2.5
GHz and a maximum sampling rate of 20 gigasamples per second.

6.6 Low Frequency Pulsating Emissions (LFP) from Drive Systems

The measurements were performed under different driving modes with current probes
located at positions A and B on the drive train, as shown in Fig. 6.4. Thus, different
emission patterns were obtained at different times. Measurements from two different
cases are presented. The objective of case I was to capture the events that occurred
during switching. A short interval of about five PWM switching cycles was considered.
To capture the events in detail, a high oscilloscope sampling rate of 250 megasamples
per second was used. The results from case I are shown in Fig. 6.5. Superposition of \( i_{cm} \)
switching spikes was observed. In Case II, the amplitude of the phase current was slowly
decreased from about 250 A down to 50 A, with a phase current frequency of about 50
Hz. This case is presented in Fig. 6.6. Low-frequency pulsating emissions in the common
mode currents with a periodicity of about 0.008 s were observed. These emissions were
termed LFP emissions. More measurements are presented in the appended papers G and
H.

6.7 Theoretical modeling of LFP emissions

A more detailed discussion of the modeling of LFP emissions has been given in papers G
and H. Representing an \( i_{cm} \) spike as a Gaussian pulse, the spike pattern for different PWM
schemes can be reconstructed by superposition [79, 80]. The spike pattern generated by
the space vector PWM (SV-PWM) scheme was considered because it is the scheme used in
the drive train on which the measurements were performed. This section briefly presents
the space vector scheme and models the effects of drive speed, load, and converter slew
rates on the amplitude of the LFP emissions.

6.7.1 Space Vector PWM (SV-PWM) scheme

The space vector hexagon shown in Fig. 6.7 represents the classical SV-PWM scheme [24,
1, 25, 2]. An arbitrary reference voltage vector (\( V_{ref} \)), defined in (2), can be represented
as a linear combination of two adjacent voltage vectors (\( V_k \) and \( V_{k+1} \)) and zero vectors
(\( V_0 \) and \( V_7 \)), as described in (3).

\[
V_{ref} = V_{0^*} \hat{d} + V_{q^*} \hat{q} \quad (6.2)
\]

\[
V_{ref} = D_k V_k + D_{k+1} V_{k+1} + [1 - (D_k + D_{k+1})] V_{0^7} \quad (6.3)
\]
Figure 6.5: Case I showing the superposition of $i_{cm}$ spikes during switching. The top plot is the $i_{cm}$ currents. The second subplot zooms in the region 0.50 s to 0.85 s of the top plot. The third subplot zooms in the region 0.250 s to 0.42 ms of the top plot. The bottom subplot is the phase current. Oscilloscope sampling rate is 250 mega samples per second, 400 times Nyquist sampling frequency.

where $D_k = \frac{T_k}{T_s}$ is the duty cycle, $T_k$ is the time spent on the $V_k$ voltage vector, $T_s$ is the PWM period and $V_d$ is the DC source voltage. The voltage vectors $V_k$ are defined as in (4).

$$V_k = \frac{2}{3} V_d \exp \left(\frac{\pi (k-1)}{3}\right)$$  \hspace{1cm} (6.4)

For half a PWM period, $D_k$ is constrained as

$$D_k + D_{k+1} + D_{0,7} = 0.5,$$  \hspace{1cm} (6.5)

where $D_{0,7}$ is the duty ratio for either $V_0$ or $V_7$ zero voltage vectors. Considering that $V_{0,7} = 0$, $V_{ref}$ is simplified as

$$V_{ref} = D_k V_k + D_{k+1} V_{k+1}.$$  \hspace{1cm} (6.6)
6.7. Theoretical modeling of LFP emissions

Using (2) to (6), the switching times \( T_k \) and \( T_{k+1} \) are obtained as

\[
\begin{pmatrix}
  T_k \\
  T_{k+1}
\end{pmatrix} = \sqrt{3} \frac{T_s}{V_{dc}} \begin{pmatrix} V_{ds} \\ V_{qs} \end{pmatrix},
\]

where \( M \) is obtained by solving (4), (5) and (6). The sector, \( k \), containing the reference voltage is obtained from the angle given in (8).

\[
\theta = \arctan \left( \frac{V_{qs}}{V_{ds}} \right).
\]

Different \( T_k \)'s are obtained for different \( V_{\text{ref}} \)'s, as in (7). The \( T_k \)'s actually determine when the converter switches fire or voltage transitions occur. This, in turn, determines when the \( i_{cm} \) spikes are generated.

### 6.7.2 Modeling of \( i_{cm} \) spikes

Modeling each \( i_{cm} \) spike as a Gaussian pulse as defined in (9), the spike pattern of a given PWM scheme can be obtained by superposition. The spike rise time and width are
Figure 6.7: Space vector scheme showing voltage vectors $V_0, V_1, ..., V_7$ and arbitrary reference voltage vector $V_{ref}$. The binary numbers 000, 001, ... represents the switch states.

varied using parameters $A$ and $C$, as shown in Fig. 6.8. The time of occurrence of the spike is varied by moving the center of the Gaussian pulse, and is equivalent to varying parameter $b$ in (9).

$$f(t) = A \exp\left(\frac{(t-b)^2}{2C^2}\right), \quad (6.9)$$

Consider a $V_{ref}$ in the sector 1, where $0 < \alpha < \frac{\pi}{3}$.

In the symmetric SV-PWM scheme [25], a feasible PWM switching cycle is the following: 

$$\{ ...000 \rightarrow 100 \rightarrow 110 \rightarrow 111 \rightarrow 111 \rightarrow 110 \rightarrow 100 \rightarrow 000... \},$$

with switching times $\{ D_1, D_2, D_3, D_4, D_5, D_6 \}$, respectively. The LFP spike pattern generated by this switching cycle is shown in Fig. 6.9. This pattern clearly depends on the duty cycles, $D_k$.

6.7.3 Reconstruction of LFP emissions

An anticlockwise rotation of a constant amplitude reference voltage $V_{ref}$ generates steady state sinusoidal phase voltages. This is obtained by varying $D_k$ and $D_{k+1}$ while respecting the constraints given in (5). Closed to the sector boundaries, either $D_k$ or $D_{k+1}$ tends to zero. These cause spike superpositions during sector boundary crossings, forming the LFP emissions. Thus, there are six LFP pulses in one complete revolution, because there are six sector boundaries.

$$T_0 = 6 T_{LFP}, \quad (6.10)$$
6.7. Theoretical modeling of LFP emissions

Figure 6.8: Gaussian pulse of height \( A \), width \( C \) and centered at time \( t = 0.5\) s. The center of the pulse corresponds to the parameter \( b \) in (9).

Figure 6.9: \( i_{cm} \) pattern for one PWM cycle, with \( V_{ref} \) in sector 1. time is expressed in terms of PWM switching periods, \( T_s = 0.0002\) s
Figure 6.10: Simulation results showing LFP emissions of periodicity 0.001 s. The results obtained using the following parameter settings: \(0 < D_k \leq 0.2, T_s = 0.0002\) s, \(T_0 = 50T_s\). The lower subplot is obtained by zooming in the response when \(38.1T_s \leq \text{time} \leq 40.0T_s\). This corresponds to case I, presented in Fig. 6.5 showing the superposition of switching spikes.

Figure 6.11: Simulation results showing LFP emissions of \(40T_s = 0.008\) s. The results obtained using the following parameter settings: \(0 < D_k \leq 0.2, T_s = 0.0002\) s, \(T_0 = 240T_s\). This a reconstruction of case II, shown in Fig. 6.6. The lower subplot is obtained by zooming in the response when \(20T_s \leq \text{time} \leq 80T_s\).
The period of the sinusoidal phase voltage $T_0$ is related to the period of the LFP emissions ($T_{LFP}$) as shown in (10). Fig. 6.10 shows simulated LFP emissions of periodicity 0.001 s. The results were obtained using the following parameter settings: $0 < D_k \leq 0.2$, $T_s = 0.0002\text{s}$, and $T_0 = 50T_s$. Fig. 6.11 presents simulation results showing LFP emissions of periodicity $T_{LFP} = 40T_s = 0.008\text{s}$, obtained using the following parameter settings: $0 < D_k \leq 0.45$, $T_s = 0.0002\text{s}$, and $T_0 = 240T_s$. This is an attempted reconstruction of Case II, shown in Fig. 6.6.

This Gaussian pulse reconstruction approach was used to investigate the effects of drive speed, load, and voltage slew rates on the magnitudes of the LFP emissions in paper G. The potential consequences, including EMI and functionality issues, are discussed in paper H. Various mitigation approaches are also discussed.
7.1 Summary of contributions

This thesis contains eight appended papers. In this section a summary of the main contribution of each paper is presented.

7.1.1 Paper A: M. Enohnyaket and J. Ekman, ”Three dimensional high frequency models for air-core reactors based on partial element equivalent circuit theory”, in Proc. IEEE Int. Symp. on EMC, Barcelona, Spain 2006

This paper presents a high-frequency model for air-core reactors with rectangular cross sections, using the PEEC approach. Each reactor turn is represented by four rectangular bars that are mutually perpendicular. The electric field couplings between all of the bars are represented by coefficients of potential, while the magnetic field couplings are represented by the partial inductances. The impedance response from the PEEC model was compared to measurements and results from commonly used lumped circuit models and showed good agreement. Further, the time required to model a realistic air-core reactor was shown to be acceptable on a regular workstation.

7.1.2 Paper B: M. Enohnyaket and J. Ekman, ”High frequency models for air-core reactors using 3D equivalent circuit theory”, in Proc. of Nordic Distribution and Asset Management Conference : NORDAC 2006

This paper enhances the PEEC models presented in paper A to characterize air-core reactors of both rectangular and circular cross-sections. Each turn was meshed into a finite number of rectangular bars. Modified analytical expressions for the coefficients of the potential and the partial inductances were used to represent the electromagnetic
couplings between bars with arbitrary relative inclinations. The measurement results are compared to the PEEC model results in the frequency domain, while the time-domain results are presented solely for the models.

7.1.3 Paper C: M. Enohnyaket and J. Ekman, ”PEEC models for air-core reactors modeling skin and proximity effects”, in Proc. IEEE Power Electronics Specialists Conference (PESC 2007), p 3034 - 3038, Orlando, FL, USA

This paper presents a partial element equivalent circuit (PEEC) model for air-core reactors that models skin and proximity effects at higher frequencies using the volume filament approach. The model created in this paper, unlike that in papers A and B, involves further discretization along the conductor cross-section to capture the nonuniformity in the current distribution due to skin and proximity effects. The modelling results are compared to measurements in both the time domain and the frequency domain and show good agreement.

7.1.4 Paper D: M. Enohnyaket and J. Ekman and Å. Wissten, "Electromagnetic fields from air-core reactors using equivalent circuit theory”, in Proc. of EMB07 - the fourth Swedish conference on computational electromagnetics: methods and applications. 2007

This paper presents an approach to computing the fields from air-core reactors at different frequencies using equivalent circuit theory. The output of the PEEC simulation gives the current distribution in the bars. Considering each bar to be an infinitesimal dipole, the field at a given point in the vicinity of the reactor is obtained as the vector sum of the field contributions from all the current segments. This approach gives a good characterization from DC up to an upper frequency limit determined by the mesh.


This paper contains a summary of the results obtained in papers A, B and C. It further contains a comparison of PEEC time domain results and measurements, showing good agreement. Some instability issues resulting from improper time stepping were considered.
7.1.6 Paper F: M. Enohnyaket and K. Hyypä, ”Characteristic signature of electromagnetic emissions from power converters”, in Proc. IEEE Vehicle Power and Propulsion Conference (VPPC 09), p. 1036-1042, Dearborn, MI, USA

This paper characterizes the sources of transients from power converters and shows the dependence of emissions from converters on the PWM switching scheme. A PSpice model of a DC motor drive circuit was used in the analysis, and a prototype was later constructed on which measurements were performed.


This paper describes the formation of low-frequency pulsating (LFP) common mode currents from space vector modulated drives. It investigates the dependence of various drive parameters such as drive speed and load on the amplitudes of the LFP emissions using theoretical models.


This paper illustrates the formation of low frequency pulsating (LFP) common mode emissions from space vector modulated drives. It establishes that common mode current spikes are formed during the switching of voltage vectors. Close to sector boundaries, these spikes superpose forming double or triple amplitude spikes constituting the LFP emissions. These pulsations occur at a frequency of $6f_0$, where $f_0$ is the fundamental frequency of the phase voltages. The LFP emissions enhance emissions at the harmonics of the switching frequency and also generates low frequency harmonics that lead to torque pulsations.
8.1 Conclusions and discussions

The work performed in this thesis can be summarized in the following questions:

1. Can full-wave 3D electromagnetic modeling feasibly be applied to the design and analysis of power electronic devices?

2. What are the major sources of electromagnetic emissions from an electric drive system? How can these sources be characterized?

Regarding the first question, the suitability of the PEEC method for full-wave electromagnetic modeling of power components has been demonstrated, using air-core reactor models. The created air-core reactor models provide the current and potential distributions at different frequencies, which is helpful both for EMI and basic functionality analysis. The largest size of problems considered in this study was about 5000 nodes. With the recent improvements in the PEEC solver, including the parallelization, problems larger than 250000 nodes can now be handled, which is quite promising [81].

Regarding the second question, the study showed emissions from drives at harmonics of the switching frequency $f_s$ and the fundamental frequency $f_0$, consistent with the existing literature [27, 69, 70, 71, 72, 73, 74, 75, 76, 26]. The study also reveals the emission of low frequency pulsating common mode (LFP) emissions from space vector modulated drives, at a frequency of $6f_0$. The LFP emissions were formed during sector boundary crossings from the superposition of common mode current spikes generated during the switching of voltage vectors. The general implications of these low frequency pulsating common mode emissions include the following:

1. Enhancement of emissions at harmonics of the switching frequency that could pose more EMI issues, including the functionality distortion of sensitive equipment in the vicinity. The amplitudes of the LFP emissions should be considered during EMI filter design.
2. Generation of low frequency harmonics that could lead to torque pulsations and mechanical oscillations in electric machines.

8.2 Future work

The PEEC method was used to create high frequency electromagnetic models for large inductors or air-core reactors. The modelling of inductors with a magnetic core was not considered, as this problem requires PEEC modeling of linear and nonlinear magnetic materials. Recent work in this area includes PEEC modeling of linear materials [82, 83, 84]. The high-frequency modelling of nonlinear magnetic material is an interesting area for future research. This extension would allow for the high frequency modelling of components, such as chokes, inductors with a core, and power transformers. Other extensions of interest include the modelling of moving grids, which enables the modelling of parts with relative motion, as in electric machines.

Regarding the characterization of electromagnetic disturbances from drive systems, the coupling between current harmonics resulting from the low frequency pulsating common mode currents and torque pulsations requires further investigation. An investigation of different mitigation approaches is also necessary. These mitigation techniques include the installation of common mode chokes on the AC side of the converters to damp the common current spikes. In designing the choke, the core material and coils have to be appropriately selected, based on the peak common mode currents and the rated current of the electric machines (load). These choices can be made through adequate material modeling. Other mitigation approaches include the enhancement of the converter topology and modulation scheme to minimize the generation of common mode emissions. On the side of the electric machines, it is of interest to optimize the geometric design of the machine components (parts) to minimize the stray or parasitic capacitances. These include, for example, capacitance between the stator and rotor windings, capacitance between the windings and the motor casing. These stray capacitances determine the magnitude of the common mode current spikes in the machine, and provides the coupling path through the machine as well. Such optimization could be achieved through appropriate parameter modeling.


Part II: Appended Papers
Three Dimensional High Frequency Models for Air-Core Reactors based on Partial Element Equivalent Circuit theory

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PAPER A: THREE DIMENSIONAL HIGH FREQUENCY MODELS FOR AIR-CORE REACTORS BASED ON PARTIAL ELEMENT EQUIVALENT CIRCUIT THEORY
THREE DIMENSIONAL HIGH FREQUENCY MODELS FOR AIR-CORE REACTORS BASED ON PARTIAL ELEMENT EQUIVALENT CIRCUIT THEORY

Mathias Enohnyaket and Jonas Ekman

Abstract

High-frequency models for air-core reactors are of interest in system simulations. For this purpose, this paper deals with three-dimensional, high frequency models for air-core reactors by using the partial element equivalent circuit (PEEC) method. The creation and solution of three-dimensional, electromagnetic reactor models using PEEC is detailed and compared to the traditional lumped circuit models presently used. By using PEEC, high frequency models are created in a systematic fashion and the resulting models offer improved accuracy at high frequencies. Further, the time complexity for modeling realistic air-core reactors is acceptable on a regular workstation. The PEEC results are compared with present models and measurements and show good agreement.

1 Introduction

Air-core reactors are usually used to introduce inductance in AC and DC distributed power systems. They are commonly used for current limiting, neutral grounding, filtering and shunting applications. An example is the 50 MVAR, 400 kV, 50 Hz shunt reactor installed in the Swedish national 400 kV grid.

The reactors are usually included in system simulations [1] as equivalent circuits of varying complexity [2, 3, 4]. The equivalent circuit models capture the dominant behavior of the reactor for low frequencies (absolute values depending on the application) while high frequency behavior is more complex to capture using these models. However, recent development within electromagnetic modeling have made it possible to model the three dimensional electromagnetic couplings between parts of the turns and thus supplying a more detailed model for the reactor.

This paper presents the use of Partial Element Equivalent Circuit (PEEC) theory for creating high frequency, electromagnetic models for air-core reactors. The PEEC models have improved spatial discretization and improved accuracy compared to the traditional lumped models. The PEEC model, like the lumped circuit models, involves the representation of the reactor by equivalent circuit parameters. But with PEEC, using
the concepts of partial inductance and partial coefficients of potential \[5, 6, 7\], a more
detailed representation of sections of a loop into discrete cells is obtained. For these
discrete cells, the equivalent circuit parameters are evaluated using closed formulas and
a resulting equation system is solved for each frequency or time instance of interest.
The method is based on an integral formulation of Maxwell’s equations thus making
the PEEC model less heavier, requiring less CPU time compared to, for example, finite
element models.

For the numerical examples, three different rectangular, air-core reactor structures are
modeled using PEEC and for two of them results are compared with the measurements
and also to the existing lumped models.

2 Basic PEEC Theory

The PEEC method is a 3D, full wave modeling method suitable for combined electro-
magnetic and circuit analysis. In the PEEC method, the electric field integral equation
is interpreted as Kirchoff’s voltage law applied to a basic PEEC cell which results in a
complete circuit solution for 3D geometries. The equivalent circuit formulation allows for
additional SPICE-type circuit elements to easily be included. Further, the models and
the analysis apply to both the time and the frequency domain. The circuit equations
resulting from the PEEC model are easily constructed using a condensed modified loop
analysis (MLA) or modified nodal analysis (MNA) formulation \[8\]. In the MNA formu-
lation, the volume cell currents and the node potentials are solved simultaneously for the
discretized structure. To obtain field variables, post-processing of circuit variables are
necessary.

This section gives an outline of the nonorthogonal PEEC method as fully detailed
in \[9\]. In this formulation, the objects, conductors and dielectrics, can be both or-
thogonal and non-orthogonal quadrilateral (surface) and hexahedral (volume) elements.
The formulation utilizes a global and a local coordinate system where the global coor-
dinate system uses orthogonal coordinates \(x, y, z\) where a global vector \(F\) is of the form
\[
F = F_x \hat{x} + F_y \hat{y} + F_z \hat{z}.
\]
A vector in the global coordinates are marked as \(r_g\). The local
coordinates \(a, b, c\) are used to separately represent each specific possibly non-orthogonal
object and the unit vectors are \(\hat{a}, \hat{b},\) and \(\hat{c}\). The starting point for the theoretical deriva-
tion is the total electric field at a conductor expressed as
\[
E^i(r_g, t) = \frac{J(r_g, t)}{\sigma} + \frac{\partial A(r_g, t)}{\partial t} + \nabla \phi(r_g, t),
\]
where \(E^i\) is an incident electric field, \(J\) is the current density in a conductor, \(A\) is
the magnetic vector potential, \(\phi\) is the scalar electric potential, and \(\sigma\) the electrical
conductivity. The dielectric areas are taken into account as an excess current with the
scalar potential using the volumetric equivalence theorem. By using the definitions of
the vector potential \(A\) and the scalar potential \(\phi\), it is possible to formulate the integral
equation for the electric field at a point \(r_g\) which is to be located either inside a conductor

\[
E^i(r_g, t) = \frac{J(r_g, t)}{\sigma} + \frac{\partial A(r_g, t)}{\partial t} + \nabla \phi(r_g, t),
\]

where \(E^i\) is an incident electric field, \(J\) is the current density in a conductor, \(A\) is
the magnetic vector potential, \(\phi\) is the scalar electric potential, and \(\sigma\) the electrical
conductivity. The dielectric areas are taken into account as an excess current with the
scalar potential using the volumetric equivalence theorem. By using the definitions of
the vector potential \(A\) and the scalar potential \(\phi\), it is possible to formulate the integral
equation for the electric field at a point \(r_g\) which is to be located either inside a conductor
Figure 1: Nonorthogonal element created by the mesh generator with associated local coordinate system.

or inside a dielectric region according to

\[ E^i(r_g, t) = \frac{J(r_g, t)}{\sigma} \]

\[ + \mu \int_{\sigma} G(r_g, r_g') \frac{\partial J(r_g', t_d)}{\partial t} dv \]

\[ + \epsilon_0(\epsilon_r - 1) \mu \int_{\sigma} G(r_g, r_g') \frac{\partial E(r_g', t_d)}{\partial t^2} dv \]

\[ + \nabla \epsilon_0 \int_{\sigma} G(r_g, r_g') q(r_g', t_d) dv. \]

Equation (2) is the time domain formulation which can easily be converted to the frequency domain by using the Laplace transform operator \( s = \frac{d}{dt} \) and where the time retardation \( \tau \) will transform to \( e^{-st} \).

The PEEC integral equation solution of Maxwell's equations is based on the total electric field, e.g. (1). An integral or inner product is used to reformulate each term of (2) into the circuit equations. This inner product integration converts each term into the fundamental form \( \int E \cdot dl = V \) where \( V \) is a voltage or potential difference across the circuit element. It can be shown how this transforms the sum of the electric fields in (1) into the Kirchoff Voltage Law (KVL) over a basic PEEC cell [7]. Fig. 2 details the \((L_p, P, \tau)\)PEEC model for the metal patch in Fig. 1). The model in Fig. 2 consists of:

- partial inductances \((L_p)\) which are calculated from the volume cell discretization using a double volume integral.

- coefficients of potentials which are calculated from the surface cell discretization using a double surface integral.
Figure 2: \((L_p, P, \tau)\)PEEC model for metal patch in Fig. 1 discretized with four edge nodes. Controlled current sources, \(I^n_p\), account for the electric field coupling and controlled voltage sources, \(V^n_L\), account for the magnetic field coupling. Further, the figure can also be interpreted as one turn for a reactor if the loop is left open at one node.

- retarded current controlled current sources, to account for the electric field couplings, given by 
  \[
  I^n_p = \frac{p_{ij}}{p_{ii}} I^i_j (t - t_{di})
  \]
  where \(t_{di}\) is the free space travel time (delay time) between surface cells \(i\) and \(j\).

- retarded current controlled voltage sources, to account for the magnetic field couplings, given by 
  \[
  V^n_L = L_{pnm} \frac{\partial I^m}{\partial t_{dnm}}
  \]
  where \(t_{dnm}\) is the free space travel time (delay time) between volume cells \(n\) and \(m\).

By using the MNA method, the PEEC model circuit elements can be placed in the MNA system matrix during evaluation by the use of correct matrix stamps. The MNA system, when used to solve frequency domain PEEC models, can be schematically described as

\[
\begin{align*}
  j\omega P^{-1} V - A^T I &= I_s \\
  AV - (R + j\omega L_p) I &= V_s
\end{align*}
\]

where \(P\) is the full coefficient of potential matrix, \(A\) is a sparse matrix containing the connectivity information, \(L_p\) is a dense matrix containing the partial inductances, \(R\) is a matrix containing the volume cell resistances, \(V\) is a vector containing the node potentials (solution), \(I\) is a vector containing the branch currents (solution), \(I_s\) is a vector containing the current source excitation, and \(V_s\) is a vector containing the voltage source excitation.

The first row in the equation system in (3) is Kirchoff’s current law for each node while the second row satisfy Kirchoff’s voltage law for each basic PEEC cell (loop). The use of the MNA method when solving PEEC models is the preferred approach since additional
active and passive circuit elements can be added by the use of the corresponding MNA stamp. For a complete derivation of the quasi-static and full-wave PEEC circuit equations using the MNA method, see for example [10].

3 Air-core Reactor Modeling

This section presents a summary of the theory behind lumped circuit and the application of PEEC for modeling air-core reactors.

3.1 Lumped modeling of air-core reactors

The lumped model basically involves partitioning the reactor into sections which are electromagnetically coupled. Each section (partition) is assumed to be electrically small and can be represented by lumped circuit parameters $L, C$ and $R$. There are two partitioning schemes, namely the inductive partitions and the corresponding capacitive partitions. Each partition consist of a given number of turns, and the inductance and capacitance are obtained using closed formulas. The lumped parameters can be used in SPICE-like simulators, like EMTDC [1], for a quasi-static analysis. If full-wave analysis is required, the corresponding circuit equations can be created and solved in an automated fashion. The coupling between the inductive partitions (magnetic field coupling) is represented by the mutual inductances while the mutual capacitances represent the capacitive coupling (electric field coupling).

Consider a reactor of $N$ turns discretized into $i$ inductive partitions and $i+1$ capacitive partitions. For the lumped models $N \gg i$, since several turns are represented in one discrete circuit element. Further, the capacitive and inductive partitions are usually shifted half a partition size with respect to each other. The circuit representation of
this reactor is shown in Fig. 3. The lumped inductances $L_{km}$ are calculated using the expressions for coaxial circular filaments \[11\] as

$$L_{km} = \frac{\mu_0 N_k N_m 2R}{\kappa_L} \left[ 1 - \frac{\kappa_L^2}{2} K(\kappa_L) - E(\kappa_L) \right]$$

(4)

where $N_k$ and $N_m$ are the number of turns in the $k$:th and $m$:th inductive partitions respectively, $R$ the radius of the inductive partitions, and $K(\kappa)$ and $E(\kappa)$ are complete elliptic integrals of the first and second kinds respectively. The term $\kappa_L$ in (4) is defined as

$$\kappa_L = \frac{2R}{\sqrt{z_{km}^2 + 4R^2}}$$

(5)

where $z_{km}$ is the spacing between the $k$:th and the $m$:th inductive partitions.

The coefficients of potential $P_{ij}$ between capacitive partitions $i$ and $j$ are calculated from the capacitive partitions using the expression for the coefficients of potential of two coaxial cylindrical cells \[4\] given by

$$P_{ij} = \frac{1}{2\pi^2 \varepsilon_0 l_i l_j} \int_{-l_i}^{l_i} \int_{-l_j}^{l_j} \frac{K(\kappa_C)}{A} dZ_j dZ_i$$

(6)

where $\varepsilon_0$ is the permittivity of free space and

$$A = \sqrt{z_{ij}^2 + 4R^2},$$

(7)

$\kappa_C = \frac{2R}{A}$, and $l_i$, $l_j$, $z_{ij}$ and $R$ are defined in Fig. 3 (b) and $K$ is the complete elliptical integral of the first kind.

### 3.2 PEEC models for air-core reactors

Section 2 detail the theory for the general (nonorthogonal) PEEC formulation. However, if some careful approximations are used, the PEEC models can be created in a very efficient manner. For example, if the air-core reactors can be approximated with co-planar windings that are interconnected with small resistances, the evaluation of the partial elements can be performed using closed formulas instead of more complex numerical (space) integration routines. The evaluation of partial inductances is detailed in \[5\] and for coefficients of potential in \[6\].

For the moment, the explored reactors consist of windings with small cross section (from $0.6 \times 0.6$ mm to $2 \times 2$ mm) and skin effects are not modeled. However, for general structures, it is necessary to account for skin effects by the use of volume filaments or surface models. Inter-layer isolation is not modeled yet but can be included directly by dielectric cells or by describing the loss mechanism by equivalent circuit models.

The time retardation in the electric- and magnetic-field couplings are taken into account by using the center-to-center distance between the surface cells (for electric field couplings) and between the volume cells (for magnetic field couplings). The time retarded EM couplings are modeled by using the coupled source formulation detailed in Sec. 2.
This results in a system of neutral delay differential equations for time domain modeling while the time retardation is translated to a phase shift for frequency domain modeling (and complex elements in the $P$- and $L_p$-matrices).

The discretization is performed based on the highest frequency of interest ($f_{\text{max}}$). A well established rule is to use 20 cells per (shortest) wavelength giving a maximal cell length of $\Delta_L \leq \frac{\lambda_{\text{shortest}}}{20f_{\text{max}}}$. Relating this criterion to the numerical examples presented in Sec. 4 that are all winded structures with sides of 50 cm or less modeled up to 5 MHz, this criterion would result in one volume cell per side. Then Fig. 2 shows the PEEC model for one turn of the reactor (except that the turns are not closed).

3.3 Lumped models versus PEEC models

The close relation between the lumped models, detailed in Sec. 3.1, and PEEC models is obvious. In fact, the lumped models can be created from the more detailed PEEC models by summing the partial inductances for the lumped turns and by reducing the coefficient of potential matrix for the lumped turns as detailed in [12].

4 Results

This section presents modeling and measurement results for two air-core reactor structures and modeling results for a realistic 750 turn air-core reactor.

4.1 90 turn reactor in the frequency domain

The first test is for a 90 turn air-core reactor winded from a round, $r=2$ mm, copper wire on a sparse wooden support. The spacing between the windings is 1 cm and the cross section of the reactor is $49 \times 58$ cm. However, small differences exists in the spacing of the windings as a result of the manual manufacturing of the reactor.

The input impedance for the structure is measured, from 10 kHz to 5 MHz, using a vector network analyzer. The input impedance is modeled using a full-wave PEEC model, $(L_p, P, R, \tau)_{\text{PEEC}}$, consisting out of 360 unknown current cells and 720 unknown voltage nodes. This discretization gives an upper frequency limit for the model at 26.0 MHz. The structure is also modeled using a lumped model consisting out of 5 self inductances, as detailed in Sec. 3.1. The results are shown in Fig. 4 and show good agreement for both the models for the first resonance around 550 kHz while above that the lumped model fail to predict the input impedance.

4.2 200 turn reactor in the frequency domain

The second test is for a 200 turn air-core reactor winded from a round, $r=0.7$ mm, copper wire on a sparse wooden support. The spacing between the windings is 3 mm and the cross section of the reactor is $50 \times 50$ cm. As for the 90 turn reactor, small differences
exists in the spacing of the windings as a result of the manual manufacturing of the reactor.

As for the 90 turn reactor, the input impedance is measured, from 10 kHz to 5 MHz. The input impedance is modeled using a full-wave PEEC model, \((L_p,P,R,T)\)PEEC, consisting out of 802 unknown current cells and 1604 unknown voltage nodes. This discretization gives an upper frequency limit for the model at 30 MHz. The result when

\[ \text{Figure 4: Input impedance (magnitude) for 90 turn reactor.} \]

\[ \text{Figure 5: Input impedance (magnitude) for 200 turn reactor.} \]
comparing the magnitude of the input impedance is shown in Fig. 5 and show good agreement for the PEEC model for all the resonances except for one around 1.85 MHz. As expected, the magnitude is not correctly modeled, especially for the higher frequencies.

4.3 750 turn reactor in both the time- and frequency- domain

To show the applicability of the method and to detail the time complexity for modeling a realistic air-core reactor, a 750 turn reactor is considered. Thus, for this structure no measurements are presented. The modeling is performed using a dual Xeon, Linux server with 4 Gb of memory. The PEEC-code is sequential and uses no acceleration techniques.

The 750 turn reactor is rectangular with a cross section of $50 \times 50$ cm, the windings are modeled as a square copper wire with cross section of $0.5 \times 0.5$ cm. Thus, the model consist of 1 500 m copper wire divided into 3 000 inductive and 6 000 capacitive segments respectively giving an upper frequency limit for the PEEC model of 30 MHz.

The PEEC model uses the electromagnetic quasi-static assumption, the partial elements are calculated using analytical formulas, and the final equation system is created using Nodal Analysis (an admittance formulation). Table 1.1 details the total time for each step performed in the code for two cases. First when analyzing the reactor in the frequency domain for 200 frequencies. Second, in the time domain for 1 500 time steps. We see clearly that time domain analysis is much faster than the frequency counterpart, as expected.

Table 1.1: Time complexity for analyzing a 750 turn air-core reactor using sequential PEEC-code

<table>
<thead>
<tr>
<th>Step</th>
<th>Time [min]</th>
<th>Time [min]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solver type</td>
<td>FD-PEEC</td>
<td>TD-PEEC</td>
</tr>
<tr>
<td>Parsing &amp; Meshing</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Calc. partial inductances</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Calc. coefficient of potentials</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Solver</td>
<td>240 †</td>
<td>15 ‡</td>
</tr>
<tr>
<td>Total</td>
<td>~ 240</td>
<td>~ 17</td>
</tr>
</tbody>
</table>

† for 200 frequencies
‡ for 1 500 time steps
5 Discussion and Conclusions

By using PEEC theory, high frequency electromagnetic models for air-core reactors can be created in a systematic way without crude approximations. The technique enables detailed studies of air-core reactors for optimization and diagnostic purposes.

Since the same PEEC models, and basically the same computer code, are used in the time and frequency domain, the structures are analyzed in both domains by a simple input parameter change (to be compared to the SPICE .AC and .tran command).

Even though the results presented in the paper are for simple structures, the extension to model the additional (loss) mechanisms are straightforward due to the equivalent circuit formulation utilized. This will be reported in future work.

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between sections of transformer windings,” in *Proc. of IEE C International Confer-

*Proc. of the IEEE International Symposium on EMC*, (Boston, MA, USA), pp. 630–
High Frequency Models for Air-Core Reactors using 3D Equivalent Circuit Theory

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Abstract

This paper presents recent advancements in creating high frequency model for air-core reactors using partial element equivalent circuit (PEEC) theory. By meshing each turn into rectangular bars, PEEC theory can be applied and the reactors can be studied in detail. Measurements results are compared to PEEC model results for the frequency domain while time domain results are presented solely for the models. It is shown that the time complexity for modeling a realistic reactor is acceptable on a regular workstation.

1 Introduction

The application of air-core reactors in power distribution systems include current limiting, neutral grounding, filtering, and shunt applications. Previous attempts to model air-core reactors include mainly lumped electrical equivalent circuit models [1],[2],[3] and [4]. The major drawbacks with the traditionally lumped models is that high frequency electromagnetic behavior is not modeled correctly and that specific parts of the windings can not be studied in detail.

In the Partial Element Equivalent Circuit (PEEC) theory, the electromagnetic behavior of a three dimensional structure is represented by electric equivalent circuits [5],[6],[7]. The PEEC method is based on an integral formulation of Maxwell’s equation, thus making the PEEC model less computational demanding compared to, for example, finite element models for certain classes of problems. The PEEC model gives a full-wave solution, with upper frequency limit determined by the discretization. The same PEEC model is used for both time and frequency domain simulations, where delay in the time domain is equivalent to a phase shift in the frequency domain.

This paper presents an approach to model the high frequency behavior of air-cored reactors, with a circular cross section, using PEEC. Each turn (circular loop) is represented by a finite number of bars with rectangular or circular cross sections. The electromagnetic coupling between the bars is modeled through mutual coefficients of potential and partial mutual inductances. The resulting electromagnetic model is accurate and robust since the partial inductances and coefficients of potential are calculated using closed formulas. This approach enables modeling in the time- and frequency domain for detail studies of various phenomenon. In this paper, PEEC model results in the frequency domain are
compared with measurements for different reactor structures. The time complexity for modeling a realistic reactor is acceptable on a regular workstation.

In Section 2 basic PEEC theory is presented, Section 3 details the air-core reactor model and partial element formulations. Section 5 presents modeling and measurement results and Section 6 finalizes the paper with discussions.

2 Basic PEEC Theory

The PEEC method is a 3D, full wave modeling method suitable for combined electromagnetic and circuit analysis. In the PEEC method, the electric field integral equation is interpreted as Kirchoff’s voltage law applied to a basic PEEC cell which results in a complete circuit solution for 3D geometries. The equivalent circuit formulation allows for additional SPICE-type circuit elements to easily be included. Further, the models and the analysis apply to both the time and the frequency domain. The circuit equations resulting from the PEEC model are easily constructed using a condensed modified loop analysis (MLA) or modified nodal analysis (MNA) formulation [8]. In the MNA formulation, the volume cell currents and the node potentials are solved simultaneously for the discretized structure. To obtain field variables, post-processing of circuit variables are necessary.

This section gives an outline of the nonorthogonal PEEC method as fully detailed in [9]. In this formulation, the objects, conductors and dielectrics, can be both orthogonal and non-orthogonal quadrilateral (surface) and hexahedral (volume) elements. The formulation utilizes a global and a local coordinate system where the global coordinate system uses orthogonal coordinates $x, y, z$ where a global vector $\mathbf{F}$ is of the form $\mathbf{F} = F_x \hat{x} + F_y \hat{y} + F_z \hat{z}$. A vector in the global coordinates are marked as $r_g$. The local coordinates $a, b, c$ are used to separately represent each specific possibly non-orthogonal object and the unit vectors are $\hat{a}$, $\hat{b}$, and $\hat{c}$, see further [9]. The starting point for the theoretical derivation is the total electric field at a conductor expressed as

$$E'(r_g, t) = \frac{J(r_g, t)}{\sigma} + \frac{\partial A(r_g, t)}{\partial t} + \nabla \phi(r_g, t),$$

where $E'$ is an incident electric field, $J$ is the current density in a conductor, $A$ is the magnetic vector potential, $\phi$ is the scalar electric potential, and $\sigma$ the electrical conductivity. The dielectric areas are taken into account as an excess current with the scalar potential using the volumetric equivalence theorem. By using the definitions of the vector potential $A$ and the scalar potential $\phi$, it is possible to formulate the integral equation for the electric field at a point $r_g$ which is to be located either inside a conductor or inside a dielectric region according to equation (2). Equation (2) is the time domain formulation which can easily be converted to the frequency domain by using the Laplace
transform operator \( s = \frac{\partial}{\partial t} \) and where the time retardation \( \tau \) will transform to \( e^{-s\tau} \).

\[
E'(r_g, t) = \frac{J(r_g, t)}{\sigma}
\]

\[
+ \mu \int_{v'\sigma} G(r_g, r_{g'}) \frac{\partial J(r_{g'}, t_d)}{\partial t} dv'
\]

\[
+ \epsilon_0 (\epsilon_r - 1) \mu \int_{v'\sigma} G(r_g, r_{g'}) \frac{\partial^2 E(r_{g'}, t_d)}{\partial t^2} dv'
\]

\[
+ \nabla \epsilon_0 \int_{v'\sigma} G(r_g, r_{g'}) q(r_{g'}, t_d) dv'.
\]

\( E_i(rg, t) = J(rg, t) \sigma(2) + \mu \int v' G(rg, r_{g'}) \frac{\partial J(r_{g'}, t) dv}{\partial t} \]

\( + \epsilon_0 (\epsilon_r - 1) \mu \int_{v'\sigma} G(r_g, r_{g'}) \frac{\partial^2 E(r_{g'}, t_d)}{\partial t^2} dv' \]

\( + \nabla \epsilon_0 \int_{v'\sigma} G(r_g, r_{g'}) q(r_{g'}, t_d) dv'. \)

\( \frac{\partial}{\partial t} \)

Figure 1: Nonorthogonal element created by the mesh generator with associated local coordinate system.

The PEEC integral equation solution of Maxwell’s equations is based on the total electric field, e.g. (1). An integral or inner product is used to reformulate each term of (2) into the circuit equations. This inner product integration converts each term into the fundamental form \( \int E \cdot dl = V \) where \( V \) is a voltage or potential difference across the circuit element. It can be shown how this transforms the sum of the electric fields in (1) into the Kirchoff Voltage Law (KVL) over a basic PEEC cell [7]. Figure 2 details the \((L_p, P, \tau)\)PEEC model for the metal patch in Fig. 1 when discretized using four edge nodes (dark full circles). The model in Fig. 2 consists of:

- partial inductances \((L_p)\) which are calculated from the volume cell discretization using a double volume integral.

- coefficients of potentials which are calculated from the surface cell discretization using a double surface integral.
• retarded current controlled current sources, to account for the electric field couplings, given by $I_{ip} = \frac{P_{ij}}{P_{ii}} I_j (t - t_{dij})$ where $t_{dij}$ is the free space travel time (delay time) between surface cells $i$ and $j$.

• retarded current controlled voltage sources, to account for the magnetic field couplings, given by $V_{nL} = L_{pnm} \frac{\partial I_m}{\partial t} (t - t_{dnm})$, where $t_{dnm}$ is the free space travel time (delay time) between volume cells $n$ and $m$.

By using the MNA method, the PEEC model circuit elements can be placed in the MNA system matrix during evaluation by the use of correct matrix stamps [8]. The MNA system, when used to solve frequency domain PEEC models, can be schematically described as

$$
\begin{align*}
    j\omega P^{-1}V - A^T I &= I_s \\
    AV - (R + j\omega L_p)I &= V_s
\end{align*}
$$

where: $P$ is the full coefficient of potential matrix, $A$ is a sparse matrix containing the connectivity information, $L_p$ is a dense matrix containing the partial inductances, $R$ is a matrix containing the volume cell resistances, $V$ is a vector containing the node potentials (solution), $I$ is a vector containing the branch currents (solution), $I_s$ is a vector containing the current source excitation, and $V_s$ is a vector containing the voltage source excitation. The first row in the equation system in (3) is Kirchoff’s current law for each node while the second row satisfy Kirchoff’s voltage law for each basic PEEC
cell (loop). The use of the MNA method when solving PEEC models is the preferred approach since additional active and passive circuit elements can be added by the use of the corresponding MNA stamp. For a complete derivation of the quasi-static and full-wave PEEC circuit equations using the MNA method, see for example [10].

3 Air-core reactor model

3.1 Reactor structure

A laboratory model of the air-core reactor was constructed by winding copper wire of diameter 0.7 mm around a cylindrical plastic support (low $\varepsilon_r$) of outer diameter 0.40 m, with a pitch of 2.5 mm.

3.2 Computational model

3.2.1 Basic setup

A corresponding PEEC model of the laboratory reactor is designed. Each turn in the PEEC model is made up of a finite number of bars with rectangular cross section. In this case 20 bars were used in one turn. The end of the first turn is connected to the start of the second turn by a small resistor. In a similar fashion, the second turn is connected to the third, the third to the fourth until the last turn, modeling a spiral winding. Figure 3 shows a sample 4 turn reactor model. In this case each turn is formed by 16 bars, just for simplicity.

![Figure 3: Schematic description for reactor PEEC model consisting of 16 bars per turn.](image)

Considering a case of 6 bars per turn, the equivalent circuit for one turn is shown in figure 4. Each bar represents a PEEC volume cell, and is used in the calculation of the partial inductance and partial coefficients of potential. The electromagnetic coupling
between the bars is represented by the partial mutual inductances and the mutual coefficients of potential. The partial inductance $L_{p_{ij}}$ is calculated from the $i^{th}$ volume cell, while the partial coefficient of potential $P_{ii}$ is obtained from the corresponding surface cells. The inductive coupling from all other volume cells is represented by $V_iL$ while the capacitive coupling is represented by $\phi_i$. Other circuit components like resistors, excess capacitances and inductances are simply included in the equivalent circuit.

![Figure 4: Equivalent circuit representation for one turn made of 6 bars.](image)

### 4 Partial element calculations for circular reactors

#### 4.1 Partial inductances

A thin filament approximation is used to obtain the partial mutual inductances between PEEC volume cells. The mutual inductance of two parallel filaments of lengths $l_i$ and $l_j$ according to [11] is given by

$$L_{p_{ij}} = 0.001 \int_{l_i} \int_{l_j} \frac{dl_i \cdot dl_j}{|r_i - r_j|}$$

(4)

where $l_i$, $l_j$ are the lengths of the filaments, while $r_i$ and $r_j$ are positions vectors of arbitrary points on the $i^{th}$ and $j^{th}$ filaments respectively. Considering the filaments with arrows in figure 3 for example, $dl_i$ and $dl_j$ would be the current directions, which corresponds to the arrow directions. For the case where the filaments are inclined at an
angle \( \alpha \), the filament \( l_j \), is replaced by the \( l'_j \) of length \( l_j \cos \alpha \), parallel to filament \( l_i \) and the center of mass of \( l_j \) and \( l'_j \) coincides. This gives \( L_{pij} \) maximum when filaments are parallel and zero when they are perpendicular. This approximation gives fairly accurate solutions, and is much faster compared to numerical integration routines.

4.2 Partial coefficients of potential

The coefficient of potential \( P_{ij} \) is given by

\[
P_{ij} = \frac{1}{4\pi\varepsilon_0 S_i S_j} \int_{S_i} \int_{S_j} \frac{1}{|r_i - r_j|} dS_i dS_j
\]

(5)

where \( r_i \) and \( r_j \) are positions vectors of arbitrary points on the \( S_i \) and \( S_j \) respectively.

\( P_{ij} \) for the two orthogonal surfaces \( S_i \) and \( S_j \) shown in figure 5 will have a maximum value \( P_{ij\text{max}} \) when \( \alpha = n\pi \) and a minimum value \( P_{ij\text{min}} \) when \( \alpha = (n + 1/2)\pi \), where \( n \) is an integer, given that \( l_j > w_j \). For all \( \alpha \), \( P_{ij} \) is approximated as

\[
P_{ij} = \cos^2 \alpha P_{ij\text{max}} + \sin^2 \alpha P_{ij\text{min}}.
\]

(6)

An exact analytical expression for \( P_{ij\text{max}} \) or \( P_{ij\text{min}} \) is given in [6] and [11]. The equation (6) is fairly accurate and it is much faster to compute compared to numerical integration routines.

\[\text{Figure 5: } S_i \text{ and } S_j \text{ are two surfaces, } l_i \text{ and } w_i \text{ are the length and width of } S_i, \alpha \text{ is the inclination of } S_j, \text{ relative to } S_i\]
5 Results

The PEEC simulations are run on a machine with a dual Intel Xeon CPU 2.8 GHz, and 3 GB RAM.

5.1 133 turns reactor

A reactor consisting of 133 turns winded copper wire with diameter of 0.7 mm was constructed using a circular plastic (low $\varepsilon_r$) support with diameter of 0.4 m. The winding separation is 2 mm giving the reactor length of approximately 0.27 m.

The 133 turns reactor is made up from 20 orthogonal, rectangular bars per turn giving a total number of lumped elements of:

- 2,660 self partial inductances and volume cell resistances,
- 7,072,940 mutual partial inductances,
- 5,320 self coefficients of potential, and
- 28,297,080 mutual coefficients of potential.

For the constructed reactor, measurements were carried out using a vector network analyzer in the frequency range 10 kHz to 5 MHz. Below presents results for the 133 turns reactor in both the time- and frequency domain with comparison with measured results for frequency domain results.

5.1.1 Frequency domain results

The frequency domain model is a $(L_p, C, R)$PEEC model which include all electric- and magnetic- field couplings between the 20 segments in the turns. The model is quasi-static and thus the phase shift in the electromagnetic field couplings are not updated for each frequency. The extension to a full-wave model is trivial in the frequency domain but not utilized in this example due to the large electrical length of the 133 reactor.

Figure 6 shows a comparison between the measurement results and the PEEC simulation results.

5.2 210 turns reactor

A 210 turns reactor was also modeled. The PEEC model consists of 20 orthogonal, rectangular bars per turn giving a total number of lumped elements of:

- 4,200 self partial inductances and volume cell resistances,
- 17,635,800 mutual partial inductances,
- 8,400 self coefficients of potential, and
Figure 6: PEEC model results for 133 turn reactor against measurements, 5 kHz to 5 MHz. The stars represents data points for the PEEC model results.

- 70,551,600 mutual coefficients of potential.

Measurements were not made for this, but the idea was to observe how good the simulation model can handle larger problems.

5.2.1 Frequency domain results

The PEEC model was excited with a current source of 1 A, and phase 0. Figure 7 presents the results from 5 kHz to 10 MHz.

5.2.2 Time domain results

The developed code allows for time domain analysis of the same model by switching the frequency domain, SPICE-like, AC analysis option

.AC LIN|LOG no_points f_start f_stop

to transient analysis by adding

.TRAN no_points t_start t_stop

For the time domain analysis the PEEC model is excited with a gaussian pulse. Figure 8 presents the timw domain results.

5.2.3 Time complexity

The time complexity for analyzing the 210 turns reactor is shown in table 2.1. It shows that the time required to analyse larger problems is acceptable on a regular workstation.
6 Discussion

There is a fairly good agreement between the measurement results and the PEEC model results. The PEEC model can characterize reactors up to 20 MHz, but measurements on the reactor so far give significant information up to 5 MHz.

In the PEEC model each turn consists of a number of bars of rectangular cross section, and the circular winding is better represented by a large number of bars. But then, the
Table 2.1: Time complexity for analyzing a 210 turns reactor, 20 bars per turn, using sequential PEEC-code

<table>
<thead>
<tr>
<th>Step</th>
<th>Time [min]</th>
<th>Time [min]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solver type</td>
<td>FD-PEEC</td>
<td>TD-PEEC</td>
</tr>
<tr>
<td>Parsing &amp; Meshing</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>Calc. partial inductances</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Calc. coefficient of potentials</td>
<td>6.0</td>
<td>6.0</td>
</tr>
<tr>
<td>Solver</td>
<td>1034 †</td>
<td>15 ‡</td>
</tr>
<tr>
<td>Total</td>
<td>~ 1037</td>
<td>~ 55</td>
</tr>
</tbody>
</table>

† for 100 frequencies
‡ for 1 000 time steps

size of the problem increases significantly with increase in the number of bars per turn. In this case, 20 bars per turn does a good characterization of the circular winding, and this is seen from the agreement with the measurement results in the presented figures.

The PEEC model reactor is a rather simple model. Skin effect and other proximity effects are not considered. We shall report in future PEEC models that include Skin effects.

Acknowledgement

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References


PEEC Models for Air-core Reactors Modeling Skin and Proximity Effects

Authors:
Mathias Enohmyakel and Jonas Ekman

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PEEC Models for Air-core Reactors
Modeling Skin and Proximity Effects

Mathias Enohnyaket and Jonas Ekman

Abstract
This paper presents a partial element equivalent circuit (PEEC) model for air-core reactors modeling skin and proximity effects at higher frequencies using the volume filament approach. Modeling results are compared to measurements in both time domain and frequency domain, and show good agreement.

Keywords: Air-core reactor, Equivalent circuit modeling, Skin effect, Proximity effect.

1 Introduction
Air-core reactors are commonly used in current limiting applications and as harmonic filters. In order to model the propagation of fast transients (∼ ns) through air-core reactors there is a need for high frequency models. For such high frequency models, it is necessary to consider skin and proximity effects. High frequency models for air-core reactors using the Partial Element Equivalent Circuit (PEEC) [1, 2, 3] approach have been reported in [4]. In the PEEC model, each turn in the reactor winding is represented by a finite number of bars, and only currents flowing along the air-reactor windings were considered. Such models are good enough for reactors with very thin conductor cross sections and at lower frequencies (a few megahertz), and does not account for skin effects. For realistic reactors it is necessary to model the current along the windings as well as the transverse currents.

Several methods of modeling skin and proximity effects for a multi-conductor geometry have been reported. For example, the volume filament approach [1, 5], which requires a 3D discretization of the conductors. There is the global surface impedance approach [6], which is more of a surface technique; and macro-models which involve a reduced decoupled RL circuits to model skin and proximity effects [7]. Other skin effects models for conductors are found in [8]. In this paper the volume filament technique is used to make a 3D model of air-core reactors accounting for skin and proximity, using the PEEC modeling approach. PEEC model results are compared with measurements in both time domain and frequency domain.

An alternative method for modeling air-core reactors, namely the lumped modeling approach, has been considered in [9, 10]. In this approach, the reactor is represented by a number of mutually coupled lumped sections, with each section consisting of a given
number of turns. A comparison with measurements showed that the lumped models are good only at frequencies less than 1 MHz, but fail to capture the high frequency variations. A more detailed distributed system which accounts for couplings between all parts of the individual winding is necessary at higher frequencies.

2 Basic PEEC Theory

The PEEC approach is more detailed described in [1, 2, 3]. Only the time domain derivation is shown here. The frequency domain derivation is similar.

Consider the electric field on a conductor given by (1)

\[ E_i(r, t) = \frac{J(r, t)}{\sigma} + \frac{\partial A(r, t)}{\partial t} + \nabla \phi(r, t), \]  

where \( E_i \) is an incident (externally) applied electric field, \( J \) is the current density in the conductor, \( A \) is the magnetic vector potential, \( \phi \) is the scalar electric potential, and \( \sigma \) the electrical conductivity. By using the basic definitions of the electromagnetic potentials as in (2) and (3),

\[ A(r, t) = \mu \int_{v'} G(r, r') J(r', t_d) dv' \]  

\[ \phi(r, t) = \frac{\nabla}{\varepsilon_0} \int_{v'} G(r, r') q(r', t_d) dv'. \]

where the Green’s function \( G(r, r') = \frac{1}{|r - r'|} \), and substituting in (1) the electric field integral equation (4), at the point \( r \) in the conductor is obtained according to

\[ E_i(r, t) = \frac{J(r, t)}{\sigma} \]  

\[ + \mu \int_{v'} G(r, r') \frac{\partial J(r', t_d)}{\partial t} dv' \]  

\[ + \frac{\nabla}{\varepsilon_0} \int_{v'} G(r, r') q(r', t_d) dv'. \]

Integrating (4) using a suitable inner product, followed by some algebraic manipulations, the Kirchhoff’s voltage law for a PEEC cell is obtained as

\[ V = RI + sL_p I + PQ, \]

where \( L_p \) contains partial inductances while \( P \) contains the coefficients of potential. The partial elements are evaluated using analytical routines for orthogonal geometries and numerical integration for non-orthogonal geometries [3, 11]. Fig. 1 (b) shows the PEEC model representation of a the conducting bar in Fig. 1 (a), and is referred to a basic PEEC cell. This type of PEEC cell is the building block of all PEEC models for conductors. The magnetic field couplings are considered through the mutual partial
inductances represented in a voltage source $V_{m}^{L}$, while the electric field couplings are considered by the mutual coefficients of potentials represented in a current source $I_{C}$.

\[11\]

\[(a)\]

Figure 1: Conducting bar in (a) and corresponding PEEC model representation (b).

3 Skin and Proximity effects

Skin and proximity effects bring about non-uniformity in the current distribution along a cross section of a conductor. The increase in current density towards the conductor surface and around edges, due to changing fields within the conductor itself only, is termed skin effect [8]. This phenomenon depends on the conductor geometry and frequency.

In this study, the volume filament (VFI) technique is applied. Skin effect is modeled by making a 3D discretization of the conductor windings into volume cells of width $\delta/2 = \sqrt{\frac{1}{4\mu\sigma f_{m}}}$, where $\delta$ is the skin dept of copper at the maximum frequency of interest $f_{m}$. An optimal way is to use a non-uniform meshing scheme, where a coarser mesh is used in the center and a finer mesh close to the edges, respecting the $\delta/2$ rule.

The current distribution in one volume filament could be influenced by changing fields in adjacent filaments. This is termed proximity effects [8]. Fig. 2 is an illustration of the current distribution in volume filaments due to skin and proximity effects. In the VFI PEEC model, the current directions in the volume cells is not assumed a priori. The electromagnetic coupling between all volume cells is considered, thus properly modeling proximity effects. The current distribution in a multi-conductor system is a combination of skin and proximity effects. Efforts are on the way to create a less expensive surface skin-effect model for reactors which requires just a surface discretization of the windings,
a method similar to that described in [6].

![Figure 2: Current distribution due to skin and proximity effects. (a) Volume cell currents in same direction. (b) currents in opposite directions (c) Only skin-effect, no proximity effect.](image)

4 Air-core reactor modeling

This section deals with the creation of the electromagnetic models for air-core reactors using PEEC cells.

4.1 Reactor structure

In order to validate the modeling approach, a simple air-core reactor model was constructed by winding 65 turns of thin copper tape of width 6.35 mm and thickness 0.076 mm around a sparse rectangular wooden support (low $\varepsilon_r$), with a constant separation of 1 cm. Unlike in [4], copper tape of large surface area was chosen in order to observe variations due to skin and proximity effects. Fig. 3 is a picture of the air core reactor model.

4.2 PEEC air-core reactor model

In the corresponding PEEC model of the reactor, each turn is represented by four rectangular bars. The end of the first turn is connected to the start of the second turn by a short circuit. In a similar fashion, the second turn is connected to the third, the third to the fourth until the last turn, modeling a spiral winding.

Each bar represents a PEEC volume cell, and is used in the calculation of the partial inductance and partial coefficients of potential. The partial inductance $L_{pi}$ is calculated from the $i$:th volume cell, while the partial coefficient of potential $P_{ii}$ is obtained from the corresponding surface cells.
4.3 Measurement setup

Impedance response over a specific frequency range is obtained directly using a vector network analyzer. For the time domain response, a low voltage impulse test is performed. A schematic of the time domain setup is described in Fig. 4. The input terminal of the reactor is excited with a fast trapezoidal pulse, of amplitude in the order of 10 V, from an impulse generator. The internal resistance of the pulse generator is 50 Ω. The output terminal of the reactor is in series with a 50 Ω resistor. The input and output pulses are observed using an oscilloscope.

5 Results

The PEEC simulations are run on a Linux machine with a dual Intel Xeon CPU 2.8 GHz, and 3 GB RAM. Using the PEEC approach, the same model can be used both in the time domain and the frequency domain.
5.1 Time domain results

The reactor was excited with a fast trapezoidal pulse of rise time 32 ns, and peak voltage level of 9.2 V from the pulse generator. The meshing used is 3 volume cells per side (cell size = 16.67 cm) and the cell count is presented in Table 3.1. The output current flows through a 50 Ω resistor while the generator has internal resistance is 50 Ω. The input voltage and output voltage were observed and recorded using an oscilloscope. Fig. 5 presents the time domain results.

5.2 Frequency domain results

The frequency domain model considers all the electric and magnetic field couplings between all volume cells. A quasi-static simulation is performed, meaning the phase shift is updated every frequency step. This is good enough due to the large electrical length of the reactor. The PEEC model was excited with a unitary current source and the meshing is the same as for the time domain test in the previous section. However, for the skin effect model each volume cell is subdivided along the width into 7 cells. This gives the cell count as presented in Table 3.1 which is a considerable increase. The frequency response was obtained for 10 kHz to 5 MHz. Fig. 6 presents the simulated and measured results in the frequency domain. The influence of skin effect on the current distribution in the reactor windings is more pronounced at higher frequencies, as shown in Fig. 7.
Figure 5: Terminal voltage response for the 65 turn tape reactor, excited at the input terminal with a fast trapezoidal pulse of rise time 32 ns, while the output is terminated through a 50 Ω resistor.

Table 3.1: Cell counts for test cases performed on 65 turn, rectangular, tape reactor.

<table>
<thead>
<tr>
<th>Part inductances</th>
<th>Coefficients of potential</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>self</td>
</tr>
<tr>
<td>Test IV-A &amp; B: Without skin effect</td>
<td>780</td>
</tr>
<tr>
<td>Test IV-B &amp; B: With skin effect</td>
<td>5720</td>
</tr>
<tr>
<td>Skin effect model by VFI-PEEC</td>
<td></td>
</tr>
</tbody>
</table>

5.3 Time complexity

The PEEC simulations are run on a machine with a dual Intel Xeon CPU 2.8 GHz, and 3 GB RAM. The time required for each step in the PEEC solver for Test IV-A and IV-B are detailed in Table 3.2. While the time for the volume filament (VFI) model is given in Table 3.3.
6 Discussion and Conclusions

There is a fairly good agreement between the measurement results and the PEEC model results. A slight shift in the resonance peaks which increases as one goes higher up in
Table 3.2: Time complexity for 65 turn, rectangular, tape reactor (time in seconds).

<table>
<thead>
<tr>
<th>Step</th>
<th>Time [s]</th>
<th>Time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solver type</td>
<td>Test IV-A</td>
<td>Test IV-B</td>
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<tr>
<td>Parsing &amp; Meshing</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Calc. partial inductances</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Calc. coefficient of potentials</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Solver</td>
<td>410 †</td>
<td>40 ‡</td>
</tr>
<tr>
<td>Total</td>
<td>420</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 3.3: Time complexity for 65 turn, rectangular, tape reactor with volume filaments for skin effect modeling (time in minutes).

<table>
<thead>
<tr>
<th>Step</th>
<th>Time [min]</th>
<th>Time [min]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solver type</td>
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<td>Test IV-B</td>
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<td>Parsing &amp; Meshing</td>
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<td>0.15</td>
</tr>
<tr>
<td>Calc. partial inductances</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Calc. coefficient of potentials</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Solver</td>
<td>235 †</td>
<td>19 ‡</td>
</tr>
<tr>
<td>Total</td>
<td>240</td>
<td>22</td>
</tr>
</tbody>
</table>

† for 100 frequencies
‡ for 1 000 time steps

frequency, is observed. This can be explained from the discrepancies between the model and the constructed reactor. As previously seen, the amplitudes are hard to capture in the modeling results leaving the measurement response more damped. Up to 5 MHz, the influence of skin effect on the measured impedance response is small. Possibly proximity effects are more important at these frequencies. The reactor model created here gives the voltage and current distribution in the reactor which could be used for example in estimating the near and far field interactions in and from the reactor, studying the mechanical stress on the reactor windings and other dynamic analysis.
Acknowledgement

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References


Electromagnetic Fields from air-core reactors using equivalent circuit theory

Authors:
Mathias Enohmyaket and Jonas Ekman

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Electromagnetic Fields from air-core reactors using equivalent circuit theory

Mathias Enolnyakot, Åke Wisten and Jonas Ekman

Abstract

This paper presents an approach to compute the fields from air-core reactors at different frequencies using equivalent circuit theory. The reactor model is represented by concentric turns of a conductor, each turn consisting of a finite number of bars of rectangular cross section. The output of the equivalent circuit simulation gives the current distribution in the bars. Considering each bar as an infinitesimal dipole, the field at a given point in the space around the reactor is considered as the vector sum of the field contribution from all the current segments at the point. This approach gives a good characterization from dc up to an upper frequency limit determined by the mesh.

1 Introduction

Air-core reactors are commonly used in medium- and high-voltage power systems for current limiting operations. The absence of a magnetic core causes the magnetic field from the reactor to spread widely in its vicinity. The magnetic fields from air-core reactors can be very strong, especially during short-circuit operations. For human safety reasons and EMC/EMI requirements, it is of importance to be able to calculate the fields from an air-core reactor at different current levels and frequencies.

This paper presents an approach to compute the fields from air-core reactors at different frequencies using a Partial Element Equivalent Circuit (PEEC) [1, 2, 3] based air-core reactor model, where the reactor is represented by concentric turns, each turn consisting of a finite number of bars of rectangular cross section. Using the current distribution in the segments(bars), and considering each segment as an infinitesimal dipole, the field at a given point in space is the vector sum of the fields from all the current segments. There has been some attempts to model the magnetic fields from air-core reactors at power frequencies, namely the current loop stack model [4]. In this approach the reactor was modeled as a series of coaxial loops, each loop considered as a magnetic dipole. The field at any point in the vicinity of the reactor is considered as the vector sum of the contributions of all the magnetic dipoles. This approach becomes limited at frequencies where the circumference of one turn is comparable to the wavelength, since the electromagnetic delay within a turn becomes significant. At such frequencies each turn needs to be represented as discrete segments. The advantage of the PEEC approach is there is no such frequency limitation.
2 Basic PEEC Theory

The derivation of PEEC theory for the time domain is shown here and further detailed in [1, 2, 3]. Consider the electric field on a conductor given by

$$E_i(r, t) = \frac{J(r, t)}{\sigma} + \frac{\partial A(r, t)}{\partial t} + \nabla \phi(r, t),$$

where \(E_i\) is an incident (externally) applied electric field, \(J\) is the current density in the conductor, \(A\) is the magnetic vector potential, \(\phi\) is the scalar electric potential, and \(\sigma\) the electrical conductivity. By using the basic definitions of the electromagnetic potentials \(A\) and \(\phi\) with the free space Green’s function \(G(r, r')\), the electric field integral equation for a conductor in free space is obtained according to

$$E_i(r, t) = \frac{J(r, t)}{\sigma} + \mu \int_{v'} G(r, r') \frac{\partial J(r', t_d)}{\partial t} dv' + \nabla \epsilon_0 \int_{v'} G(r, r') q(r', t_d) dv'.$$

Integrating (2) using a suitable inner product, followed by some algebraic manipulations, the Kirchoff voltage law for a PEEC cell is obtained as

$$V = RI + \frac{\partial i}{\partial t} L_p + PQ,$$

where \(RI\) is the voltage drop over an equivalent resistance, \(\frac{\partial i}{\partial t} L_p\) the voltage drop related to the partial inductances, and \(PQ\) related to the coefficients of potential. The partial elements are evaluated using analytical routines for orthogonal geometries and numerical integration for non-orthogonal geometries [3, 5, 6, 7]. Figure 1(b) shows the equivalent circuit representation of the conducting wire in Figure 1(a), and is referred to as a basic PEEC cell. The PEEC cells are the building blocks of PEEC models. The magnetic field couplings are considered through the mutual partial inductances represented in a voltage source \(V_{Lmm}\) while the electric field couplings are considered by the mutual coefficients of potentials represented in a current source \(I_{i,jp}\).

3 Field computation - Infinitesimal dipole approach.

The length of the current segments, \(l\), in the PEEC model is always chosen such that \(l << \lambda_{min}\), where \(\lambda_{min}\) is the minimum wavelength of interest. This allows us to approximate the current segments as infinitesimal dipoles. The magnetic vector potential at a point \(r\) in space due to an infinitesimal dipole on the z-axis is given by

$$A(r, t) = \frac{1}{4\pi \epsilon_0} \frac{l_0 e^{-jkr}}{r},$$

assuming constant current \(l_0\) in the segment, \(k\) is the wave number, and omitting \(e^{j\omega t}\). The magnetic field is thus obtained as \(H = \frac{1}{\mu} (\nabla \times A(r, t))\), while the electric field is
derived from the magnetic field as \( \mathbf{E} = \frac{1}{\mu_0} \nabla \times \mathbf{H} \). The magnetic fields, in the spherical coordinate system, due to the dipole, according to [8] and [9] is

\[
H_\phi = jkI_0 l \sin(\theta) \left( 1 + \frac{1}{jk^2r} \right) e^{-jk^2r}
\]

while the electric field is given by

\[
E_r = \eta I_0 l \cos(\theta) \left( 1 + \frac{1}{jk^2r} \right) e^{-jk^2r}
\]

\[
E_\theta = j\eta I_0 l \sin(\theta) \left( 1 + \frac{1}{jk^2r} - \frac{1}{(kr)^2} \right) e^{-jk^2r}.
\]

The magnetic field components \( H_r = H_\theta = 0 \) while the electric field component \( E_\phi = 0 \).

The magnetic field for the \( dc \) case, \( H_\phi^{dc} \) given in (7), is obtained by taking the limiting value of \( H_\phi \), when \( k \) tends to zero, which is in agreement with Biot-Savart law.

\[
H_\phi^{dc} = \frac{I_0 l \sin(\theta)}{4\pi r^2}
\]

Since the PEEC methods gives the solution down to \( dc \), the field calculations are valid from \( dc \) to the upper frequency limit given by the meshing.
4 Test cases

This section presents two cases where the field evaluation approach is used to determine the fields around more complex structures in combination with PEEC modeling. For each case, a frequency domain simulation is performed and the current distribution in the volume cells obtained from the PEEC simulation at each frequency step is post-processed as infinitesimal dipoles to obtain the fields as detailed in Section (3). The following sub-sections presents the two cases.

4.1 Half-wavelength dipole

The field pattern for a 20 cm half-wavelength dipole oriented at the x-axis at resonance (750 MHz) is determined. Figure (2) and (3) shows the field patterns, along a circle of radius 300 cm in the xy-plane, with the dipole at the center. This result agrees with the existing theoretical solution for the far field of a half-wavelength dipole [8].

![Electric field from a x-directed dipole (20 cm) at resonance (750MHz) where (a) and (b) represent the real and imaginary parts respectively.](image)

4.2 Rectangular air-core reactor

The air-core reactor used in this test has a cross section of 50 × 50 cm and a height of 65 cm. It was formed by winding 65 turns of copper tape of width 6.35 mm and thickness 0.076 mm around a sparse wooden support. In the corresponding PEEC model, each turn is represented by four interconnected bars or PEEC volume cells (neglecting the pitch angle). The PEEC model is excited with a current source, and ran for a few frequencies.

Figure (4) and (5) show the field pattern for the 65 turn reactor. Unlike the dipole case, there do not exist any analytical expressions which can characterize the fields from...
such a complex structure. In any case, from the modeling results, it is seen that the field pattern is strongest along a vertical axis through the center of the reactor, which is expected. There are plans to perform field measurements on the 65 turn laboratory reactor to verify the model results.

Figure 3: H field from a x-directed dipole (20 cm) at resonance (750MHz) where (a) and (b) represent the real and imaginary parts respectively.

Figure 4: H field from a 65 turns tape reactor 50 × 50 cm at 1 Hz where (a) and (b) represent the real and imaginary parts respectively.
5 Discussions and conclusion

Using the infinitesimal dipole approach in combination with the PEEC based model, the fields from large complex structures, like air-core reactors, can be characterized. The approach can predict the fields at different current levels and frequencies, thus providing useful input for EMC/EMI studies.

Acknowledgement

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References


Analysis of Air-Core Reactors from DC to Very High Frequencies using PEEC Models

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Analysis of Air-Core Reactors from DC to Very High Frequencies using PEEC Models

Mathias Enohnyaket and Jonas Ekman

Abstract

The Partial Element Equivalent Circuit (PEEC) approach has been shown to be useful in the analysis of electromagnetic problems for microelectronics. However, recent trends show a continuous increase in the utilization of power electronic devices in power systems, and as a consequence there is a need to characterize the effects of the high frequency noise resulting from the electronic switching, on other components. This paper presents the application of PEEC theory for the creation of three dimensional, electromagnetic models for air-core reactors. The electromagnetic field couplings are separated in mutual partial inductances and mutual coefficients of potential giving a correct solution from DC to a maximum frequency determined by the meshing. The PEEC models are validated by comparing simulation results, for both time and frequency domain analysis, against measurements and other established modeling methods, and show good agreement. The model created by PEEC theory, could be helpful in the design and diagnostics of air-core reactors and other power system components.

Keywords: PEEC, air-core reactor, numerical modeling, electromagnetic modeling, equivalent circuits, partial inductances, coefficients of potential.

1 Introduction

Common applications of air-core reactors in power distribution networks includes damping of transient currents, neutral grounding of three-phase devices, filtering applications, and shunting applications. There is an increasing utilization of modern power electronic devices in power systems, and present trends in power electronics show that the operational frequencies of power electronic devices now cover from power frequencies to the megahertz range [1]. It is important to understand the effects of electronic switching transients (in the nanosecond range) on other circuit components. As a consequence creating computer models of other circuit components like air-core reactors (inductors), capacitors and transformers accounting for the high frequency behaviour (up to the megahertz range) are new challenges. This paper presents a high frequency electromagnetic model for air-core reactors using the partial element equivalent circuit (PEEC) approach.

Several lumped air-core reactor models have been proposed. For example, the planar filament current loop stack model [2] for studying power frequency magnetic field distri-
bution around large air-core reactors, and the equivalent circuit lumped model approach [3] for studying the impulse voltage distribution for single layered air-core reactors. Air-core reactors can also be modeled using simple lumped modeling techniques developed for modeling power transformer windings [4, 5]. These lumped models are usually made up of a series of coaxial sections, each section consisting of a number turns. Such models work well at low frequencies where the voltage distribution along the turns in each section can be considered linear. For higher frequencies, current and voltage distributions are non-linear [6, 7], and thus a more detailed distributed model accounting for the electromagnetic coupling is necessary.

The PEEC air-core reactor model presented in this study considers in detail the electromagnetic couplings between parts of the individual windings, making it accurate even at high frequencies. In fact, each turn in the PEEC reactor model is represented by a finite number of interconnected bars. The electromagnetic coupling between the bars is represented by mutual partial inductances and the mutual coefficients of potential. The model parameters, mainly the partial inductances, the coefficients of potential, and the resistances are computed from the geometry of the bars using analytical routines. The constructed PEEC model is for example solved by a general purpose circuit solver [8] or by a dedicated solver assembling the circuit equations to obtain the current and voltage distribution in the windings (nodal voltages and volume cell currents) [9]. A main advantage from the circuit approach is that both time and frequency domain analysis can be performed using the same model.

To show the validity of the approach, several types of laboratory, air-core reactors have been constructed. The first type are reactors winded by thin, round, copper wire on rectangular and circular support. The second type are reactors winded on rectangular support from a thin copper tape for studying skin effect. The PEEC model results have been compared to measurements done on the laboratory, air-core reactor models, and using other established modeling methods and show good agreements.

The PEEC approach is an integral method, and unlike differential equation-based methods, it gives rise to fewer unknowns and is very suitable for the geometries under study [10]. Though the resulting matrices are dense, the use of faster solvers [11] and today’s computer capabilities, the time complexity for PEEC simulations is acceptable. As for other integral equation based methods, spurious resonances may occur [10]. It has been observed that for PEEC simulation this most likely is a result from poor geometric meshing. This artifact has been extensively studied in the literature, and several measures to suppress them have been suggested [12], [13].

The paper is organized as follows. Section 2 presents basic PEEC theory, while Section 3 presents the creation of a PEEC model for air-core reactors. The modeling of skin effect is considered in Section 3.3, while Section 4 deals with the model validation, involving the comparison between model results and measurements. The paper is finished with discussions and conclusions in Section 6.
2 Basic PEEC Theory

In the PEEC method, the electric field integral equation (EFIE) is interpreted as Kirchhoff’s voltage law applied to a basic PEEC cell which results in a complete circuit solution for 3D geometries. PEEC theory is presented here in brief and a more detailed derivation is given in [14] - [15]. The PEEC approach to create electromagnetic models involves the following phases:

- Equivalent circuit interpretation of the EFIE;
- Meshing;
- Matrix formulation: Obtaining circuit equations for the meshed structure;
- Matrix solution: Solving the circuit equations to obtain currents and potentials in the meshed structure;
- (Optional). Post-processing of current and potentials to obtain field variables.

2.1 Equivalent circuit interpretation of EFIE

Consider the electric field on a conductor given by

\[
E'(r, t) = \frac{J(r, t)}{\sigma} + \frac{\partial A(r, t)}{\partial t} + \nabla\phi(r, t),
\]

where \(E'\) is an incident (externally) applied electric field, \(J\) is the current density in the conductor, \(A\) is the magnetic vector potential, \(\phi\) is the scalar electric potential, and \(\sigma\) the electrical conductivity. By using the basic definitions of the electromagnetic potentials as in (2) and (3),

\[
A(r, t) = \mu \int_{v'} G(r, r')J(r', t_0)dv'
\]

\[
\phi(r, t) = \frac{\nabla}{\epsilon_0} \int_{v'} G(r, r')q(r', t_0)dv'.
\]

where the Green’s function \(G(r, r') = \frac{1}{|r-r'|}\), and substituting in (1) the electric field integral equation, (3), at the point \(r\) in the conductor is obtained according to

\[
E'(r, t) = \frac{J(r, t)}{\sigma}
+ \mu \int_{v'} G(r, r')\frac{\partial J(r', t_0)}{\partial t}dv'
+ \frac{\nabla}{\epsilon_0} \int_{v'} G(r, r')q(r', t_0)dv'.
\]
Expanding the current density \( J = J^C + J^P \), where the free current density \( J^C = \sigma E \), and the polarization current density \( J^P = \epsilon_0 (\epsilon_r - 1) \frac{\partial E}{\partial t} \), the EFIE is re-written as

\[
E^i(r, t) = \frac{J(r, t)}{\sigma} + \mu \int_{vt'} G(r, r') \frac{\partial J(r', t_d)}{\partial t} dv't' + \epsilon_0 (\epsilon_r - 1) \mu \int_{vt'} G(r, r') \frac{\partial^2 E(r', t_d)}{\partial t'^2} dv't' + \frac{\nabla}{\epsilon_0} \int_{vt'} G(r, r') q(r', t_d) dv't'.
\]

The third term in the right-hand side of (4) vanishes for ideal conductors \( (\epsilon_r = 1) \), thus permitting the separation of the ideal conductor and ideal dielectric properties.

Assuming an ideal conductor consisting of \( k \) sub-conductors, and further partitioning each sub-conductor into \( n_\gamma \) volume cells, each of constant current density \( J_{\gamma nk} \), where \( n_\gamma = n_x, n_y, n_z \) for partitions in the x-, y-, or z-direction. Further defining pulse functions as in (5)

\[
P_{\gamma nk} = \begin{cases} 
1, & \text{inside the } nk:\text{th volume cell} \\
0, & \text{elsewhere} 
\end{cases}
\]

and taking a weighted volume integral over each \( v_{\gamma nk} \) volume cell, the second term in the right-hand side of (4) represent the inductive voltage drop \( v_L \) over the conductor as

\[
v_L = \sum_{k=1}^{K} \sum_{n=1}^{N_{nk}} \frac{\mu}{4\pi a_{\gamma nk}} \int_{vt} \int_{v_{\gamma nk}} \frac{\partial I_{\gamma nk}(r_{\gamma nk}, t_{\gamma nk})}{\partial t} dv_{\gamma nk} dv't'
\]

where \( I_{\gamma nk} = \frac{I_{\gamma nk}}{a_{\gamma nk}} \). The inductive voltage drop could be further expressed as

\[
v_L = \sum_{k=1}^{K} \sum_{n=1}^{N_{nk}} L_{p_{\gamma vt_{\gamma nk}}} \frac{\partial}{\partial t} I_{\gamma nk}(t - \tau_{vt_{\gamma nk}})
\]

where \( \tau_{vt_{\gamma nk}} \) is the center to center delay between the volume cells \( vt \) and \( v_{\gamma nk} \) and \( L_{p_{\gamma vt_{\gamma nk}}} \) are partial inductances which are generally defined for volume cells \( v_\alpha \) and \( v_\beta \) as

\[
L_{p_{\alpha \beta}} = \frac{\mu}{4\pi a_{\alpha} a_{\beta}} \int_{v_\alpha} \int_{v_\beta} \frac{1}{|r_\alpha - r_\beta|} dv_\alpha dv_\beta.
\]

The \( L_{p_{\alpha \beta}} \) terms are referred to as the self partial inductance while the \( L_{p_{\alpha \beta}} \) is the mutual partial inductance representing the inductive couplings between the volume cells.

From the fourth term of the right-hand side of (4), the capacitive voltage over the \( m:th \) volume cell is obtained. Extracting \( S_{mk} \) surface cells from the \( m:th \) volume cell to
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give a surface representation of the charge distribution over the volume cell, and using pulse functions defined as

\[ p_{mk} = \begin{cases} 
1, & \text{inside the } mk:th \text{ surface cell} \\
0, & \text{elsewhere} 
\end{cases} \]  \hspace{1cm} (9)

and the following finite difference approximation

\[ \int \frac{\partial}{\partial \gamma} F(\gamma) dv \approx a \left[ F \left( \gamma + \frac{l_m}{2} \right) - F \left( \gamma - \frac{l_m}{2} \right) \right] \]  \hspace{1cm} (10)

the capacitive voltage over the \( m^\text{th} \) volume cell is obtained as

\[ v_C = \sum_{k=1}^{K} \sum_{m=1}^{M_k} \left[ q_{mk}(t_{mk}) \frac{1}{4\pi\epsilon_0} \int_{S_{mk}} \frac{1}{|r - r'|} dS' \right] \]

\[ - \left[ q_{mk}(t_{mk}) \frac{1}{4\pi\epsilon_0} \int_{S_{mk}} \frac{1}{|r - r'|} dS' \right] \] \hspace{1cm} (11)

where the vectors \( r^+ \) and \( r^- \) are associated with the positive and negative end of the cell respectively [16]. From (11) the coefficient of potential is defined as

\[ p_{ij} = \frac{1}{S_i S_j} \frac{1}{4\pi\epsilon_0} \int_{S_i} \int_{S_j} \frac{1}{|r_i - r_j|} dS_j dS_i \]  \hspace{1cm} (12)

The \( p_{ii} \) terms are referred to as the self coefficient of potential while the \( p_{ij} \) is the mutual coefficient of potential representing the capacitive couplings between the surface cells.

From the first term of the right-hand side of (4), the resistive voltage drop over the \( m:th \) volume cell is obtained, from which resistances are defined as

\[ R_i = \frac{l_\gamma}{a_\gamma \sigma_\gamma} \]  \hspace{1cm} (13)

where \( l_\gamma \) is the length of the volume cell in the \( \gamma \) direction, \( a_\gamma \) is the cross section of the volume cell normal to the \( \gamma \) direction, and \( \sigma_\gamma \) is the conductivity.

This interpretation of the EFIE, allows for a systematic approach to construct equivalent circuit representations of electromagnetic problems for mixed conductor-dielectric structures. Further, the PEEC model, allows active and passive circuit elements to be added to the analysis of the electromagnetic problem. Figure 1 shows the PEEC model of a conducting bar. The magnetic field couplings are considered through the mutual partial inductances represented in a voltage source \( V_{L_{mn}} \), while the electric field couplings are considered by the mutual coefficients of potentials represented in the current sources \( I_{P} \) and \( I_{P} \). Each node is connected to infinity by the corresponding self capacitance \( \frac{1}{C_i} \) and \( \frac{1}{P_{ij}} \) as shown in the figure.
2.2 Meshing of structure

Two meshing schemes are required for PEEC analysis. First, a volume cell mesh to model the current distribution and second a surface mesh to model the charge distribution, as explained in the previous section. From the volume cells, partial inductances given in (8) and DC resistances given in (13) are calculated. From the surface cell mesh, coefficients of potential given in (12) are calculated.

The maximum cell size in the mesh is required to be less than $\frac{\lambda_{\text{min}}}{20}$, where $\lambda_{\text{min}}$ is the minimum wavelength of interest (corresponding to the highest frequency in the excitation).

2.3 Matrix formulation

This phase involves the formulation of circuit equations from the equivalent circuit representation of the meshed structure. If the complete equivalent circuit is expressed in a SPICE-compatible .cir-file, the formulation and solution of the circuit equations can be performed directly in freeware SPICE-like solvers. However, for the full-wave case, when time retardation is included, special solvers have to be used [17]. The circuit equations are formulated from the equivalent circuit representation of the conducting bar shown in Fig. 1 by applying Kirchhoff’s voltage law on the inductive loop and enforcing Kirchhoff’s current law at each node. This results in the following circuit equations

$$-A\Phi(t) - R_i(t) - L_p \frac{\partial i_L(t)}{\partial t} = v_s(t) \quad (14)$$

$$P^{-1} \frac{\partial \Phi(t)}{\partial t} - A^T i_L(t) = i_s(t)$$

Figure 1: Conducting bar (a) and corresponding PEEC model (b).
where $P$ is the full coefficient of potential matrix, $A$ is a sparse matrix containing the connectivity information, $L_p$ is a dense matrix containing the partial inductances, $R$ is a matrix containing the volume cell resistances, $\Phi$ is a vector containing the node potentials (solution), $i_L$ is a vector containing the branch currents (solution), $i_s$ is a vector containing the current source excitation, and $v_s$ is a vector containing the voltage source excitation [18]. The first row in the equation system in (14) is Kirchhoff’s voltage law for each inductive loop or basic PEEC cell while the second row satisfy Kirchhoff’s current law for each node.

2.4 Matrix solution

This phase involves solving the equation system in (14) for the potential and current distribution in the meshed structure. As shown in the previous section, the modified nodal analysis (MNA) method [19] was adopted. In this approach, the nodal potentials and volume cell currents are solved at once and the system coefficient matrix have two dense blocks (upper right and lower left). The MNA method also allows simple inclusion of additional active and passive circuit elements with the electromagnetic model.

In the solution of (14), the time derivatives can, for example, be calculated by a backward Euler scheme [20] as shown here for the $j$:th node potential

$$\frac{\partial \Phi_j(t)}{\partial t} = \frac{\Phi^n_j - \Phi^{n-1}_j}{\Delta t}$$

where $\Delta t$ is the time step separating the two discrete time instances $n$ and $n-1$. Discretizing (14) in time gives

$$\begin{bmatrix}
-A & -(R + L_p \frac{1}{\Delta t}) \\
P^{-1} \frac{1}{\Delta t} & A^T \\
\end{bmatrix}
\begin{bmatrix}
\Phi^n \\
i_L^n \\
\end{bmatrix}
= \begin{bmatrix}
v_s^n - L_p \frac{1}{\Delta t} i_L^{n-1} \\
i_s + P^{-1} \frac{1}{\Delta t} \Phi^{n-1} \\
\end{bmatrix}$$

when written in a matrix fashion with the sub-matrices as detailed in the previous section.

In a quasi-static (QS) solution of (14), only the potentials and currents at the $n$:th and the $n-1$:th time steps are used in the evaluation of the derivatives. While, for a full-wave (FW) solution accounting for the time retardation in the electromagnetic couplings, a history of currents and node potentials is needed. The time step ($\Delta t$) should be carefully chosen since extremely small $\Delta t$ can lead to numerical problems. A case showing this behavior is presented in the model validation section.

2.5 Postprocessing

The node potentials and volume cell currents can be post-processed to obtain electromagnetic field variables. This is shown in [21] for antenna problems and in [22] for printed circuit board problems.
3 Air-core Reactor Model Creation

This section deals with the creation of the electromagnetic models for air-core reactors using PEEC:s.

3.1 Geometry description

In the presented model, each turn (rectangular or circular) of the reactor is made up of a finite number of bars with rectangular cross section in one plane (i.e. pitch angle neglected). The end of one turn is connected to the beginning of the next turn by a short circuit. In this way, the complete reactor winding is created. Figure 2 shows a sample 4 turn reactor model. In this case, each turn is formed by only 6 bars, for simplicity.

![Figure 2: Schematic description for 4 turn reactor model formed by 6 bars (volume cells in the PEEC model) per turn.](image)

The PEEC model for one turn is shown in Fig. 3, for the case of 6 bars per turn (corresponding to one turn of the geometry in Fig. 2).

The effects of ignoring the pitch angle of the turns has been studied in [23], and is not expected to be a major source of error.

3.2 Partial Element Evaluation

As mentioned previously, magnetic field couplings between the bars are represented by partial mutual inductances and the electric field couplings by mutual coefficients of potential. These so called partial elements are the foundation of the PEEC model and have to be calculated with great care. There exist analytical formulas for partial inductances and coefficients of potential for orthogonal structures in parallel or perpendicular orientations only [24, 25]. These are suitable for calculation of so called Manhattan-type of geometries - orthogonal block parallel or perpendicular. Therefore, these routines can be used when modeling reactors with a rectangular cross section (when 4 perpendicular bars represent one turn) [26].

To represent circular turns, the bars need to be inclined at arbitrary angles, as seen in Fig. 2 for 6 interconnected bars with an inclination of 60 degrees. For these types of problems, non-orthogonal geometries with arbitrary orientations, as well as orthogonal
geometries which are neither in parallel nor perpendicular orientations, numerical integration routines are used to evaluate the partial elements [15]. Instead of using numerical integration that are susceptible to numerical errors and time consuming, the partial elements for the reactor model are evaluated in a special, more efficient, way as shown in the following sub-sections.

### 3.2.1 Partial inductances

The partial inductances, seen as the $L_{pi}$s in Fig. 3, are calculated from the corresponding volume cell. The inductive/magnetic field couplings from all volume cells are represented by the partial mutual inductances of the form $L_{ij}$. In quasi-static simulations these can be translated into the well known SPICE-type coupling factor $K$ while for full-wave models the time retarded couplings are modeled through the voltage sources $V_{iL}$.

The partial inductance between two volume cells was given in (8) and can be expanded to show the current directions as

$$L_{pij} = \frac{\mu}{4\pi} \frac{1}{a_i a_j} \int_{l_i} \int_{l_j} \int_{a_i} \int_{a_j} \frac{dl_i \cdot dl_j}{|r_i - r_j|} da_i da_j$$

where $l_i$, $l_j$ are the lengths of the cells, $a_i$, $a_j$ are the cross sectional areas, while $r_i$ and $r_j$ are position vectors of arbitrary points in the $v_i$ and $v_j$ volume cells, respectively. Exact analytical formulas for $L_{pij}$ are given for example in [14, 25, 27, 28], for the case of parallel rectangular volumes. Considering the volume cells with arrows in Fig. 2 for example, $dl_i$ and $dl_j$ would be the current directions. For the case where the cells are inclined at some arbitrary angle $\alpha$ as shown in Fig. 4, the volume $v_j$, is replaced by
$v'_j$ of length $|l'_j| = l_j \cos \alpha$, parallel to volume $v_i$ and the centers of mass of $v_j$ and $v'_j$ coincide. This gives $L_{p_{ij}}$ maximum when volumes are parallel and zero when they are perpendicular. This approximation gives fairly accurate solutions, and is much faster compared to numerical integration routines.

3.2.2 Coefficients of potential

The partial coefficient of potentials, seen as the $P_n$s in Fig. 3, are obtained from the corresponding surface cells. The capacitive/electric field couplings from all surface cells are represented by the mutual coefficients of potentials of the form $P_{ij}$. In quasi-static simulations these can be translated directly into mutual capacitances [29] while for full-wave models the time retarded couplings are modeled through the current sources $I_{ip}$.

Here the evaluation of the coefficients of potential expression given in (12), for orthogonal surfaces lying on parallel planes is considered. The coefficient of potential for the two orthogonal surfaces $S_i$ and $S_j$ shown in Fig. 5 will have a maximum value $P_{ij_{\text{max}}}$ when $\alpha = n\pi$ and a minimum value $P_{ij_{\text{min}}}$ when $\alpha = (n + 1/2)\pi$, where $n$ is an integer, given that $l_j > w_j$. For all $\alpha$, can be $P_{ij}$ is approximated as

$$P_{ij} = \cos^2 \alpha P_{ij_{\text{max}}} + \sin^2 \alpha P_{ij_{\text{min}}}. \quad (18)$$

An analytical expressions for $P_{ij_{\text{max}}}$ and $P_{ij_{\text{min}}}$ are given in [24, 25]. The (18) is fairly accurate and much faster to compute compared to numerical integration routines. Therefore, this approach is used for the circular, air-core models in this paper.
Skin and proximity effects

Skin and proximity effects bring about non-uniformity in the current distribution along a cross section of a conductor. The increase in current density towards the conductor surface and around edges, due to changing fields within the conductor itself only, is termed skin effect \[30\]. This phenomenon is noticeable for conductors/bars were the cross section (width or thickness) is larger than the Skin depth, \(\delta\), defined as

\[
\delta = \sqrt{\frac{1}{4\pi\mu_0\sigma f_m}} \tag{19}
\]

and \(f_m\) is the maximum frequency of interest.

In this study, the volume filament (VFI) technique is applied \[14, 31\], where skin effect is modeled. The idea is to make a 3D discretisation of the interiors of the bars making up the reactor windings with maximum with and thickness \(\delta/2\). However, the VFI approach is expensive in terms of cell count, and an optimal way would be to use a non-uniform meshing scheme, where a coarser mesh is used in the center of the cell (with less current distribution) and a finer mesh close to the edges, respecting the \(\delta/2\) rule.

The current distribution in one volume filament can be influenced by changing fields in adjacent filaments. This is termed proximity effects \[30\] and can be well pronounced in reactor-type of structures. Figure 6 is an illustration of the current distribution in bars due to skin and proximity effects. The current distribution in a multi-conductor system is a combination of skin and proximity effects.
Figure 6: Current distribution due to skin and proximity effects. Volume cell currents in same direction (a). Currents in opposite directions (b). Only skin-effect, no proximity effect (c).

Table 5.1: Cell counts for test cases.

<table>
<thead>
<tr>
<th>Part inductances</th>
<th>Coefficients of potential</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>self</td>
</tr>
<tr>
<td>Test A: 90 turn, rectangular, wire reactor</td>
<td>360</td>
</tr>
<tr>
<td>Test B: 200 turn, rectangular, wire reactor</td>
<td>800</td>
</tr>
<tr>
<td>Test C: 133 turn, circular, wire reactor</td>
<td>2 660</td>
</tr>
<tr>
<td>Test D: 65 turn, rectangular, tape reactor</td>
<td>780</td>
</tr>
<tr>
<td>Test D: 65 turn, rectangular, tape reactor</td>
<td>5 720</td>
</tr>
</tbody>
</table>

4 Model Validation

In order to validate the PEEC reactor modeling approach, air-core reactor models were constructed by winding copper wire or tape around a sparse wooden or plastic support. This section presents the geometric description of the various test cases, as well as a comparison between the PEEC models and measurements in both time and frequency domain. In one case the PEEC model results were compared with a lumped modeling approach. A case showing the voltage distribution along the reactor winding is presented, but could not be compared to measurements as only terminal voltage measurements were made.

The time complexity for a few interesting cases is also presented. The PEEC simulations are run on a Linux machine with a dual Intel Xeon CPU 2.8 GHz, and 3 GB RAM.
4.1 90 turn, rectangular, wire reactor

The first test is for a 90 turn, rectangular, air-core reactor winded from a round, \( r=2 \) mm, copper wire on a sparse wooden support. The spacing between the windings is 10.0 mm and the cross section of the reactor is 49×58 cm. The input impedance for the structure was measured, from 10 kHz to 5 MHz, using a vector network analyzer.

4.1.1 PEEC model

The PEEC model consists of 4 bars per turn, giving a total number of partial elements according to Table 5.1. This meshing gives an upper frequency limit for the model at 26.0 MHz using the \( \lambda/20 \)-rule. The model is excited with a unitary current source and a full-wave (\( L_p, P, R, \tau \))PEEC simulation is performed. The input impedance is obtained directly from the voltage at the input node.

4.1.2 Lumped model

The structure is also modeled using a simple lumped model. The lumped model basically involves partitioning the reactor into sections, each consisting of several turns, which are electromagnetically coupled. Each section (partition) is assumed to be electrically small and can be represented by lumped circuit parameters \( L, C, \) and \( R \). There are two partitioning schemes, the inductive partitions and the corresponding capacitive partitions. Each partition consist of a given number of turns, and the inductance and capacitance are obtained using closed formulas. Consider a reactor of \( N \) turns discretized into \( i \) inductive partitions and \( i + 1 \) capacitive partitions. For the lumped models \( N \gg i \), since several turns are represented in one discrete circuit element. Further, the capacitive and inductive partitions are usually shifted half a partition size with respect to each other.

The model used for this test consists of only 5 self inductances, obtain by partitioning the reactor windings into 5 lumped elements, as detailed in [26], and 6 capacitances. The results for modeled input impedance, using the lumped model and the PEEC model, compared to measured values are shown in Fig. 7 and show good agreement for both models. Both models predict the first resonance around 550 kHz well while above that the lumped model fail to predict the input impedance correctly.

4.2 200 turn, rectangular, wire reactor

The second test was done on a 200 turn, rectangular, air-core reactor winded from a round, \( r=0.7 \) mm, copper wire on a sparse wooden support. The spacing between the windings is 3 mm and the cross section of the reactor is 50×50 cm. As for the 90 turn reactor, the input impedance is measured, from 10 kHz to 5 MHz. The PEEC model is constructed as in Section 4.1 which gives a total number of partial elements according to Table 5.1. This mesh gives an upper frequency limit for the model at 30 MHz. Figure 8 presents the comparison between the the PEEC model results and the measurements. There is good agreement between the PEEC model and the measurements for all the resonances except for one around 1.85 MHz, which is mostly likely from irregularities in
the measurements and/or a geometrical mismatch between the computational model and the constructed lab model. Figure 8 shows an overestimation of the input impedance at some of the resonance points, for example around 3.25 MHz.

The time required for each step in the PEEC solver for the 200 turn, rectangular, wire reactor is detailed in Table 5.2.
Table 5.2: Time complexity for Test B: 200 turn, rectangular, wire reactor for 100 frequencies.

<table>
<thead>
<tr>
<th>Step</th>
<th>Time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parsing &amp; Meshing</td>
<td>5</td>
</tr>
<tr>
<td>Calc. partial inductances</td>
<td>2</td>
</tr>
<tr>
<td>Calc. coefficient of potentials</td>
<td>4</td>
</tr>
<tr>
<td>Solver</td>
<td>267</td>
</tr>
<tr>
<td>Total</td>
<td>∼ 278</td>
</tr>
</tbody>
</table>

4.3 133 turn, circular, wire reactor

A reactor consisting of 133 turns wound copper wire with diameter of 0.7 mm was constructed using a circular plastic (low $\varepsilon_r$) support with diameter of 40.0 cm. The winding separation is 2.0 mm giving the reactor length of approximately 27.0 cm. In the corresponding PEEC model each turn is made up from 20 orthogonal, rectangular bars giving a total number of partial elements according to Table 5.1.

A circular winding is better represented by a large number of bars. But then, the size of the problem increases significantly with increase in the number of bars per turn. In this case, 20 bars per turn does a good characterization of the circular winding, and this is seen from the agreement with the measurement results in the presented Fig. 9.

As opposed to the rectangular reactor model, with four bars per turn, the circular reactor model is more detailed and demands for more resources. The time required for each step in the PEEC solver for the 133 turn, circular, wire reactor is detailed in Table 5.3.

Table 5.3: Time complexity for 133 turn, circular, wire reactor

<table>
<thead>
<tr>
<th>Step</th>
<th>Time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parsing &amp; Meshing</td>
<td>4</td>
</tr>
<tr>
<td>Calc. partial inductances</td>
<td>4</td>
</tr>
<tr>
<td>Calc. coefficient of potentials</td>
<td>7</td>
</tr>
<tr>
<td>Solver</td>
<td>500</td>
</tr>
<tr>
<td>Total</td>
<td>∼ 515</td>
</tr>
</tbody>
</table>
4.4 65 turn, rectangular, tape reactor

The reactor was constructed by winding 65 turns of thin copper tape of width 6.35 mm and thickness 0.076 mm around a sparse 48 cm × 50 cm rectangular wooden support (low \( \varepsilon_r \)), with a constant separation of 10.0 mm. Unlike the previous examples, copper tape of large surface area was chosen in order to observe variations due to skin and proximity effects. Impedance response over a specific frequency range is obtained as in the previous cases above. For the time domain response, a low voltage impulse tests have been performed. The input terminal of the reactor is excited with a fast trapezoidal pulse of different rise times, and peak voltage of 10 V, from an impulse generator. The input and output pulses are observed using an oscilloscope. Two separate PEEC models were made. In one, the current along each bar (one turn has 4 bars) is assumed constant and each bar is represented by three volume cell. Each turn is thus modeled by 12 bars giving a total of 780 volume cells as shown in Table 5.1. In the other, the non-uniform current distribution across the tape width due to skin effect is modeled, by further partitioning each bar into 7 volume cells along the width. For this case, the voltage distribution along the reactor winding at different frequencies is studied, though it could not be compared to measurements. The results are presented in the following subsections.

4.4.1 Frequency domain results

The PEEC model was excited with a a unitary current source at the input terminal. In modeling skin effect, each bar (volume cell) is subdivided along the width into 7 cells giving a (VFI)PEEC model. This gives a total number of partial elements according to Table 5.1. The frequency response was obtained for 10 kHz to 5 MHz. Figure 11 presents the simulated impedance response for two different PEEC-based codes (PowerPEEC [32]
and our implementation), the (VFI)PEEC model, and measurements. In this case, no significant difference is observed in the impedance response when skin effect is modeled. Skin-effect might be more influential at frequencies higher than 5 MHz, \( \delta = 29.2 \, \mu\text{m} \). An example showing the current distribution in the tape at 75 MHz is seen in Fig. 12.

To further show the application of the PEEC-based solver, the voltage distribution in the tape reactor at 4 MHz is shown in Fig. 13. The corresponding voltage distribution along the winding is also shown in the graph in Fig. 14. Since only terminal measurements were made, the voltage distribution could not be compared with measurements.
4.4.2 Time domain results

The reactor was excited with fast trapezoidal pulse of ns rise time($t_r$), and peak voltage level of 10.0 V, from the pulse generator. The reactor input terminal is connected to the pulse generator which has an internal resistance of 50 Ω, while the output terminal is
either connected directly to the 2MΩ oscilloscope, or through a 50 Ω resistor. A schematic of the time domain setup is described in Fig. 15. The input voltage and output voltage were observed and recorded at the oscilloscope. Figure 16 presents the measured and modeled time domain response for a pulse of rise time 32 ns, peak voltage 9.2 V, and pulse width 26 µs, with Rout = 50Ω. The responses are in good comparison.

The case with the output terminal of the reactor connected directly to a 2 MΩ, which is considered here as open, seems more interesting, because of the reflections at the open end. This is seen in Fig. 17 (a) and (b) for pulses of rise times 11.4 ns and 200 ns, respectively.
In the investigation, it was shown that the time step size, $\Delta t$, from (16) was influencing the damping of the responses. Very small time steps can lead to very large amplitudes, possibly due to the $L_p \frac{1}{\Delta t}$ and $P^{-1} \frac{1}{\Delta t}$ terms in the time-discretization of the matrix equation shown in (16). Thus the time steps has to be adequately chosen to obtain a satisfactory response. A basic rule of $\Delta t = \frac{t}{10}$ was adopted. Figure 18 shows the dependence of the time step size, for a pulse of rise time 11.4 ns.

The PEEC simulations are run on a machine with a dual Intel Xeon CPU 2.8 GHz, and 3 GB RAM. The time required for each step in the PEEC solver for the 65 turn, rectangular, tape reactor tests are detailed in Table 5.4.

5 Further work/target application

The presented work is conducted in order to verify the accuracy for PEEC-based modeling of air-core reactor structures. So far, only simple lab, air-core reactors, as presented in this paper, have been considered. However, the target of this work is to be able to fully model the complex, multi-layer, air-core reactors up to very high frequencies that are in use in power systems. An example of this type of air-core reactor is the line trap shown in Fig. 19, that contain 3 concentric layers and three different types of windings (cross section of wire and number of turns).
Figure 17: Time domain response for the open ended 65 turn, rectangular, tape reactor for a pulse rise time 11.4 ns (a) and 200 ns (b).
Figure 18: Time domain response for the open ended 65 turn, rectangular, tape reactor for a
pulse rise time 11.4 ns using different time steps.

<table>
<thead>
<tr>
<th>Step</th>
<th>Time [s]</th>
<th>Time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solver type</td>
<td>FD-PEEC</td>
<td>TD-PEEC</td>
</tr>
<tr>
<td>Parsing &amp; Meshing</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Calc. partial inductances</td>
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<td>3</td>
</tr>
<tr>
<td>Calc. coefficient of potentials</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Solver</td>
<td>410 †</td>
<td>40 ‡</td>
</tr>
<tr>
<td>Total</td>
<td>~420</td>
<td>~50</td>
</tr>
</tbody>
</table>

† for 100 frequencies
‡ for 1000 time steps

6 Discussions and Conclusions

The application of PEEC theory for the creation of 3D, electromagnetic models for aircore reactors has been presented. The obvious benefit of the PEEC-solution is the circuit based formulation valid from DC to an upper frequency limit decided by the mesh. PEEC reactor modeling results have been compared with measurements and showed fairly good
agreement. A slight shift in resonance peaks, which increases as one goes higher up in frequency, was observed. This can be explained from the discrepancies between the model and constructed reactor. For the test cases, up to 5 MHz, the influence of skin effect on the impedance response is negligible. The reactor model created here gives the voltages and currents at different points in the reactor which could be used for example estimating the near and field interactions in and from the reactors, studying the mechanical stress on the reactor windings, and other dynamic analysis. From the time complexity, it is seen that the time required to analyze larger problems is acceptable on a regular workstation. From the comparisons with lumped models it is seen that a more detailed distributed equivalent circuit model is necessary for high frequency modeling of reactors.

Acknowledgment

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Characteristic Signature of Electromagnetic Emissions from Power Converters

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Characteristic Signature of Electromagnetic Emissions from Power Converters

Mathias Enohnyaket and Kalevi Hyypää

Abstract

Switching operations in power converters controlling electric machines in a hybrid drive train constitute a major source of electromagnetic related disturbances. The emissions from power converters seem to have a characteristic pattern or signature and can be picked up at different locations in the vehicle. This study aims at investigating the signature of emissions from power converters using an H-bridge driving a dc motor. PSpice model of the motor and drive circuit was made and current, voltage, and field measurements were performed on a constructed prototype. Current transients and oscillations generated during voltage transitions have been investigated using the PSpice model. A correlation between the common mode currents and the magnetic field emissions was observed.

Keywords: Power converters, common mode currents, current transients, time domain measurements, magnetic field emission.

1 Introduction

Fast switching operations in power converters commonly used in motor drive circuits in hybrid drive system, constitute a major source of electromagnetic (EM) related disturbances. The fast switching operations generates high frequency ringing or noise which is injected into the motor and other systems, triggering resonances due stray capacitances and inductances. The high frequency signal generates EM emissions into the vicinity and can be picked up at different locations in the vehicle. High levels emissions might distort CAN communication and other sensitive equipment such as sensors on board. High frequency emissions from power converters at switching frequency and higher order harmonics have been reported in several issues [1, 2, 3, 4, 5, 6, 7]. The source of these emissions and their dependence on different drive circuit parameters is not well understood. At the converter output, common mode current \(i_{\text{cm}}\) transients or spikes are generated during voltage transitions, due to stray capacitances. In the presence of stray inductances, \(i_{\text{cm}}\) ringings are generated, as discussed in section 2. The magnitude of the current spikes are dependent on the voltage gradient \(\frac{\Delta V}{\Delta t}\) and the stray capacitances [8, 9, 10]. The converter output current \(i_o\) has ripple resulting from the PWM scheme [11, 12]. In the case of a dc converter, the sum of the current to the motor \(i_{\text{m}}\) and the current from the motor \(i_{\text{m}}\) is ideally zero, neglecting stray capacitances. But due
to imbalance in the currents, possibly due to spatial displacement of the cables and the presence of stray capacitances, there is a non-zero resultant, which is seen as $i_{cm}$ currents. The $i_{cm}$ currents constitute a major source of magnetic (H) field emissions from the motor drive circuit [13]. The $i_{cm}$ current usually contains components of the current ripple due to PWM switching, and the spikes generated during voltage transitions. This projects some characteristics of the converter, and the events occurring in it. Analyzing the output current and voltage quality from the drive circuit provides useful information on the ringing resulting from the switching operations and the eventual emissions.

This paper investigates the characteristic signature of emissions from power converters using an H-bridge and a dc motor, analyzes the source of the emissions and discusses ways of damping them. A PSpice model of the motor drive circuit was first made and a prototype later constructed. A current probe of bandwidth 75 MHz and voltage probes of bandwidth 300 MHz were used to analyze the output current and voltages. An antenna of bandwidth 20 Hz to 500 kHz was used to measure the fields in the vicinity. It was observed that the ripple and transients in the output current were picked up by the field probes. Using the PSpice model, the source of the current transients was investigated.

Section 2 discusses the modeling of the drive circuit. Section 3 discusses the measurements done on the constructed prototype. Sections 4 and 5 round off with some discussions and conclusions.

# 2 Drive circuit modeling

The drive circuit model is an H-bridge. Power MOSFETs are used as switching devices. The drive circuit is designed to switch a 24V dc input, controlling a dc motor. The H-bridge is switched diagonally, using a trapezoidal pulse train ($v_P(t)$) of 0-5 V. The PSpice model of the drive circuit is shown in Fig. 1. The average output voltage ($V_0$) and motor current ($I_0$) are controlled by controlling the duty cycle of $v_P(t)$.

## 2.1 Modeling ripple and transients in output current

Figure 2 shows the converter output voltage across the motor, ($v_0(t)$), the control pulse train $v_P(t)$, and the output current to the motor $i_0(t)$, when the stray capacitances and inductances are ignored. Ripple due to the PWM scheme could be seen in $i_0(t)$. The output current, $i_0(t)$, does not contain transients or spikes due to voltage transitions, because the stray capacitances ($C_s$) and inductances ($L_s$) were neglected. $C_s$ could be modelled by placing a capacitor across the motor, as in Fig. 1, while neglecting $L_s$. The chosen value of $C_s$ is an estimate of the stray capacitances in the motor winding which is assumed to be the dominant contributor. The line to ground stray capacitances of the motor cables is represent $C_{m1}$ and $C_{m2}$. The common mode current ($i_{cm}$) is modelled as the sum of the currents through $C_{m1}$ and $C_{m2}$, meaning $i_{cm}(t) = i_{cm1}(t) + i_{cm2}(t)$. This estimate of $i_{cm}$ is equivalent to having a rogowski coil or current probe around both cables to the motor [13]. Spikes generated during voltage transitions could be seen in
Figure 1: PSpice schematic of H-bridge drive circuit. The stray capacitance between the motor cables is $C_s$, while the line to ground capacitances are $C_{m1}$ and $C_{m2}$ for each cable.

$i_{cm}(t)$, as shown in Fig. 3, where $M_1, M_2, M_3$ and $M_4$ are MOSFETs, while $v_{DS}$ are the respective drain source voltages. The $i_{cm}(t)$ spikes seem to depend on the slew rate ($\frac{\Delta V}{\Delta t}$), and the stray capacitances to ground.

Including $L_s$, as shown in Fig. 1, would generate ringings in both $i_{cm}(t)$ and $i_{Ls}(t)$, where $i_{Ls}(t)$ is the current through $L_s$. The frequency and amplitude of the oscillations depend on the chosen values of $L_s$ and $C_s$. For more valuable results, $L_s$ and $C_s$ should be computed from the geometry of the drive circuit. Figure 4 and Fig. 5 show the ringings when $L_{s1} = L_{s2} = 5\mu H$, $C_s = 350pF$ and $C_{m1} = C_{m2} = 75pF$. When the value of $L_{s1}$ and $L_{s2}$ is increased to $10\mu H$, the frequency of the oscillations is decreased and the amplitudes are reduced. This is shown in Fig. 6 and Fig. 7.

The $i_{cm}(t)$ ringing constitutes a major source of the magnetic (H) field emissions from the drive circuit. This could be seen from the correlation between the H field emissions and $i_{cm}(t)$, presented in section 3. Thus minimizing these ringings might reduce the EMI issues. This could be achieved through the use of current limiting snubbers described in [11]. On the other hand, snubber components are passive components and would involve energy loss as heat in the circuit, which might affect the efficiency.
3 Measurements on H-bridge prototype

From the drive circuit model a prototype was built. The 24 V dc input was supplied by a standard 24 V dc supply. The step pulse $v_P(t)$ is obtained from a pulse generator. The driving mode of the motor was controlled by varying the duty cycle $v_P(t)$. Duty cycle of 0.5 is standstill, greater than 0.5 is forward, and less than 0.5 is reverse. Current probe of bandwidth 75 MHz, and a 300 MHz voltage probe were used to analyze the current and voltages. Magnetic field loop antennas (10 Hz - 500 kHz, 10 kHz to 30 MHz) were used for field measurements. The measured data is stored in a 2 GHz oscilloscope.

Figure 8 shows the motor current and the fields picked up during a few switching cycles. Some ringing could be seen in the motor current with frequency of 250 kHz. The ringings were suppressed when decoupling capacitors were connected across the 24 V dc supply, as shown in Fig. 9. The ringings might have resulted from the stray capacitances and inductances introduced by the 24 V power supply. This follows from the PSpice model results of the voltage across the motor when the impedance of the power supply and the wires to it are modelled. The ringing is damped with the installation of decoupling capacitors, as shown in Fig. 10. For the rest of the measurements presented in this section, the decoupling capacitors are maintained.

3.1 Correlation between the common mode currents and H field emissions.

The H field measurements are given in millivolts. The induced voltage on the H field probe is proportional to the field intensity and the frequency. Since we are only looking for
correlations between the H field and the currents, the unit of the H field can be arbitrary. Measurements on the constructed prototype show that transients in the $i_{cm}$ are picked up by field probes. This is seen from the correlation between the $i_{cm}$ transients and the transients in the H field emissions shown in Fig. 11. The generation of common current transients during voltage transitions is shown in Fig. 12 and Fig. 13. The transients seen at $t = 60\mu s$ and $t = 180\mu s$ in Fig. 13 results from switching the $0 - 5V$ control voltage ($v_P(t)$). Transients due to the $0 - 5V$ voltage source were not seen in the PSpice model, since the $0 - 5V$ voltage source was modelled as an ideal voltage source. But on the prototype, the $v_P(t)$ input line lies about 1.0 cm close to the output lines to the motor, and the stray capacitances between the lines cause EM couplings. This allows transients in the $v_P(t)$ input line to perturb the motor currents. Observe that voltage transients in the $v_P(t)$ input line also perturb the H fields at $t = 60\mu s$. The current ripple in $i_0$ due to the PWM scheme is seen in Fig. 14 and Fig. 15. The common mode currents shown in

---

**Figure 3:** PSpice model results showing the generation of $i_{cm}$ spikes during voltage switchings. $M_1, M_2, M_3$ and $M_4$ are MOSFETs, while $v_{DS}$ are the respective drain source voltages.
Figure 4: PSpice model results when \( L_{s1} = L_{s2} = 5 \mu H \), \( C_s = 350 \text{pF} \) and \( C_{m1} = C_{m2} = 75 \text{pF} \), showing the generation of \( i_{cm} \) ringing during voltage switchings. The output current to the motor \( i_0 \) is almost free of ringing.

Figure 5: PSpice model results when \( L_{s1} = L_{s2} = 5 \mu H \), \( C_s = 350 \text{pF} \) and \( C_{m1} = C_{m2} = 75 \text{pF} \), showing the generation of \( i_{cm} \) ringing during voltage switchings. Obtained by zooming in the region 0.0007s to 0.000726s in Fig. 4.
Figure 6: PSpice model results showing $i_{cm}$ ringing during voltage switchings, when $L_{s1} = L_{s2} = 10\mu F$, $C_s = 350pF$, and $C_{m1} = C_{m2} = 75pF$.

Figure 7: PSpice model results showing $i_{cm}$ ringing during voltage switchings, when $L_{s1} = L_{s2} = 10\mu F$, $C_s = 350pF$, and $C_{m1} = C_{m2} = 75pF$. Obtained by zooming in the region 0.0007s to 0.000726s in Fig. 7. Here also the output current to the motor $i_0$ is almost free of ringing. The frequency of the oscillations is decreased.
Figure 8: Measured motor current, H field and the control pulse train \( v_p \), without decoupling capacitors. Note that the \( H \) field is given in millivolts because the \( H \) field probe generates a voltage proportional to the field. The exact value of the field is not important in this study.

Figure 9: PSpice model showing placement of decoupling capacitors (\( C_{d1}, C_{d2}, C_{d3}, \) and \( C_{d4} \)) across the H-bridge in the constructed prototype. \( R_s \) and \( L_{sm} \) represents the impedance of the power supply and the wires to it.

Fig. 12 seem clear of any PWM current ripple. In fact, at the point of measurement, the \( i_0^+ \) and \( i_0^- \) cables were brought very close to fit in the 2 mm diameter current probe. At
other points, for example at the motor input, the cables have over 6 cm separations. The ripple would cancel out when $i_0^-$ and $i_0^+$ are placed very close to each other, for example twisted. A small spatial displacement in the cables generates a resultant current ripple in $i_{cm} = i_0^- + i_0^+$, which is eventually radiated. This accounts for the correlation between the current ripple and ripple in the H field emissions.

Figure 11: Measurements showing correlation between common mode current transients and transients in H field emissions.
Figure 12: Measurements showing correlation between common mode transients and voltage transitions. Each voltage transition generates transients in $i_{cm}$ and is picked up in the $H$ field.

Figure 13: Measurements showing correlation between common mode transients and voltage transitions. Obtained by zooming in the region 0.00000s to 0.00002s of Fig. 12.
Figure 14: Measurements showing correlation between current ripple in $i_o$ and ripple in $H$ field emissions.

Figure 15: Measurements showing correlation between current ripple in $i_o$ and ripple in $H$ field emissions. Obtained by zooming in the region 0.0000s to 0.00002s of Fig. 14. The transients seem to agree more with $i_{cm}$. 
4 Discussions

Correlations between the current and H field emissions has been observed. The spread of the emissions would depend on the routing of the current cables. The correlation between current and H field follows from Ampere’s law given in (1).

\[ \nabla \times \mathbf{H} = \mathbf{J}, \]  

(1)

where \( \mathbf{J} = \mathbf{J}^C + \mathbf{J}^D \), with \( \mathbf{J}^C = \sigma \mathbf{E} \) being the conduction current density while \( \mathbf{J}^D = \frac{\partial \mathbf{D}}{\partial t} \) is the displacement current density. From (1), it follows that time varying currents would generate time varying H fields.

The stray capacitance and inductance, \( C_s \) and \( L_s \), used in the PSpice modeling were not appropriately estimated, but were rather good guesses. This explains why the ringings in the model and the measurements do not have an exact match. But the modeling did show that the presence of the stray capacitances and stray inductances cause the ringings.

5 Conclusions

Analyzing the output current quality of a given drive circuit, useful information about the drive circuit parameters could be obtained, for example the switching frequency of the power converters, stray capacitances and inductances causing ringings. Correlation between the current and the H field emissions have been shown.

The voltage gradient during transitions (\( \frac{\Delta V}{\Delta t} \)) alongside the parasitics (\( C_s \) and \( L_s \)) seem to determine the emission levels of a given drive circuit. If the parasitics are efficiently estimated, appropriate snubber components (\( L_z, C_z \), and \( R_z \)) could be installed to damp ringings in currents and voltages, which would eventually damp emissions. The great challenge is the computation of parasitics for a complex drive circuit.

References


Parameter Characterization of Low Frequency Pulsating Emissions from Space Vector PWM drives

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Parameter Characterization of Low Frequency Pulsating Emissions from Space Vector PWM drives

Mathias Enohnyaket, Kalevi Hyyppä, and Jerker Delsing

Abstract

Power converters in hybrid electric drives constitute a major source of electromagnetic disturbances. Recent studies have established that the space vector PWM scheme commonly employed in drive systems, generates low frequency pulsating (LFP) emissions, at a frequency of 6f₀, where f₀ is the fundamental frequency the phase voltages. The switching of voltage vectors generates common mode current (i_{cm}) spikes due to the presence of stray capacitances and inductances. Across sector boundaries, the i_{cm} spikes superpose forming spikes of double or triple amplitude which constitute the LFP emissions. These pulsating emissions could pose EMC issues, and functionality issues like torque pulsations and speed fluctuations that could affect the reliability of the drive. This paper investigates the effects of drive speed, load, and converter slew rates, on the amplitude of the LFP emissions, using theoretical models.

Keywords: Low frequency pulsating emissions, EMI, torque pulsations, common mode currents, time domain measurements, drive system.

1 Introduction

The increasing demand for clean energy systems in vehicles has propelled a rapid development of electric and hybrid electric vehicles (HEV). Power converters play a central role in hybrid electric drive systems and constitute a major source of electromagnetic disturbances. PWM switching of DC/AC converters generates common mode current spikes (i_{cm}) due to the presence of stray capacitances. These spikes account for the emissions from PWM drives at harmonics of the switching frequency (f_c) and harmonics of the fundamental frequency f₀, mostly reported in the literature [1, 2, 3, 4, 5, 6, 7, 8].

Amongst the PWM schemes, the space vector scheme is mostly preferred for its flexible speed control capabilities [9, 10, 11, 12]. However recent studies have reported some issues related to the space vector scheme. In [8], it was shown that the amplitude of current ripples at switching frequency (f_c) are influenced by the placement of active vectors within each half carrier or PWM period. Issues related to the crossing of sector boundaries in the space vector hexagon has been reported [13, 14, 1, 15]. In [14], the formation of common mode current spikes due to sector boundary crossing was mentioned. In [13], the generation of large torque pulsations due to sector boundary crossing was reported.
The generation of low frequency pulsating (LFP) common mode emissions during sector boundary crossing reported in [15], shall be considered in this paper. The LFP emissions were formed by the superposition of common mode current spikes generated during the switching of voltage vectors. In the space vector PWM scheme, a given reference vector is obtained as a linear combination of the adjacent voltage vectors \( (V_k, V_{k+1}) \), with duty cycles \( D_k \) and \( D_{k+1} \), respectively [10, 9]. The duty cycles determine the time interval between the \( i_{cm} \) spikes. Closed to sector boundaries, either \( D_k \) or \( D_{k+1} \) tends to zero causing a superposition of \( i_{cm} \) spikes to occur. These pulsations enhance emissions at harmonics of switching frequency \( (f_s) \); create low frequency magnetic fields, and when injected into electric motors could lead to torque pulsations and speed fluctuations [14, 13, 16, 17]. This paper investigates the effects of drive speed, load, and converter slew rates, on the amplitudes of the LFP emissions, using theoretical models.

Section 2 presents some measurements from an HEV showing LFP emissions. Section 3 presents the theoretical modeling of LFP emissions using Gaussian pulses, and models the effects of different drive parameters. Section 5 rounds off with some discussions and conclusions.

2 Measurements on HEV

This section presents some measurements from an HEV showing LFP emissions.
2.1 Measurement setup

Fig. 1 shows a schematic of the hybrid drive system on which the measurements were performed. The electric machine is a three phase synchronous machine. The phase current was measured at location A, with a Tektronix current probe (model TCP404XL), of bandwidth dc to 2 MHz, and peak current 750 A DC. Common mode currents were measured at location B, with a Powertek Rogowski coil (model CWT 6B) of bandwidth 0.1 Hz to 16 MHz, with peak current rating of 1.2 kA, and 8 kA/µs. The measurements were recorded on a Tektronix digital oscilloscope (TDS7254), of bandwidth 2.5 GHz, and maximum sampling rate of 20 giga samples per second.

2.2 Some results

The measurements were performed under different driving modes, with the current probes located at the positions A and B on the drive train, as shown in Fig. 1. Measurements from two different cases are presented. The objective of case I was to capture the events occurring during switching. A short interval of about five PWM switching cycles was considered. In order to capture the events in detail, a high oscilloscope sampling rate of 250 megasamples per second, was used. Results from case I, shown in Fig. 2, show superposition of $i_{cm}$ switching spikes. In Case II, the amplitude of the phase current is slowly decreased from about 250 A down to 50 A, with a phase current frequency of about 50 Hz. This case is presented in Fig. 3. Low frequency pulsating (LFP) emissions with periodicity of about 0.008 s in the common mode currents are observed.

3 Theoretical Modeling

Representing an $i_{cm}$ spike as a Gaussian pulse, the spike pattern for different PWM schemes can be reconstructed by superposition [18, 19]. The pattern generated by the space vector PWM (SV-PWM) scheme is considered, since it is the scheme used in the drive train on which the measurements were performed. This section briefly presents the space vector scheme and models effects of drive speed, load, and converter slew rates, on the amplitude of the LFP emissions.

3.1 Space Vector PWM (SV-PWM) scheme

The space vector hexagon shown in Fig. 4 represents the classical SV-PWM scheme [10, 9, 11, 12]. An arbitrary reference voltage vector ($V_{ref}$) defined in (2) can be represented as a linear combination of two adjacent voltage vectors ($V_k$ and $V_{k+1}$), and zero vectors ($V_0$ and $V_7$), as described in (3),

$$V_{ref} = V_{dq} \hat{d} + V_{dq} \hat{q}$$  \hspace{1cm} (1)

$$V_{ref} = D_k V_k + D_{k+1} V_{k+1} + [1 - (D_k + D_{k+1})] V_0, 7$$  \hspace{1cm} (2)
where $D_k = \frac{k}{6}$ is the duty cycle, $T_k$ is the time spent on the $V_k$ voltage vector, $T_s$ the PWM period, and $V_d$ is the dc source voltage. The voltage vectors $V_k$ are defined as

$$V_k = \frac{2}{3} V_d \exp \frac{j\pi(k-1)}{3}.$$  

(3)

For half a PWM period, $D_k$ is constrained as

$$D_k + D_{k+1} + D_{0.7} = 0.5,$$  

(4)

where $D_{0.7}$ is the duty ratio for either $V_0$ or $V_7$ zero voltage vectors. Considering that $V_{0.7} = 0$, $V_{ref}$ is simplified as

$$V_{ref} = D_k V_k + D_{k+1} V_{k+1}.$$  

(5)

Using (2) to (6), the switching times $T_k$ and $T_{k+1}$ are obtained. The $T_k$s determine when the converter switches fire or voltage transitions occur. This, in turn, determines the interval between the $i_{cm}$ spikes.

---

**Figure 2:** Case I showing the superposition of $i_{cm}$ spikes during switching. The top plot is the $i_{cm}$ currents. The second subplot is obtained by zooming in the region 0.50 s to 0.85 s of the top plot. The third subplot is zooming in the region 0.250 s to 0.42 ms of the top plot. The bottom subplot is the phase current. Oscilloscope sampling rate is 250 mega samples per second, which is 400 times Nyquist sampling frequency.
Figure 3: Case II showing LFP emissions with periodicity 0.008 s. The second subplot is obtained by zooming in the region 0.005 s to 0.016 s of the top plot. The third subplot is zooming in the region 0.008 s to 0.011 s of the top plot. The bottom plot is the phase current. Oscilloscope sampling rate is 1.25 mega samples per second, twice Nyquist frequency.

3.2 Modeling of $i_{cm}$ spikes

The spike rise time and amplitude are varied using parameters $A$ and $C$, as shown in Fig. 5. The time of occurrence of the spike is varied by moving the centre of the Gaussian pulse, and is equivalent to varying parameter $b$ in (9).

$$f(t) = A \exp\left(\frac{-(t-b)^2}{2C^2}\right),$$

Consider a $V_{ref}$ in the sector 1, where $0 < \alpha < \frac{\pi}{3}$. In the symmetric SV-PWM scheme [11], a feasible PWM switching cycle is the following:

$$\{\ldots, 100 \rightarrow 110 \rightarrow 111 \rightarrow 111 \rightarrow 110 \rightarrow 100 \rightarrow 000 \ldots\},$$

with switching times $\{D_1, D_2, D_7, D_7, D_2, D_1\}$, respectively. The $i_{cm}$ spike pattern generated by this switching cycle is shown in Fig. 6.
3.3 Reconstruction of LFP emissions

An anticlockwise rotation of a constant amplitude reference voltage $V_{\text{ref}}$ generates steady state sinusoidal phase voltages. This is obtained by varying $D_k$ and $D_{k+1}$ while respecting the constraints given in (5). Close to sector boundaries, either $D_k$ or $D_{k+1}$ tends to zero. These cause spike superpositions during sector boundary crossings, forming double or
Figure 6: $i_{cm}$ pattern for one PWM cycle, with $V_{ref}$ in sector 1. Time is expressed in terms of PWM switching periods, $T_s = 0.0002s$.

Figure 7: Simulation results showing LFP emissions of periodicity 0.001 s. The results obtained using the following parameter settings: $0 < D_k \leq 0.2$, $T_s = 0.0002s$, $T_0 = 50T_s$. The lower subplot is obtained by zooming in the response when $38.1T_s \leq \text{time} \leq 40.0T_s$. This corresponds to case I, presented in Fig. 2 showing the superposition of switching spikes.
Figure 8: Simulation results showing LFP emissions of $40T_s = 0.008s$. The results obtained using the following parameter settings: $0 < D_k \leq 0.2$, $T_s = 0.0002s$, $T_0 = 240T_s$. This a reconstruction of case II, shown in Fig. 3. The lower subplot is obtained by zooming in the response when $20T_s \leq \text{time} \leq 80T_s$.

triple amplitude spikes which constitute the LFP emissions. The six sector boundaries give rise to six LFP pulses in one complete revolution.

$$T_0 = 6T_{LFP}$$  

(7)

The period of the sinusoidal phase voltage $T_0$, is thus related to the period of the LFP emissions ($T_{LFP}$) as in (10). Fig. 7 shows simulated LFP emissions of periodicity 0.001 s. The results were obtained using the following parameter settings: $0 < D_k \leq 0.2$, $T_s = 0.0002s$, $T_0 = 50T_s$. Fig. 8 presents simulation results showing LFP emissions of periodicity $T_{LFP} = 40T_s = 0.008s$, obtained using the following parameter settings: $0 < D_k \leq 0.45$, $T_s = 0.0002s$, $T_0 = 240T_s$. This was a reconstruction attempt of case II, shown in Fig. 3.

4 Parameters affecting the LFP emissions

In this section, the dependence of the amplitudes of the LFP emissions on the voltage slew rates of the converter switches, the drive speed, and load is investigated using the theoretical models developed in section 3.
4.1 Effects of Voltage slew rates on LFP emissions

Close to the boundary between the $k$th sector, and the $(k + 1)$th sector, $D_k$ becomes comparable to the width of the gaussian pulse ($C$), as described in (8). The rise time $t_r$ is an estimate of the voltage slew rate, for a fixed dc source voltage, and is approximately equal to the pulse width ($C$) of the generated common mode current spike [2]. In the boundary region, marked by $\theta$ in Fig. 9, $D_k$ can be represented as a fraction of the maximum duty cycle $D_{k,\text{max}}$, as in (8) and (9), assuming that PWM period $T_s = 1$. When $D_k = C$, the angular width of the LFP emissions, $\theta = \theta_c$ can expressed as a fraction of the sector angle as in (10). Using (8) to (10), the expression for $\theta_c$ in terms of $t_r$ and $D_{k,\text{max}}$ given in (11), is obtained. The width of the LFP emissions in seconds is obtained as $d_{\text{LFP}} = \theta_c/2\pi * T_0$ and is given in (12).

![Figure 9: Space vector hexagon showing the boundary region, where $D_k \leq C \sim t_r$, denoted by $\theta$](image)

\[
D_k \leq C \sim t_r, \quad D_k T_s = C
\]  
(8)

\[
D_k = \frac{D_{k,\text{max}}}{x}
\]  
(9)

\[
\theta_c/2 = \frac{\pi}{3}/x
\]  
(10)

\[
\theta_c/2 = \frac{\pi}{3} \frac{t_r}{D_{k,\text{max}}}
\]  
(11)

\[
d_{\text{LFP}} = \frac{1}{6} \frac{1}{D_{k,\text{max}} T_s} T_0 t_r
\]  
(12)
The effects of slew rates is described by (13) when $D_{k_{\text{max}}}$, $T_0$, and $T_s$ are held constant. Thus the larger the rise time, the larger the width of the LFP emissions. This is shown in Fig. 10 and Fig. 11, where $C \sim t_r = 0.02s$ and $C \sim t_r = 0.035s$, respectively.

\begin{equation}
    d_{\text{LFP}} \propto t_r
\end{equation}

Figure 10: Reconstruction of case II with $C \sim t_r = 0.02s, d_{\text{LFP}} \sim 10T_s, T_{LFP} = 40T_s = 0.008s$ and $T_0 = 240T_s$. Parameter settings: $0 < D_k \leq 0.45$, $T_s = 0.0002s$. The lower subplot is a zoom of the region $20T_s < t < 80T_s$.

4.2 Effects of drive speed on LFP emissions

The drive speed is assumed proportional to the fundamental frequency of the phase voltages ($1/T_0$). From (11), when $\theta_c$ is fixed, $d_{\text{LFP}}$ is given by

\begin{equation}
    d_{\text{LFP}} = \frac{\theta_c}{2\pi} T_0.
\end{equation}

This is seen in the simulated prediction of LFP emissions when $T_0 = 0.6s$, shown in Fig. 10, with $d_{\text{LFP}} \sim 100T_s$.

4.3 Effects of load on LFP emissions

The load or torque output is assumed proportional to the amplitude of the phase current. This is simulated by varying the maximum duty cycle $D_{k_{\text{max}}}$ in the interval
Figure 11: Reconstruction of case II with \( C \sim t_r = 0.035s, d_{LFP} \sim 20T_s, T_{LFP} = 40T_s = 0.008s \) and \( T_0 = 240T_s \). Parameter settings: \( 0 < D_k \leq 0.45, T_s = 0.0002s \).

Figure 12: Simulated prediction of LFP emissions with \( T_0 = 3000T_s = 0.6s \) and \( d_{LFP} \sim 100T_s \). Parameter settings: \( 0 < D_k \leq 0.45, T_s = 0.0002s \). The lower subplot is a zoom of the region \( 0 < t < 1200T_s \).
From (12) it is observed that $d_{LFP}$ is proportional to $D_k \max$, when $T_0$, $t_r$, and $T_s$ are held constant. This is an indication of light load instabilities. This is shown in Fig. 13, when $D_k \max = 0.05$. It is observed that the LFP pulses overlap in this case, unlike in the previous cases where was set to 0.2 or 0.45.

5 Discussions and Conclusions

Drive systems employing the space vector PWM scheme could emit Low Frequency Pulsating (LFP) emissions at a frequency of $6f_0$, where $f_0$ is the fundamental frequency of the phase voltages. The LFP emissions is built up from double or tipple amplitude common mode current spikes, formed from the superposition of common mode spikes generated during sector boundary crossings. Measurements from an HEV showing the LFP emissions were presented. Using simple theoretical models the effects of parameters like the voltage slew rates, drive speed and drive load on the LFP emissions of have been investigated. The coupling of these pulsations to torque pulsations requires further investigations, however, relationships between current harmonics and torque pulsations have been deveped in [16, 17]. Mitigation approaches shall be investigated in a future work.
References


Generation of Low Frequency Pulsating Emissions from PWM Power Converters

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Generation of Low Frequency Pulsating Emissions from PWM Power Converters

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Abstract
In the course of investigating EMI emissions from power converters controlling synchronous machines in a hybrid electric vehicle, low frequency pulsating common mode emissions were found on the ac link. The common mode emissions were pulsating at a frequency of \(6f_0\), where \(f_0\) is the fundamental frequency of the phase voltages. Amplitudes close to ten percent of the rated current (750 A) were measured. These pulsations enhance emissions at harmonics of switching frequency; create low frequency magnetic fields, and when injected into electric motors could lead to torque pulsations and speed fluctuations. This paper presents measurements showing the low frequency pulsations, and investigates the source using simple theoretical models. It was observed that the switching of voltage vectors generates common mode current (\(i_{cm}\)) spikes due to the presence of stray capacitances. In the space vector scheme, a given reference vector in sector \(k\), is obtained as a linear combination of the \(V_k\) and \(V_{k+1}\) voltage vectors, with duty cycles \((D_k)\) and \((D_{k+1})\), respectively. Close to sector boundaries, either \(D_k\) or \(D_{k+1}\) tends to zero, causing a superposition of \(i_{cm}\) spikes. The measurement data and modelling clearly indicate that the superposition of \(i_{cm}\) spikes generates the low frequency pulsating emissions.

Keywords: Power conversion, current measurements, Electromagnetic Compatibility, time domain measurements, Pulse width modulated power converters, low frequency pulsating emissions, space vector modulation.

1 Introduction

Power converters constitute a central part of modern electric drive systems. There exist several different Pulse Width Modulation (PWM) schemes for controlling power converters, for example, the sinusoidal PWM scheme, the random PWM scheme, and the space vector PWM scheme [1, 2, 3]. In order to achieve flexible speed control requirements, the space vector approach is mostly preferred [1, 2, 3, 4]. PWM schemes usually involve switching high voltages and currents. With the recent advancement in semiconductor technology, switching devices, such as MOSFETS, now allow for operation in the megahertz range. These high switching rates generate common mode currents (\(i_{cm}\)) and voltages due to the presence of stray capacitive and inductive couplings in the drive
system [5]. These, in turn, generate electromagnetic (EM) emissions [6, 5], induce shaft voltages, and create bearing currents [7, 8, 9, 10], causing switching losses and torque pulsations in electric motors [11, 12, 13, 14, 15, 16, 17, 18, 19].

Emissions from PWM drives are normally expected at harmonics of the switching frequency \( kf_c \) and harmonics of the fundamental frequency \( mf_0 \), where \( k \) and \( m \) are integers [20, 21, 22, 23, 24, 25, 26, 27, 28, 29]. The harmonics generated by a given PWM scheme has some dependence on the placement of zero vectors. In [29], it was shown that the amplitude of current ripples at carrier or switching frequency \( f_c \) are reduced by centering the active vectors within each half carrier or PWM period. The switching of voltage vectors, either active or zero vectors, generates \( i_{cm} \) transients or spikes, due to the presence of stray capacitances and inductances. Neglecting the stray inductances, the magnitude of \( i_{cm} \) depends on the capacitance to ground or common mode capacitance \( C_m \) and the voltage slew rate \( dV/dt \), as given below

\[
i_{cm} = C_m \frac{dV}{dt}.
\]

The common mode capacitance \( C_m \) depends on the proximity to metal objects in the vicinity and the cable configuration. For EMI analysis, \( i_{cm} \) is thus a suitable quantity, since it is sensitive to changes in the vicinity through \( C_m \), is non-zero only when \( dV/dt \) is non-zero, and accounts for a large part of the emissions from the drive [21].

Issues related to the crossing of sector boundaries in the space vector hexagon has been reported [11, 12, 20]. For example, in [12], the formation of common mode current spikes due to sector boundary crossing was mentioned. In [11], the generation of large torque pulsations due to sector boundary crossing was reported. However, these issues have not been formally characterized. This study illustrates the generation of Low Frequency Pulsating (LFP) common mode emissions at a frequency of \( 6f_0 \) from space vector PWM drives, from the superposition of \( i_{cm} \) spikes formed during the switching of either active or zero voltage vectors, close to sector boundaries. These common mode pulsations enhance emissions at harmonics of the switching frequency \( f_c \) by a factor 3; create low frequency emissions; when injected into electric motor could cause torque pulsations and speed fluctuations that could deteriorate the functionality of the drive. The study focuses on the characterization of these pulsating emissions by analyzing the common mode currents. Measurements from an HEV demonstrating the existence of LFP emissions are presented, and the source is characterized using simple theoretical models. A discussion on the potential consequences of the LFP emissions is presented.

The paper is organized as follows. Section 2 presents the measurements from an HEV showing LFP emissions, and a discussion on measurement errors. Section 3 presents theoretical modeling of the generation of \( i_{cm} \) switching spikes using Gaussian pulses and a reconstruction of the LFP emissions. Potential consequences of the LFP emissions are discussed in section 4. The paper is completed with some discussions and conclusions in section 5.
2 Measurements on HEV

This section presents the measurement setup for the common mode current measurements, discusses potential sources of errors, and presents some measurement results. The results from cases I to IV are shown in Fig. 2, Fig. 3, Fig. 5 and Fig. 6, respectively. Zooming in the LFP pulses, it could be observed that a periodic superposition of $i_{cm}$ spikes occurs. The period of the LFP emissions ($T_{LFP}$) seems to be correlated to the period of the phase current ($T_0$). An investigation of the trends in the LFP emissions, using theoretical models, is presented in section 3.

2.1 Measurement setup

The common mode current measurements were performed on an HEV prototype. Fig. 1 shows a schematic of the hybrid drive system on which the measurements were performed. The electric machine is a three phase synchronous machine. The phase current was measured at location A, with a Tektronix current probe (model TCP404XL), of bandwidth dc to 2 MHz, and peak current rating of 750 A DC. The common mode current was
Table 8.1: Results from different measurement cases. \( T_0 \) and \( T_{LFP} \) are the periods of the phase current \( i_0 \) and the LFP emissions, respectively, while \( A_0 \) and \( A_{LFP} \) are the respective amplitudes. The different cases are further explained in the text.

<table>
<thead>
<tr>
<th>Cases</th>
<th>( T_0 ) (s)</th>
<th>( A_0 ) (A)</th>
<th>( T_{LFP} ) (s)</th>
<th>( A_{LFP} ) (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I: At switching</td>
<td>–</td>
<td>100 A</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>II: Medium freq</td>
<td>0.02 s</td>
<td>200 A</td>
<td>0.008 s</td>
<td>14 A</td>
</tr>
<tr>
<td>III: High freq</td>
<td>0.015 s</td>
<td>300 A</td>
<td>0.004 s</td>
<td>14 A</td>
</tr>
<tr>
<td>IV: Light Load</td>
<td>0.05 s</td>
<td>50 A</td>
<td>–</td>
<td>14 A</td>
</tr>
<tr>
<td>V: Low speed</td>
<td>0.60 s</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

measured at location B, with a Powertek Rogowski coil (model CWT 6B) of bandwidth 0.1 Hz to 16 MHz, with peak current rating of 1.2 kA and 8 kA/\( \mu \)s. The measurements were recorded on a Tektronix digital oscilloscope (TDS7254) with a bandwidth of 2.5 GHz and maximum sampling rate of 20 giga samples per second.

### 2.2 Results

The measurements were performed under different driving modes, with the current probes located at positions A and B on the drive train, as shown in Fig. 1. Thus, different emission patterns are obtained. The measurements from four different cases, case I to IV, are presented in Table 8.1, where \( T_0 \) and \( T_{LFP} \) are the periods of the phase current \( (i_0) \) and LFP emissions, respectively, while \( A_0 \) and \( A_{LFP} \) are the respective amplitudes. These cases are selected based on the frequency and amplitude of the phase currents, which represents the speed and torque characteristics of the drive.

The objective of case I was to capture the events during switching. A short interval of about five PWM switching cycles was considered. In order to capture the events in detail, a high oscilloscope sampling rate of 250 megasamples per second, was used. In Case II, the amplitude of the phase current is slowly decreased from about 250 A down to 50 A, with a phase current frequency of about 50 Hz. In case III, the amplitude of the phase current is fairly constant, with a frequency of about 66 Hz. Case IV presents a light load condition, with a phase current amplitude of about 50 A and frequency of about 25 Hz. There is a sudden jump in the current from 50 A to about 200 A. In these cases, the frequencies 66 Hz, 50 Hz, and 25 Hz are referred to as high, medium, and low frequencies, respectively. Case V could not be performed due to memory limitations of the oscilloscope, but the corresponding response could be predicted using the models in section 3.
Figure 2: Case I showing the superposition of $i_{cm}$ spikes during switching. The top plot is the $i_{cm}$ currents. The second subplot is zooms in the region 0.50 s to 0.85 s of the top plot. The third subplot is obtained by zooming in the region 0.250 s to 0.42 ms of the top plot. The bottom subplot is the phase current. Oscilloscope sampling rate is 250 mega samples per second, which is 400 times Nyquist sampling frequency.

2.3 Measurement errors

Possible sources of error in these measurements include the following:

- Choice of probes and oscilloscope.
- Aliasing and oscilloscope sampling rate.
- Impedance matching at the probe oscilloscope interface.

The probe bandwidth must be large enough to accurately measure the events being investigated. In this case, a dc to 2 MHz current probe was used to measure the phase currents, while a 0.1 Hz to 16 MHz Rogowski coil was used in measuring the $i_{cm}$ currents.
Figure 3: Case II showing LFP emissions with periodicity 0.008 s. The second subplot is obtained by zooming in the region 0.005 s to 0.016 s of the top plot. The third subplot zooms in the region 0.008 s to 0.011 s of the top plot. The bottom plot is the phase current. Oscilloscope sampling rate is 1.25 mega samples per second, twice Nyquist frequency.

Figure 4: FFT of common current ($i_{cm}$) for case II.
Figure 5: Case III, showing LFP emissions with a periodicity of 0.0040 s. The top plot is the $i_{cm}$ currents. The second subplot zooms in the region 0.005 s to 0.018 s of the top plot. The third subplot zooms in the region 1.0 ms to 2.5 ms. The bottom plot is the phase current. Oscilloscope sampling rate is 12.5 mega samples per second, twenty times the Nyquist frequency.

Figure 6: Case IV, showing LFP emissions during light load conditions. The top plot is the $i_{cm}$ currents. The second subplot zooms in the region 0.005 s to 0.016 s of the top plot. The third subplot zooms in the region 0.001 s to 0.002 s. The bottom plot is the phase current. Oscilloscope sampling rate is 1.25 megasamples per second, twice the Nyquist frequency.
The phase currents typically vary from a few hertz to a few hundred hertz, while $i_{cm}$ spikes of rise times in the order of 3.0 $\mu$s were expected from the converters. The converter switching frequency was 5 kHz. As a requirement from the probe specification, the bandwidth of the oscilloscope should be at least five times the bandwidth of the probes. In this case, a 2.5 GHz oscilloscope was used.

Aliasing is a distortion of the measured signal being measured shifted to a frequency lower than the actual frequency [20]. When the oscilloscope is not sampling fast enough, aliasing occurs. According to the Nyquist sampling criterion, the sampling rate of the oscilloscope should be at least twice the highest frequency components of the measured signal [30]. In these measurements, the converters were switched at 5 kHz, with a pulse rise time of about 3 $\mu$s. Thus, an oscilloscope sampling rate, or Nyquist frequency, of at least $\frac{1}{2}$ mega samples per second is required. In the four measurement cases presented in section 2.2, the oscilloscope sampling rate was at least 1.25 mega samples per second. Thus, no aliasing effects are expected in the data. The sampling rate for each measurement is presented along with the measurements, for clarity.

For accurate data acquisition, the impedance of the probe and oscilloscope must be matched at the interface. This requires the input impedance of the oscilloscope port to match with the line impedance of the coaxial cable from the probe. In this case, the 2 MHz Tektronix probe comes along with a dedicated 50 ohm interface cable. Similarly, the 16 MHz Rogowski coil has a dedicated 50 ohm interface cable.

### 3 Theoretical Modeling of LFP emissions

PWM schemes exhibit strong coupling to the generated common mode voltages and currents [1, 8]. It was shown in [5] that voltage transitions generate $i_{cm}$ spikes. Representing an $i_{cm}$ spike as a Gaussian pulse, the spike pattern for different PWM schemes can be reconstructed by superposition [31, 32]. The spike pattern for the space vector PWM (SV-PWM) scheme is of greater interest in this study as it is the scheme used in the drive train on which the measurements were performed. Using a similar approach, the spike pattern for different PWM schemes can be predicted. This section briefly presents the space vector scheme and models the $i_{cm}$ spike pattern formed during different load conditions.

#### 3.1 Space Vector PWM (SV-PWM) scheme

SV-PWM has been described in detail in [1, 2, 3, 4] but the basics are briefly discussed here for completeness. In order to avoid short circuiting the dc supply in the power converter topology, shown in Fig. 7, eight switching states are feasible. This yields eight voltage vectors, presented in Fig. 8, with $V_0(000)$ and $V_7(111)$ being the zero voltage vectors. In the conventional space vector scheme, any arbitrary reference voltage vector can be represented as a linear combination of two adjacent voltage vectors ($V_k$ and $V_{k+1}$) and zero vectors.
Figure 7: A three phase converter topology.

Figure 8: Space vector scheme showing voltage vectors \( V_0, V_1, \ldots, V_7 \) and arbitrary reference voltage vector \( V_{ref} \). The binary numbers 000, 001, ... represents the switch states.

Given a reference voltage \( V_{ref} \) in the \( dq \)-reference frame, as described in (2), \( V_{ref} \) is expressed as a linear combination of voltage vectors as in (3),

\[
V_{ref} = V_{d0} \hat{d} + V_{q0} \hat{q}
\]  

\[
V_{ref} = D_k V_k + D_{k+1} V_{k+1} + [1 - (D_k + D_{k+1})]V_{0,7}
\]  

\[
V_k = \frac{2}{3} V_d \exp\left(-\frac{j \pi (k-1)}{3}\right)
\]

where \( D_k = \frac{T_k}{T_s} \) is the duty cycle, \( T_k \) is the time spent on the \( V_k \) voltage vector, with \( T_s \) as the PWM period, and \( V_d \) is the dc source voltage. The voltage vectors \( V_k \) are defined in (4). For half a PWM period, \( D_k \) is constrained as
\[ D_k + D_{k+1} + D_{0.7} = 0.5, \]  

where \( D_{0.7} \) is the duty ratio for either \( V_0 \) or \( V_7 \) zero voltage vectors. Considering that \( V_{0.7} = 0 \), \( V_{ref} \) is simplified as

\[ V_{ref} = D_k V_k + D_{k+1} V_{k+1}. \]  

Using (2) to (6), the switching times \( T_k \) and \( T_{k+1} \) are obtained as

\[
\begin{pmatrix}
T_k \\
T_{k+1}
\end{pmatrix} = \frac{\sqrt{3} T_s}{V_{dc}} M \begin{pmatrix}
V_{ds} \\
V_{qs}
\end{pmatrix},
\]

where \( M \) is obtained by solving (4), (5) and (6). The sector, \( k \), containing the reference voltage is obtained from the angle given in (8).

\[ \theta = \arctan \frac{V_{qs}}{V_{ds}}. \]  

Different \( T'_s \)s are obtained for different \( V'_{ref} \)s, as in (7). The \( T'_s \)s actually determine when the converter switches fire or voltage transitions occur. This, in turn, determines when the \( i_{cm} \) spikes are generated.

### 3.2 Modeling of \( i_{cm} \) spikes

Modeling each \( i_{cm} \) spike as a Gaussian pulse defined in (9), the spike pattern of a given PWM scheme can be obtained by superposition. The spike rise time and width are varied using parameters \( A \) and \( C \), as shown in Fig. 9. The time of occurrence of the spike is varied by moving the centre of the Gaussian pulse, and is equivalent to varying parameter \( b \) in (9).

\[ f(t) = A \exp\left(-\frac{(t-b)^2}{2C^2}\right). \]  

Consider a \( V_{ref} \) in the sector 1, where \( 0 < \alpha < \frac{\pi}{3} \). In the symmetric SV-PWM scheme [3], a feasible PWM switching cycle is the following: \{...

\[ ...000 \rightarrow 100 \rightarrow 110 \rightarrow 111 \rightarrow 111 \rightarrow 110 \rightarrow 100 \rightarrow 000... \],

with switching times \( \{D_1, D_2, D_3, D_4, D_5, D_6, D_7\} \), respectively. The \( i_{cm} \) spike pattern generated by this switching cycle is shown in Fig. 10. This pattern clearly depends on the duty cycles, \( D'_k \)s.

For a large \( D_k \), when \( D_k \geq C \), where \( C \) is the width of the Gaussian pulse, the following special cases can be identified:

1. When \( D_k, D_{k+1}, \) and \( D_{0.7} \) are all large, no spike superposition occurs. In this case, there are three spikes during the first half of the PWM period and three
spikes down during the second half of the period, and each spike would have unity amplitude. This configuration yields the best case from this PWM scheme from an EMC perspective. An example is shown in Fig. 10.

2. When $D_k$ tends to zero, with $D_{k+1}$ large, a double amplitude spike and unity amplitude spike are obtained during each half of the PWM period. The double amplitude spikes result from a superposition of the $k$th and $(k-1)$th pulses. This can be observed in Fig. 11.
3. When $D_k$ and $D_{k+1}$ both tend to zero at the same time, as in light load conditions, all three spikes generated in the first half period would superpose to form a spike with three times the amplitude. A similar situation occurs in the second half PWM period. This represents the worst case for this PWM scheme.

3.3 Reconstruction of LFP emissions

Anti-clockwise rotation of $V_{ref}$ generates the three sinusoidal phase voltages. This is obtained by varying $D_k$ and $D_{k+1}$ with respect to the constraints given in (5). Close to the boundary between the $k$th sector, and the $(k+1)$th sector, either $D_k$ or $D_{k+1}$ tends to zero. This causes spike superpositions at the sector boundary crossings, which forms the LFP emissions. Thus, there are six LFP pulses in one complete revolution, since there are six sector boundaries. The period of the output sinusoidal voltage $T_0$, is related to the period of the LFP emissions ($T_{LFP}$), as described in (10).

$$T_0 = 6T_{LFP}$$

Thus, varying the frequency of the reference voltage also varies the periodicity of the LFP emissions. The measurement cases presented in Table 8.1 are simulated using the parameter settings presented in Table 8.2. The simulated results corresponding approximately to cases II, III, and IV are presented in Fig. 13, Fig. 14, and Fig. 15, respectively.
Figure 12: Simulation results showing LFP emissions of periodicity 0.001 s. The results obtained using the following parameter settings: $0 < D_k \leq 0.2$, $T_s = 0.0002 s$, $T_0 = 50T_s$. The lower subplot is obtained by zooming in the response when $38.1T_s \leq \text{time} \leq 40.0T_s$. This corresponds to case I, presented in Fig. 2 showing the superposition of switching spikes.

Figure 13: Simulation results showing LFP emissions of $40T_s = 0.008 s$. The results obtained using the following parameter settings: $0 < D_k \leq 0.2$, $T_s = 0.0002 s$, $T_0 = 240T_s$. This is a reconstruction of case II, shown in Fig. 3. The lower subplot is obtained by zooming in the response when $20T_s \leq \text{time} \leq 80T_s$. 
Figure 14: Simulation results showing LFP emissions of $20T_s = 0.004s$. The results obtained using the following parameter settings: $0 < D_k \leq 0.2$, $T_s = 0.0002s$, $T_0 = 120T_s$. This a reconstruction of case III, shown in Fig. 5. The lower subplot is obtained by zooming in the response when $20T_s \leq \text{time} \leq 80T_s$.

Figure 15: Simulation results for light load conditions, a reconstruction of LFP pattern in case IV, shown in Fig. 6 with a period of 0.005 s. The results are obtained using the following parameter settings: $0 < D_k \leq 0.05$, $T_s = 0.0002s$, $T_0 = 150T_s$. The lower subplot is obtained by zooming in the response when $20T_s \leq \text{time} \leq 80T_s$. 
Figure 16: Simulation results showing LFP emissions of periodicity 0.6 s. The results obtained using the following parameter settings: \(0 < D_k \leq 0.2\), \(T_s = 0.0002\) s, \(T_0 = 3000 T_s\). This is a prediction of case V, in Table 8.1. The lower subplot is obtained by zooming in the response when \(0 \leq \text{time} \leq 1200 T_s\).

Table 8.2: Modeling LFP emissions corresponding approximately to the measured cases. The maximum duty cycle \(D_k\) models the maximum amplitude of \(V_{ref}\). \(T_0\) is the fundamental frequency of the phase voltages.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Period ((T_0))</th>
<th>(D_k) max</th>
<th>Period ((T_{LFP}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>I: At switching</td>
<td>0.06 s</td>
<td>0.2</td>
<td>0.01</td>
</tr>
<tr>
<td>II: Medium</td>
<td>0.048 s</td>
<td>0.45</td>
<td>0.008 s</td>
</tr>
<tr>
<td>III: High speed</td>
<td>0.024 s</td>
<td>0.45</td>
<td>0.004 s</td>
</tr>
<tr>
<td>IV: Light Load</td>
<td>0.03 s</td>
<td>0.05</td>
<td>0.005 s</td>
</tr>
<tr>
<td>V: Low speed</td>
<td>0.60 s</td>
<td>0.45</td>
<td>0.1 s</td>
</tr>
</tbody>
</table>

The model is used to predict the LFP emissions at low frequency, corresponding to case V in Table 8.2, and is shown in Fig. 16. The simulated cases are rather simple and do not exactly match the corresponding measurement cases. Unlike the measurements, the amplitude and frequency of the reference voltage for a given case are fixed throughout the simulation. Frequency and amplitude variations are only made between cases. Although the models are rather simple, they give a good characterization of the source of the LFP emissions.
Figure 17: Common current (i_{cm}) coupling path in ac drive system.

4 Potential consequences of LFP emissions

The EMI and functionality related issues resulting from the LFP emissions are discussed in the following sub-sections.

4.1 EMI issues

The EMI issues include the following:

- The superposition of i_{cm} spikes would increase the spike amplitude by a factor of three in the worst case, as shown in Fig. 2. These would enhance the emissions at harmonics of the switching frequency. This is seen in the harmonic spectrum of the i_{cm} currents in case II, shown in Fig. 4. The enhanced emission levels should be considered in the design of EMI filters.

- The continuous superposition of i_{cm} spikes across sector boundaries generates low frequency emissions, which are more challenging to damp using EMI filters.

4.2 Functionality issues

The functionality related issues include the following:
The enhanced harmonics of the switching frequencies could perturb sensitive equipment like the CAN bus in a vehicle environment.

The coupling path of the common mode currents in the drive system is shown in Fig. 17. When these pulsating currents are injected into the motor, most of the high frequency content would be damped by the stator winding inductance, while the low frequency components would still be significant. This could lead to torque pulsations, speed fluctuations, and subsequent mechanical oscillations. Further analysis is required to characterize the coupling between the LFP common mode currents and torque pulsations, however such torque pulsations due to current harmonics have been reported in [11, 12, 13, 14, 15, 16, 17, 18, 19].

These pulsations could generate acoustic noise depending on the frequency content [33].

5 Discussions and conclusions

The PWM drive in the HEV generated low frequency pulsating (LFP) common mode currents. Common mode currents with an amplitude of about ten percent of the phase current were measured. Modeling results show that the low frequency pulses were caused by the SV-PWM scheme, from the continuous superposition of common mode current spikes or $i_{cm}$ spikes, generated during the switching of voltage vectors across sector boundaries. In the SV-PWM scheme, each reference voltage vector, $V_{ref}$, is formed as a linear combination of the adjacent voltage vectors, $V_k$ and $V_{k+1}$, with duty cycles, $D_k$, $D_{k+1}$, respectively. The duty cycles determine the interval between the $i_{cm}$ spikes. For a $V_{ref}$ closed to sector boundaries, or close to the region where $\theta_c/2\pi T_0 \leq C$, either $D_k$ or $D_{k+1}$ tends to zero, causing a superposition of $i_{cm}$ spikes. Since there are six sectors in the space vector hexagon, the periodicity of the LFP emissions is one-sixth the period of the phase voltages. The dependence of the LFP emissions on drive parameters like load, speed, voltage slew rates have been addressed in [34]. In the worst case, the amplitude of the LFP emissions could be three times the individual switching spike amplitude. Unlike in the model, the periodicity of the LFP emissions in the measurements seem to be about 4.5 times the periodicity of the phase currents. This could be due to the complexity of the drive system in the HEV, compared to the simple three phases of the AC-DC drive system assumed in the model.

The amplitude of the switching spikes can be addressed by minimizing the stray capacitances. This could be obtained by carefully designing the cables and connectors, thus requiring adequate estimation of stray capacitances in the system. Here, PEEC electromagnetic modeling tools are suitable [35, 36]. Regarding the superposition of switching spikes caused by the PWM scheme, approaches in which the spikes generated by the PWM scheme are cancelled out would be suitable. This calls for rigorous control of timing in the software implementation of the PWM scheme. Here, new approaches based on formal reactive programming may be efficient [37]. Recent work presented in [8], in which a multilevel inverter topology with a modified SV-PWM scheme that yields
zero voltage vectors with no common mode voltage was proposed, indicates that this may be possible. Although the approach in [8] only considers the common mode voltage generated from the switching of zero vectors, spikes generated by switching non-zero vectors also have a significant contribution on the LFP emissions.

References


