X-ray Microtomography and Digital Volume Correlation for Internal Deformation and Strain Analysis

Fredrik Forsberg

Luleå University of Technology
Department of Applied Physics and Mechanical Engineering
Division of Experimental Mechanics

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Fredrik Forsberg
Dedicated to the memory of my dear mother,
Britt-Marie Forsberg
The research presented in this thesis has been carried out at the Division of Experimental Mechanics at Luleå University of Technology (LTU) in Sweden. It has been performed during the years 2002-2008. The work has been economically funded by the Swedish Research Council (VR).

First of all, I would like to thank my supervisor Prof. Mikael Sjödahl, whom also is co-author to Paper C. He has in an encouraging way supported me and given me guidance in my research. He has reviewed most of my written work, often with short notice, but always very thoroughly and efficiently. I would also like to thank Prof. Emeritus Nils-Erik Molin, whom was the head of our division during my first years as a PhD-student. Thanks for many great advices and words of wisdom. I would also like to acknowledge Istvan Sárady for helping me with the manufacturing of the calibration phantoms that are used for the quality assurance of the x-ray microtomography system. Furthermore, I would like to thank all of my colleagues at the division of experimental mechanics, both seniors and fellow research students, for making the time very enjoyable and for creating a positive and constructive atmosphere. Especially, I would like to thank Dr. Erik Olsson – whom I was sharing office with for many years - not just for being a good help, through all the discussions we have had through the years, but also for being a good friend and having lots of humour.

I have had the opportunity to spend three month, during August-November 2007, at the Swiss Federal Laboratories for Materials Testing and Research (EMPA) in Dübendorf, Switzerland, and perform collaborative research that was very fruitful. I would like to thank the three foundations The Royal Swedish Academy of Science, Nordeas Norrlandstiftelsen and The Wallenberg foundation for the financial support that made this visit possible. I would also like to thank all the friendly people at the Electronic/Metrology Laboratory at EMPA, for their kind hospitality during my visit. Especially I would like to thank Dr. Erwin Hack, René Mooser and Peter Wyss, co-authors on the Papers B, C and D, for a productive collaboration. Especially the work at the synchrotron facility SLS (Swiss Light Source) is worth remembering, which was very intensive but also very joyful. Also, thanks for all the nice chats during the coffee breaks at EMPA, when we were discussing social and cultural differences between our countries, and I got addicted to Swiss chocolate. Also, I would like to acknowledge Martin Arnold at the Wood Laboratory at EMPA for his valuable contributions on Paper C.
I would also like to thank Dr. Stephen Grantham, former research student at the Cavendish Laboratory at the University of Cambridge, UK, for the nice collaboration we had when performing the silo flow experiment (Paper A).

I would like to express my appreciation to Dr. Clive Siviour, co-author of Paper E, at the Department of Engineering Science, University of Oxford, UK. It has been great fun to work with you and to have you here during your annual visits in Luleå. Thanks also for hosting me and Mikael during our visit in Oxford, which was very pleasant.

I would like to thank Dr. Jonas Danvind at the division of Wood Physics at Luleå University of Technology, co-author of paper D, for a fruitful collaboration and many helpful discussions. Furthermore, I want to thank Dennis Johansson, at the same division, for supplying us with the pine wood samples that were used in the experiments that are described in Paper B, C and D.

Also, I would like to take the opportunity to acknowledge Dr. Henrik Turbell for sharing his cone-beam reconstruction software with us. Without his generous contribution a great deal of this work wouldn’t have been possible.

My deepest gratitude to my mother (posthumous), father and sister for all the support they have given me through the years. Finally, I would like to express my gratitude to Lisa, my beloved common law wife for her endless support during my work on the thesis. Without your support I couldn’t have managed. I also want to thank our children Elsa and Leo, just for being who they are. Thank you!

Fredrik Forsberg
Luleå, November 2008
A material that is exposed to mechanical load or experience a variation in its immediate environment (temperature, pressure, humidity etc.) will to some extent be affected by these new conditions, which is reflected through structural movements in the material. In order to measure engineering properties related to these structural changes, such as for example deformation and strain, we need to gain information about them that are precise and reliable. There exist many different methods for such measurements, which in most cases are based on the pure surface response due to the deforming mechanism. As long as the material structure is reasonably homogeneous the surface information may be enough but as the complexity of the material structure increase it gets more important to obtain information from the inside of the material.

Here, a method for full 3D imaging and quantitative analysis of internal deformation and strain in inhomogeneous materials is presented. 3D structural information from the deforming material is obtained through use of x-ray microtomography. The deformation of the structure is analysed with a 3D pattern recognition technique called digital volume correlation, which is a 3D extension of digital image correlation. A thorough theoretical description of both image formation through x-ray microtomography as well as 2D and 3D structural deformation analysis is given. Complimentary, more practical aspects of the different x-ray imaging systems used in the research are described together with the different methods used for image quality assurance.

Four different applications are presented. The first is an example of how rapid processes such as internal granular flow can be imaged and analysed with this kind of methods. The temporal resolution needed to resolve the process yields a sacrifice of spatial information and the analysis is carried out in 2D with digital image correlation. Secondly, the deformation and strain in 3D micro-scale wood structure exposed to three-point-bending is measured by use of synchrotron x-ray microtomography and digital volume correlation. Thirdly, the 3D structural swelling in wood microstructure due to water exposure is analysed using the same methods. Finally, the motion and induced strain in a granular material due to compaction is measured in 3D. The results show good agreement with corresponding 2D measurements, carried out for comparison. The experiments show that the method successfully can be used for analysis of various kinds of deformations and materials and that the results are trustworthy.
This thesis consists of a summary and the following five papers:


Apart from the appended papers listed above there are also a number of publications not covered in this thesis.


- F. Forsberg and M. Sjödahl “Tomographic 3D-DSP: Measurement of internal deformations,” Advances in


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A. Measurements of granular flow in a silo using Digital Speckle Radiography
B. Full 3D strain measurements on wood exposed to three-point-bending:
   Analysis by use of digital volume correlation applied to synchrotron
   radiation \( \mu \)CT image data
C. 3D micro-scale deformations of wood in bending: Synchrotron radiation
   \( \mu \)CT data analyzed with digital volume correlation

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D. 3D micro-scale analysis of swell behaviour in Scots pine wood using Synchrotron Radiation Micro Computed Tomography and Digital Volume Correlation

E. 3D deformation and strain analysis in compacted granular sugar using x-ray microtomography and digital volume correlation
Part I

Summary
1. INTRODUCTION

When a material is exposed to mechanical load or experience variations in temperature, pressure or chemical composition it will be affected by these new conditions. The material will recognise new forces that are acting, and the material structure will abduct to the new load situation, and deform in an appropriate way. This structural deformation will have a great impact on the characteristic quantities of the material. It is therefore of great importance that materials used for various sorts of construction such as engineering materials, biomaterials and geomaterials are properly designed for their application and able to hold against various loads. This design process however requires that one has a profound understanding for the different deformation processes that might lead to a degrading of the material, and a possible failure. In order to measure engineering properties related to these structural changes, such as for example deformation and strain, we need to gain information about them that are precise and reliable. There exist many different methods for such measurements that all have there advantages and disadvantages. Some requires contact with the investigated material, while other are non-contact methods. A few measures in just one single point on the material surface while other gathers full field information in both 2D and 3D. What method to use for monitoring the deformation process depends on the application. In most cases these methods are based on the pure surface response due to the deforming mechanism, with the use of cameras and optical sensors. As long as the material structure is reasonably homogeneous the surface information may be enough but as the complexity of the material structure increase it gets more important to obtain information from the inside of the material.

In this thesis, a method for full 3D imaging and quantitative analysis of internal deformation and strain in inhomogeneous materials is presented. The investigated material is imaged in 3D using x-ray microtomography (or micro-computed tomography, μCT) - first in unloaded state and then after deformation. The two sets of 3D image data are then analysed by a 3D pattern recognition technique called Digital Volume Correlation (DVC), which results in a 3D displacement field that describes the intermediate deformation process within the material. Finally, from the obtained displacements it is possible to determine the 3D strains in the material due to the structural deformation.
1.1 Thesis outline

This thesis consists of two separate parts. The first part (Part I) is a summary that contains ten chapters that together summarises the most relevant work in this thesis, but also treat certain aspects in the work that has not been reflected in the published papers. Part II consist five appended papers that together illuminate the scientific achievements of this thesis.

The first chapters in the summary describe the two-dimensional techniques that together form the fundament for the three-dimensional imaging and deformation analysis. In x-ray microtomography, for example, the investigated object is reconstructed based on a set of two-dimensional x-ray projection images, acquired from different angles during a rotation of the object. The same close relationship with 2D methods holds for DVC, which descend from the two-dimensional method called Digital Image Correlation (DIC). In DIC the deformation in a plane is analysed from a pair of digital images describing the 2D structure before and after deformation. At an early stage of this research project some initial experiments were carried out using these 2D techniques for measurements of inner deformations based on x-ray projection images. Therefore it is natural that the first chapters in the summary describe these two-dimensional techniques.

In Chapter 2, the principles behind 2D x-ray imaging will be covered. The chapter starts with a historical perspective and continues with a description over the characteristic processes for image formation. Thereafter we will look at the special case when acquiring the image data digitally, which is called digital radiography. This is carried out with respect to the digital radiography system that has been used within the project, and which forms the fundament in the microtomography system.

Chapter 3 describes the correlation method used for two-dimensional measurements of deformations, Digital Image Correlation. The chapter starts with a description of the features within the image data, on which the correlation technique is based upon, called speckle patterns. Then a historical review of correlation based methods in 2D is given. Thereafter the DIC-algorithm used in this research is described together with information about the accuracy aspects of this algorithm. Sometimes the method is referred to as Digital Speckle Photography (DSP), or Electronic Speckle Photography (ESP) which are the names that historically has been used for this particular algorithm.

Chapter 4 describes an example on how the techniques described in the two preceding chapters can be combined and used for measurements of rapid deformation processes inside inhomogeneous materials. The measurements that are presented here describe the internal flow behaviour in a silo, filled with alumina-powder. The measurements are carried out in a plane,
positioned through the centre of the silo. This experiment is thoroughly described in Paper A.

In the following sections the three-dimensional techniques and measurements are described. A profound theoretical description of the full measurement methodology is given.

Chapter 5 describes the 3D image formation by use of x-ray microtomography. The chapter starts with a mathematical description of the x-ray projections used for tomographic reconstruction, and continuous with the theory that was started in Chapter 2. Then follows theory describing the reconstruction algorithms that are used to retrieve 3D image data from the acquired projection data, both using parallel beam tomography as well as cone-beam tomography (The two different beam shapes that are used in this thesis). Then a more practical view of microtomography is given, where the two different imaging systems are described. The first is a Synchrotron radiation microtomography (SR\(\mu\)CT) system and the second is a lab based cone-beam tomography system. The chapter ends with a description of different methods used for image quality assurance and image artefact correction.

Chapter 6 gives a profound description of the DVC-algorithm used for 3D deformation analysis. A novel approach where the displacements are described as Chebyshhev polynomials is described. Furthermore, theory for the calculation of different 3D strains is presented. The DVC-algorithm is also thoroughly described in Paper B. The chapter ends with a comparison study between DVC and DIC. Here the result in a 2D cross-section through the complete 3D displacement data, obtained with DVC, is compared with the results from DIC analysis, carried out on the image data from the corresponding cross-section. These results are also presented in Paper E.

In Chapter 7 three different experiments are presented where 3D deformation and strains are measured in inhomogeneous materials using the x-ray microtomography in combination with DVC. In the first experiment, the deformation and strain in 3D micro-scale wood structure exposed to three-point-bending is measured. Here synchrotron x-ray microtomography is used for the imaging which results in a spatial resolution of approximately 1\(\mu\)m, which allows imaging of the complex cellular structure of the wood material. This experiment is profoundly described in Paper B and C. In the second experiment, the 3D structural swelling in wood microstructure due to water exposure is analysed using the same methods as in the first experiment (Paper D). Finally, the motion and induced strain in a bed of granular sugar due to compaction is measured in 3D, by cone-beam microtomography and DVC (Paper E).
In Chapter 8 the results and findings of this research are discussed and concluded and in Chapter 9 an outline for future work is discussed. In Chapter 10, finally, the appended papers are summarised.
During an early stage of this research project some initial experiments were carried out using Digital Image Correlation (DIC), also called Digital Speckle Photography (DSP), for measurements of inner deformations based on 2D x-ray images. Although these images only describe a projection of the investigated object, and there is a lack of information along the depth-direction, there are other advantages with the two-dimensional technique. For example, 2D imaging with x-ray can be performed with a high frame rate which allows rapid deformation processes to be investigated. An example of such measurements is the silo flow investigations that are described in Chapter 4, and in Paper A.

Also, when performing the three-dimensional measurements, x-ray microtomography is used to reconstruct the three-dimensional geometry of the investigated object. This is done based on a set of two-dimensional x-ray projection images, acquired from different angles during a rotation of the object.

The two-dimensional x-ray imaging hence forms a fundament for the three-dimensional investigations and therefore deserves a proper introduction, which is given in this section.

2.1 Image formation

Digital Radiography is based on conventional radiography, or X-ray imaging, an imaging method that can be found at every hospital around the world. The discovery of x-rays was presented by Röntgen\(^1\) in 1895 and for this he was awarded the first Nobel Prize in Physics, in 1901.

A short introduction to conventional radiography is given below. However for a more profound description of this and related techniques please take a look in a text book that treats translucent imaging, for example ref. [2].

In conventional radiography the object to be investigated is placed in a beam of x-rays. Let us assume that the emitted x-rays have the intensity \( I_0 \). These x-rays all enter the object. If we assume the object to be homogenous, the transmitted intensity of x-rays that reaches the detector, \( I_D \), along one path or line is given by Beer’s law

\[
I_D = I_0 e^{-\mu L},
\]  

where \( \mu \) is the linear attenuation coefficient and \( L \) is the length the rays travelled inside the object. The linear attenuation is closely related to the
density of the object. If the ray instead traverses \( n \) regions along the path \( L \) through the object in the \( x \)-direction, all with different linear attenuation \( \mu(x) \), then we can write

\[
I_D = I_0 e^{-\int_{x}^{L} \mu(x) dx}.
\]  

(2.2)

The image formation is based upon the differential absorption of the x-rays in the object. The x-rays travelling along a path through dense matter will attenuate more than those along a path through light matter. The contrast in the so obtained image is defined as

\[
C = \frac{I_1 - I_2}{I_1 + I_2}, \quad I_1 > I_2,
\]  

(2.3)

where \( I_1 \) and \( I_2 \) are the intensities in two different detection points. Note that the contrast, through its definition, is positive and ranges between 0 and 1.

Let’s take a look at a practical example. Bone tissue in our body mainly consists of Calcium, which is a fairly heavy element compared to Hydrogen, Carbon and Oxygen, which is the main constituents in the soft tissue (fat and muscles etc.). The bone tissue will thus attenuate more x-rays than the soft tissue and we’ll obtain high contrast between these two tissues.

In a radiographic image, Figure 2.1(a), we see a grayscale map corresponding to the attenuation values in the object. The intensity in the image is inversely proportional to the transmitted intensity at the detector. Bright areas therefore correspond to high

![Figure 2.1](image)

(a)

(b)

Figure 2.1. A radiographic image of a bone (a). The attenuation values are mapped to 256 grayscale values, from 0 to 255. The two-dimensional attenuation surface is shown in (b).
attenuation. The two-dimensional attenuation surface is shown in Figure 2.1(b). The attenuation values can then be related to density values of the imaged constituents. However the radiographic image only shows the projection of the true three-dimensional attenuation distribution of the object. Since we don’t have any spatial resolution in the depth-direction we can’t determine the exact position of an object along the optical axis, nor its size. This uncertainty behaviour is shown in Figure 2.2. Two objects A and B with different size and location along the optical axis give rise to images of equal size. The reversed situation may also happen. Two images B and C of different size are projections of object B and C - two objects of equal size but situated at different positions along the optical axis. The objects A and C however produce images that can be related to each other, since these two objects both lie in the same plane, orthogonal to the optical axis. So by making sure that the image information originates from objects confined to the same plane, orthogonal to the optical axis, we can reduce the uncertainty effect. This can be done by use of a contrast agent. For example by introducing a sheet of small x-ray opaque particles, see Figure 2.3. This gives us a simple geometrical relationship between the imaged structure and the corresponding object points. This relationship is given by

\[ M = \frac{S_i}{S_o} = \frac{D_{s-d}}{D_{s-o}}, \]  

(2.4)

where \( M \) is the magnification, \( S_i \) and \( S_o \) the sizes of the image and object, respectively, and \( D_{s-d} \) and \( D_{s-o} \) the source-to-detector distance and source-to-object distance, respectively.

This method is used in the DSR-experiments and will be explained in further detail in Section 4.1.
2.2 Digital radiography

Conventional x-ray images were captured by use of x-ray film as recording media. Nowadays it is common to use x-ray imaging plates, where the image initially is recorded analogue on the plate and then in a second step transferred to digital form through a scanning laser system connected to a computer\(^3\). Both these methods have the advantage of good spatial resolution. However, the acquisition involves several intermediate steps before the final image has been developed. If the radiographic images instead are captured digitally with a CCD and transferred through an image processor to a host computer or server it will open up for more sophisticated real time image processing and analysis.

The digital radiography system that is used in this work is schematically shown in Figure 2.4. The system consists of a microfocus x-ray source (L7901-01), an x-ray detection unit (C7876-10) and an external image processor (Argus-20) that all comes from Hamamatsu Photonics. The image acquisition is controlled from a host computer. The x-ray tube has a voltage range from 20 to 100 kV and the tube current ranges from 0 to 250 \(\mu\)A. The effective source spot size is 5-7 \(\mu\)m, depending on the x-ray output effect. The detector, finally, consists of an image intensifier and a 17 mm CCD, with 600 x 800 pixels. The spatial resolution of the detector is 4.5 lp/mm [4]. Furthermore, the effective field of view (FOV) of the detector is 72 x 54 mm. However, since the x-ray beam is cone shaped, the effective FOV will depend on where along the axis between the source and detector the specimen is positioned

\[
FOV = \frac{1}{M} \frac{FOV_{\text{det}}}{D_s-d} \frac{D_s-o}{D_{s-d}}, \tag{2.5}
\]

where \(M\) is the magnification, from (2.4), and \(D_{s-d}\) and \(D_{s-o}\) are the source-to-detector distance and source-to-object distance, respectively. The theoretical pixel size, which gives the theoretical upper limit for the spatial resolution, is given by the FOV divided with the number of pixels along the corresponding directions, and hence scaled with the FOV in Equation (2.5). The theoretical isotropic pixel size is 90 \(\mu\)m\(^2\) at the detector and 45 \(\mu\)m\(^2\) if a 2x-magnification is used, i.e. when the object is positioned right between the x-ray source and the detector.

The detector allows real-time visualisation of the investigated object and is capable of capturing image sequences with a frame rate of 25 frames/s. High temporal resolution is required when for example investigating objects that either move or transform in a rapid pace. The image processor is used for filtration purposes. It can for example be used for multi-frame-averaging (temporal averaging), in order to reduce the image noise. Another possibility is to subtract the background, in order to either compensate for uneven illumination or remove static objects that we don’t want in the image. The
image processor is controlled either directly, on the image processor unit, or through software, from the host computer. The most concrete difference in a digital radiographic image from an image collected with a conventional system is that they have inversed grayscale values, as we can see in Figure 2.5. In the digital detector the image is created when the photons are summed up for each individual pixel and bright areas therefore correspond to high transmission of x-rays.

Figure 2.5. An image captured with digital radiography, showing the same bone as in Figure 2.1(a). The grayscale of the digital image is inversed, and thus represent the transmission of x-rays.
3. DIGITAL IMAGE CORRELATION

In this chapter the Digital Image Correlation (DIC) method used in both Paper A and E is described. It was mainly used at an early stage of the thesis, for experiments of the type described in Paper A. However it has also been used at a final stage of the thesis, to confirm the 3D measurements that have been carried out with the Digital Volume Correlation (DVC) algorithm, as described in Paper E.

The DIC-technique is not at all new. The pioneer work was carried out in the early 1980’s. The algorithm that is described in this chapter is called Digital Speckle Photography (DSP), or Electronic Speckle Photography (ESP). It was proposed approximately ten years later by Sjödahl and Benckert and was the subject of Sjödahl’s doctoral thesis. Later on the algorithm has also been implemented for use in a stereo imaging system based on white light, for measurements of both in-plane and out-of-plane displacements as well as shape. It has also been used in combination with in-line and stereoscopic x-ray systems for measurements of internal displacements.

The DVC-algorithm, for full three-dimensional analysis of deformation and strain, presented in this thesis is described in Chapter 6 and Paper B.

Before the two-dimensional algorithm is described, in Section 3.3, we will start by looking at the features that the technique is based upon – the speckles. Then a historical review to two-dimensional image correlation is given, in Section 3.2

3.1 Speckle patterns

The granular pattern shown in Figure 3.1(a) is called a speckle pattern. What characterises a speckle pattern is a high spatial frequency distribution within a given bandwidth and that it’s random. Each region of the pattern is therefore unique. There are two classes of speckle patterns that usually are referred to as speckle patterns.

Laser speckles are created when light from a coherent laser is reflected diffusely by a rough surface, see Figure 3.1(b). Each point in space senses a unique optical wave due to the diffuse scattering at the surface. The speckle pattern will therefore be different for different points in space.

Another type of speckle formation is white light speckles. In this case the speckle pattern comes as the natural relief of the surface - the speckles behave as if they were glued onto the surface.
Figure 3.1. (a) A speckle pattern. (b) Monochromatic light of wavelength $\lambda$ is diffusely scattered by an optically rough surface. (c) The speckle formation using an x-ray system is based on the differential absorption of x-rays within the object structure.

In this thesis we use speckle patterns generated with the use of an x-ray system. The speckles occur as intensity variations due to differential absorption within the material, which is illustrated in Figure 3.1(c). Variations in the object structure therefore imply variations in the recorded speckle patterns and the speckles may therefore be categorised as white light speckles.

3.2 Speckle Photography – a historical review

Speckle Photography (SP) is the great ancestor of all correlation based methods used to measure surface deformations. It is profoundly discussed in both text books and review articles\textsuperscript{14-17}.

Speckle Photography uses information coded in the bulk movement of a speckle pattern as the object deforms. Traditionally a photographic double exposure is collected where two speckle patterns, collected before and after deformation, are registered on top of each other. The result when developing this double exposure is called a specklegram. It can be seen as two images, which are overlapped and slightly displaced. When a narrow laser beam shines through the specklegram a diffraction halo is produced on an image screen,
placed behind the specklegram. The diffraction halo is modulated with Young’s fringes and by measuring the pitch and orientation of these fringes the local object displacement is determined. However since the displacement is coded in fringes there will be a 180° directional ambiguity of the displacement. The total deformation field is obtained by probing the specklegram for an arbitrary number of positions. It is a quite neat and straight-forward method that has gained a lot of attention and found numerous applications through the years – ever since the first pioneering works in the late sixties and early seventies18-20.

However, there are several non-optimised and time-consuming steps along the way to obtain a full deformation field. One is processing the photographic film and another is the laser scanning procedure that needs to be carried out in order to get a field measurement. Therefore the technique has slowly but surely evolved and found automated solutions for these tiresome steps. The first full automation of the method, where all the steps are performed digitally in a computer was taken in the early 1980’s, by Peters and Ranson3 – Digital Image Correlation (DIC), or Digital Speckle Photography (DSP), was born.

### 3.3 Digital image correlation/Digital speckle photography

Digital images, collected with solid state detectors, permit a more efficient data management. A large number of images can be captured sequentially with an appropriate frame rate which results in measurements of better time resolution. This together with the tremendous increase in computational power during the last decades has resulted in deformation field measurements that can be carried almost instantaneously. In the literature, a number of DIC-systems have been presented by many researchers, including Sjödahl et al.7, 21, Chen and Chiang22-24, Noh and Yamaguchi25, and Sutton et al.6, 26. In experimental mechanics it is most often used to investigate material behaviour and to test constitutive models27, 28. It is also in regular use within the field of fluid mechanics, under the name Particle Image Velocimetry (PIV)29.

#### 3.3.1 The DSP-algorithm

Now we will look at the correlation procedure used in Digital Speckle Photography. The method is discussed step-by-step, however sometimes more briefly. A much more extensive description is given in the chapter “Digital Speckle Photography” by Sjödahl in a book on speckle related techniques edited by Rastogi30.

In order to measure the deformation field of a deforming surface with DSP two images of the surface has to be collected, one before and one after deformation. The images are then first stored in two arrays, \(h_1\) (the reference image) and \(h_2\) (the displaced image) and later divided into smaller elements, subimages. One image can now be seen as a grid of such subimages and an image with \(n\) subimages can be written as
The algorithm requires that the images have a random speckle pattern since the idea is that each subimage shall be a carrier of entirely unique information. In each of these subimages the deformation can be approximated to be constant in size and direction.

By looking at how much the pattern in one specific subimage in $h_2$ has been translated relative to the pattern in the corresponding subimage in $h_1$, the displacement of this subimage is achieved. By iterating this procedure for all subimages a grid of displacements is created, where each individual displacement is connected to a specific subimage. This grid describes the whole deformation field. The displacement calculation is done with a cross-covariance routine which, in mathematical terms, is the inverse Fourier transform of the spectrum from one of the subimages complex conjugated multiplied with the spectrum of the other subimage. This is written as

$$c(p,q) = \mathcal{F}^{-1} \{ H_{1s}^* H_{2s} \},$$

where $p$ and $q$ are matrix indices, the $\mathcal{F}$ indicates a Fourier transform and $h_{1s}$ and $h_{2s}$ are subimages from the reference image and the displaced image, respectively. In the spatial domain $c(p,q)$ is the convolution between the two subimages. The principle of DSP is shown in Figure 3.2. If the two corresponding subimages contain a certain degree of overlap the function $c(p,q)$ will contain a peak value. The height of this correlation peak gives the statistical resemblance between the two subimages while the position gives the mean displacement of the features, to size and direction. By repeating this correlation procedure for all of the subimages the full deformation field is obtained. The deformation field shown in Figure 3.2 describes a shrinking of the imaged surface. If the speckle movement is too large, we’ll get a zero overlap between the subimages and they are said to be decorrelated. In this case there won’t be a correlation peak at all and $c(p,q)$ will be uniformly zero. On the other hand, if the speckles remain still between the two subimages we’ll get a perfect match, and the correlation value will be 1. The displacement is in units of pixels and is denoted $u$ and $v$ for the $x$- and $y$-direction, respectively. A large displacement will give rise to a poor overlap of the subimages, as already mentioned, which will result in a low signal to noise ratio in the correlation surface. It is therefore important to limit the displacement between the subimages by choosing an appropriate time interval between the captures. If the cross-correlation procedure is repeated with a new choice of the displaced subimage that corresponds to the previously calculated displacement, while the reference subimage remains fixed, the degree of correlation will be better. So the algorithm includes an iterative
Figure 3.2. The principle of the DSP-algorithm. Two subimages, $h_{1s}$ and $h_{2s}$, from the reference image and deformed image, respectively, are cross-correlated. The position of the peak value in the resulting correlation function gives the deformation in the corresponding subimage, to size and direction. By repeating the procedure for all subimages the full deformation field is obtained. The displacement vector inside the circle corresponds to the calculations made on the two subimages above.
process where the procedure above is repeated until a maximum overlap and correlation is obtained. So far the displacement is restricted to integer pixel values since the calculations depends on images captured with a solid state detector, where everything is represented in discrete pixels. Therefore is the described solution only correct in the special case where the real displacements happen to coincide with the discrete detection points. To avoid this problem and to achieve better resolution in the correlation an interpolation routine is applied on the current solution. This routine is based on the fact that \( c(p,q) \) can be considered as a harmonic function since it has been Fourier transformed. The continuous cross-correlation surface is written as

\[
c_{\text{cont}}(x, y) = \frac{1}{P} \sum_{k=0}^{P-1} \sum_{l=0}^{P-1} c(p,q) \frac{\sin(\pi(x-p)) \sin(\pi(y-q))}{\sin(\frac{\pi(x-p)}{P}) \sin(\frac{\pi(y-p)}{P})},
\]  

(3.3)

where \( P \) is chosen to be odd. The higher order of \( P \) and the more terms that are included in the series the better the resolution will be in the correlation surface. The peak position in \( c_{\text{cont}}(x, y) \) describes a more exact, non integer pixel valued, displacement.

The algorithm is now almost optimized for its task but still it suffers from the fact that the two subimages have been collected at different locations, and as a result of that a perfect overlap can’t be obtained. By using the Fourier shift theorem to shift the deformed subimage by non-integer pixel values a maximum overlap is obtained. This can only be done if the images have been properly sampled so that all the information is retained. Applying the pixel-shift algorithm therefore cancels the problem occurring from the fact that the sampling has taken place at different locations. The pixel-shift algorithm is written as

\[
H_{\text{shift}}(p,q) = H(p,q) e^{2\pi i (pu +qv)/M},
\]

\[
h_{\text{shift}}(u,v) = \mathcal{F}^{-1}\left\{ H_{\text{shift}}(p,q) \right\},
\]  

(3.4)

where \( H \) is the Fourier transform of the subimage \( h \), and \( u \) and \( v \) are the displacements calculated from Equation (3.3). \( h_{\text{shift}} \) is the new displaced subimage that have a perfect overlap with the reference image. For perfectly correlated images the use of the pixel-shift algorithm will result in the equivalence of an auto-correlation, which cancels the noise problem due to non-overlapping parts of the subimages.
3.2.2 Errors in DSP

When using DSP there are two different types of errors involved - random errors and systematic errors. The random errors originate from difficulties in determining the exact position of the correlation peak. Sjödahl\textsuperscript{31} have made an extensive investigation of the accuracy in DSP. He proposes an expression for empirical estimation of the random error. This expression is written as

\[ e = k \frac{\sigma^2}{B} \left( \frac{1-\delta}{\delta} \right)^{1/2}, \]  

where \( e \) is the random error, \( k \) is a constant, \( \sigma \) is the average speckle size, \( \delta \) is the mean correlation value and \( B \) is the subimage size. It is assumed that the average speckle size remains constant throughout each of the two speckle patterns. The parameter \( k \) depends on the subimage size and is therefore constant during a correlation procedure, but may alter between different measurements.

The second type of error is the systematic error, which most common is the result of bad sampling. In order to obtain reliable results it is important that the Nyquist sampling criterion is fulfilled. If we fail to resolve all the spatial frequencies in the speckle pattern we’ll encounter aliasing-problems. When using speckle metrology with a digital detector and a circular aperture, it is stated that\textsuperscript{32} problems caused by undersampling can be prevented by using mean speckle sizes above 2 pixels. This value is an optimum since it minimises the random error at the same time as systematic errors are prevented.
4. Analysis of 2D Deformations

We will here combine the techniques Digital Radiography and Digital Speckle Photography, discussed in the former two chapters. The result is a technique called Digital Speckle Radiography (DSR). Here, an experiment where the method is used to measure the granular flow behaviour in a bin and hopper configuration is presented. These measurements are described thoroughly in Paper A.

4.1 Investigation of the granular flow in a silo

In two dimensions, we are restricted to measure the deformation field in a single plane within the deforming object, in order to get accurate quantitative results. This is carried out by using various so called seedings or contrast agents during the imaging procedure. The seeding typically consists of small x-ray opaque particles – normally Tungsten – spread out in a thin sheet within the examined material. The speckles in the x-ray images are therefore directly related to the seeding particles. This implies that our measured deformation field describes the deformation of the seeded layer. However, we assume that the particles behave just like the object material.

Displacement field analysis of this kind is also regularly used in Particle Image Velocimetry (PIV), with which DSR shares similarities. In PIV a transparent fluid has a random distribution of particles introduced into it, then using laser light, a single sheet of these particles are illuminated and imaged digitally. This allows the flow in the illuminated sheet to be measured.

In DSR, a similar single plane is imaged within the specimen, but this time the specimen may be optically opaque.

Using optical image data, researchers have performed analysis of shear band formation in sand due to compressive loads, and for granular flow analysis, similar to the one presented here.

The idea of combining radiographic images with DIC is not entirely new. In the late eighties, Russel and Sutton carried out an experiment where they measured the strain in fiber-reinforced composite laminate with use of this method. They however based their measurements on speckle patterns generated from x-ray opaque markers attached on the object surface. So from that point of view, the strain fields measured were not obtained from the internal structure.

The method has also been applied in the medical sciences when Bay measured two-dimensional strain fields in trabecular bone tissue. However it is
more often used for image registration to match features found in images collected with different imaging techniques - for example match CT-data with MRI-data. Another medical research field is to localise lesions within the body, mainly tumours by use of a techniques that shares similarities with image correlation.

DSR has also been used to study internal displacements of opaque specimens in ballistic events, using flash X-rays.

A drawback of using a contrast agent is that the technique no longer is a non-contact method in the sense that the object must be prepared before doing the actual measurements. The preparation with a seeding material also puts some restrictions on possible applications of the technique.

### 4.1.1 Experimental procedure

In order to determine the actual length scale in the measurements, a calibration routine is necessary. The calibration routine used in these measurements was performed by placing a very fine metal grid in the defined object plane and capturing a radiographic image of it. The period of the grid can be calculated using an automated fine grid analysis. The calibration routine takes the 2-dimensional Fourier transform of the grid pattern to separate the horizontal and vertical components of the grid. The phase can then be calculated and unwrapped to give a measure of the number of pixels per period of the grid to sub-pixel accuracy. The pitch of the grid is measured accurately with a microscope, allowing a conversion factor from pixels to millimetres to be calculated.

The silo used consists of a bin and hopper configuration, as shown in Figure 4.1. During the experiment, measurements were carried out for two different outlet diameters - 2.5 mm and 5.0 mm. However, here we’ll only look at the results from the latter one. See Paper A for results from the entire experiment. The material of interest is an Al₂O₃-powder (Alumina) of average grain size 50 μm. This material is seeded with a thin sheet of Tungsten particles, also with the average grain size 50 μm. We assume that the flow behaviour of the two materials is equivalent. The seeding is inserted in a plane through the centre of the bin, orthogonal to the optical axis.

The measurements are carried out in a region within the bin, marked out with a rectangle in Figure 4.1. The physical dimensions of this region are 11.25 by 8.79 mm. The field of view may however be chosen freely.

The x-ray tube voltage and current is set to 43 kV and 30 μA, respectively, which result in images of optimum contrast.
The measurements begin as the outlet is opened and the powder starts to flow. Sequential images are captured with a frame rate of 20 frames/s, which yield that the time period between two successive frames is 0.05 s. In total a number of 230 images are collected within a sequence, thus covering 11.5 seconds of flow.

4.1.2 Results and discussion

Figure 4.2(a)-(c) shows the measured velocity field at three different times during the initial stages of release. In Figure 4.2(a) the velocity field 0.05 s after release is presented, with an initial motion in the lower central part of the image clearly evident. Figure 4.2(b) shows the velocity field after 0.10 s. In this instance, the flow in the lower central region is so rapid that the speckles become blurred, causing a region of de-correlation to arise in which no measurements can be obtained. In Figure 4.2(c) the velocity field after 0.35 s is presented. The flow in the central region has now slowed, enabling the full field to be measured. A distinct column of flow is occurring by this time, suggesting the presence of plug flow behaviour. Further into the flow process the top region begins to collapse and a “cone of flow” arises.

Using the calibration method, discussed earlier, a scaling factor of 18 μm/pixel is obtained. From the measured displacements we then get the velocities since we know the time elapsed between two successive frames. We obtain a velocity-scaling where 1 mm of displacement corresponds to a velocity of 20 mm/s. The vectors in Figure 4.2(a)-(c) are however scaled a factor 6 in order to enhance the appearance. A vector of length 1 mm therefore corresponds to a velocity of 3.33 mm/s.
Figure 4.2. The velocity field at the time 0.05 s (a), 0.10 s (b) and 0.35 s (c) after release.
The results contain random errors that originate from the correlation procedure, as we know from Section 3.2. These are given by Equation (3.5). In this case we use subimages of the size 32×32 pixels (B=32). The average correlation value is estimated to 0.82 and the average speckle size is approximately 3 pixels, which results in $k=0.5$. With these values we get a random displacement error of 0.07 pixels, which corresponds to a physical displacement error of 1.2 μm.

With regard from what we know about systematic errors, also discussed in Section 3.2, we can conclude that with average speckle sizes of 3 pixels we manage to prevent such undersampling-related errors.
Now, we will extend our imaging procedure to cover all three dimensions. This is done using x-ray microtomography, or micro-computed tomography (μCT), which is a type of computed tomography (CT) method used for small scale imaging. By the use of x-ray tomography we will retrieve the object information along the depth-direction that was lost in ordinary radiography. The term “tomography” is derived from the Greek word “tomos”, meaning “section”. Tomography is, from that point of view, equivalent with “sectional imaging” or “slice imaging”.

We will here go through the basic principles behind computed tomography. A thorough presentation of x-ray tomography methods can be found in the standard book by Kak and Slaney. We will start by deriving a relationship for reconstruction of one single slice through the investigated object. This is carried out for the case where the x-ray beam is parallel, which is the beam geometry in synchrotron x-ray microtomography. Thereafter, we will derive a relationship for reconstruction of the whole 3D volume using cone beam tomography. The theoretical reconstruction methodology will then be followed by complementary practical descriptions of the microtomography systems used in the research. In the final part of this section we will go through image quality assurance methods and techniques for correction of image artefacts in microtomography.

### 5.1 Projections

The basic idea of computed tomography is to, from a number of measured projections, retain the full geometrical information about the object. The projections are reversely projected back to the object space, and

![Figure 5.1. The relation between two objects and their projections, seen from three different angles.](image)
the combined result is the searched two-dimensional object function (in a slice). Figure 5.1 shows the relation between two objects and their projections, seen from three different angles. The more projections that are collected and backprojected the more exact solution for the object function is obtained. In the case with parallel beams of x-rays passing through the object, as in Figure 5.1, only projections acquired along a 180° trajectory are required to reconstruct the object, since the projections from the opposite directions will contain equal information. However this will not be the case when the x-ray beam is divergent, since the magnification of the imaged features will depend on the rotation, due to variations of the source-to-object (or feature) distance (see Equation (2.4)). The exact solution is therefore attained when projections from the full circular trajectory is collected.

An expression for the intensity of the detected x-rays after propagation through an object containing \( n \) features was given in Equation (2.2). Now, if the object is confined to a slice in the \( xy \)-plane we need to rewrite this equation

\[
I_D = I_0 e^{-\int_{L} \mu(x,y) \, dl},
\]

where \( \mu(x,y) \) is the two dimensional linear attenuation and \( dl \) is a length increment, and the integration is carried out along the path \( L \) from the source to the detector. From Equation (5.1) we can write,

\[
\int_{L} \mu(x,y) \, dl = -\ln \frac{I_D}{I_0}.
\]

So the measured intensities at the detector can be seen – if they are written on the form \( -\ln(I_D/I_0) \) – as the line integration values of the attenuation coefficient. The line integral on the left hand side of Equation (5.2) is called the Radon transform of the function \( \mu(x,y) \)

\[
P(\theta, t) = \int_{L} \mu(x,y) \, dl.
\]

The parameters \( \theta \) and \( t \) are the projection angle and the radial position of the ray, as shown in Figure 5.2. These are the line integration parameters, coupled to the spatial coordinates \( x \) and \( y \) through the relationship

\[
x \cos \theta + y \sin \theta = t.
\]

The expressions above are however only valid under the assumption that all x-rays used for imaging are monochromatic, i.e. have the same wavelength. However all x-ray sources is more or less polychromatic and hence the x-ray beams have a certain bandwidth. Although the synchrotron x-rays have a far narrower bandwidth than those produced in x-ray tubes. The linear attenuation \( \mu(x,y) \) is in general a function of photon energy and ideally
Equation (5.3) should also include integration over the x-ray energy spectrum. The monochromatic approximation gives rise to an effect called \textit{beam hardening}, which will be discussed in further detail in Section 5.6.

The source and detector are rotated around the object (z-axis) and for each new value of projection angle $\theta$ a 1D projection is acquired. The full rotation results in projection data for all values of $\theta$ and $t$.

One important factor that decides the quality of the reconstructed image is the number of projections collected from a 360° rotation. If the sampling is too sparse the reconstruction will be of poor quality and contain aliasing artefacts due to the angular undersampling, which also is discussed in Section 5.6. A method to control the projection data before reconstruction is to plot the full amount of acquired 1D projections side-by-side in an angular sequence. The obtained distribution of projection data as a function of projection angle is called a sinogram, and shown in Figure 5.3. Here, 364 projections have been captured equiangular during one rotation, so that the angular step between two subsequent projections approximately is 1°. If too few projections are collected there will be discrete steps between the separate subsequent projections, and the smooth appearance will be lost. By studying the sinogram one can also detect if the studied object have moved during scanning, which induce a discontinuous movement of the features, for a limited number of angles. This unwanted motion results in unsharpness in the reconstructed slice.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure5.2}
\caption{A beam of x-rays propagating through a cross-section of the object.}
\end{figure}
Figure 5.3. In a sinogram the projections are plotted as a function of the projection angle, $\theta$.

5.2 Parallel-beam imaging

5.2.1 The Fourier slice theorem

Next we will formulate the expression for the reconstructed object, based on the acquired projection data. The mathematical expression for the projections was given by Equation (5.3), where the linear attenuation $\mu(x,y)$ was representing the object. For the reconstruction, we will, however, represent the object through the more general notation, and use $f(x,y)$ instead of $\mu(x,y)$, since these algorithms can be generalised and used for reconstruction of other quantities. We will therefore slightly rewrite Equation (5.3)

$$P(\theta,t) = \int \! f(x,y) \, dl.$$  

The Fourier slice theorem, which is visualised in Figure 5.4, states that the one-dimensional Fourier transform of a parallel projection $P(\theta,t)$ is equivalent with a line $k$ through the origin at angle $\theta$ in the two-dimensional Fourier transform of the object $f(x,y)$. Mathematically this can be written as

$$F_1[p](\theta,k) = \int_{-\infty}^{\infty} P(\theta,t) e^{-2\pi i kt} \, dt,$$

$$= F_2[f](k_x, k_y).$$

Therefore, by capturing projection data at all angles during a 180° rotation of the object (or imaging system) we get information at all points in the two
Figure 5.4 The Fourier slice theorem connects the one dimensional Fourier transform of a parallel projection with the two dimensional Fourier transform of the object along a radial line.

dimensional spectrum of the object. Using Equation (5.6) we can finally put up the following expression for the two dimensional object function \( f(x,y) \)

\[
f(x,y) = \mathcal{F}_2^{-1} \{ \mathcal{F}_2 \{ f \} \}(x,y).
\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{F}_2 \{ f \}(k_x, k_y) e^{i2\pi(ak_x + yk_y)} dk_x dk_y.
\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{F}_2 \{ P(\theta, k) \} e^{i2\pi(ak_x + yk_y)} dk_x dk_y.
\]

The results from using the Fourier slice theorem is a neat conceptual model for retrieving the object function in the imaged plane. However, when it comes to solving this problem practically there is another method that is more suitable for discrete data handling and implementation – this method is called Filtered backprojection (FBP).

5.2.2 Filtered backprojection

The FBP-algorithm can be derived from Equation (5.7) above, by the use of a polar parameterisation, where the integration variables are changed from \( dk_x dk_y \) to \( |k| dk \theta \). By writing out all Fourier transforms and rearranging the results we get
The expression inside parentheses at the last line in Equation (5.8) is recognised as a ramp filter, i.e. a high-pass filter that in the Fourier domain has the shape of a ramp. By denoting the filter \( g(t) \) we can write

\[
\frac{1}{2} \int_{-\infty}^{\infty} |k| e^{i2\pi kt} dk \quad (5.9)
\]

By substituting Equation (5.9) in Equation (5.8) we finally get

\[
f(x, y) = \int \int_{0-\infty} P(\theta, t)g(x \cos \theta + y \sin \theta - t) dtd\theta,
\]

\[
= \int \int_{0-\infty} (P \ast g)(\theta, x \cos \theta + y \sin \theta) d\theta,
\]

(5.10)

where \( \ast \) denotes convolution. This is the fundamental formulation of the Filtered back projection algorithm, for parallel rays, and the name can fairly easily be understood from this expression. First the projection \( P \) is filtered through the convolution with \( g \). Then the resulting filtered projections are backprojected (a reversed projection process) or smeared out along the scanning direction, at angle \( \theta \). This is carried out for all projection angles and results in the full reconstructed slice \( f(x, y) \). If this is done without first filtering the projection data the reconstructed image will get very blurry. There is a lot of literature covering this subject, and additional reading can be found in ref.[3, 42, 43].

The parallel beam reconstruction is used when performing synchrotron x-ray microtomography, which is further discussed in Section 5.5.1.

5.2.3 Creating 3D data

A slice has a spatial extension along the \( z \)-axis, which together with the reconstructed image in the \( xy \)-plane constitutes a three dimensional volume, although the third dimension is much smaller than the other two. The reconstructed slice is therefore divided in small volume elements, voxels, which is the three-dimensional version of pixels.
When the reconstruction is completed in the first slice a new slice is scanned and reconstructed. This is done by either moving the object or the source-detector system a small amount along the rotational axis. The whole volume of the object can in this manner be reconstructed - slice-by-slice – and represented as a stack of individual slices. Figure 5.5(a) shows a stack of such slices describing a granular material. Here, 6 slices are shown, evenly distributed in the full imaged region, which totally consist 51 slices. Figure 5.5(b) shows a 3D rendering of the full data set (based on all 51 slices). A high sampling rate along the z-direction yields an accurate 3D representation of the examined specimen. In microtomography the reconstructed voxels are typically isotropic, and the spatial resolution in the z-direction is equal to the one in the x- and y-direction (within a slice).

5.4 Cone-beam imaging

The cone-beam geometry is the natural shape of the beam as it leaves an x-ray tube. A cone-beam tomography algorithm is advantageous to use since it allows reconstruction of the 3D space directly from 2D projection data, instead of reconstructing the volume slice by slice. In other words, we no longer collimate the beam and can therefore make full use of the emitted energy.

In order to reconstruct three-dimensional volume data from two-dimensional projections we use a two-dimensional filtered backprojection algorithm. There exist several different techniques for cone-beam reconstruction. The one that is used here is based on the pioneer work by Feldkamp, Davis and Kress. It is most often referred to as the Feldkamp- or FDK-algorithm. Detailed information about this algorithm and other cone-beam techniques can be found in the work by Turbell.
5.4.1 The Feldkamp-algorithm

In analogy with Equation (5.5) we can now write down an expression for the two dimensional projection through the three dimensional object \( f(x, y, z) \)

\[
P(\theta, a, b) = \frac{1}{L} \int_{L} f(x, y, z) dl ,
\]

(5.11)

where \( a \) and \( b \) are the horizontal and vertical detector coordinates, respectively. As before \( L \) is the geometrical length that a ray propagates through the object on its way to the detector and \( \theta \) is the projection angle. The detector coordinates \( a \) and \( b \) are related to \( x, y, z \) and \( \theta \) through

\[
a(x, y, \theta) = R \frac{-x \sin \theta + \cos \theta}{R + x \cos \theta + y \sin \theta} ,
\]

\[
b(x, y, z, \theta) = z \frac{R}{R + x \cos \theta + y \sin \theta} ,
\]

(5.12)

where \( R \) is the radius of the source trajectory. The cone-beam geometry is shown in Figure 5.6. Note that the source-detector system here rotates around the object or world coordinate system. However in the actual measurements the source-detector system remains fixed while the object is the one being rotated. The relative angular movement is however the same which is the only significant parameter in the reconstruction. Also, notice that the detector coordinate system here has been shifted so that the \( b \)-axis coincides with the \( z \)-direction. This operation is purely mathematical and the virtual detector is

\[\text{Figure 5.6. The cone-beam geometry}\]
related to the physical detector through a simple scaling factor.

The reconstruction procedure is fundamentally the same as when using parallel x-rays. However, the increased geometrical complexity yields that a number of additional weighting factors are required in the reconstruction algorithm. The filtering process is given by

$$\tilde{P}(\theta, a, b) = \left( \frac{R}{\sqrt{R^2 + a^2 + b^2}} \right) P(\theta, a, b) \ast g(a),$$  \hspace{1cm} (5.13)

where $\ast$ denotes convolution and $g(a)$ is the ramp filter, from Equation (5.9).

The factor in front of the projection is called a pre-weight factor and can be related to the beam geometry through the relationship

$$\frac{R}{\sqrt{R^2 + a^2 + b^2}} = \cos \phi \cos \psi,$$  \hspace{1cm} (5.14)

where $\phi$ and $\psi$ are the fan angle and cone angle, respectively, shown in Figure 5.6. The pre-weighted and filtered projections $\tilde{P}(\theta, a, b)$ given by Equation (5.13) are finally backprojected into the three-dimensional object space $f(x, y, z)$

$$f(x, y, z) = \int_0^{2\pi} \frac{R^2}{(R + x \cos \theta + y \sin \theta)^2} \tilde{P}(\theta, a(x, y, \theta), b(x, y, z, \theta)) d\theta.$$  \hspace{1cm} (5.15)

### 5.5 The used x-ray microtomography systems

X-ray microtomography, or micro-computed tomography ($\mu$CT), is fundamentally based on the same principles as conventional computed tomography. However, in general, there are some differences that are worth mentioning. Microtomography is, as the name reveals, used for small scale imaging. There are large differences between the different systems available and it is therefore hard to give values for characteristic imaging parameters such as FOV and spatial resolution. Stock, however, makes an attempt to list all the commercial systems presently available (2008), with information about FOV and voxel size. The specimen diameter is generally in the range 0–150 mm (most often in the lower regimes). The lab based systems used for investigations of specimens with the diameter ~10 mm can be expected to produce a highest theoretical resolution between 1 and 10 $\mu$m. The synchrotron based $\mu$CT systems generally produce higher spatial resolution and better image quality than the lab based systems.

The method is rapidly developing and has become a popular tool for studies on biologic-, geologic- and engineering materials. Generally, the projections that are captured in microtomography have a high spatial resolution, due to the use of high-resolution detectors in combination with x-ray sources of small size, with improved spatial coherence, compared to those
used in conventional computed tomography systems. This in combination with a dense angular sampling results in the high spatial resolution of the reconstructed volumes. Furthermore, the projection data is generally two dimensional, of roughly the same size in both dimensions. This yields that all projection data required for reconstruction of the 3D geometry of a specimen is acquired during one single rotation. Also, instead of rotating the source-detector system when collecting our projections, as normally is the case in conventional computed tomography, we now rotate the specimen. The effect of these two procedures is the same since the relative motion between the specimen and source-detector system is unchanged.

In this research, experiments have been carried out using two different x-ray μCT imaging systems – one based on synchrotron x-rays and one cone-beam tomography system. The synchrotron x-ray imaging is carried out using a parallel beam of x-rays of very high coherence, compared to x-rays from an x-ray tube, and allows high resolution imaging with very good image quality. The experiments are performed at a synchrotron facility (See Figure 5.7) where there is a limited number of imaging systems available. Prior to any experiments one needs to make an application for beamtime, containing a description of the research you (and your research group) want to carry out. Also, assuming your application for beamtime is accepted, all the experiments need a careful planning since the time at the beamline normally is very limited. Lab based cone-beam tomography systems on the other hand generally don’t come up to the same standard when it comes to spatial resolution and image quality, but has no restrictions in availability and can be used more spontaneous.

5.5.1 The synchrotron microtomography system

At a synchrotron facility, as shown schematically in Figure 5.7, electrons are accelerated in a synchrotron (the inner circle in Figure 5.7) and then injected into a storage ring (the outer circle in Figure 5.7). Here, synchrotron radiation is produced as the electrons are forced along the circular path by use of very strong magnets. When the electron path is bent the particle looses energy through emission of x-rays that are deflected into the beamlines, which extend tangentially from the storage ring. In the beamline the x-ray beam goes through a system of x-ray optical devices, such as for example slits, attenuators and crystal monochromators, which control the bandwidth, beam dimensions, photon flux and beam collimation. What is included in this system of x-ray optical devises depends on the actual application. Finally, the x-rays enter the end station where the experiments are carried out, which in Figure 5.7 is the centre block at each beamline. Finally, at the other side of a large concrete wall, safe from the radiation, the personal can observe and control the experiment.
Figure 5.7. Schematic of a synchrotron facility. The electrons are accelerated in the synchrotron ring (the inner ring) and injected to the storage ring (the outer ring), where they are forced into the circular path by use of strong magnets. The interaction with the magnetic field causes the electrons to loose energy, which is emitted as x-rays that are guided into the tangential arms, called beamlines. Courtesy of EPSIM 3D/JF Santarelli, Synchrotron Soleil, through Wikimedia Commons.

Figure 5.8. The synchrotron microtomography system. The synchrotron x-rays enter through the pipe that is visible at left hand side in (a). The specimen is positioned in the beam path right in front of the detector (b). The specimen manipulator (c) allows the specimen to be translated and rotated with high accuracy.

These measurements have been carried out at the TOMCAT beamline at the Swiss Light Source (SLS), located at the Paul Scherrer Institute (PSI) in Villigen, Switzerland. Figure 5.8(a)–(c) shows the different components of the microtomography system. The x-rays enter the end station through a pipe,
which is visible at the left hand side in Figure 5.8(a). The synchrotron x-ray beam has the photon energy range 8-45 keV and the bandwidth is approximately 2-3 %. The specimen is placed in front of the x-ray detection unit at the end of the pipe, as shown in Figure 5.8(b). The positioning and rotation of the specimen is carried out by a specimen manipulator, shown in Figure 5.8(c). This unit consists of a three axis translation stage for positioning, a two axis translation stage for centering and, finally, a rotation stage. The positioning and centering stages have a reproducibility of less than 1.0 and 0.1 μm, respectively, and the rotation stage has a “runout” less than 1 μm at 100 mm. The detection chain consists of a 20 μm YAG-Ce-scintillator, an optical microscope and a low-noise fast-readout CCD of 2048 x 2048 pixels and 16 bit dynamic range. The magnification of the microscope ranges from 1.25x to 20x, depending on the objective used. The field of view (FOV) can be varied from 0.72 x 0.72 mm² (20x magnification) up to 11.4 x 11.4 mm² (1.25x magnification), with isotropic voxel sizes ranging from 0.35 μm³ up to 5.6 μm³. The optimum spatial resolution at 10% MTF is 1.04 μm, obtained with the 10x-objective⁴⁹. The acquisition time for a full scan depends on the exposure time (and thus the beam energy) and the number of projections acquired. In our experiments, the acquisition times ranged from approximately 5 to 12 minutes. The reconstruction is carried out on a Linux cluster with a parallel filtered backprojection algorithm, similar to the one presented in Section 5.2.2, and takes approximately 10 minutes. The projection data is here two-dimensional (and not one dimensional as in Section 5.2.2) but since parallel beams are used each row in the projection image is related to its own slice through the scanned object. Hence, the object can be reconstructed slice-by-slice from the corresponding rows in the projections. The use of 2D projections is beneficial and more time effective since the full amount of projection data is acquired during one single 180°-rotation of the specimen (instead of doing 2048 individual scans along the z-axis, using a 1D detector array with 2048 pixels).

5.5.2 The cone-beam microtomography system

The cone-beam microtomography system has been built based on the components of the digital radiography system that was described in Section 2.2. However, between the microfocus x-ray source and the x-ray detector unit, we have now placed a motorised rotation stage. This stage, from Linos Photonics (RT 120 ST), is controlled either directly from the rotation stage controller or from the host computer through use of LabView. The rotation stage has a microstep resolution of 0.002 degrees and an absolute positioning accuracy of 0.05 degrees. The system is schematically shown in Figure 5.9.

The system has been mounted on an optical table, which allows a flexible but rigid setup.
The field of view is the same as for the digital radiography system and given by Equation (2.5) where \( \text{FOV}_{\text{det}} \) is the detector field of view, which is \( 72 \times 54 \, \text{mm} \).

During a scan typically 500–600 projections are captured, equiangular distributed over 360 degrees. The number of projections depends on the dimensions of the investigated specimen in the projection data. If too few projections are acquired the reconstructed images will suffer from aliasing artefacts due to angular undersampling, which will be further discussed in the following section. The exposure time for each acquired projection is 2.56 s.

The reconstruction is carried out with a Feldkamp cone-beam reconstruction algorithm\(^{44, 45}\), on a standard PC with dual CPUs (Pentium4 XEON 2.2 GHz processors) and 2 GB of RAM. It takes approximately 60–90 minutes to reconstruct a volume with the dimensions \( 640 \times 640 \times 500 \, \text{voxels} \), which is the maximum size, depending on the number of projections used.

The spatial resolution in the reconstructed data varies with the magnification, which is given by Equation (2.4). However there are also other parameters that will influence the resolution, which will be further discussed in the following section.

![Figure 5.9. The cone-beam microtomography system.](image-url)
5.6 Image quality assurance and artefact correction

X-ray microtomography is rapidly developing and has become a popular tool for studies of various materials. However there are a lot of image quality aspects that need to be taken into account. Most of these have been inherited from conventional CT (clinical CT) but some of them are new. These are often related to the cone-beam geometry, which is the dominating geometry in microtomography, and to the new type of materials that are investigated. For example, the materials investigated with microtomography often contain sharp edges. This kind of structures generally requires more projection images than normally are used in clinical applications, where sharp edges less frequently appear. Otherwise undersampling artefacts will degrade the reconstruction, as will be discussed further below. Further information on methods for image quality assurance in computed tomography, of which many also can be adopted to microtomography, is given in ref. [50].

The calibration procedure of an x-ray tomography system is not trivial, and depends on what the system is used for. Several different system evaluations have been presented, both for cone-beam tomography systems\textsuperscript{51-56} as well as synchrotron x-ray tomography systems\textsuperscript{57, 58}. Here different image quality parameters are investigated and various image artefacts identified and corrected. In conventional CT it is common to use various calibration phantoms for evaluation of the image quality. These are specially designed specimens where material density and structure are pre-defined and customised to evaluate certain image quality parameters. However the phantoms used for calibration of CT are usually too large for use in a microtomography system. Recently, however, Du et al. presented a quality assurance phantom specialised for performance evaluation of microtomography systems\textsuperscript{59}.

In this section, the most common image quality parameters in microtomography are presented and exemplified together with information about the performance of the cone-beam tomography system used in the research.

5.6.1 Spatial resolution and influence of noise

The x-ray projection data will contain a certain amount of random noise that in the end will degrade the reconstructed data. There are two main sources contributing to noisy projection data. The first is the detector noise due to stochastic signal errors that occur somewhere in the detection chain or during transmission of signal. The other source to random noise is scattered x-ray photons, i.e. x-rays that don’t travel through the investigated specimen along a straight line. These will impinge the detector and give rise to a signal that falsely will be related to the integrated linear attenuation along the straight path from the x-ray source to the detection point. The noise distribution in
the projection data will be transferred to the reconstructed image data. Moreover, the noise term will be amplified due to the use of a high-pass filter (the ramp filter) in the reconstruction process. Therefore it is important to reduce the noise term prior to reconstruction. One way to reduce the number of scattered x-rays on the detector is to use collimators. A collimator is an aperture made of a dense material that will restrict the beam so that only the field of view is irradiated. The collimator absorbs the x-rays that don’t contribute to the image formation since these otherwise might be scattered towards the detector, and result in additional noise. The noise term in the acquired projections can also be reduced through temporal and spatial averaging. The temporal average is an average over a number, $N_T$, of subsequent projection frames. The total exposure time will hence increase a factor $N_T$. The result is an increased signal-to-noise ratio where the static intensity distribution, describing the object, will be enhanced while the fluctuating noise distribution will be suppressed. This method is efficient as long as the imaged structure is static in time. If the structure moves during the acquisition, this will result in a certain motion blurring in the averaged projection data, which will give rise to motion artefacts in the reconstructed data. Spatial averaging on the other hand is carried out on individual frames. Here the intensity in each pixel is taken as an average over the intensities in the neighbouring pixels. This method is less sensitive to structural movements during scanning but the spatial averaging implies that image information is smeared out, which results in a reduced spatial resolution.

In the cone-beam microtomography system used in this research several of the methods above have been implemented to improve the noise characteristics. First, the signal-to-noise ratio of the detector is 61 dB. Second, a thick lead plate with a rectangular aperture in the centre is used as a beam collimator. Finally, all projections are taken as an average over a certain amount of individual frames. Each captured with a frame rate of 25 frames/s, and an exposure time of 0.04 s. Typically, the average is taken over 64 frames which result in a total exposure time of 2.56 s.

Figure 5.10(a)-(c) shows the phantom object for the evaluation of the spatial resolution in the xy-plane. It is a 2.4 mm thick cylindrical PVC plate with the diameter 10 mm. It consists of six groups with holes of varying sizes. In each group there are five holes and the gap between two neighbouring holes is equal to the diameter of the hole. Each group of holes corresponds to a certain spatial frequency, measured in line pairs per unit length (lp/mm). The diameter of the holes range from 0.11 mm to 0.50 mm which corresponds to spatial frequencies in the range from 1.00 lp/mm to 4.50 lp/mm, as described in Figure 5.10(b). As can be seen in the 3D drawing of the phantom in Figure 5.10(a), the holes propagate all the way through the 2.4 mm thick plate. Figure 5.10(c) shows the surface of the phantom, acquired with optical microscopy.
Figure 5.10. The spatial resolution calibration phantom. (a) shows a 3D drawing of the phantom. (b) shows a cross-section through the phantom drawing in the xy-plane together with information about the hole diameters and the corresponding spatial frequencies. (c) shows a microscopic image of the physical phantom. It is visible that the smallest holes have been merged together.

The manufacturing of the spatial resolution phantom have been achieved by selective evaporation of the polymer material by a diode pumped Nd:YAG laser, emitting 532 nm visible green light. However due to the high temperatures during the manufacturing process the boundary region of the holes was affected more than planned, which resulted in slightly larger diameters. This phenomenon is clearly visible in Figure 5.10(c) where the smallest holes, in group $P$, $Q$ and $R$, are merged together.

Figure 5.11(a)-(b) shows the influence of noise in the reconstructed data and the improvement when the exposure time is extended. Figure 5.11(a) shows a reconstructed slice taken through the centre of the spatial resolution phantom, described in Figure 5.10(a)-(c), based on projection data acquired
Figure 5.11. Reconstructions of the spatial resolution calibration phantom based on projections acquired with the exposure time 0.04 s (a) and 2.56 s (b). The large noise content in (a) results in a poor contrast and it is hard to resolve the smaller features. In (b) the contrast is improved and the smaller holes are sharper.

with 0.04 s exposure time, i.e. using single frame projections. We can clearly see that there are irregular noise fluctuations present in the reconstructed image. The contrast between the holes and the PVC surface is poor and it is hard to resolve the smaller structures. By visual inspection, it is hard to separate the holes in group \( O \) from each other while this is possible for the holes in group \( N \). From this we conclude that the resolution, measured in spatial frequency, lies between 2.0 lp/mm and 3.0 lp/mm. Figure 5.11(b) shows the corresponding reconstructed slice when using the exposure time 2.56 s, which is the standard setting of the system. Here we can clearly resolve the holes in group \( O \), which correspond to the spatial frequency 3.0 lp/mm. Furthermore, the smallest features also appear to be resolved although there are no real holes in group \( R \) due to the manufacturing errors that occurred, as previously mentioned. This calibration experiment was carried out with 4.4x-magnification and with the tube voltage and tube current 25 kV and 100 \( \mu \)A, respectively.

In order to get a quantitative measure of the spatial resolution by using this kind of calibration phantom one need to plot the intensity profiles along a line through the centre of the holes in each of the groups. From these intensity profiles one can get contrast values, using Equation (2.3), for the corresponding spatial frequencies. These values will give the modulated transfer function (MTF) which is a curve describing the contrast, from 0 to 1 (or 0-100%), as function of spatial frequency. The typical measure for spatial resolution is obtained at 10% MTF, which corresponds to the point at the MTF-curve where the contrast has the value 0.1 (10%).
The use of periodic structures for measurements of spatial resolution is very illustrative but not very effective for quantitative measurements. The accuracy of the MTF-curve will depend on the number of groups on the calibration phantom. Moreover, the smallest structures are difficult to produce.

The spatial resolution at 10% MTF is a measure of the low contrast resolution and gives the lower bound of the spatial resolution. The upper bound, assuming the imaging conditions and the contrast are ideal, is given by the theoretical voxel size, which is obtained by dividing the three dimensional field of view by the number of voxels along the corresponding directions. The field of view will depend on the magnification, as described for the two dimensional case in Equation (2.5), and hence the spatial resolution also will depend on the degree of magnification.

For example a theoretical isotropic voxel size of 45 μm is obtained if a 2x-magnification is used. This is obtained by placing the axis of rotation right between the x-ray source and the detector, so that and object-to-detector distance equals the object-to-detector distance.

5.6.2 Image artefacts

Generally there will be several different image artefacts present in the reconstructed data, which can be described as distortions and misrepresentations that degrade the imaged structure. These artefacts need to be identified and corrected for in order to get reconstructed images of high quality and without distortions. The recognition of image artefacts is not always easy but there are some good literatures that can be consulted. For example, Barrett and Keat give a survey of various artefacts in CT and Davis and Elliot thoroughly describes image artefacts in microtomography. Figure 5.12(a)-(e) show a number of common image artefacts. The imaged structure describes a cross-section through a hollow Perspex cylinder filled with granular sugar. The image in Figure 5.12(a) represents the ideal or reference structure, obtained after system calibration and artefact correction. The artefacts in Figure 5.12 are ring artefacts (b), double contours (c), distortions (d) and undersampling artefacts (e). Figure 5.13(a)-(f) shows a sequential improvement of image quality due to correction of the artefacts from Figure 5.12(b)-(e). In Figure 5.13(a) the uncorrected reconstruction of the slice is shown. (b) shows the structure after correction for ring artefacts. (c) shows the structure from (b) after correction for shifted axis of rotation (alignment of the scanning geometry). (d) shows the structure from (c) after correction for skew of the axis of rotation (alignment of the scanning geometry). (e) shows the structure from (d) after correction for undersampling artefacts. In (f) finally the corrected structure, from (e), is compared with the uncorrected structure, from (a). This good vs. bad reconstruction clearly shows the benefits of a thorough identification and correction procedure for image artefacts.
Ring artefacts

A detector array must be calibrated in order to give reliable projection data during the imaging procedure. In each acquired projection, one specific detector cell records an intensity value that, through the tomographic reconstruction, can be coupled to a specific position along the ray path. A detector cell that has a sensitive profile that deviates from the general sensitivity profile of the array will induce a read out error for each acquired projection. In a sinogram where the projection data along a detector row is described as function of projection angle, $\theta$, (see Section 5.1) such cells with deviating sensitivity profiles will be visible as straight lines that are constant for all values of $\theta$. When reconstructed this error yields a circular trace of constant values around the centre of the reconstructed slice, a so called ring artefact – as shown in Figure 5.12(b).

Calibration of the recorded projection images can be carried out through a flat and dark field correction (FDFC)

\[ I_c(\theta) = \frac{I(\theta) - I_d}{I_f - I_d}. \]  

(5.16)

Here $I(\theta)$ and $I_c(\theta)$ is the recorded and corrected projection at the angle $\theta$, respectively. $I_d$ is the dark field recorded with the x-ray shut off, and $I_f$ is the flat field recorded with the x-ray on, but without sample. The dark and flat field describes the dark current (minimum intensity value) and the full transmitted intensity (maximum intensity value) at each detector cell, respectively. Apart from correcting anomalous sensitivity profiles in the detection chain, the FDFC-procedure also compensates for inhomogeneous x-ray distributions. Further, to reduce random errors in the dark and flat field recordings these should both be taken as time averages over an extended exposure time. In the correction procedure carried out here the flat and dark field both were captured as time averages of 320 individual frames, each with an exposure time of 2.56 s. The x-ray flux on the detector cells depends on case specific parameters such as the FOV, the tube voltage and the tube current. Therefore, it is important to perform a new FDFC-procedure in connection with every experiment carried out with a new set of imaging parameters. Generally, however, the flat and dark field correction only reduces the ring artefacts up to a certain degree – it doesn’t eliminate the problem entirely. Several different approaches for further ring artefact reduction have been suggested through the years. These ranges from hardware based solutions with the sample or detector unit being moved during acquisition to software based methods of various kind. A filtration procedure is applied either prior to reconstruction, during reconstruction or on the reconstructed data.

For the cone-beam microtomography system used here a method for sinogram filtration, proposed by Raven, has been implemented.
Figure 5.12. Image artefacts in a reconstructed slice. Reference (a). Ring artefacts (b). Double edges due to shift of axis of rotation (c). Distortions due to skew of the axis of rotation (d). Undersampling artefacts (e).
Figure 5.13. Sequential improvement of image quality due to artefact correction. Uncorrected (a). Corrected for ring artefacts (b). Corrected for ring- and AOR shift artefacts (c). Corrected for ring-, AOR shift- and AOR skew artefacts (d). Corrected for ring-, AOR shift-, AOR skew- and undersampling artefacts (e). Good vs. bad (f)
The sinogram data is processed by the use of a one-dimensional Butterworth filter in the spectral domain. The result from this operation is visible in the sinogram data from granulated sugar, in Figure 5.14. In Figure 5.14(a) the original sinogram data is shown, in which horizontal streaks can be found. The streaks are due to defective or miscalibrated detector elements that leave traces independent of projected angle. After performing the filtration these streaks are heavily reduced, as shown in Figure 5.14(b). Reconstruction based on the filtered sinogram data results in an almost complete removal of the ring artefacts, as shown in Figure 5.13(b).

**Artefacts due to geometrical misalignment**

Misaligned scanner geometry is a common source to severe image artefacts. Typical artefacts arising from geometrical misalignments are image distortions and double contours.

The ideal scanning geometry is showed in figure 5.15. The line between the source focus spot and the centre of the detector array is called the central ray. The axis of rotation (AOR) is positioned along this line, parallel with the centre column of the detector. Further, the plane spanned by the central ray and the centre detector row is referred to as the midplane.

An easy method to get a rapid and quite good alignment of the AOR is to use a narrow metal rod, and monitor the projection images as the rod is rotated. Assuming that the centre of the rod lies exactly at the AOR one can determine whether there are any misalignments of the AOR present. If the rod is positioned correctly at the AOR then the projected image of it will be centred at the detector and independent of direction. Otherwise, the sampled projections will move along an elliptic path. Another helpful tool when
aligning the microtomography system is the laser cross hair, i.e. two orthogonal laser discs that illuminates the system from above with the centre positioned at the AOR.

A more thorough alignment of the microtomography system, however, requires high precision mechanics and stable mounting. In fact it is almost impossible to entirely eliminate deviations from this ideal geometry, i.e. misalignments. Artefacts due to misaligned imaging setups are therefore common. However, the nature and magnitude of the artefacts depend on the type of misalignment.

The AOR can be shifted transversally and longitudinally. It can also be tilted towards the detector plane or skewed in the plane parallel with it. Furthermore, there are six different possible misalignment states for the detector unit – a translation and a rotation in all three directions. Generally these misalignments result in artefacts that are similar to the ones from AOR misalignments. Source misalignments, finally, consists of three different shifts – one in each direction. A vertical shift of the source will give rise to a vertical shift of the midplane. As a result the sharpest reconstructed slice, which is the one lying in the midplane, will move up or down relative the centre row of the detector. The fact that all slices apart from the one at the midplane have a certain portion of smear comes from the, so called, cone beam effect, which will be briefly discussed later. A horizontal transversal shift of the source can be interpreted as a transversal shift of the AOR together with a slight rotation (slant) of the detector. It can however be treated in a similar way as the AOR transversal shift misalignment.

In Figure 5.15 two different misalignments of the AOR is presented – the transversal shift of AOR (red arrow) and the skew of AOR (blue arrow).

Figure 5.15. Geometrical misalignment during scanning.
The transversal shift of the AOR will imply that the position of the rotation axis when projected at the detector is shifted from the centre column. This results in double contour artefacts, as shown in Figure 5.12(c). These double contour artefacts are the result of a 2 pixel shift of the AOR in the projection data. However, these can efficiently be eliminated by virtual correction of the AOR shift during backprojection. Figure 5.13(c) shows the image quality improvement for the same structure after correction for AOR shift.

AOR skew is a rotation of the AOR in the plane parallel with the detector plane, as shown in Figure 5.15. This misalignment results in structural distortions of the reconstructed slice, as shown in Figure 5.12(d). The skew of the AOR in the projection data result in use of incorrect weighting factors during the backprojection. Moreover the skew will also lead to a cross-talk between the reconstructed slices\(^66\). Figure 5.13(d) shows the granular sugar structure after elimination of the AOR skew. There is intense ongoing research in this field. Typically, the various misalignment are determined from different geometrical models analysed by point object trajectories in the projection data\(^67-71\). The analysis of the point trajectories results in a transformation matrix that can be used for virtual correction of the various geometrical misalignments.

**Aliasing artefacts due to angular undersampling**

If an object is reconstructed based on too few projections the result will contain undersampling artefacts. These will be visible as streak patterns that are extending from sharp edges, in the radial direction towards the boundary. Hence, the importance of undersampling grows if the investigated structures contain a lot of sharp edges.

Figure 5.16 shows the frequency domain of a reconstructed slice of the object. Each radial line \(k_r\) corresponds to the one-dimensional Fourier transform of the projection (in 1D) at angle \(\theta_r\), as described in Figure 5.4. Furthermore, each sampling point in the frequency domain corresponds to a sample point in the spatial domain. From Figure 5.16 it is easy to see that the resolution in the frequency domain will depend on the sampling frequency at the detector, given by the number of detector pixels, as well as on the number of projections acquired. Furthermore one can see that the azimuthal resolution, i.e. the resolution in the \(\theta\)-direction, decrease with distance from the origin. This explains why angular undersampling has a great impact on structures that contain a lot of edges, which are regions described by high spatial frequencies.

An estimate for the minimum number of projections, \(N_p\), that should be used in order to avoid such artefacts is given in the standard book by Kak and Slaney\(^42\)

\[
N_p = \frac{\pi}{2} D, \quad (5.17)
\]
where $D$ is the maximum dimension of the investigated specimen along the projection rows, measured in pixels, which in Figure 5.16 correspond to the number of sampling points along a radial line.

Equation (5.17) gives the approximate number of projections required in order to have a worst case sampling frequency in the angular direction (in the peripheral parts of the frequency spectrum of Figure 5.16 equal to the sampling frequency in the radial direction.

Figure 5.12(e) shows the impact of undersampling for the case with reconstructed cylindrical sample with granular sugar structure. The slice has here been reconstructed based on 109 projections and one can see a radial streak pattern with an isotropic extension in the peripheral parts of the cylinder. These are the result of having a homogeneous distribution of sugar crystal which induces an isotropic distribution of many small sharp edges. In this case the specimen is described with 340 pixels at the detector ($D=340$), which according to Equation (5.17) yields that a number of 534 projections are required to avoid undersampling artefacts. In Figure 5.13(e) the undersampling artefacts from Figure 5.13(d) has been corrected using a number of 545 projections for the reconstruction. We can see that the radial streaks now have vanished.

**Beam hardening**

Another type of reconstruction error that are common in x-ray CT is *Beam hardening*. Beam hardening arise from the fact that the energy spectrum of the polychromatic x-ray beam from an x-ray tube is shifted towards higher energies as it propagates through an object. This is because x-rays of lower energy are more likely to be attenuated than the more energetic (harder) ones. If the energy spectrum, however, is very narrow and the x-ray beam close to
monochromatic, which is the case with synchrotron radiation, the beam hardening effect will be negligible.

There are two kinds of beam hardening artefacts. The first, and most common, is the so called cupping artefact, where reconstructed structures in the centre of the object appear to be less dense than they really are. The attenuation profile along a line in the centre of the slice through a homogeneous object is cup-shaped, instead of constant. The second kind of beam hardening effect is called streak artefact and imply streak formation between two, or more, highly attenuating features. Theoretically, the reconstruction formalism is only exact under the condition of monochromatic photons. Therefore some sort of correction procedure is necessary during the reconstruction, to account for the polychromatic effects. There exist various correction methods to reduce the beam hardening phenomena. Most common is the use of hard ware filters in order to reduce the number of low-energy (soft) x-rays, which results in a narrower x-ray energy spectrum. It is also common to use a linearization procedure where the non-linear dependence between object thickness and registered attenuation is mapped into a linear relationship, which simulates the monochromatic case. Here wedge shaped calibration phantoms are used in order to determine the nonlinear relationship between object thickness and attenuation. An alternative is to perform a series of measurements on a phantom with a variable thickness (obtained by use of different number of thin plates). Ideally a calibration phantom is used for each material that is to be examined by the system. However, depending on application, some deviations in material density can be allowed and still give sufficient results. When the object contains more than one material the calibration procedure gets slightly more complex and requires additional information. Furthermore, there are approaches where the energy dependence of the linear attenuation is modelled. These methods either require information about the energy spectrum of the x-ray source or use approximate models for the energy spectrum.

Here, the beam hardening correction is carried out mainly through pre-filtering by use of a 0.4 mm Al plate. Also, some initial investigations of the applicability of using the energy-based model approach proposed by Van de Casteele et al. have been made. However, there is a need of further investigations in this field.

Cone beam artefacts

The most popular method for cone-beam reconstruction is the Feldkamp backprojection algorithm, described in Section 5.4.1. This technique is based on the methods used in conventional CT and the cone-beam is approximated by a set of tilted fan-beams. A fan-beam is a divergent x-ray beam collimated into one plane that irradiates a slice through the specimen. When the tilted
fan-beam approximation is used for the cone-beam geometry a certain
degrading of image quality will occur along the direction of the rotation axis.
The image quality degradation is visible as blur that tends to increase with
distance from the midplane (see Figure 5.15). The cone-beam error decrease
with the cone angle and can hence be reduced to an acceptable level by
increasing the source-to-object distance. Moreover, it also depends on the
geometry and heterogeneity of the investigated specimen. Rapid variations
of the mean linear attenuation will induce more cone-beam artefacts.

An alternative to the Feldkamp-algorithm, which manages to give an exact
representation of the cone-beam geometry and hence avoid the problem with
cone-beam artefacts are the backprojection algorithm proposed by Defrise and
Clack. However due to the simplicity of the Feldkamp-algorithm it is still
the most popular method used for cone-beam reconstruction and the one
used in this thesis.
6. **Digital volume correlation**

Digital volume correlation (DVC) is used for correlation analysis of volumetric image data, typically acquired with x-ray microtomography. The technique is used to determine the three-dimensional structural deformation of the investigated object. The calculations are based upon two sets of volumetric data collected from the object before and after deformation. Just as for the two-dimensional algorithm we divide the initial data sets into smaller regimes, called subvolumes or correlation windows. Each subvolume contains three-dimensional randomly distributed features - analogue with the speckle patterns in the 2D case. Within these subvolumes we calculate the three-dimensional displacement by finding the position and orientation of the same pattern of features in the deformed volume. Then by repeating the procedure for the whole volume we achieve the full three-dimensional deformation field.

3D deformation analysis by use of DVC was first proposed by Bay *et al.* for strain calculations in trabecular bone tissue exposed to uni-axial compression load. The technique has since then been further developed, through studies that mainly have been based on the same type of bone structure and loading. Furthermore, in recent years, the technique has also been compared with finite element analysis. Therefore, the 3D analytical framework now can be complemented with fully comparable experimental results, which yields better understanding, especially in inhomogeneous regions, where it’s hard to predict the behaviour of the material by use of simulation tools.

Here, a novel approach for the DVC procedure is presented. Instead of describing the displacements, in the conventional way, as constants or linear functions within each correlation window we choose to approximate them with Chebyshev polynomials. This yields a continuous description of the displacements throughout the analyzed region.

Although the fundamental principle is the same as for the two-dimensional technique, described in Section 3.2, the approach is rather different. We will take a look at how the algorithm works and at some of its features.

6.1 **The DVC-algorithm**

6.1.1 *Minimisation of the 3D cross-covariance function*

Two volumetric data sets, $V_1$ and $V_2$, describing the 3D inhomogeneous structure of an object before and after deformation, are analysed with DVC.
Figure 6.1. At a local scale, within the analysed global region (region of interest, ROI), a box-shaped subvolume is distorted into a general shape. The deformation process is described with respect to the deformed state, i.e., an Eulerian description is used. The position vectors of the initial box and the deformed shape is therefore \( x_p \) and \( \mathbf{x}_p \), respectively. The local displacement of the box due to the deformation is denoted \( u_p \).

Initially, each of the volumes \( V_1 \) and \( V_2 \) – also denoted the reference volume and the deformed volume, respectively – are divided into a set of subvolumes. Each of these subvolumes contains a unique set of random 3D features. If no deformation would have occurred in the investigated object then every subvolume \( S_2 \) in the deformed volume would have been an exact copy of the corresponding subvolume in the reference volume \( S_1 \). However, due to the deformation of the object the structure in each deformed subvolume \( S_2 \) will be distorted into a general shape and moved away from its original position – as shown in Figure 6.1. A robust way to calculate the deformation between the two regions \( S_1 \) and \( S_2 \) is to minimise the three-dimensional discrete covariance function

\[
 f(\mathbf{u}; \mathbf{x}) = 1 - \frac{\sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{k=1}^{P} \tilde{S}_1(x_{ijk} - u_{ijk}) \tilde{S}_2(x_{ijk})}{\left[ \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{k=1}^{P} \tilde{S}_1^2(x_{ijk} - u_{ijk}) \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{k=1}^{P} \tilde{S}_2^2(x_{ijk}) \right]^{1/2}}, \tag{6.1}
\]
where $\mathbf{x}_{ijk} = [x,y,z]$ is the discrete position vector, $\mathbf{u}_{ijk} = [u(x_{ijk}), v(x_{ijk}), w(x_{ijk})]$ is the local deformation vector and a correlation window of size $M \times N \times P$ voxels is used. Furthermore, $\tilde{S}_1$ and $\tilde{S}_2$ are structural copies of $S_1$ and $S_2$, respectively, but with the global greyscale variation removed. Moreover, the global greyscale variation is typically equivalent with the global mean of greyscale values in the investigated structures, i.e. $\tilde{S}_i = S_i - \tilde{V}_i$.

We here use the approach were we describe the deformation process with respect to the deformed state, which generally is referred to as the Eulerian description of the variation within the body. The position vectors of the reference subvolume and the deformed subvolume is therefore denoted $\mathbf{x}_p$ and $\mathbf{x}_p$, respectively, with the local displacement vector $\mathbf{u}_p$.

### 6.1.2 Chebyshev approximation of deformations

To make the minimisation in Equation (6.1) solvable we need to make an approximation of the deformation within the correlation window, an approximation that puts a continuity constraint on the deformation. For the two-dimensional method, described in Section 3.3 we assumed the deformation within the correlation window to be constant, which is the simplest and most commonly used technique. A constant value is often sufficient as long as the deformations are of primary interest, the deformations are small and the correlation window can be made small with sufficient reliability. One inherited advantage of such an approximation is that the three unknowns, $u$, $v$ and $w$, are found from the location of the covariance peak without the need of a non-linear optimisation routine. However, including also the deformation gradients in the minimisation has proven to result in a more reliable and accurate algorithm, although on the cost of programming complexity. Here we have chosen to use Chebyshev polynomials to represent a general deformation up to a given order $n$. The three deformation components can therefore be written as

$$
\begin{align*}
    u(x) &= \sum_{p=0}^{n} \sum_{q=0}^{n} \sum_{r=0}^{n} a_{pqr} T_p(x) T_q(y) T_r(z), \\
    v(x) &= \sum_{p=0}^{n} \sum_{q=0}^{n} \sum_{r=0}^{n} b_{pqr} T_p(x) T_q(y) T_r(z), \\
    w(x) &= \sum_{p=0}^{n} \sum_{q=0}^{n} \sum_{r=0}^{n} c_{pqr} T_p(x) T_q(y) T_r(z),
\end{align*}
$$

(6.2)

$$
    p + q + r \leq n,
$$
where \( a_{pq} \), \( b_{pq} \) and \( c_{pq} \) are the Chebyshev components and \( T_p \), \( T_q \) and \( T_r \) are the Chebyshev base functions of the first kind, of order \( p \), \( q \) and \( r \), respectively, defined as

\[
T_p(x) = \cos(p \cos^{-1} x),
\]

\[
T_q(y) = \cos(q \cos^{-1} y),
\]

\[
T_r(z) = \cos(r \cos^{-1} z).
\]

The first five Chebyshev polynomials, up to order four, is given in Equation (6.4) and plotted in Figure 6.2.

There are several reasons why the approximation given by Equation (6.2) is convenient. Firstly, the form of the approximation makes it straightforward to extend the algorithm to higher dimensions. Secondly, Chebyshev polynomials are known to be close to the minimax polynomial, which has the smallest maximum deviation from the true function. Thirdly, Chebyshev approximations can be truncated to lower orders without significantly deteriorate the approximation. Finally, since Chebyshev polynomials are confined to the interval \([-1,1]\) the minimisation becomes numerically robust.

The variables \( u, v \) and \( w \) in Equation (6.1) are in general continuous, which means that the reference subvolume \( S_1 \) needs to be continuous also. Prior to the covariance calculation, therefore, a trivariate spline of given order, \( m \), is constructed from the discrete values in \( S_1 \). The minimisation procedure used

\[
\begin{align*}
T_0(x) &= 1 \\
T_1(x) &= x \\
T_2(x) &= 2x^2 - 1 \\
T_3(x) &= 4x^3 - 3x \\
T_4(x) &= 8x^4 - 8x^2 + 1
\end{align*}
\]
Table 6.1. The number of Chebyshev components, \( Q \), required for the minimisation of an expansion of order \( n \).

<table>
<thead>
<tr>
<th>( n )</th>
<th>( 0 )</th>
<th>( 1 )</th>
<th>( 2 )</th>
<th>( 3 )</th>
<th>( 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q )</td>
<td>3</td>
<td>12</td>
<td>30</td>
<td>60</td>
<td>105</td>
</tr>
</tbody>
</table>

Here is a large-scale algorithm based on an interior-reflective Newton Chebyshev components satisfying the condition \( p + q + r \leq n \) as indicated in Equation (6.2) and gives the number of variables to optimise for a given expansion. Table 6.1 gives the value of \( Q \) for expansions up to order \( n=4 \). As a result of the minimisation we get the set \{ \( a_{pq} \), \( b_{pq} \), \( c_{pq} \) \} that best matches the features in \( \tilde{S}_1 \) with the features in \( \tilde{S}_2 \) as well as the correlation value,

\[
\gamma(\mathbf{u}; \mathbf{x}) = 1 - f(\mathbf{u}; \mathbf{x}),
\]

as a measure of the significance of the fit.

### 6.1.3 Method to ensure convergence

One significant problem with the procedure described above is that we need to know \textit{a priori} that the deformation falls within the covariance peak in order for the minimisation to converge. For an image pair with features at the Nyquist sampling rate this restriction means the deformation needs to be confined within \( \pm 2 \) pixels. In general deformations may exceed that narrow range by a factor of ten or more and something needs to be done to ensure convergence of the minimisation. One approach would be to provide a qualified guess within the first correlation window and analyse the rest of the areas using overlapping windows. Such an approach would, however, include a hands-on step that will prevent full automation. We have therefore made the choice to perform the correlation in two steps. In the first step the covariance volume \( C(\Delta \mathbf{x}) \) between \( \tilde{S}_1 \) and \( \tilde{S}_2 \) is expressed as

\[
C(\Delta \mathbf{x}) = \mathcal{F}_3^{-1} \left\{ \mathcal{F}_3 \{ \tilde{S}_1 \} \ast \mathcal{F}_3 \{ \tilde{S}_2 \} \right\}, \tag{6.6}
\]

where \( \mathcal{F}_3 \) and \( \mathcal{F}_3^{-1} \) indicates the forward and inverse three-dimensional Fourier transform, respectively, and \( \ast \) denotes complex conjugation. The advantage with Equation (6.6) is that the full three-dimensional discrete covariance volume is obtained in contrast to Equation (6.1) that only gives the covariance value for a given set of variables. However, \( \mathbf{u} \) is here restricted to constant values within the subvolume and will therefore only provide a coarse sampling of the deformation. Nevertheless, picking the discrete position of the maximum value within \( C(\Delta \mathbf{x}) \) will provide a good initial guess for the
fundamental Chebyshev components $a_{000}$, $b_{000}$, and $c_{000}$ and will ensure the optimization to be within the global minimum peak.

6.1.4 Mapping of results

In general the deformation within the subvolume represented by Equation (6.2) will provide a local set of deformations within a larger body. Figure 6.3 shows a schematic figure of the analysis in one single subvolume, where the displacement $\mathbf{u}(\mathbf{x})$ in point $\mathbf{x}$ is denoted by a red vector.

To get the deformation within the whole body the procedure needs to be repeated for a large amount of different subvolumes throughout the body, as described above. Furthermore these subvolume calculations are carried out with an overlap. The distance between the centres of two neighbouring subvolumes is given by half the subvolume dimension, i.e. $M/2$, $N/2$ and $P/2$ for the $x$-, $y$- and $z$-direction, respectively. This yields that every point within the body (apart from points close to the boundary) is represented by eight different subvolumes, as shown in Figure 6.3(b) and 6.4. To ensure continuity between the different sets of deformations, an approach developed by Sjödahl and Oreb\textsuperscript{87} is used. The method is based on a “centre-of-mass” weighting of the different contributions. The final expression for the deformation components in a general point $\mathbf{x}$ within the body is given by

$$
\mathbf{u}(\mathbf{x}) = \frac{\sum_{i=1}^{K} \mathbf{u}(i) \left[ \delta_{\text{sum}} - \delta^2(i) \right]}{(K-1)\delta_{\text{sum}}^2},
$$

(6.7)

$$
\delta(i) = d(i)/\gamma(i),
$$

where $\delta(i)$ is a scaled version of the distance $d(i)$ between the centre of the $i$:th subvolume and position $\mathbf{x}$, obtained from division with the correlation value $\gamma(i)$. In Figure 6.4 the displacement $\mathbf{u}(i)$ and distance $d(i)$, in each of the subvolumes, is represented by a red and black vector, respectively. Furthermore, $K$ in Equation (6.7) represents the number of overlapping subvolumes for a given point $\mathbf{x}$ and $\delta^2 = \sum \delta^2(i)$. Equation (6.7) will hence favour subvolumes with centres close to the chosen point $\mathbf{x}$ and those having high correlation values.
Figure 6.3. The point $\mathbf{x}$ experience the displacement $\mathbf{u(x)}$, due to the structural deformation of the body, and is analysed at a local scale in individual subvolumes (a). The displacement analysis is repeated for all subvolumes, through out the body. Due to an overlap of the subvolumes the displacement of $\mathbf{x}$ is analysed in a number of different subvolumes (b).

Figure 6.4. Individual representation of the subvolume members of the cluster from Figure 6.3(b). To ensure continuity of the deformation analysis at a global scale, the displacement in each point is obtained as a weighted average of the displacements from the contributing subvolumes, represented by red vectors. The weight in each point depends on the correlation value for that subvolume as well as on the local distance from the subvolume centre — here represented by black vectors.
6.2 Calculating the 3D strains

The concept of DVC as described above deforms the original volume to fit into the grid of the deformed structure, i.e. the Eulerian description is used. We therefore chose to calculate the so-called Almansi’s strain tensor given by

\[
\varepsilon_{ij} = \frac{1}{2} \left[ \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} - \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \right],
\]

where standard tensor notations has been used. \(u_\alpha\) is a general expression for the displacement components, where \(\alpha\) can take any of the values 1, 2 and 3, and where the components \(u_1, u_2\) and \(u_3\) correspond to \(u, v\) and \(w\).

With unabridged notations Equation (6.8) turns into

\[
\varepsilon_{xx} = \frac{\partial u}{\partial x} - \frac{1}{2} \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial x} \right)^2 \right],
\]

\[
\varepsilon_{yy} = \frac{\partial v}{\partial y} - \frac{1}{2} \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right],
\]

\[
\varepsilon_{zz} = \frac{\partial w}{\partial z} - \frac{1}{2} \left[ \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right],
\]

for the normal strains and

\[
\varepsilon_{xy} = \frac{1}{2} \left[ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \left( \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \right) \right],
\]

\[
\varepsilon_{xz} = \frac{1}{2} \left[ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} + \left( \frac{\partial u}{\partial x} \frac{\partial w}{\partial z} + \frac{\partial w}{\partial x} + \frac{\partial w}{\partial z} \right) \right],
\]

\[
\varepsilon_{yz} = \frac{1}{2} \left[ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} + \left( \frac{\partial v}{\partial y} \frac{\partial w}{\partial z} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} \right) \right],
\]

for the shear strains. Hence we have a complete description for the full strain tensor, which can be presented in matrix form as

\[
\varepsilon_{ij} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix},
\]

for each point in the volume.

Note that the strain tensor is symmetric, i.e. \(\varepsilon_{ij} = \varepsilon_{ji}\). This enables us to make a principal strain formulation, i.e. formulate the strain tensor as an orthogonal
set of eigenvectors. These vectors give the directions where we only have normal strain components (no shear strain) and thus pure tension and compression deformations are described. The principal strains are the eigenvalues of the strain tensor, i.e. $\varepsilon_{ij} = eI = e\delta_{ij}$, and if we denote the principal strains as $e_1$, $e_2$ and $e_3$, we can write

$$
\varepsilon_{ij} = \begin{pmatrix} e_1 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{pmatrix}, \quad (6.12)
$$

where $e_1$, $e_2$ and $e_3$ are given by the roots of the determinant function

$$
|\varepsilon_{ij} - e\delta_{ij}| = 0. \quad (6.13)
$$

Solving Equation (6.13) gives us the full set of principal strains (eigenvalues) and the corresponding directions (eigenvectors) associated with the pure tension/compression state of the body, and where no shear is present. However we can also use these results to calculate the exact opposite – the maximum shear strain

$$
\varepsilon_{max} = \frac{e_3 - e_1}{2}, \quad (6.14)
$$

where $e_1$ and $e_3$ are the minimum and maximum principal strain, respectively.

To form the various strain components above we need to calculate the deformation gradients throughout the body. One way to do this is to numerically differentiate the deformation components, which involves choosing an appropriate gauge length that will influence the accuracy and resolution of the estimate. The numerical differentiation can be done either on the global displacement fields given by Equation (6.7) or on the displacements at subvolume level given by Equation (6.2). In the latter case the global displacement gradient distribution is obtained using the mapping procedure described in Equation (6.7). Another approach is to use the gradient data in $g_f$, used for the minimisation of Equation (6.1). These calculations are also carried out at subvolume level, and the full strain fields are formed by use of Equation (6.7). In the latter approach the deformation gradients are expressed as

$$
\frac{\partial u_j(x)}{\partial x} = \sum_{p=1}^{n} \sum_{q=1}^{n} p x_{pq} u_p U_{p-1}(x) T_q(y) T_r(z)
$$

$$
\frac{\partial u_j(x)}{\partial y} = \sum_{p=1}^{n} \sum_{q=1}^{n} q x_{pq} T_p(x) U_{q-1}(y) T_r(z) \quad (6.15)
$$
\[
\frac{\partial u(x)}{\partial z} = \sum_{p=0}^{n} \sum_{q=0}^{n} \sum_{r=1}^{n} \alpha_{pq} T_p(x) T_q(y) U_{r-1}(z)
\]

where \( T_p, T_q \) and \( T_r \) again represent Chebyshev base functions of the first kind, of order \( p, q \) and \( r \), respectively, and \( n \) is the order of the approximation. However, here we also have a contribution of \( U_{p-1}, U_{q-1} \) and \( U_{r-1} \) which represents the Chebyshev base functions of the second kind, of order \( p-1, q-1 \) and \( r-1 \), respectively. As before the \( u_i \) represents one of the three components of the local deformation vector \( u \) and the \( \alpha_{pq} \) represents either \( a_{pq}, b_{pq} \) or \( c_{pq} \). When expressed through Equation (6.15) the strain estimate will be independent of the choice of gauge length, but might become numerically unstable if the strains within the subvolume are small and additional smoothing might need to be introduced.

### 6.3 A comparison with DIC

As well as performing DVC analysis of the 3D reconstruction of a material, it is also possible to perform 2D DIC on planes cut through the body. We can therefore take two-dimensional subsets of the 3D displacement fields obtained with DVC and compare those with the results obtained with the DIC-algorithm, described in Section 3.3. Through this comparison we get a quality measure of the DVC-algorithm since DIC is a well established technique for this kind of deformation analysis, with an accuracy that is profoundly documented\(^7\), \(^{21}\), \(^{31}\), \(^{32}\).

Figure 6.5(a)-(d) shows a comparison between results obtained with the two techniques. Here the investigated specimen is a cylindrical bed of granular sugar that has been compacted through a compressive load. This analysis is a part of the study presented in Paper E.

The investigated region has the dimensions 6.72 x 6.72 x 8.21 mm\(^3\). Figure 6.5(a) shows the 3D w-displacement field from DVC, for half the investigated region. The cut has been made at \( x=3.36 \) mm, i.e. the yz-plane at the geometrical centre of the specimen along the x-direction. Figure 6.5(b) shows a vector plot describing the v- and w-displacements in the yz-cross-section at \( x=3.36 \) mm, where the cut was made in (a), obtained from DVC. The vector plot is overlaid onto the continuous w-displacement field. In Figure 6.5(c) the results from DIC analysis in the same plane is shown. The DIC analysis has been performed with correlation parameters comparable with those used for DVC: the correlation windows were 32 x 32 pixels in size and there was a 16 pixels overlap between them in both directions. It is important to note that the vectors in Fig. 6.5 (b) represent a subset of the full result, which has been
Figure 6.5. The DVC-results compared with results from DIC, describing the displacements along the z-axis in a cylindrical granular sugar material that is compacted through compression. The analysis have been carried out in the yz-plane at x=3.36 mm (the geometrical centre of the specimen). (a) shows (b) and (c) shows vector representations of the v- and w-displacements for DVC and DIC, respectively. In (b) the vector plot have been overlaid on the continuous w-displacement field, obtained by DVC. In (d) the results from the two methods are compared for two columns along the z-direction - at y=3.36 mm (the centre) and at y=6.35 mm (right hand side border).

Comparison of the vectors in Figure 6.5(b) and (c) shows that the w-displacements are fully comparable with each other. This is more apparent in Figure 6.5(d), which shows the results for the w-displacement along the z-direction at two different y-positions. The first is taken at y=3.36 mm, which is the centre column of the vector field in (b) and (c). Globally, this represents the absolute centre of the cylinder. In (d) these results are denoted I. The
second set of data is taken at \( y=6.35 \) mm, which corresponds to the border region at the right hand side in (b) and (c). These results are in (d) denoted II. These graphs show that the results from DVC and DIC give highly comparable representations of the \( w \)-displacement in both these regions. The fluctuations of \( w \) observed in the results obtained by DVC also show up in the DIC results. Generally, the DVC method presents slightly lower displacement values, especially in the upper regimes of the cylinder. However, these discrepancies are small, and in several regions the two methods give close to identical results. By comparison of the vector fields in (b) and (c) it is also observed that the results from the DIC-analysis report a slightly larger displacement in the \( y \)-direction. The lack of DIC-results in the uppermost region in Figure 6.5(c) and (d) is due to the large displacements, which resulted in de-correlation. A possible reason to why DVC manages to describe the large top-region displacement, while DIC runs into de-correlation, might be that the DVC analysis is carried out based on additional information from a third dimension. The increased amount of data about the structure and the displaced features yield a better statistical representation of the deformation process.

In correlation based analysis, the spatial resolution is normally described by the size of the correlation windows - the smaller the windows, the higher the resolution. Furthermore, the sizes of the correlation windows are limited by the features used in the correlation. However, with the continuous representation of the displacements with Chebyshev-polynomials the resolution is no longer strictly dependent on the size of the correlation windows.

The conclusion is that the results in a 2D plane, taken from the 3D displacement field from DVC is fully comparable with those obtained for the same plane with DIC.
7. Analysis of 3D deformations and strain

7.1 Deformation and strain in micro-scale wood structure exposed to three-point-bending

The scope for this work was to determine the full three-dimensional deformation and strain in a small wood specimen, exposed to three-point-bending (TPB). 3D Image data, describing the micro-architecture of the wood specimen during a TPB load sequence, was acquired with synchrotron x-ray microtomography (SRμCT)\textsuperscript{49, 88}, described in Section 5.5.1, and analysed with DVC. This experiment is thoroughly described in Paper B and C.

Wood is a multi scale material with characteristic features at several hierarchical levels – from annual rings at the cm-level, and visible for the bare

![Figure 7.1. Micro-scale wood structure, rendered from the reconstructed data, where the most significant features has been marked out. (a) Tracheid lumen, (b) Tracheid cell wall, (c) Radial bordered pit, (d) Uniseriate heterogeneous ray, (e) ray parenchyma cell. The full visualized volume is 112 x 112 x 56 μm\(^3\).](image-url)
eye, down to fibrils at the nano-scale and cellobiose molecules at the Å-scale\(^8\). It is far from homogeneous – especially at the lower spatial scales, as shown in Figure 7.1 – and therefore properties such as fracture toughness is hard to predict and incorporate into models. Many fractures that occur in macro-scale wood structures can be explained from the micro-scale behaviour of the material. Figure 7.1 shows a rendering of the micro-scale cellular wood structure in a region that measures 112 x 112 x 56 μm\(^3\), based on the reconstructed data from the experiments carried out here. The coordinate system describes the orientation within the wood structure with \(T\), \(L\) and \(R\) denoting the tangential, longitudinal, and radial direction, respectively. However, for the correlation procedure the conventional \(xyz\)-notation is used, as described by Figure 7.1. Furthermore, characteristic features in the wood structure such as the tracheid lumen (a), tracheid cell wall (b), bordered pits (c), radial oriented uniseriate rays (d) and ray parenchyma cells (e) are marked in Figure 7.1. These features constitute a delicate transportation network for water and nutrition.

Figure 7.2 shows how the micro-scale wood structure is related to the macro-scale wood, in a cross-section of a tree. The box in the circular close-up represents micro-scale wood structure, such as the one presented in Figure 7.1. Note, however, that this is local approximation since the TLR-coordinate system at a macro scale is cylindrical. \(R\) is the radial direction of the tree, orthogonal to the annual rings. \(T\) represents the tangential direction, given by the tangent of the annual rings, and \(L\) is the longitudinal direction of the tree. For more information on wood anatomy and interactions in wood consult for example the standard book of Siau\(^8\).
On wood, deformation analysis by use of correlation based methods has exclusively been two-dimensional. The analysis has typically been based on surface images captured with conventional optical photography or microscopy. The method has been used for hygro-mechanical analysis to determine displacement and strain fields due to swelling and shrinking of wood\textsuperscript{89, 91}. In fracture toughness experiments on wood, DIC analysis has been used to determine strain fields due to three-point-bending\textsuperscript{92}, as well as strain fields at crack tips\textsuperscript{93}. CT was proposed as a tool for wood structure analysis in the mid-eighties\textsuperscript{94, 95}. However, the poor spatial resolution of these systems only allowed macro-scale wood structures to be reconstructed – at the scale of annual rings – which was not suited for image correlation analysis. During the last decade, however, the development of CT techniques and the improved spatial resolution have permitted the micro-scale structure to be imaged\textsuperscript{88, 96}. Although the cellular structure that is found at this scale show influences of the annual growth and therefore a certain periodicity it can still be considered as reasonably stochastic. The new CT technology has therefore open up the possibility to use correlation based methods to better understand how the wood structure beyond the surface responds to different load types.

7.1.1 Experimental procedure

The specimen and the load device

A bar-shaped wood specimen with the dimensions of 1.57 x 3.42 x 0.75 mm\textsuperscript{3} (visible in Figure 7.3(c)) was made out of Scots pine (\textit{Pinus sylvestris}). It was cut from a larger piece by use of a sharp scalpel and calibrated by use of very fine sandpaper.

The three-point-bending experiment was carried out using a small scale load device, shown in Figure 7.3(a)-(c), which has been constructed from two metal cylinders connected through four 1.0 mm carbon fiber reinforced plastic (CFRP) rods. Figure 7.3(a) and (b) shows a schematic 3D drawing of the load device, viewed from the front (a) and from the side (b). Moreover, in Figure 7.3(b) the device has been made semi-transparent so that hidden structures appear. The rod-cylinder configuration is held together through use of very strong adhesives. These are applied in the eight slots that are visible in Figure 7.3(a). The idea with the slots is to increase the interface between the metal cylinder bodies and the CFRP rods. Examples of such resin filled slots are visible in the photo in Figure 7.3(c). The reason for using carbon fibers as frame material in the construction, apart from their strength, is that they are only weakly absorbing the x-rays, leaving no visible shadows in the reconstructed data. The top of the load device holds a screw mechanism that is used for the loading. The screw mechanism contains a M3 screw that is coupled to a 2.0 mm steel cylinder, through a Ø 2.5 mm steel bearing, as shown in the upper part of Figure 7.3(b). The steel cylinder is in contact with the wood specimen, as shown in Figure 7.3(c). The wood specimen lies on
two 1.0 mm CFRP supports, positioned at opposite sides. At the bottom, the load device has a Ø 3.15 mm connector pin for easy mounting to the SRμCT specimen manipulator, described in Section 5.5.1. A full rotation of the top screw should ideally represent a 500 μm displacement of the wood/steel interface, which comes from the threading pitch of the M3. Therefore a 90° rotation ideally corresponds to a 125 μm applied displacement. However during these experiments we have only registered an 89 μm displacement, for the case with a 90° rotation. The 36 μm deviation is believed to be due to a weakness in the construction of the load device, which becomes more pronounced as the load increases. However since the wood/steel interface is visible in all the acquired image data sets it is possible to determine the effective applied displacement between the different load states. Hence we don’t suffer from the reduced efficiency of the load device.

**Scanning the specimen**

The TPB-experiments has been carried out at the TOMCAT beamline at the Swiss Light Source (SLS), described in Section 5.5.1.
Figure 7.4. The geometry and loading of the wood specimen (a). The dimensions of the wood specimen are $1.57 \times 3.42 \times 0.75 \text{ mm}^3$. The load, $F_b$, is applied at point $p_1$. The specimen rests on two carbon fiber supports, at positions $p_2$ and $p_3$. $F_{r2}$ and $F_{r3}$ denote the reactive forces in point $p_2$ and $p_3$, respectively. (b), (c) and (d) show a sequence of reconstructed LR-slices through the centre of the specimen and TPB-load configuration, at two subsequent load states, (b) and (c), and after failure (d).

The synchrotron x-ray energy was 11 keV and the object-to-detector distance was 7.0 mm. The detector optics was set to 4x-magnification and the field of view (FOV) was $3.58 \times 1.22 \text{ mm}^2$, corresponding to $2048 \times 701$ pixel. The spatial resolution of the detector chain under these conditions is $2.15 \mu\text{m}$ at 10% MTF. A number of 2501 projections were captured for each scan of the specimen, equiangular distributed over $180^\circ$. The exposure time was 300 ms and the total scan time was approximately 12 minutes. The reconstruction time, for each data set, was approximately 10 minutes. The reconstructed volumes have the dimensions $3.58 \times 3.58 \times 1.22 \text{ mm}^3$, corresponding to $2048 \times 2048 \times 701$ voxels. Hence the voxel dimension is $1.753 \mu\text{m}^3$.

The specimen is scanned first in unloaded state and then again at three different load states, until it finally fractures. Each subsequent scan is carried out with an increased load, caused by rotating the top screw $90^\circ$ and thus an applied displacement of $89 \mu\text{m}$.

The geometry and loading of the wood specimen is described schematically in Figure 7.4(a). The load, $F_b$, is applied at point $p_1$ using the 2.0 mm steel rod. Furthermore, the wood specimen rests on two 1.0 mm carbon fiber reinforced plastic (CFRP) rods, at positions $p_2$ and $p_3$. $F_{r2}$ and $F_{r3}$ denote the reactive forces in point $p_2$ and $p_3$, respectively. The bundles of gray force vectors illustrate the fact that the contact regions are not strict mathematical points. The forces are instead distributed over some limited area of the specimen. Furthermore, Figure 7.4(b), (c) and (d) show a sequence of reconstructed longitudinal-radial-slices from the specimen, taken through the centre of the TPB-load configuration, at three subsequent stages along the
load cycle - (b) and (c) in first and second loaded state, respectively, and (d) after failure. One can see that the longitudinal wood structure bend as the load increases and that the wood structure in the three-contact regions are deformed non-elastically. The two loaded wood structures presented in Figure 7.4(b) and (c) are also the ones that have been used for the correlation analysis.

7.1.2 Correlation procedure

A region $R_c$ of size $0.448 \times 2.69 \times 0.56$ mm$^3$, described by $256 \times 1536 \times 320$ voxels in the original image data, is analysed using DVC. A 2 x 2 binning was performed on the image data, along the x- and y-direction, prior to the correlation analysis. Thus, using limited computational resources, a larger physical region has been analyzed at the cost of decreased spatial resolution. In Figure 7.5(a) and (b) the wood structure in region $R_c$ is shown for the two different load states.

The extension of $R_c$ in the y-direction makes it interesting since it covers a region that contains all three contact regions – the tip of the steel cylinder and the two CFRP-supports, which are visible in Figure 7.5(b).

The subvolume size used in the correlation analysis is $64^3$ voxels, which correspond to $0.224 \times 0.224 \times 0.112$ mm$^3$.

![Figure 7.5 3D renderings of the reference volume data (a) and deformed volume data (b) in region $R_c$. In (b) all three contact points are visible.](image-url)
7.1.3 Results and discussion

3D deformations due to the TPB-load

Figure 7.6(a) shows the full 3D \( v \)-displacement field in the region \( R_c \), describing the displacements in the \( y \)-direction. Figure 7.6(a-i) shows a 2D \( yz \)-slice from the centre (\( x=0.224 \) mm) of the 3D field in (a). Moreover, the \( v \)-displacements in this plane are also visualised with a vector field. From the results in Figure 7.6(a) and (a-i) one can see compression and tension bands in the upper and lower region, respectively. These two bands are separated by a neutral band, with displacements near zero, in the middle. Although the general appearance of the \( v \)-displacement field agrees with what is expected from a sample exposed to three-point-bend there is also irregular behaviour that may be due to the anisotropy of the wood structure. Furthermore, Figure 7.6(a-i) also shows that the centres of the compression and tension, in the \( v \)-displacement field, is shifted towards the right hand side and located approximately at \( y=1.5 \) mm. This non-symmetric behaviour agrees with the

![Figure 7.6 The 3D v-displacement field in region R_c (a), and the 2D centre yz-slice from the same region (a-i). Moreover, in (a-i), the 2D v-displacement field is also presented as a vector field. The appearance with a compression and tension band in the upper and lower region respectively, separated by a neutral band in the middle, agrees with what is expected for a specimen exposed to three-point-bend.](image)
Figure 7.7. The \( w \)-displacement in region \( R_c \) overlaid on the wood structure in deformed state (a). The 2D displacement field in the front \( yz \)-plane, denoted (i), is shown (without wood structure) in (a-i). The maximum \( w \)-displacement is approximately 68 \( \mu \)m and located right beneath the steel tip.

fact that there is a certain lack of symmetry, of the three contact regions, which can be seen in Figure 7.4(a).

Figure 7.7(a) shows the \( w \)-displacement field of \( R_c \) overlaid on the wood structure in deformed state. A maximum deformation of 68 \( \mu \)m is located right beneath the steel tip, and corresponds to 76 % of the applied displacement. The deformation decreases with distance from the centre, down to 0 \( \mu \)m in the two support regions. Figure 7.7(a-i) shows the 2D \( w \)-displacement field (without wood structure) on the front face of the 3D field, denoted (i). Also here, one can see that the loading is slightly non-symmetric, in agreement with Figure 7.4(a). If we follow the maximum \( w \)-displacement, in the top centre region, as it propagates downwards we find that the displacement is reduced to approximately 58 \( \mu \)m as we reach the bottom. This 85 % reduction in deformation is probably due to inelastic effects that absorb the applied bending force as it propagates downwards in the wood structure.

3D Strains

Figure 7.8(a), (b) and (c) show the \( \varepsilon_{yy} \), \( \varepsilon_{yz} \) and \( \varepsilon_{zz} \) strain distributions, respectively. As described in Section 6.2, \( \varepsilon_{yy} \) and \( \varepsilon_{zz} \) represent the normal strain in \( y \)- and \( z \)-direction, respectively, while \( \varepsilon_{yz} \) represents the shear strain.
Figure 7.8. The 3D strain distributions $\varepsilon_{yy}$ (a), $\varepsilon_{yz}$ (b) and $\varepsilon_{zz}$ (c) in region $R_c$. $\varepsilon_{yy}$ and $\varepsilon_{zz}$ represent the normal strain in $y$- and $z$-direction, respectively, while $\varepsilon_{yz}$ represent the shear strain in the $yz$-plane.

in the $yz$-plane. As predicted from the $v$-displacement field in Figure 7.6(a) one can find two regions with strain concentrations in the centre of $\varepsilon_{yy}$, along the $y$-direction. These two, found in the upper and lower centres describes the compressive and tensile strain, respectively. The region with tensile strain corresponds to the region where the failure eventually takes place – as seen in Figure 7.4(d).

The shear strain that is introduced between the compression band, in the upper region, and the tension band, in the lower region, is described by $\varepsilon_{yz}$. The shear strain has strong concentrations of both positive and negative shear
half way through the region along the $z$-direction – where compression meets tension (the neutral band in the $v$-displacement field).

The $\varepsilon_{zz}$-distribution, shown in Figure 7.8(c), contains a concentration of compressive strain in the centre, right beneath the upper contact region, as expected from the $u$-displacement field in Figure 7.7. The strongest minimum (compressive strain) is however found in the boundary region at the left hand side. From the $u$-displacement field one finds that there is a slight discontinuity in the corresponding region. Moreover, one also finds well pronounced strain concentrations in the corresponding region, in both $\varepsilon_{yy}$ and $\varepsilon_{yz}$. However, no structural defect or crack formation that matches this region geometrically can be found from visual inspection of the wood structure, in Figure 7.5(a) and (b). A possible explanation of this discontinuity and the high strain levels can be found if we search beyond the region $R_c$. In Figure 7.4(b) and (c), which both show longitudinal-radial slices through the reconstructed full specimen (in different load states), one can find a well defined crack that starts at the edge of the specimen, on the right hand side, approximately in the centre along the radial direction ($z$-direction). Although the crack cannot be distinguished in the region $R_c$, there is a possibility that it affects the 3D strain fields in Figure 7.8.

### 7.2 Swelling behaviour of wood microstructure exposed to water

Drying and wetting of wood causes swelling and shrinkage on all structural levels of wood. Most often these wood structure deformations are studied at the macro-scale, and assumed to also describe the swelling and shrinkage at the micro-scale. However, there may be local differences on the micro level that is not reflected in the higher levels due to averaging effects. These differences might cause stress concentration that influences the macro behaviour of wood.

Murata\textsuperscript{90, 97} has used DIC to analyse the swelling of wood at the cellular level based on confocal scanning laser microscopy images of the surface. A drawback with all surface imaging methods is that the investigated specimen need a lot of preparation in advance in order to give trustworthy information. It is very hard to produce a surface usable for investigations without also introducing distortions from the micro-machining tools. An exposed surface is also susceptible to humidity changes in the surrounding air, which may influence the studied properties of the wood. From this perspective it is easy to see that non-destructive imaging techniques are getting more popular.

The rather homogeneous structure of the wood imaged with conventional CT systems, due to the limitation of spatial resolution mentioned above, has not allowed the use of methods like DIC for analysis of internal structures. An approach to move the correlation based deformation analysis to regions
beyond the surface was however proposed by Danvind and Synnergren.\textsuperscript{91} Here, an experiment where two cross-sections of drying wood were imaged sequentially with optical digital imaging and CT at the same time, complementary. The optical images with higher spatial resolution contained features that could be used for image analysis with DIC. The results from the surface were then assumed to also hold for the adjacent inner regions imaged with CT.

Here, the 3D swelling in wood micro structure, due to 12 h water exposure, has been analysed with DVC. The 3D imaging has been carried out using the same microtomography system as in the previously described three-point-bend experiment. The experiment is thoroughly described in Paper D.

### 7.2.1 Experimental procedure

As in the previous experiment these measurements are also carried out on a Scots pine (\textit{Pinus Sylvestris}) wood sample. The sample is approximate Ø 1 mm and 10 mm long. It was glued to an Ø 3.15 mm metallic pin that was used to attach the specimen to the SR\textsubscript{μ}CT controller stage at the beamline. The specimen and pin adapter configuration is shown in comparison with a standard sized match in Figure 7.9(a). Furthermore, Figure 7.9(b) shows a schematic 3D drawing of the specimen with a close up on the imaged region. The cross-section of this region is, in dry state, approximately elliptic with minor and major axis 0.9 mm and 1.3 mm, respectively.

The specimen is scanned four times, first in dry state and then again after being placed 5.5h, 9.0h and 12 h in water. The x-ray photon energy was 9.4 keV.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure79}
\caption{(a) The specimen and the Ø 3.15 mm pin adapter used to mount the specimen to the SR\textsubscript{μ}CT controller stage is compared with a standard sized match. (b) A schematic 3D drawing of the specimen and pin configuration with a close up on the investigated region.}
\end{figure}
keV, and the object-to-detector distance was 2.0 mm. An objective with 10x-
magnification was used and the field of view (FOV) was 1.43 x 0.63 mm², 
corresponding to 2048 x 901 pixel, which results in the theoretical voxel size 
0.73 μm³. The spatial resolution of the detector chain under these conditions is 
1.05 μm at 10% MTF. A number of 1501 projections were captured for 
each scan of the specimen, equiangular distributed over 180°. The exposure 
time was 480 ms and the total scan time was approximately 12 minutes. The 
reconstruction time was approximately 10 minutes and the reconstructed 
volume contained 901 slices, each with the dimensions 2048 x 2048 pixels. 
The physical dimensions of the reconstructed data were 1.43 x 1.43 x 0.63 
mm³, as shown in Figure 7.9(b).

7.2.2 Correlation procedure

In these measurements the x-, y- and z-coordinate system, used for the DVC 
analysis, corresponds to the tangential, radial and longitudinal direction in the 
wood structure, respectively. The analysis is carried out in a region that 
measures 0.715 x 0.715 x 0.447 mm³, described by 512 x 512 x 160 voxels. 
Figure 7.10(a) and (b) shows the rendered wood structure in this region for 
the dry state and after 12 h water immersion. The DVC-analysis is carried out 
based on subvolumes of the size 64³ voxels or 89.4 x 89.4 x 179 μm³.

Figure 7.10. (a) and (b) shows renderings of the reconstructed 3D wood structure inside 
the correlated region R, for the dry state and after 12 h water immersion. The region R, 
has the dimensions 0.715 x 0.715 x 0.447 mm³, described by 512 x 512 x 160 
voxels.

7.2.3 Results and discussion

Figure 7.11(a)-(f) shows the displacement fields u, v and w, corresponding to 
x-, y- and z-directions respectively, due to the swelling of the pine wood after 
12 h exposure to water. Figure 7.11(a), (c) and (e) shows the three 
displacement fields (only) whilst Figure 7.11(b), (d) and (f) show them 
overlaid on the wood structure in wet state. The u-displacement field, Figure 
7.11(a) and (b), describes the displacement in the tangential direction of the
Figure 7.11. The three-dimensional $u$-, $v$- and $w$-displacement fields. In (a), (c) and (e) the pure displacement fields are shown whilst in (b), (d) and (f) the displacement fields, overlaid on the wood structure in wet state are shown. The $u$-displacement field, shown in (a) and (b), describes the structural swelling of the wood in the $x$- (tangential) direction and range between $-22.4 \, \mu m$ and $24.5 \, \mu m$. The $v$-displacement field, shown in (c) and (d), describes the radial swelling of the wood structure. $v$ range between $-8.63 \, \mu m$ and $14.1 \, \mu m$. The $w$-displacement field, shown in (e) and (f), describes the structural motion in the longitudinal direction. It is mostly close to zero. However, in some regions one find radical movements that occurs very locally. These range from $-12.8 \, \mu m$ to $15.5 \, \mu m$. 
wood structure, and ranges between \(-22.4 \mu m\) and \(24.5 \mu m\). Furthermore, the \(u\)-displacement seems to have a linear variation along the \(x\)-direction while being rather constant with respect to the \(y\)- and \(z\)-direction.

The \(v\)-displacement field, in Figure 7.11(c) and (d), describes the radial swelling of the wood structure, in the \(y\)-direction, and ranges between \(-8.63 \mu m\) and \(14.1 \mu m\). The spatial relationship of \(v\) seems to be mostly linear in the tangential direction, which is expected. However the displacement field also reveal large structural movements in the radial direction that occur very locally near a resin canal, which is visible at the border of \(R_c\) (at \(y=0\) mm) in Figure 7.10(b). The resin canal seems to have a rather strong influence on the structural behaviour in the radial direction while it does not seem to have any significant effect on the tangential behaviour. Moreover, the \(u\)- and \(v\)-displacements has been used to estimate the global strain in the tangential and radial direction, respectively. The tangential and radial strain is \(6.56\%\) and \(3.18\%\), respectively, to compare with \(7.7\%\) and \(4\%\) that is found in literature\(^98\).

### 7.3 Motion in a bed of granular sugar due to compaction

The behaviour of granular materials and powders under compaction is an important area of study with relevance to a range of industrial, civil engineering and geophysical applications including production of ceramics, metals, pharmaceuticals and explosives\(^99\mbox{-}101\).

It is important to develop experimental capability for better investigating the behaviour of granular beds under compaction. A key aspect of this development is the ability to measure local displacement fields within a specimen.

There exist a number of publications where structure or motion within granular materials has been analysed by x-ray computed tomography (CT) or microtomography in combination with various image processing techniques\(^99\mbox{-}102\mbox{-}105\). A number of authors have used tracer particles to obtain information about the movement of the bed\(^102\mbox{-}104\mbox{-}106\); however, such measurements only give pointwise information.

The use of DIC for measurements of deformations and flows in granular structures was introduced in Section 4.1. However, that discussion was restricted to 2D imaging and did not cover the case when the granular structure is reconstructed using microtomography.

McDonald et al.\(^100\) combined DIC and X-ray microtomography for analysis of internal deformation and particle movements during loading by a punch into a die containing a mixture of aluminium and tin powder. Correlation analysis was performed, based on both 2D slices and 3D volumes of the reconstructed image data, thus giving much improved data density and spatial resolution compared to the previous pointwise measurements. The emphasis of this
study was the flow field of particles around the indenter, rather than the compaction behaviour of the powder, which will be the focus in the experiment presented here.

Here, the 3D motion in a bed of granular sugar due to compressive forces is analysed by DVC and cone-beam x-ray microtomography. The experiment is thoroughly described in Paper E.

### 7.3.1 Experimental procedure

Figure 7.12 shows the loading device used in the experiment, where the sugar (c) is encapsulated in a hollow Perspex cylinder (b). The inner- and outer diameter of the cylinder are 7.00 mm and 10.1 mm, respectively. The compaction of the material is achieved by applying a compressive load in the axial (z) direction with a 6.95 mm diameter solid cylinder, made out of brass (a).

The 3D imaging of the sugar structure was carried out using the cone-beam x-ray tomography system, described in Section 5.5.1. In these experiments the voltage and current were held at 25 kV and 100 µA, respectively. The source-to-specimen and source-to-detector distances were 72 mm and 320 mm,
respectively, resulting in a 4.44x magnification of the specimen and a 16.2 x 12.2 mm² FOV. During the experiment 545 projections were captured, distributed at equal angles over 360 degrees. The exposure time for each acquired projection was 2.56 s.

The reconstructed data has dimensions 7.98 x 7.98 x 8.87 mm³, described by 342 x 342 x 380 voxels and the region includes the granular sugar as well as the innermost parts of the Perspex cylinder. Figure 7.13 shows a volume rendering of the reconstructed granular sugar.

7.3.2 Correlation procedure

The correlation analysis is carried out in a region \( R_c \), with dimensions 6.72 x 6.72 x 8.21 mm³, described by 288 x 288 x 352 voxels. The theoretical spatial resolution, determined by the dimension of the cubic voxels, is thus 23.3 \( \mu \)m.

The shape of the region has been optimised to describe as much of the cylindrical specimen as possible, with the given subvolume size, as shown in Figure 7.14(a)–(c). Here, the reconstructed cylindrical specimen is shown (a) together with a schematic representation of \( R_c \), describing the shape and location of the region within the reconstructed cylinder. The homogeneous structure of the Perspex cylinder that holds the sugar is avoided in order to prevent de-correlation, which apart from degrading the results also leads to longer calculation times. Figure 7.14(c), finally, shows a volume rendering of the reconstructed granular sugar inside the region \( R_c \). The analysis is carried out using subvolumes of size 0.75³ mm³, corresponding to 32³ voxels.

Figure 7.14. (a) shows a volume rendering of the reconstructed granular sugar inside the Perspex cylinder. Furthermore, a schematic of the correlated region, \( R_c \), is shown (b) together with a rendering of the reconstructed data inside this region (c).
7.3.3 Results and discussion

The w-displacement field, describing the motion of the sugar along the z-axis, parallel to the applied load, is shown in Figure 7.15(a)-(d). (a) to (c) describe subsets of the w-displacement field, obtained by cuts in the yz-plane through the region Rc at x-positions 1.68 mm, 3.36 mm and 5.04 mm, respectively. In (d) the full w-displacement field is shown. The displacements range from 55 to 240 μm in the negative z-direction. As expected, the largest displacements occur in the top region, where the force is applied. This region is rather well defined and the large movements in the sugar decline rapidly with distance from the top face. The smaller sugar crystals can move more freely than the

![Figure 7.15](image_url)
bigger ones. This figure also shows how edge effects can affect the material behaviour. As expected, there is a variation of displacement across the top face; regions near the walls of the tube move less due to the friction between the sugar crystals and the walls. At the bottom region, the sugar crystal displacement reaches its minimum of 55 \( \mu m \). The base of the sugar filled cylinder, where the displacement is zero, is outside the field of view in these measurements.
8. CONCLUSIONS

Here a method for full 3D quantitative measurements of deformation and strain in inhomogeneous materials has been presented. 3D imaging (volume imaging) of the investigated material is performed before and after deformation, by use of x-ray microtomography. The image data, describing the undeformed and deformed 3D microstructure of the material, is analysed by use of digital volume correlation (DVC) and results in 3D deformation and strain fields which describe the material response during deformation.

A cone-beam x-ray microtomography system has been built and customised for the purpose. Image quality aspects of this system have been thoroughly investigated and correction schemes for various image artefacts have been implemented.

The DVC-analysis is carried out through cross-covariance optimisation of the two-sets of image data, sequentially performed in subsets of the full measurement volume, called subvolumes or correlation windows. The presented DVC-algorithm uses a novel approach where the displacements are approximated with Chebyshev polynomials, which yield a number of benefits. For example it allows a straightforward expansion of the algorithm to higher dimensions. Also, the order of approximation can be truncated without any significant deterioration of the results.

Three different experiments, reflecting the applications of the method is presented. These are carried out on two different materials – wood and granular sugar – and describe three different types of deformation processes.

In the first experiment micro-scale wood structure is analysed when exposed to three-point-bending. The 3D imaging is here carried out using synchrotron radiation microtomography (SRμCT), which allows imaging with a spatial resolution of approximately 1 μm. DVC-analysis of the acquired image data results in deformation and strain fields, which describe the full 3D response of the wood structure due to bending. In the second experiment the same type of wood material is exposed to water and the structural swelling of wood is analysed using the same equipment as in the previous one. Results from correlation analysis of wood in dry state and after 12 h water immersion are presented.

The obtained deformation and strain fields in these two experiments on wood mostly describe a structural behaviour that agrees with what is expected from similar macro-scale experiments. However the DVC-algorithm also manages
to resolve anomalous effects that most probably relates to the micro-scale anisotropy of the wood structure.

Finally, the 3D motion in a bed of granular sugar due to compaction has been investigated. The imaging is here carried out with cone-beam microtomography and the spatial resolution is approximately 23 μm.

The DVC-results are here compared to results from a well established two-dimensional image correlation algorithm, which has an accuracy that is profoundly documented. This comparison show that the two techniques present comparable, and in some regions identical, results.
9. FUTURE WORK

The imaging with microtomography generally results in large 3D arrays, depending on imaged region and resolution. The large amount of data is computational demanding for the further analysis. Especially the image data acquired with SRμCT, with reconstructed volumes of the approximate size $2048 \times 2048 \times 1000$ voxels, depending on application. This is the reason why the correlation analysis has been carried out in special regions of interest, instead of in the full 3D imaged region. There is always a trade-off between spatial resolution and size of analysed region. However these rather large data sets have also driven the development and optimisation of the algorithm, from a computational perspective.

In its present formulation the DVC-algorithm successfully describe continuous structural deformations. However due to the weighted averaging over subvolumes, described in Section 6.1.4, problems occur when representing discontinuous deformations, typically near crack formations. Future development of the DVC-algorithm will therefore initially be focused on an improved description of displacements near cracks and discontinuities\textsuperscript{107}.

Also of interest is to implement a rescaling procedure so that the displacement calculations are performed at several spatial scales (multi grid). This will allow an adaptive representation of displacements, which becomes important in regions with large strain, where there are rapid variations of the deformation, typically near cracks.

A step further would be to increase the spatial resolution in the correlation analysis by treating each of the (present) features in the investigated material as a correlation window (subvolume), where the shape, size and location of each correlation window corresponds to a unique feature\textsuperscript{108}. A typical example would be to treat each of the sugar crystals in the compaction experiment, described in Section 7.3 and Paper E, as a subvolume and analyse its deformation between scans. This would, however, require that each feature contains a set of randomly distributed sub-features that can be used for the correlation. This approach results in an analysis concentrated to the material itself, while the spaces between the features of interest are avoided. It would allow deformation and strain to be measured within each feature. This information can then be used to build material specific constitutive models from the cell scale and up.

For this to be possible, the local correlation analysis must be carried out complimentary, as an extension of the present correlation algorithm. In this way the global scale correlation analysis supplies the local analysis with
boundary conditions that ensure convergence in the minimisation based correlation. A complete transfer from the global scale to the micro scale may hence be performed.

An increased resolution in the correlation analysis requires that the microtomography can be performed with sufficient spatial resolution in order to resolve the sub-features required for the correlation analysis. At the TOMCAT beamline at SLS the present limit for spatial resolution is reconstructed voxels of the size $0.35 \, \mu m \ [48]$, which should be enough to resolve the sub-features. Moreover, recently, several cone-beam $\mu$CT systems have been introduced, which according to the vendors, have spatial resolution comparable with the one obtained with synchrotron $\mu$CT.$^{46}$

Combining these high resolution methods would allow analysis on micro-scale wood with full cellular resolution to be carried out, where each correlation window in shape and size correspond to a wood cell. Here, the correlation analysis is carried out based on smaller features such as bordered pits, parenchyma cells etc. In a granular material the sub-features could be inhomogeneous structures, air or water enclosures, micro-cracks etc.

Finally, it would be interesting to find new materials and applications for the technique that can offer entirely new challenges, which is a requirement for further improvements and understandings.
10. SUMMARY OF APPENDED PAPERS

The appended papers are listed according to their content in Table 10.1 below. Thereafter follows a brief summary, conclusions and division of work for each paper.

Table 10.1. Content of appended papers.

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Paper A: Measurements of granular flow in a silo using Digital Speckle Radiography

By: S. G. Grantham and F. Forsberg

Summary: The flow behaviour of an opaque powder contained in a silo was investigated with DSR. The powder that was studied is an alumina powder, with an average particle size of 50 μm. The measurements were performed in a plane located in the centre of the silo, defined by Tungsten particles, with the same grain size as the alumina. Radiographic image sequences were digitally captured and sequentially correlated with use of DSP. Flow rate and strain measurements were carried out for two different sizes of outlet.

Conclusions: The experimental results show the applicability of DSR to the study of flow behaviour in opaque materials. A clear difference in flow behaviour, for the two different outlets, is observed. A dynamical velocity field describing the flow, from release and onwards, is
obtained by placing the measured velocity fields in subsequent order.

**Division of work:** Forsberg and Grantham performed the experiments. Grantham wrote the paper.

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**Paper B:**

*Full 3D strain measurements on wood exposed to three-point-bending: Analysis by use of digital volume correlation applied to synchrotron radiation μCT image data*

**By:** F. Forsberg, M. Sjödahl, R. Mooser, E. Hack, P. Wyss

**Summary:**

A thorough description of the DVC-algorithm in which the displacements are approximated with Chebyshev polynomials is given. An application of DVC, where deformation and strain in micro-scale wood structure exposed to three-point-bend is measured. The size of the wood specimen was 1.57 x 3.42 x 0.75 mm³. 3D image data was acquired with synchrotron radiation micro-computed tomography (SRμCT) and a region with dimensions 0.448 x 2.69 x 0.56 mm³ was analysed with DVC. The region covers all three contact points. Based on the measured displacement data various 3D strain distributions are calculated.

**Conclusions:**

The displacement fields show a structural behavior that reminds of what is expected for a specimen exposed to three-point-bend. However there are also anomalous effects present in the displacement fields that can be coupled to characteristic features in the cellular structure of the wood. Furthermore, 3D strain calculations based on the obtained displacement data shows a concentration of tensile strain in the region where the specimen eventually collapses.

**Division of work:** Forsberg, Mooser, Hack and Wyss performed the experiments. Sjödahl made the initial implementation of the DVC-algorithm. Forsberg have modified the DVC-algorithm. Forsberg carried out the DVC-analysis. Sjödahl wrote about the DVC methodology. Forsberg wrote the rest of the paper.

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**Paper C:**

*3D micro-scale deformations of wood in bending: Synchrotron radiation μCT data analyzed with digital volume correlation*
**By:** F. Forsberg, R. Mooser, M. Arnold, E. Hack, P. Wyss

**Summary:** The three-point-bend experiment, from Paper A, is here described more thoroughly with a focus on the anisotropic wood structure. Analysis with DVC is carried out in two selected regions of different shape, spatial scale and geometrical position. The wood structure in region 1 contains a crack that propagates along the longitudinal direction of the wood, through the whole specimen. The crack divides the sample in two parts that react differently to the applied load. Region 2 is the one presented in Paper A.

**Conclusions:** The results for region 1 shows how differently the two parts, divided by the crack, behave due to the applied force. One part moves radically while the other is more stable. The stable part seems to be the main support, and responds more or less elastically to applied load. The obtained displacement fields give a strong indication on where the crack is located. Region 2 has a representative beam shape which strong resemblance to the wood sample itself. The displacement field along the beam represents the typical bending mechanism with a compression band in the upper half and a tension band in the lower part. However there is also irregular behaviour that might be due to the micro-scale anisotropy of the wood.

**Division of work:** Forsberg, Mooser, Hack and Wyss performed the experiments. Forsberg carried out the DVC-analysis. Arnold wrote the parts discussing the TPB-deformation from a wood anatomical perspective. Forsberg wrote the rest of the paper.

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**Paper D:** 3D micro-scale analysis of swell behaviour in Scots pine wood using Synchrotron Radiation Micro Computed Tomography and Digital Volume Correlation

**By:** J. Danvid, F. Forsberg, R. Mooser, E. Hack, P. Wyss

**Summary:** Here 3D quantitative analysis of the swell behaviour in wood structure at the micro scale is presented. The full three-dimensional wood structures of a pine wood sample, approximate Ø 1.0 mm and length 0.6 mm, was reconstructed at cellular level, with a spatial resolution of 0.7 μm, using SRμCT – first in dry state and then after
5.5 h, 9.0 h and 12 h water immersion. In the wood structure obtained from the two intermediate scans (5.5 h and 9 h) one finds that many of the cell lumens are filled with water. The DVC had too low correlation for calculating displacements when comparing wood without water in cell lumen and deformed wood with capillary water. However in the image data after 12 h water immersion the wood cells have been emptied from water and the structure is comparable with the structure in dry state. Hence, the structural swelling, due to the 12 h water exposure, was analysed with digital volume correlation. This was carried out in a region that measures 0.7 x 0.7 x 0.4 mm³.

Conclusions: The results describe a structural behaviour that in many respects reflects what is expected, based on macro scale analysis, and the magnitude of the measured swelling correlates well with what is found in literature. However, also unexpected phenomena can be found within the results. The radial swelling is, for example, strongly influenced by the presence of a resin channel, while the tangential swelling seems unaffected.

Division of work: Forsberg, Mooser, Hack and Wyss performed the experiments. Forsberg carried out the DVC-analysis. Danvind and Forsberg wrote the paper.

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Paper E: 3D deformation and strain analysis in compacted granular sugar using x-ray microtomography and digital volume correlation

By: F. Forsberg, and C. R. Siviour

Summary: This paper presents results from the compaction of a cylindrical bed of sugar, with the dimensions Ø7.0 mm and height 8.2 mm, using cone-beam x-ray microtomography to evaluate the internal structure, and Digital Volume Correlation to calculate 3D displacement information from these data.

Conclusions: The maximum displacement parallel with the compressive force is 240 μm and obtained in the top region of the bed, and induces approximately 7 % compressive strain. The magnitude of sugar crystal displacements decrease with increased distance from the top interface. From the 3D deformation field it is
possible to identify a shear region within the sugar material. The results show good agreement when compared with a well established 2D image correlation technique.

**Division of work:** Forsberg and Siviour performed the experiments. Forsberg carried out the DVC-analysis. Siviour performed the strain calculations. Forsberg and Siviour wrote the paper.
11. REFERENCES


Part II

Papers
Paper A

Measurements of granular flow in a silo using Digital Speckle Radiography
Measurement of granular flow in a silo using Digital Speckle Radiography

S.G. Granthama,*, F. Forsbergb

a Cavendish Laboratory, Department of Physics, University of Cambridge, Madingley Road, Cambridge, CB3 0HE, UK
b Division of Experimental Mechanics, Luleå University of Technology, SE-971 87 Luleå, Sweden

Received 19 August 2002; accepted 6 July 2004
Available online 18 September 2004

Abstract

In this paper, the flow of a powder through a silo is investigated using Digital Speckle Radiography (DSR). This technique allows displacement measurements to be made on the sub-mm scale to an accuracy of 0.06 μm and a spatial resolution of 26 μm. The method performs an image cross-correlation on a random seeding of X-ray opaque material as it flows out of the silo with the powder. The flow is captured digitally using a continuous X-ray source and an image intensifier and Charge Coupled Device (CCD) camera to give real time measurements. The powder used is Al2O3 with an average particle size of 50 μm and the seeding material is tungsten powder, also with an average particle size of 50 μm. Clear flow behaviour is observed for two different sizes of outlet and the flow rate and strains occurring during the flow are also investigated.

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Keywords: Radiography; Image correlation; Displacement measurements; Silo

1. Introduction

The field of particulate flow in bins and hoppers is vast and spans several decades of research, both experimentally and theoretically. The study of particulate flow in these types of system is of direct relevance to the food, chemical and pharmaceutical industries. Understanding the characteristics of the flow and avoiding blockages and excessive strains on the walls are of obvious importance [1]. A great deal of theoretical work has been performed on such problems, although this will not be discussed here [2,3]. Experimental methods of measuring the flow in a hopper have traditionally been based on radiographic studies, where the flow is stopped, an X-ray taken, and then the flow is allowed to continue before being stopped again [4]. This method allows a qualitative study of the changes in powder density, and hence slip-directions to be established. A further improvement on this technique is to place a regular grid of X-ray opaque markers in a plane in the silo. This allows a crude quantitative measure of the displacements during the flow [5]. Displacement measurements can also be made using photography to track a number of particles as they flow through the system [6–8]. This is restricted however to a silo with transparent walls. Continuous measurement of the displacements in an opaque silo during a discharge, without disruption to the flow and to sub-mm accuracy and spatial resolution can be achieved using DSR. This is a new technique that utilises an X-ray system and the computer analysis traditionally used in speckle metrology. The granular material in the hopper must be seeded with a flat plane of X-ray opaque grains of a similar particulate size, and then using a continuous X-ray source and X-ray Charge Coupled Device (CCD) camera, a digital image can be captured as the material is released from the silo under the force of gravity. A sequence of 220 frames is obtained from a single experiment at a framing rate of 20 frames/s and an

Abbreviations: Al2O3, Aluminum Oxide; CCD, Charge Coupled Device; φ, Diameter; σ, Speckle size; e, Random error; u, Displacement in x-direction; v, Displacement in y-direction; n, Stepsize (n=1 in this paper); u/φ, Principle strain in x-direction; v/φ, Principle strain in y-direction; u/v, Components of shear strain in x- and y-direction.

* Corresponding author. Tel.: +44 1223 337322; fax: +44 1223 350266.
E-mail address: sgg21@cam.ac.uk (S.G. Grantham).
image cross-correlation can be applied to successive images. The image correlation technique was developed to study the surface displacements of a sample with a random pattern on the surface. The random pattern is traditionally produced by shining a laser onto the surface to create a laser speckle pattern [9], or by using the natural relief or features of the surface as in white light speckle photography [10]. This type of analysis is also regularly used in Particle Image Velocity (PIV) [11] with which DSR shares similarities. In PIV a transparent fluid has a random distribution of particles within it, using laser light a single sheet of these particles are illuminated and imaged digitally to allow the flow to be measured. In DSR a similar single plane is imaged within the specimen, but this time the specimen can be opaque. The technique was developed to study the internal displacements of opaque specimens in ballistic events using flash X-rays [12,13,14,15].

2. Theory

The algorithm used for the image correlation was devised by Sjödahl [16] and calculates the digital cross covariance between the features in sub-images from a reference and deformed random pattern. The position of the peak within the covariance surface gives the mean relative translation between the two sub-images to size and direction. By further reconstructing the peak using Fourier components and performing a shift routine, the noise is reduced and sub-pixel accuracy can be measured in the displacements [17]. The normalised height of the cross-covariance peak, the so-called correlation value, will range between $1$ and $0$ where $1$ represents perfect correlation and zero no correlation. The correlation value is intimately linked to the accuracy of the algorithm and will be high for a direct translation, but will decrease for distortions and rotations. Random noise will also decrease the correlation value. Repeating the calculations for a large number of sub-images gives the complete deformation (velocity) field at a spatial resolution limited by the size of the sub-images used and the pitch between the sub-images.

3. Experimental

The X-ray source used in these experiments was a microfocus X-ray tube, which possesses high spatial coherence, compared to a conventional X-ray tube. This is achieved by having an extremely small source spot size of the order 5 μm in diameter. The tube has a voltage range of 20–100 kV and the tube current ranges from 0 to 250 μA. The anode material is Tungsten and the photon energy of the characteristic X-rays is 59.3 keV, although these experiments were performed at approximately 43.0 kV so only bremsstrahlung X-rays are contributing to the measurements.

The detector consisted of an image intensifier and a CCD camera to enable digital, real-time imaging. The input window of the detector is 72 × 54 mm and the camera used is a 17 mm CCD with 0.48 Mpixel. The silo used consists of a bin and hopper configuration as shown in Fig. 1 and is placed between the source and detector. The hopper outlet had two different sizes in these experiments of 5 mm and 2.5 mm and the powder used was 50 μm Al₂O₃ with 50 μm particle size tungsten distributed in a flat plane vertically through the central axis, orthogonal to the optical axis. The images are only captured in the bin region, the flow through the hopper section is not in the field of view, although it could be if desired.

The degree of error involved in these measurements can be estimated using Eq. (1), which is the expression for the error of the algorithm used [18]. This is an empirical estimation based on the ingoing parameters and the random error has not been specifically evaluated in this paper.

$$e = k \sigma^2 \left(1 - \frac{\sigma}{\delta}\right)^{1/2},$$

(1)

When using a sub-image of 32 × 32 pixels, the valid expression for $k$ is 0.030$\sigma^{-1/3}$. The speckle size is given by $\sigma$, so for an average speckle size of approximately 3 pixels, the average correlation factor for an experiment is
0.82, therefore a typical random error in a measurement of this type is therefore 0.04 pixels of displacement.

3.1. Scaling factor

The measurements calculated are in pixels, so to convert to millimetres an accurate scaling factor must be obtained. This is obtained by placing a metal grid in the plane of the lead filings and calculating the period of the grid using an automated fine grid analysis [19]. This allows the horizontal and vertical components of the grid to be separated using a two-dimensional Fourier transform. The phase can then be calculated and unwrapped to give a gradient in pixels/radian and an accurate measure of the number of pixels per period of the grid to sub-pixel accuracy. Using a microscope, the pitch of the grid can be accurately measured in millimetres and hence a scaling factor in pixels/mm can be calculated.
Using this method, a scaling factor of 1 pixel = 1.63 μm has been calculated. Applying this to the above error in pixels gives an error corresponding to 0.06 μm.

3.2. Strain calculations

The strain can subsequently be calculated from the displacement measurements. To do this, the components of the Langrangian strain tensor given in Eqs. (2a)-(2c) [20] must be considered.

\[
\gamma_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2,  
\]  

(2a)

\[
\gamma_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2} \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2,  
\]  

(2b)

\[
\gamma_{xy} = \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} + \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} \right).  
\]  

(2c)
The partial differential components of the $u$ and $v$ displacements need to be calculated. A direct differentiation of $u$ and $v$ will amplify the errors in the measurements, so a better approach is to treat each set of 4 nearest neighbours as an array and average the displacement gradients in these arrays [21]. This is demonstrated in Eqs. (3a)–(3d).

\[
\frac{\partial u}{\partial x}(i,j) = \frac{u(i,j) + u(i+1,j+1) - u(i,j) - u(i+1,j)}{2n},
\]

\[
\frac{\partial v}{\partial y}(i,j) = \frac{v(i,j) + v(i,j+1) - v(i,j) - v(i+1,j)}{2n},
\]

\[
\frac{\partial v}{\partial x}(i,j) = \frac{v(i,j+1) + v(i+1,j+1) - v(i,j) - v(i+1,j)}{2n},
\]

\[
\frac{\partial u}{\partial y}(i,j) = \frac{u(i+1,j) + u(i+1,j+1) - u(i,j) - u(i+1,j+1)}{2n}.
\]

Fig. 6. 0.30 s after powder release.

Fig. 7. 0.35 s after powder release.
The principle strains and maximal shear strains can then be calculated from the data.

4. Results

The results from the first eight successive correlations from the 5 mm funnel can be seen in Figs. 2–9. The arrows represent displacement and are scaled by a factor of 6 to make the movement more visible. The displacements measured are those occurring in the 0.05 s between successive frames.

From these results it can be seen that initially the flow is very localised near the hopper region where the opening is located. After 0.15 s the flow in this region has become so rapid that the speckles in the image have...
become too blurred and there is a region of de-correlation so measurements cannot be made. Above this region, in the main bin, the flow can be seen beginning higher than in the first image. By the third image, 0.20 s into the flow, the same behaviour is visible, but with the displacements beginning higher still and the sides are also beginning to collapse into the central region. In the fourth image the formation of a plug flow is clearly visible and this continues through the subsequent images until a steady state of plug flow is achieved with an apparently constant rate of flow. The localised shearing and principle strains due to this plug flow can also be seen and are illustrated in Figs. 10 and 11. The example shown here is for the correlation...
between images 5 and 6 of the sequence, that is 0.30 s into the flow. The shearing, as located, is very localised to the boundary between the flow and the relatively stationary powder. The strains within the plug flow are effectively zero, which also suggests that there is continuous steady flow in this region. The large areas of strain across the base of the two figures occur because the random pattern has flowed out through the hopper causing de-correlation in this region between the two images in this region.

Fig. 12 shows the displacements at 8.50 s into the flow. In this figure, there is de-correlation down the central section. This is due to the rapid flow rate and loss of most of the markers from the hopper. The sides can now clearly be seen slipping as well and beginning to flow from the hopper as well in a funnel flow.

A smaller 2.5 mm outlet can also be placed at the base of the hopper causing a slower rate of flow. The mean displacement in the central vertical section can now be measured and plotted and is illustrated in Fig. 13. Since the time between correlated frames is always 0.05 s, the degree of displacement is directly proportional to the velocity and hence the rate of flow.

From Fig. 13 it can be seen that the displacements are much greater between successive frames for the 5 mm outlet, which is expected given the faster rate of flow. It can be seen that the maximum displacement, and hence velocity, reached for the larger diameter outlet begins to decrease after at approximately 0.75 s after release. This is in agreement with the observed displacement fields since the flow begins to widen at this time reducing the flow rate. For 2.5 mm outlet the maximum rate of flow does not reach the same magnitude as the 5 mm case as expected. Similar plug flow behaviour dominates the majority of the flow, which is restricted to 11 s in both cases, except for the latter part of the flow. At 5.50 s into the flow, the displacements deviate from plug flow. This is slightly more pronounced at 10.00 s and can be seen in Fig. 14.

This suggests that there is an area around the hopper allowing faster flow than in the main bin and there is clearly a symmetrical boundary occurring between the two different regions of flow. This appears to be the onset of arching which will occur if the outlet diameter is...
below a critical size for the height of bin, angle of hopper and frictions between the powder and the silo walls [22].

5. Conclusions

The experimental results shown here demonstrate the applicability of Digital Speckle Radiography to the study of granular flows in opaque powders in silos. These experiments are performed on a small scale with accuracy of the order of 0.06 μm. The scale of these experiments was determined by the nature of the microfocus X-ray source and the size of the electronic detector. The image correlation process could easily be carried out on a larger scale given larger detection system. The flow behaviour can be observed from these displacement measurements, as can the flow rate and the strains. The method is limited to looking a single plane through the silo, although this plane could be moved in subsequent experiments. By applying two different sizes of outlet at the base of the hopper, a clear difference in the flow behaviour has been observed.

Acknowledgements

S. Grantham thanks Professor J.E. Field for his advice and encouragement. The Cambridge research is supported by QinetiQ and the Engineering and Physical Sciences Research Council, UK.

F. Forsberg thanks the Swedish Research Council (V.R.).

References


Paper B

Full 3D strain measurements on wood exposed to three-point-bending: Analysis by use of digital volume correlation applied to synchrotron radiation μCT image data
Full 3D strain measurements on wood exposed to three-point-bending: Analysis by use of digital volume correlation applied to synchrotron radiation μCT image data

F. Forsberg¹, M. Sjödahl¹, R. Mooser², E. Hack² and P. Wyss²

¹ Division of Experimental Mechanics, Luleå University of Technology, SE-971 87, Luleå, Sweden
² Electronics/Metrology Laboratory, Swiss Federal Laboratories for Materials Testing and Research (EMPA), Überlandstrasse 129, CH-8600, Dübendorf, Switzerland

Abstract
A micro-scale three-point-bend experiment on wood has been carried out. The full 3D strain field of the micro-scale wood structure has been determined by use of digital volume correlation, based on reconstructed 3D image data acquired with synchrotron radiation micro-computed tomography. The wood specimen that measures 1.57 x 3.42 x 0.75 mm³ was scanned in different load states along the three-point-bend load cycle, from unloaded state to failure. Based on the reconstructed 3D wood structure, from two intermediate load states, a region with the dimensions 0.448 x 2.69 x 0.56 mm³ was investigated. The correlation algorithm is based on a Chebyshev polynomial description of the displacements, which allows a continuous representation of the 3D deformation fields. Based on the measured displacement data various 3D strain distributions are calculated. The displacement fields show a structural behavior that reminds of what is expected for a specimen exposed to three-point-bend. However there are also anomalous effects present in the displacement fields that can be coupled to characteristic features in the cellular structure of the wood. Furthermore, 3D strain calculations based on the obtained displacement data shows a concentration of tensile strain in the region where the specimen eventually collapses. The experimental results show that the use of x-ray based tomography with high spatial resolution in combination with digital volume correlation successfully can be used to perform 3D strain measurements on wood, at the micro-scale.

Key words: Digital volume correlation; Digital image correlation; Synchrotron; X-ray tomography; Wood; Microstructure; Three-point-bend; 3D displacement field; Deformation; Strain.
Three-point-bending (TPB) experiments, for fracture toughness analysis, are most often carried out on macro-scale structures, typically on surfaces. This is quite natural since these are the circumstances under which most of the ordinary life fractures are observed. However for complex and inhomogeneous structures these kinds of measurements only give a poor prediction of what is going on inside the material.

Wood is a multi scale material with characteristic features at several hierarchical levels – from annual rings at the cm-level, and visible for the bare eye, down to fibrils at the nano-scale and celllobiose molecules at the Å-scale. It is far from homogeneous – especially at the lower spatial scales - and therefore properties such as fracture toughness is hard to predict and incorporate into models. Many fractures that occur in macro-scale wood structures can be explained from the micro-scale behaviour of the material.

The use of full-field imaging techniques is beneficial when performing fracture toughness experiments, since these permit analysis of large regions without physical interference with the examined specimen. Various methods have been proposed for investigations through use of the three-point-bend formalism – both using interferometry [1-6] as well as correlation based techniques [7-10]. The correlation based methods have gained increased interest in the last decade with the development of high resolution CCD cameras together with the increase of computational resources. The fundamental idea is that two images of the investigated specimen, taken before and after a deformation of the structure, is analysed through cross-correlation. It is of great importance that the imaged structure contains random features –
either natural or synthetic – that the correlation analysis is based upon. In two-dimensions, based on planar image data, this technique is called digital image correlation (DIC) [11] or electronic speckle photography (ESP) [12]. In three-dimensions, based on volumetric 3D image data - typically acquired with computed tomography (CT) – the technique is referred to as digital volume correlation (DVC).

Digital volume correlation was first presented by Bay et al. [13] for measurements of 3D strain fields in trabecular bone tissue exposed to uni-axial compression loads. This kind of bone tissue is ideal for DVC analysis due to a random and isotropic micro-architecture which generates fully stochastic 3D features of high contrast. The method has been considered to have a great potential for non-contact measurements of strength in different bone types and similar research around osteoporoses. Applications like these have therefore been the driving force for the development of the method [14-16]. This kind of applications has furthermore exclusively dealt with compression loads. This is quite natural since the experimental setup in these cases is circular symmetric, which is the preferred geometry for CT-imaging.

DVC was proposed after the introduction of micro-computed tomography (μCT) – a small scale CT technique with improved spatial resolution compared to conventional CT systems, typically in the range 5-50 µm [17]. DVC is continuously under development and has during the last decade been applied to a number of different materials, such as alumina foam and argillaceous rock [18, 19].

On wood, deformation analysis by use of correlation based methods has exclusively been two-dimensional. The analysis has typically been based on surface images captured with conventional optical photography or microscopy. The method has been used for hygro-mechanical analysis to determine displacement and strain fields due to swelling and shrinking of wood [20, 21]. In fracture toughness experiments on wood DIC analysis has been used to determine strain fields due to three-point-bending [22], as well as strain fields at crack tips [23]. Theoretically this kind of correlation based strain analysis could have been applied to wood structures already in the mid-eighties, when CT was proposed as a tool for wood structure analysis [24]. However, in reality, the poor spatial resolution of these systems only allowed macro-scale wood structures to be reconstructed – at the scale of annual rings – which was not suited for correlation analysis. During the last decade, however, the development of CT techniques and the improved spatial resolution have permitted the micro-scale structure to be imaged [25]. Although the cellular structure that is found at this scale show influences of the annual growth and therefore a certain periodicity it can still be considered as reasonably stochastic. The new CT technology has therefore open up the possibility to use correlation based methods to better understand how the wood structure beyond the surface responds to different mechanical loads.

The scope for this work is to determine the full three-dimensional strain field in a small wood specimen, exposed to three-point-bending. Image data, describing the full three-dimensional micro-architecture of the wood specimen during a TPB load sequence, is acquired with synchrotron radiation micro-computed tomography (SRμCT) [25-27]. This CT technique is based on the use of synchrotron x-rays, i.e. a parallel beam of x-rays with a very narrow band width. These semi-coherent x-rays allow magnification optics to be used and hence the spatial resolution can reach values below 1 µm. Finally, the full 3D strain field of the deforming wood structure is determined by use of digital volume correlation.

In this paper we describe a novel approach for the DVC procedure. Instead of describing the displacements, in the conventional way, as constants or linear functions within each correlation window we here choose to approximate them with Chebyshev polynomials. This yields that the displacements are described continuously throughout the analyzed region. A thorough description of the digital volume correlation methodology is given in the following section.
Digital volume correlation

Consider a three-dimensional box $I_1$ consisting of a discrete number of greyscale values evenly distributed within the box. Consider next the box $I_2$, which is an exact copy of $I_1$ only deformed by some external force such that locally a box shaped region has been distorted into a general shape, and moved away from its original position – as shown in Fig.1. A robust way to calculate the deformation between the two boxes $I_1$ and $I_2$ is to minimise the three-dimensional discrete covariance function expressed here as,

$$f(u, x) = 1 - \frac{\sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{k=1}^{P} s_1(x_{i,j,k} - u_{i,j,k})s_2(x_{i,j,k})}{\left( \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{k=1}^{P} s_1(x_{i,j,k} - u_{i,j,k})^2 \right)^{3/2}},$$

where $x_{i,j,k} = [x, y, z]$ is the discrete position vector, $u_{i,j,k}$ is the local deformation vector and a correlation window of size $M \times N \times P$ voxels is used. Furthermore, $s_1$ and $s_2$ are structural copies of $I_1$ and $I_2$, respectively, but with the global greyscale variation removed, i.e. $s_i = I_i - G_i$. Typically $G_i$ is equivalent with the mean value of $I_i$.

To make the minimisation in Eq. (1) solvable we need to make an approximation of the deformation within the correlation window, an approximation that puts continuity constrain on the deformation. The simplest and most commonly used technique is to assume the deformation within the correlation window to be constant [28-30]. A constant value is often sufficient as long as the deformations are of primary interest, the deformations are small and the correlation window can be made small with sufficient reliability [29]. One inherited advantage of such an approximation is that the three unknowns, $u$, $v$ and $w$, are found from the location of the covariance peak without the need of a non-linear optimisation routine. However, including also the deformation gradients in the minimisation has proven to result in a more reliable and accurate algorithm [11, 31], although on the cost of programming complexity. If the strains within the correlation window can be considered to be infinitesimal a constant deformation gradient within the correlation window gives a sufficient approximation. For finite strains, however, also the deformation gradients need to be at least one time differentiable and higher orders should be included in the approximation. To be able to approximate a general deformation up to a given order, $n$, we choose to express the deformation components as,

$$u(x) = \sum_{p=0}^{n} \sum_{q=0}^{n} \sum_{r=0}^{n} a_{pqr} T_p(x) T_q(y) T_r(z)$$

$$v(x) = \sum_{p=0}^{n} \sum_{q=0}^{n} \sum_{r=0}^{n} b_{pqr} T_p(x) T_q(y) T_r(z)$$

$$w(x) = \sum_{p=0}^{n} \sum_{q=0}^{n} \sum_{r=0}^{n} c_{pqr} T_p(x) T_q(y) T_r(z)$$

$$p + q + r \leq n$$
Figure 1. A box within the region of interest (ROI) is distorted into a general shape. The deformation process is described with respect to the deformed state, i.e. an Eulerian description is used. The position vectors of the initial box and the deformed shape is therefore $x_p = u_p$ and $x_p$ respectively. The local displacement of the box due to the deformation is denoted $u_p$.

where $a_{pq}$, $b_{pq}$ and $c_{pq}$ are the Chebyshev components and $T_p$, $T_q$ and $T_r$ are the Chebyshev base functions of the first kind, of order $p$, $q$ and $r$, respectively. There are several reasons why the approximation given by Eq. (2) is convenient. Firstly the form of the approximation makes it straightforward to extend the algorithm to higher dimensions. Secondly, Chebyshev polynomials are known to be close to the minimax polynomial that has the smallest maximum deviation from the true function. Thirdly, Chebyshev approximations can be truncated to lower orders without significantly deteriorate the approximation. Finally, since Chebyshev polynomials are confined to the interval $[-1,1]$ the minimisation becomes numerically robust. The variables $u$, $v$ and $w$ in Eq. (1) are in general continuous, which means that the image $s_\gamma$ needs to be continuous also. Prior to the covariance calculation, therefore, a trivariate spline of given order, $m$, is constructed from the discrete values in $s_\gamma$. Eq. (1) is then minimised using a non-linear minimisation routine based on the interior-reflective Newton method [32]. For the minimisation we also need to provide the $1 \times Q$ gradient vector, $g_f = \partial \hat{f} / \partial \hat{a}_j$, where $a_j$ are either $a_{pq}$, $b_{pq}$ or $c_{pq}$, and the $Q \times Q$ Hessian matrix, $H_f = \partial^2 \hat{f} / \partial \hat{a}_j \partial \hat{a}_j$. The parameter $Q$ is the number of all Chebyshev components satisfying the condition $p + q + r \leq n$ as indicated in Eq. (2) and gives the number of variables to optimise for a given expansion. If for example $n$ is set to 2 then 30 variables needs to be optimised. As a result of the minimisation we get the set \{ $a_{pq}, b_{pq}, c_{pq}$ \} that best fits the features in $s_\gamma$ with the features in $s_\gamma$ as well as the correlation value,

$$\gamma(u; x) = 1 - \hat{f}(u; x),$$

as a measure of the significance of the fit.

One significant problem with the procedure described above is that we need to know a priori that the deformation falls within the covariance peak in order for the minimisation to converge. For an image pair with features at the Nyquist sampling rate this restriction means
the deformation needs to be confined within ±2 pixels. In general deformations may exceed that narrow range by a factor of ten or more and something needs to be done to ensure convergence of the minimisation. One approach would be to provide a qualified guess within the first correlation window and analyse the rest of the areas using overlapping windows. Such an approach would, however, include a hands-on step that will prevent full automation. We have therefore made the choice to perform the correlation in two steps. In the first step the covariance volume $C(\Delta x)$ between $I_1$ and $I_2$ is expressed as,

$$C(\Delta x) = \text{IFFT}\left(\text{FFT}(s_2) \cdot \text{conj}(\text{FFT}(s_1))\right),$$

where $\text{FFT}$ and $\text{IFFT}$ indicates the forward and inverse three-dimensional Fourier transform operator, respectively, and $\text{conj}$ denotes complex conjugation. The advantage with Eq. (4) is that the full three-dimensional discrete covariance volume is obtained in contrast to Eq. (1) that only gives the covariance value for a given set of variables. However, $u$ is here restricted to constant values within the sub-volume and will therefore only provide a coarse sampling of the deformation. Nevertheless, picking the discrete position of the maximum value within $C(\Delta x)$ will provide a good initial guess for the fundamental Chebyshev components $a_{000}$, $b_{000}$, and $c_{000}$, and will ensure the optimization to be within the global minimum peak.

In general the deformation within the sub-volume represented by Eq. (2) will provide a local set of deformations within a larger body. To get the deformation within the whole body the procedure needs to be repeated for a large amount of different sub-volumes throughout the body. Furthermore these sub-volume calculations are carried out with an overlap. The distance between the centres of two neighbouring sub-volumes is given by half the sub-volume dimension, i.e. $M/2$, $N/2$ and $P/2$ for the x-, y- and z-direction, respectively. This yields that every point within the body (apart from points close to the boundary) is represented by eight different sub-volumes. The problem then arises to get continuity between the different sets of deformations. Our approach was developed by Sjödahl and Oreb [33] to solve the problem of stitching different interferograms together and is based on a “centre-of-mass” weighting of different contributions. The final expression for the deformation components in a general point $x$ within the body is given by;

$$u(x) = \sum_{i=1}^{K} \frac{\sum \delta^2 - \delta^2(i)}{(K-1)\delta^2_{\text{mean}}},$$

where $\delta(i) = d(i)/\gamma(i)$ is the scaled distance $d(i)$ between the position of the $i$:th sub-volume centre and point $x$ and the correlation value $\gamma(i)$ is given by Eq. (3). In Eq. (5) $K$ represents the number of overlapping sub-volumes for a given point $x$ and $\delta^2_{\text{mean}} = \sum_{i=1}^{K} \delta^2(i)$. Eq. (5) will hence favour sub-volumes with centres close to the chosen point $x$ and those having high correlation values. The stitching of the final deformation as described by Eq. (5) is not mathematically rigorous in the same sense as for example cubic splines are rigorous, but have proven to be sufficient for the measurements we have performed.

In general strains are of more importance than are deformations and some care needs to be taken in calculating the strains. The concept of DVC as described above deforms the original volume to fit into the grid of the deformed structure, which in continuum mechanics generally
is known as an Eulerian description of the variation within the body. We therefore chose to
calculate the so-called Almansi’s strain tensor given by \[34\];

\[
\epsilon_{ij} = \frac{1}{2} \left[ \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} - \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \right],
\]

(6)

where standard tensor notations has been used. To form the strain tensor we need to calculate
also the deformation gradients throughout the body. One way to do this is to numerically
differentiate the deformation components as given by Eq. (5). That approach involves
choosing an appropriate gauge length that will influence the accuracy and resolution of the
estimate. Another approach is to differentiate the deformation components as given by Eq. (2)
and mapping the final estimate using Eq. (5), where the components of \( \mathbf{u} \) is replaced by the
deformation gradients are expressed

\[
\frac{\partial u_i}{\partial x} = \sum_{p=0}^{n} \sum_{q=0}^{n} \sum_{r=0}^{n} px_{pq} T_p(x) r_{pq} T_r(z)
\]

(7)

\[
\frac{\partial u_i}{\partial y} = \sum_{p=0}^{n} \sum_{q=0}^{n} \sum_{r=0}^{n} qx_{pq} T_p(x) r_{pq} T_r(z)
\]

\[
\frac{\partial u_i}{\partial z} = \sum_{p=0}^{n} \sum_{q=0}^{n} \sum_{r=0}^{n} rx_{pq} T_p(x) r_{pq} T_r(z)
\]

where \( T_p, T_q \) and \( T_r \) again represent Chebyshev base functions of the first kind, of order \( p, q \)
and \( r \), respectively, and \( n \) is the order of the approximation. However, here we also have a
contribution of \( U_{p-1}, U_{q-1} \) and \( U_{r-1} \) which represents the Chebyshev base functions of the
second kind, of order \( p-1, q-1 \) and \( r-1 \), respectively. As before the \( u_i \) represents one of the
three components of the local deformation vector \( \mathbf{u} \) and the \( x_{pq} \) represents either of \( a_{pq} \),
\( b_{pq} \) or \( c_{pq} \). When expressed through Eq. (7) the strain estimate will be independent of the
choice of gauge length, but might become numerically unstable if the strains within the sub-
volume are small and additional smoothing might need to be introduced.

Here, the former approach is used. However the strain calculation is performed at subvolume
level. Based on these results the full region strain fields are finally formed by use of Eq. (5).
Regardless of approach in calculating the deformation gradients, local principal strains and
maximum shear strain can finally be calculated from the eigenvalue solution of \( \epsilon_{ij} \) as
described in any textbook on continuum mechanics, see for example Ref. [34].

**Experimental procedure**

The wood specimen has the dimensions 1.57 x 3.42 x 0.75 mm\(^3\) and is made out of Scots pine
(Pinus Sylvestris). It was cut out from a larger piece by use of a sharp scalpel and the
dimensions finally adjusted by use of very fine sandpaper.
The three-point-bending experiment was carried out using a small scale load device, shown in
Fig. 2, which has been constructed from two metal cylinders connected through four 1.0 mm
CFRP rods. The reason for using carbon fibers as frame material in the construction, apart
from their strength, is that they are only weakly absorbing the x-rays, leaving no visible
shadows in the reconstructed data. The top of the load device holds a screw mechanism that is used for the loading. The screw mechanism contains a M3 screw that is coupled to a 2.0 mm steel cylinder, through a Ø 4.0 mm steel bearing. The steel cylinder, finally, is in contact with the wood specimen, as shown in Fig. 2. A full rotation of the top screw corresponds to a 356 μm displacement of the steel/wood interface. At the bottom, the load device has a Ø 3.15 mm connector pin for mounting to the SRμCT-control stage, at the beamline. The specimen is scanned first in unloaded state and then again at three different load states, until it finally collapses. Each subsequent scan is carried out with an increased load, caused by rotating the top screw 90 degrees and thus an applied displacement of 89 μm. The TPB-experiments has been carried out at the TOMCAT beamline at the Swiss Light Source (SLS), located at the Paul Scherrer Institute (PSI) in Villigen, Switzerland. The synchrotron x-ray energy was 11 keV and the object-to-detector distance was 7.0 mm. The detector consists of a 20 μm YAG-Ce-scintillator, an optical microscope with 4x-magnification, and a low-noise fast-readout CCD of 2048 x 2048 pixels and 16 bit dynamic range. The field of view (FOV) was 3.58 x 1.22 mm², corresponding to 2048 x 701 pixel². The spatial resolution of the detector chain under these conditions is 2.15 μm at 10% MTF [26]. A number of 2501 projections were captured for each scan of the specimen, equiangular.

Figure 2. The wood specimen positioned in the load device during the three-point-bend experiment.

Figure 3. The geometry and loading of the wood specimen (a). The dimensions of the wood specimen are 1.57 x 3.42 x 0.75 mm³. The load, $F_b$, is applied at point $p_1$. The specimen rest on two carbon fiber supports, at positions $p_2$ and $p_3$. $F_{r2}$ and $F_{r3}$ denote the reactive forces in point $p_2$ and $p_3$, respectively. (b), (c) and (d) shows a sequence of reconstructed LR-slices through the centre of the specimen and TPB-load configuration, at two subsequent load states, (b) and (c), and after failure (d).
distributed over 180 degrees. The exposure time was 300 ms and the total scan time was approximately 12 minutes. The reconstruction time, for each data set, was approximately 10 minutes and the reconstruction was carried out on a Linux cluster using a parallel beam filtered back projection algorithm. The reconstructed volumes have the dimensions 3.58 x 3.58 x 1.22 mm$^3$, corresponding to 2048 x 2048 x 701 voxels. Hence the voxel dimension is 1.75$^3$ μm.  

The geometry and loading of the wood specimen is described schematically in Fig. 3 (a). The load, $F_b$, is applied at point $p_1$ using the 2.0 mm steel rod. Furthermore, the wood specimen rests on two 1.0 mm carbon fiber reinforced plastic (CFRP) rods, at positions $p_2$ and $p_3$. $F_{r2}$ and $F_{r3}$ denote the reactive forces in point $p_2$ and $p_3$, respectively. The bundles of gray force vectors illustrate the fact that the contact regions are not strict mathematical points. The forces are instead distributed over some limited area of the specimen. Furthermore, Fig. 3 (b), (c) and (d) shows a sequence of reconstructed longitudinal-radial-slices from the specimen, taken through the centre of the TPB-load configuration, at three subsequent stages along the load cycle - (b) and (c) in first and second loaded state, respectively, and (d) after failure. One can see that the longitudinal wood structure bend as the load increases and that the wood structure in the three-contact regions are deformed non-elastically. The two loaded states of the wood structure presented in Fig. 3 (b) and (c) are also the ones that have been used for the correlation analysis and will hereafter be referred to as the reference state and the deformed state, respectively.

After reconstruction the 3D image data was post processed with the image processing software ImageJ in order to get a rough alignment between the four image data sets. The rough alignment compensates for rotational and translational deviations that occur during the loading process. A set of characteristic features found within the carbon CFRP rod supports were used as landmarks for this alignment.

Furthermore, all reconstructed volumes were aligned with the tangential (T), longitudinal (L) and radial (R) direction of the wood structure, as shown in Fig. 4. For the correlation analysis, however, the conventional x, y and z-notation is used. The reconstructed three-dimensional wood structure, shown in Fig. 4, comes from a region that measures 112 x 112 x 56.0 μm$^3$. A region $R_t$ of size 0.448 x 2.69 x 0.56 mm$^3$, described by 128 x 768 x 320 voxels, is analysed using digital volume correlation. The position and size of this region within the wood specimen is schematically shown in Fig. 5.

Figure 4. The reconstructed 3D cellular structure in a region of size 112 x 112 x 56.0 μm$^3$. 
Figure 5. Schematic layout of the wood specimen and the position and size of the correlated region, $R_c$. The physical dimension of $R_c$ is $0.448 \times 2.69 \times 0.56$ mm$^3$, described by $128 \times 768 \times 320$ voxels.

For this analysis a 2 x 2 binning was used in the xy-slices of the reconstructed volume data. Thus, using limited computational resources (a PC with two Intel 1.67 GHz processors and 2 GB of RAM), a larger physical region has been analyzed on the cost of decreased spatial resolution.

The sub-volume size used in the correlation analysis is $64^3$ voxels, corresponding to a physical region of the size $0.224 \times 0.224 \times 0.112$ mm$^3$.

The extension of $R_c$ in the y-direction makes it interesting since it covers a region that contains all three contact regions. The region $R_c$ was shifted 52.4 $\mu$m in the z-direction in the deformed data set in order to cover as much of the displaced structure as possible and prevent decorrelation. Therefore, all of the wood structure found in the reference volume, shown in Fig.6 (a), is also present in the deformed volume, shown in Fig.6 (b). The tip of the steel bar is visible in the upper region of the deformed volume. Furthermore, the two CFRP-supports are, due to the shift of the deformed $R_c$, clearly visible in the bottom region in Fig. 6 (b).

Figure 6. 3D renderings of the reference volume data (a) and deformed volume data (b) in region $R_c$. 

Results and discussion

Fig. 7 (a) shows the full 3D v-displacement field in the region $R_c$, describing the displacements in the y-direction. Fig. 7 (b) shows a 2D yz-slice from the centre ($x=0.224$ mm) of the 3D field in (a). Moreover the v-displacements in this plane is also visualised with a vector field. From the results in Fig. 7 (a) and (b) one can see compression and tension bands in the upper and lower region, respectively. These two bands are separated by a neutral band, with displacements near zero, in the middle. Although the general appearance of the v-

![Figure 7](image)

Figure 7. The 3D v-displacement field in region $R_c$ (a), and the 2D centre yz-slice from the same region (b). Moreover, in (b), the 2D v-displacement field is also presented as a vector field. The appearance with a compression and tension band in the upper and lower region respectively, separated by a neutral band in the middle, agrees with what is expected for a specimen exposed to three-point-bend.

![Figure 8](image)

Figure 8. The 3D w-displacement field in the region $R_c$, describing the displacements in the direction parallel with applied force. The displacements range from 0 μm in the two lower contact regions to approximately 68 μm right beneath the upper contact region, where the force is applied.
displacement field agrees with what is expected from a sample exposed to three-point-bend there are also irregular behaviour that may be due to the anisotropy of the wood structure. There are bands with increased density and stability, formed by the longitudinal junction of tracheids. On the other hand there are also weaker regions in the longitudinal centres of the tracheids, which can be considered as hollow ellipsoidal shapes, where the density has its local minima. Furthermore, Fig. 7 (b) also shows that the centres of the compression and tension, in the v-displacement field, is shifted towards the right hand side and located approximately at $y=1.5$ mm. Therefore, both the minimum and maximum values of the v-displacement are found at the left hand side of the analysed region. This non-symmetric behaviour agrees with the fact that there is a certain lack of symmetry, of the three contact regions, which can be seen in Fig. 3 (a).

Fig. 8 shows the 3D w-displacement field describing the structural deformation in the direction of the applied force. It ranges from 0 $\mu$m in the two CFRP-support regions to 68 $\mu$m right beneath the tip of the steel bar. The reason why the measured maximum displacement is less than the 80 $\mu$m applied displacement is probably due to non-elastic deformation in the contact regions – a phenomenon that can be observed in Fig. 3 (b) – (d). Another proof for this non-elastic behavior is the very rapid decrease in w-displacement that is observed in both of the two support regions.

Fig. 9 (a), (b) and (c) shows the $\varepsilon_{yy}$, $\varepsilon_{yz}$ and $\varepsilon_{zz}$ strain distributions, respectively. $\varepsilon_{yy}$ and $\varepsilon_{zz}$ represents the normal strain in y- and z-direction, respectively, while $\varepsilon_{yz}$ represent the shear strain in the yz-plane. As predicted from the v-displacement field in Fig. 7 (a) one can find two regions with strain concentrations in the centre of $\varepsilon_{yy}$, along the y-direction. These two, found in the upper and lower centres describes the compressive and tensile strain, respectively. The region with tensile strain corresponds to the region where the failure eventually takes place – as seen in Fig. 3 (d).

Furthermore, the shear strain that is introduced between the compression band, in the upper region, and the tension band, in the lower region, is described by $\varepsilon_{yz}$. The shear strain has strong concentrations of both positive and negative shear half way through the region along the z-direction – where compression meets tension (the neutral band in the v-displacement field).

The $\varepsilon_{yz}$-distribution, shown in Fig. 9 (c), contains a concentration of compressive strain in the centre, right beneath the upper contact region, as expected from the w-displacement field in Fig. 8. The strongest minimum (compressive strain) is however found in the boundary region at the left hand side. From the w-displacement field one finds that there is a slight discontinuity in the corresponding region. Moreover, one also finds well pronounced strain concentrations in the corresponding region, in both $\varepsilon_{yy}$ and $\varepsilon_{yz}$. However, no structural defect or crack formation that matches this region geometrically can be found from visual inspection of the wood structure, in Fig. 6 (a) and (b). The explanation of the discontinuity and the high strain levels are instead found when we search beyond the region $R_c$. In Fig. 3 (b) and (c), which both show longitudinal-radial slices through the reconstructed full specimen (in different load states), one can find a well defined crack that starts at the edge of the specimen, on the right hand side, approximately in the centre along the radial direction (z-direction). Although the extension of this crack seems to be rather limited, from the fact that it cannot be distinguished in the region $R_c$, there are undisputable effects of it in the 3D strain fields, in Fig. 9.
Figure 9. The 3D strain distributions $\varepsilon_{yy}$ (a), $\varepsilon_{yz}$ (b) and $\varepsilon_{zz}$ (c) in region $R_c$. $\varepsilon_{yy}$ and $\varepsilon_{zz}$ represent the normal strain in y- and z-direction, respectively, while $\varepsilon_{yz}$ represent the shear strain in the yz-plane.

Conclusions
A micro-scale three-point-bend experiment on a wood specimen has been investigated by use of synchrotron radiation µCT and digital volume correlation. The full 3D wood specimen has been reconstructed in different load state along the three-point-bend load cycle, from unloaded state to failure. Based on the 3D cellular structure of the wood in two of the intermediate load states the full unrestricted deformation field has been determined by use of digital volume correlation. Furthermore, the obtained displacement data has been used to calculate a number of strain distributions for the analysed region that can be used to understand the micro-scale processes that take place in the material when exposed to three-point-bend.
The v-displacement field, which represent the deformation in the longitudinal direction, shows that the specimen contains two bands – one compression band in the upper region and one tension band in the lower region - a behaviour that is expected for specimens exposed to three-point bend. However, also anomalous deformation patterns are resolved that can be traced to the micro-architecture of the wood structure.

The measured w-displacement field is rather symmetric with a maximum of approximately 68 μm in the upper contact region, and with a radial decrease down to 0 μm in the two supports. From the displacement field one can also suspect a certain amount of non-elastic deformation near the contact regions, which would explain the difference between measured maximum displacement, 68 μm, and the applied displacement, 80 μm.

The strain field $\varepsilon_{yy}$, which describes the normal strain field in the longitudinal direction, show high levels of tensile strain in the lower centre region – the region where the specimen eventually collapse. Furthermore, high strain concentrations located at the right hand border of the analysed region reveals a crack that is found at the edge of the specimen - not visible in the analysed region.

In general we can conclude that the complex cellular structure of wood at the μm-scale is well suited for digital volume correlation analysis. Finally, the use of synchrotron radiation μCT in combination with digital volume correlation opens up the possibility to perform 3D strain measurements on wood, at the micro-scale.

Acknowledgements

The authors would like to thank Samuel McDonald at the Swiss Light Source (SLS) at the Paul Scherrer Institute (PSI), Villigen, Switzerland, for helping us at late hours while performing the synchrotron experiments at the TOMCAT beamline.

F. Forsberg would like to thank The Swedish Research Council (VR) for the funding of his research project. Furthermore, he would like to acknowledge the three foundations The Royal Swedish Academy of Science, Nordeas Norrlandstiftelse and The Wallenberg foundation for financial support during his stay at the Swiss Federal Laboratories for Materials Testing and Research (EMPA), in Dübendorf, Switzerland, through scholarships.

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3D micro-scale deformations of wood in bending: Synchrotron radiation μCT data analyzed with digital volume correlation
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F. Forsberga, R. Moosera, M. Arnolds, E. Hackb, P. Wyssb

a Division of Experimental Mechanics, Department of Applied Physics and Mechanical Engineering, Luleå University of Technology, SE-971 87 Luleå, Sweden
b Electronics/Metrology Laboratory, Swiss Federal Laboratories for Materials Testing and Research (Empa), Überlandstrasse 129, CH-8600 Dübendorf, Switzerland

ABSTRACT

A micro-scale three-point-bending experiment with a wood specimen was carried out and monitored by synchrotron radiation micro-computed tomography. The full three-dimensional wood structure of the 1.57 × 3.42 × 0.75 mm³ specimen was reconstructed at cellular level in different loading states. Furthermore, the full three-dimensional deformation field of the loaded wood specimen was determined by digital volume correlation, applied to the reconstructed data at successive loading states. Results from two selected regions within the wood specimen are presented as continuous displacement and strain fields in both 2D and 3D. The applied combination of synchrotron radiation micro-computed tomography and digital volume correlation for the deformation analysis of wood under bending stress is a novel application in wood material science. The method offers the potential for the simultaneous observation of structural changes and quantified deformations during in situ micro-mechanical experiments. Moreover, the high spatial resolution allows studying the influence of anatomical features on the fracture behaviour of wood. Possible applications of this method range from bio-mechanical observations in fresh plant tissue to fracture mechanics aspects in structural timber.

1. Introduction

Mechanical properties of engineering materials are assessed by a wide variety of experiments. These experiments are typically carried out on a rather high spatial scale, which is quite natural since many observable daily life fractures and structural changes take place at this scale. However, to gain a more fundamental understanding for the behaviour of materials under load, it is desirable to carry out the experiment at a lower scale—the scale of the natural structure of the material. Experimental work of this kind is most often based on pure surface information, which also is quite natural since this is what we see—either with our eyes or through a camera. Most materials, however, are far from homogeneous and can not be appropriately described from surface analysis alone. Even though, for example, a crack may start at the surface, most of the critical regions, which will determine the eventual crack propagation, lie somewhere within the bulk material—hidden for pure surface methods.

Based on this we formulate the motivation for the work presented here—to obtain three-dimensional (3D) experimental results from a wood specimen at a spatial scale corresponding to the natural cell structure of the material. The specimen will be stressed using a three-point bending (TPB) setup and imaged by use of synchrotron radiation micro-computed tomography (SRμCT). The image data describe the full 3D micro-architecture of the wood specimen. The deformation of the wood structure due to the induced load will be determined by use of digital volume correlation (DVC) applied to the acquired image data.

Wood as a natural, highly structured, but inhomogeneous composite is a particularly interesting material for such studies. Although we mostly observe wood at a macroscopic level, the characteristic properties of wood are highly dependent on the cellular architecture at the micro-scale. In wood from gymnosperms (softwoods) properties like density, water resistance and strength are all strongly coupled to micro-structural parameters such as length, diameter and shape of their main cell types, the tracheids (Fig. 1). Typical dimensions of softwood tracheids are 2–4 mm in length, 30–50 μm in diameter and 5–10 μm for cell wall thickness. Wood from angiosperms (hardwoods) consists of different cell types and is structured more complex.

Various methods are used for imaging the wood structure at the cellular level. Optical microscopy is historically the most frequently used method for this kind of investigations. It is normally applied to two dimensional surfaces. However, in transmission mode (transmission light microscopy) it is also possible to gain a...
tions of 3D cellular wood structures by use of SR imaging. The images presented in the former publication were so-called images from micro-computed tomography (μCT), and the one used here is very closely related, by Trtik and co-workers and the one used here is very closely related. Although the imaging method proposed by TRTik et al. (2007) explicitly suggests in situ micro-mechanical experiments.

The first fracture toughness analysis on a wood specimen, carried out by TPR experiments, is, to the author’s knowledge, a study by King et al. (1999). Here, they used a tensile testing machine, normally used for testing of metal specimens. The results were force/displacement curves for specimens of different wood orientation, in dry and wet state. There is however no structural analysis in this kind of experiments.

In mechanical material tests the use of optical methods and other full-field imaging techniques are beneficial, since they permit the observation of large surfaces without physical interference with the examined specimen. Moreover they offer a far better spatial resolution than many of the conventional methods used to measure strain, such as for example measurements by use of strain gauges. A lot of experimental work with pure optical methods has been carried out on various materials—mainly based on Moiré-interferometry (Du et al., 1992; Guo et al., 1993; Schultheisz et al., 1998), but also on other close related optical phenomena (Bruck and Rosakis, 1993; Durig et al., 1991; Lee and Rosakis, 1993).

A link between imaging and deformation analysis is given by correlation based methods. Two-dimensional correlation analysis, applied to ordinary planar images, is called digital image correlation (DIC) (Sutton et al., 1983), or digital speckle photography (DSP). DIC methods have gained increased interest in the last decade with the development of high resolution CCD cameras together with the increase of computational resources. Furthermore, these methods have a more or less straightforward experimental procedure which makes them rather easy to implement. The fundamental idea is that the deformation field of the investigated specimen is obtained through cross-correlation between two images of it, taken before and after load changes. It is of high importance that the investigated structure contains random features, or speckles, that the correlation analysis can be based upon. DIC has been applied for investigation of TPR-specimens in several experimental studies (Hild and Roux, 2006; Rethore et al., 2008; Yao et al., 2006).

DIC as a tool for analysis of wood structures under deformation was first proposed by Cho et al. (1991), based on digitized video frames. The same methodology has later also been used at a microscopic level for crack propagation analysis (Thuvander et al., 2000). Most of the applications have been surface methods based on optical image data. However in 2001, Danvind and Synnergren used the technique for studies of processes taking place inside the wood, using conventional CT (Danvind and Synnergren, 2001). This work that mainly covered the shrinkage process of drying wood eventually resulted in a PhD-thesis (Danvind, 2005).

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Full 3D deformation analysis by use of DVC was first proposed by Bay et al. (1999). This 3D extension of DIC was used for strain calculations in trabecular bone tissue exposed to uni-axial compression load. The technique has since then been further developed, through studies that mainly have been based on the same type of bone structure and loading (Smith et al., 2002; Verhulp et al., 2004). Furthermore, in recent years, the technique has also been compared with finite element analysis (Zauel et al., 2006). Therefore, the 3D analytical framework now can be complemented with fully comparable experimental results, which yields better understanding, especially in non-homogeneous regions, where it’s hard to predict the behavior of the material by use of simulation tools.

Most of the experimental work based on volume correlation, presented in scientific journals, has been conducted based on image data from micro-computed tomography (μCT). However,
Lenoir et al. (2007) recently conducted DVC-based strain measurements on SRµCT data of argillaceous rock exposed to tri-axial compression—although at a rather coarse scale compared to what is presented here.

Until now, however, volume correlation analysis has exclusively been applied to image data from compression experiments. This is quite natural since the symmetry of that sort of experimental setup is more suited for computed tomography. However, with this experimental work it is shown that also TPB experiments can be analyzed using the same methodology. In this sense the combination of SRµCT and DVC for the deformation analysis of wood under bending stress is a novel application.

2. Experimental procedure

2.1. Wood specimen

A bar-shaped wood specimen with the dimensions of 1.57 × 3.42 × 0.75 mm$^3$ (tangential × longitudinal × radial) was made out of Scots pine (Pinus sylvestris). It was cut from a larger piece by use of a sharp scalpel and calibrated by use of very fine sandpaper. Fig. 1 shows a rendering of the analyzed wood structure. The displayed region has the physical dimensions 147 × 175 × 77 μm$^3$. The coordinate system describes the orientation within the wood structure with T, J and R denoting the tangential, longitudinal, and radial direction, respectively. However, for the correlation procedure the conventional xyz-notation is used, with alignment to the wood structure as described in Fig. 1. Furthermore, characteristic features in the wood structure such as the tracheids (a), bordered pits (b) and radial oriented uniseriate rays (c) are marked in Fig. 1. These features constitute a delicate transportation network for water and nutrition. For more information on wood anatomy and interactions in wood consult for example the standard book of Siau (1995).

2.2. Loading device and CT scanning

The experimental setup of the TPB experiment is shown in Fig. 2. The loading device (Fig. 2a) was constructed from two metal cylinders connected through four 1.0 mm carbon fiber reinforced plastic (CFRP) rods. The CFRP rods hold the configuration together through the use of very strong adhesives. The reason to use CFRP as plastic (CFRP) rods. The CFRP rods hold the configuration together due to this 90°rotation of the screw is 89 μm. Due to some deformation in the loading device this is somewhat less than the expected 125 μm from the rotation of the M3 screw. The fourth and last scan was carried out after that the specimen has fractured. The subsequent 3D deformation analysis was carried out between the first loaded state (‘reference state’) and the second loaded state (‘deformed state’).

The measurements were carried out at the TOMCAT beamline at the Swiss light source (SLS), PSI Villigen, Switzerland. The photon energy used was 11.0 keV and the object-to-detector distance was 7.0 mm. Further, the detection chain consists of a 20 μm YAG-Ce-scintillator, an optical microscope (with magnification range between 1.25× and 20×) and a low-noise fast-readout CCD of 2048 × 2048 pixels and 16 bit dynamic range. In the experiment a magnification factor of 4 was used and the field of view (FOV) was 3.58 × 1.22 mm$^2$, corresponding to 2048 × 701 pix$^2$. The spatial resolution of the detector chain under these conditions is 2.15 μm at 10% MTF (Stampfoni et al., 2002). A number of 2501 projections were captured for each scan of the specimen, equiangular distributed over 180°. The exposure time was 300 ms and the total scan time was approximately 12 min.

The reconstruction of the 3D image was carried out on a Linux cluster using a parallel beam filtered back projection algorithm. The reconstruction time was approximately 10 min. The reconstructed volume contained 701 slices, each with the dimensions 2048 × 2048 pixels$^2$. The physical dimensions of the reconstructed data were 3.58 × 3.58 × 1.22 mm$^3$ and thus the isotropic voxel dimension is 1.75 × 1.75 × 1.75 μm$^3$.

After reconstruction, the image processing software ImageJ was used for alignment of the volume data. Each of the data sets from

![Fig. 2](image)

Fig. 2. The photo (a) shows the actual experimental setup with the specimen positioned in the loading device, in front of the detector unit. (b) shows the geometry and loading of the specimen. The bending force, $F_b$, acts in point $P_3$, and the reactive forces $F_{r2}$ and $F_{r3}$ act in point $P_2$ and $P_3$, respectively. The bundles of gray vectors denote the fact that the forces are distributed over some limited area of the specimen, rather than in a strict mathematical point.
different load states had an angular as well as a translational deviation from the corresponding unloaded state data set. This deviation arises from the loading process, where the top screw of the load device is rotated—which also slightly affects the whole load set-up. A set of characteristic features found within the carbon CFRP rod supports were used as landmarks for this alignment. In addition, the wood structure coordinate system was aligned with the coordinate system used for the correlation analysis, as described earlier. After compensation of the angular and translational deviation and wood structure-alignment, the datasets were cropped so that only the wood specimen itself was present. This step reduced the original storage size of each data set by roughly a factor of 5, from 5.89 GB to 1.15 GB, which makes the further treatment easier. The handling of the large data sets has been carried out on a 64-bit Linux workstation, using eight Intel 2.66 GHz processors and 16 GB of RAM. The reduction of image data was necessary since the correlation software only were accessible on a PC with less computational resources—two Intel 1.67 GHz processors and 2 GB of RAM.

2.3. Correlation procedure

Two different regions have been analyzed within the bent wood specimen. Fig. 3a and b give a geometrical description of where within the wood specimen these two regions, denoted R1 and R2, are found. The reason for selecting special regions for the correlation analysis, instead of using the full reconstructed specimen, is simply due to limited computational resources. In Fig. 3a and b a pre-existing crack is also indicated, which is running all the way through the wood specimen in the longitudinal direction. This crack is radial oriented and was probably induced during the drying of the wood. Region R1 contains the wood structure between the loading steel rod and the crack at a high spatial resolution to resolve local variations better. Region R2 is representing a small bending beam running along the crack and including the area of the CFRP rod supports. The physical dimensions of region R1 are 0.895 × 0.895 × 0.224 mm³, which in the image data is described by 512 × 512 × 128 voxels. The dimensions of R2 are 0.448 × 2.685 × 0.559 mm³ corresponding to a region of 256 × 1536 × 320 voxels in the original image data. A 2 × 2 binning was, however, performed on the R2 image data, along the tangential and longitudinal direction, prior to the correlation analysis. Therefore, using limited computer resources, a larger physical region has been analyzed on the cost of decreased spatial resolution. Here, the volume correlation procedure is described only briefly. For a thorough description of the methodology please consult reference Forsberg et al. (2009).

During the correlation procedure the correlated volumes are divided into smaller regions—sub-volumes or correlation windows. The displacements within each of these sub-volumes are found through minimization of the cross-covariance function between the reference- and the deformed sub-volume data. The result gives the displacement field that deforms the original volume to fit into the grid of the deformed structure. The displacements are here represented continuously as Chebyshev polynomials. The full set of results from all sub-volumes is finally mapped together to one continuous displacement field, covering the whole investigated volume.

A requirement for the correlation procedure to work is that the volume data contain random 3D features of roughly the same size in all dimensions. Although there is a certain periodicity in the wood structure, as can be seen in Fig. 1, it can still be considered as reasonably random—especially on the micro-scale. Also, the extension of the tracheids in the longitudinal direction (y-direction) makes these features far from isotropic, which yield a reduced accuracy along this direction. However, as can be seen in Fig. 1, the structure in the longitudinal direction also contains features far smaller than the tracheids such as uniseriate rays and bordered pits. These features compensate the lack of structure due to tracheids along this direction, and ensure sufficient accuracy.

The sub-volume size used in the correlation analysis is 64³ voxels, which in R1 and R2 correspond to 0.112³ mm³ and 0.224 × 0.224 × 0.112 mm³, respectively. During an initial stage in the analysis of R1 it was however required to perform the calculations in sub-volumes three times that size, i.e. 192³ voxels, or 0.336³ mm³. The reason for this is to extend the search region for the displaced 3D texture, which becomes necessary in regions near the crack, found in R1.

The strains, denoted ε, have been calculated based on the Almansi’s strain formulation (Fung, 1994). Furthermore, we have in this report chosen to concentrate on the normal strain in the y-direction, εyy, which is given by

\[
\varepsilon_{yy} = \frac{\partial v}{\partial y} - \frac{1}{2} \left( \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right). \tag{1}
\]

where u, v and w are the deformation components in the x-, y- and z-direction, respectively. In regions where the displacements are small, the square terms can be assumed to be negligible and Eq. (1) is reduced to the v-gradient with respect to y. However, generally, all terms need to be taken into account. Moreover, the deformation gradients are calculated through numerical differentiation of the deformation components at sub-volume level. The 3D strain in the full analysed region is finally obtained, from these results, through a mapping procedure, as described in ref (Forsberg et al., 2009).

![Fig. 3](image-url)  
Fig. 3. (a) and (b) show the position of the two correlated regions, R1 and R2, within the wood specimen, from two different angles. Also indicated is a radial crack that runs all the way through the specimen along the longitudinal direction.
3. Results and discussion

Two complementing types of results are available from the analysis of the CT data of the wood specimen in bending. First, one can visually observe the structural changes in the wood specimen based on reconstructed 3D images. Second, deformations can be quantified for the whole specimen volume based on the results of the DVC calculations.

3.1. Structural observations

Fig. 4a–d show the reconstructed wood structure in two orthogonal planes in the reference and deformed state. The planes have been chosen so that they coincide with the centre of the steel rod through which the force is applied. Both elastic and plastic (permanent) deformations can be observed. In Fig. 4a and c, where the radial–longitudinal plane is shown, one can see that the wood structure is elastically bent downwards as the load increases. Furthermore, one can see all three contact points—the two CFRP support rods in the bottom and the steel rod at the top—and how the surrounding wood structure in these regions is deformed permanently. The tracheids close to these contact points collapse under the excessive compression perpendicular to the grain, which is particularly apparent in Fig. 4b and d, where the wood structure in the radial–tangential cross-section is shown. From this angle one can also see the radial oriented crack located approximately in the center of the specimen. This crack, which is running along the longitudinal direction all the way through the sample, widens with increasing load.

The wood structure in R1, in the reference state, is shown in Fig. 5. In Fig. 5a a volume rendering of the full 3D region is presented, while (a-i), (a-ii) and (a-iii) show three different cross-sections through the specimen. In the reconstructed wood structure many anatomical features are visible in good detail. Similar to Fig. 1, in the TR cross-section (a-ii) the rather simple, honeycomb-like cell structure of the earlywood tracheids is visible. Particular wood-specific features are the radial oriented wood rays in the TL plane (a-i) and the circular pits (connecting openings between tracheids) in the RL plane (a-iii).

3.2. Deformations

The results of the DVC analysis are the displacement components, which are denoted u, v and w and describe the displacement in the x-, y- and z-direction, respectively. Each of these displacement component fields is continuous and describe the displacement in every point inside the region. In Fig. 6a the full three-dimensional w-displacement field in R1 is shown. In Fig. 6 (a-i), (a-ii) and (a-iii) the two dimensional w-distribution in three orthogonal planes at position i, ii and iii in (a) are shown, respectively. These planes correspond geometrically to the ones shown in Fig. 5a-i, a-ii and a-iii.

From these results it is visible that there is a region of maximum w-displacement in the center of the xy-plane, right under the point where the load is applied. The w-displacement at this position is approximately 60 μm, which is about 67% of the w-displacement applied by the loading screw. The clearly visible indentation under the spherically shaped tip of the steel rod is confirming the already visually recognized excessive compression perpendicular to the grain. Similar indentations can be found near the lower CFRP support rods (see Fig. 4). These findings indicate that the loading device is not only introducing pure bending in the wood specimen but also stresses perpendicular to the grain, which is a particular weak loading direction in the anisotropic behavior of wood. For the interpretation of the results, this has to be taken into account. For future applications it is thus important to include also the mechanical point of view in the design of such experiments (e.g. apply higher span-depth ratio and improve load introduction).

The two parts on opposite sides of the crack react differently to the increased load. This can be seen in Fig. 6a, a-i and a-ii where the w-displacement on the left-hand side of the correlated region is much smaller than on the right-hand side. This difference in behavior is caused by the crack on the under-side of the specimen, which is dividing the bending specimen in two separate parts. In fact, the crack is clearly impeding the stress transfer from the steel rod to the left-hand side of the specimen.

Finally, the w-displacement field in the yz-plane shown in Fig. 6a-iii describes the downwards bending-motion over the height of the specimen, with maximum displacement in the upper

![Fig. 4. The wood structure in two orthogonal planes through the centre of the specimen, just beneath the point where the bending force is applied. (a) and (c) show the radial–longitudinal wood structure at the reference and deformed states respectively, while (b) and (d) show the radial–tangential cross-section. In (c) and (d) one can see how the wood structure is elastically bent downwards as the load increases. In (b) and (d) the permanent deformation of the wood structure close to the contact points and the widening of the radial oriented crack under increasing load can be observed.]
centre caused by the above mentioned additional compression perpendicular to the grain. However, as it can be seen in Fig. 6a-iii, this behavior is somewhat disturbed near the centre, where we find a local minimum of displacement. This irregularity is probably due to anatomical features (e.g., coincidence of overlapping tracheid ends and wood rays) leading to a local stiffening of the wood structure, as shown in Fig. 5a-iii, or may be influenced by edge effects of the nearby crack.

From these observations we conclude that the spatial resolution of our method is sufficiently high to resolve abnormal effects in the displacement field due to inhomogeneities in the microstructure of the wood or structural defects like cracks.

It is clear from Fig. 6 that, from a three-point-bend perspective, the structural response of the specimen, due to the increased load, is better represented by the region on the right hand side. Therefore, R2, which covers almost this entire region, gives an essential description of the significant processes that are of main interest in this study. The 3D w-displacement field in R2 is shown in Fig. 7a, on top of the wood structure of the deformed state. Similar to region R1, the maximum deformation of 68 μm is located right beneath the steel tip, and corresponds to 76% of the applied displacement. The deformation decreases with distance from the centre, down to 0 μm in the two support regions. Fig 7b shows the 2D w-displacement field (without wood structure) on the front face of the 3D field. Here, one can see that the loading is slightly non-symmetric, which agrees with the geometry described in Fig. 2b. Under the assumption that the specimen can be regarded as an elastic beam and that the curvature of the bent beam is negligible, the deformation in a cross-section (xz-plane) should be constant. If we exclude the three contact regions where we find large in-elastic deformations, this seems to be a fairly good approximation. If we follow the maximum, in the top centre region, as it propagates downwards we find that the displacement is reduced to approximately 58 μm as we reach the bottom. This 85% reduction in defor-
information is probably due to inelastic effects that absorb the applied bending force as it propagates downwards in the wood structure. Of particular interest in a bending situation are the \( v \)-displacements, which correspond to the deformations in the longitudinal direction of the wood fibers. In Fig. 8a the full 3D \( v \)-displacement field is shown for R2 together with a sequence of 2D fields (a-i), (a-ii) and (a-iii), corresponding to the \( x \)-positions i, ii and iii, respectively. As expected in the linear (elastic) phase of a bending experiment, the deformations indicate a compression movement (displacements towards center) on the upper side and a tension movement (displacements away from center) on the lower side of the bending beam. As in the earlier example with the \( w \)-displacement, there are some obvious irregularities in the displacement fields, which are most likely caused by local anatomical features.

For the interpretation of these results in engineering applications, it is useful to calculate (dimension-less) strain fields from the absolute displacement data. In Fig. 8b this is done regarding the normal strains in \( y \)-direction for the same examples as in Fig. 8a. The highest compression strains appear in the center of the upper side of the beam right under the loading point, while in the center of the lower side we observe the highest tension strains. In mid-height of the beam the neutral strain axis is indicated and in the region of the CFRP supports the longitudinal strains are also almost zero. These strain fields agree with the expected strains from beam bending theory.

Again, there are some apparent irregularities in the strain fields which, however, should be interpreted with caution, keeping in mind the small numerical values of the strains and the given spatial resolution of the CT images. However, on the right hand end of the beam one can see a sharp discontinuity at mid-height, where no structural defect or crack formation that matches this region geometrically is found from visual inspection of the wood structure. The discontinuity and the high strain levels are instead probably due to a shear crack formation, visible also in Fig. 4a and c.

4. Conclusions

We have conducted an in-situ CT micro-mechanical test in the form of a TPB experiment on a small-scale wood specimen. The 3D wood structure has been reconstructed at cellular level, at several different stages during a loading cycle, by use of synchrotron radiation micro computed tomography. The full 3D deformation field, between two successive scans under a different loading, has been determined by use of digital volume correlation. Data from two selected regions of different shape, spatial scale and geometrical location have been analyzed. The results are presented as continuous displacement and strain fields—in both 2D and 3D.
The first region, where the analysis was carried out with full resolution, contains a large crack that introduces a crack-related response to the increased load—clearly shown in the obtained displacement fields. Further, the high spatial resolution allows the structural analysis at a scale where µm-sized irregularities are found in the displacement field, which can be traced to local variations in the cellular wood structure.

The second region includes the two CFRP supports that the specimen rests upon with its center right beneath the interface. A maximum of approximately 68 µm is reached in the upper contact region. The deformation decreases with distance from the centre down to 0 µm in the two supports. One can see clear evidences of inelastic structural behavior in all three contact regions—which also is verified by inspection of the reconstructed structure. The obtained υ-displacement fields correspond to the deformations in the longitudinal direction of the wood fibres, which are of particular interest in a bending situation. Here, one finds a compression and tension band in the upper and lower edges, respectively—a behaviour that is expected for specimens exposed to TPB. The resulting strain fields agree well with the situation known from macro-scale TPB experiments. However, mens exposed to TPB. The resulting strain fields agree well with

References


Paper D

3D micro-scale analysis of swell behaviour in Scots pine wood using Synchrotron Radiation Micro Computed Tomography and Digital Volume Correlation
3D micro-scale analysis of swell behaviour in Scots pine wood using Synchrotron Radiation Micro Computed Tomography and Digital Volume Correlation

J. Danvind\(^1\), F. Forsberg\(^2\), R. Mooser\(^3\), E. Hack\(^3\) and P. Wyss\(^3\)

\(^1\) Division of Wood Technology, Luleå University of Technology, Skellefteå campus, SE-951 87, Skellefteå, Sweden
\(^2\) Division of Experimental Mechanics, Luleå University of Technology, SE-971 87, Luleå, Sweden
\(^3\) Electronics/Metrology/Reliability Laboratory, Swiss Federal Laboratories for Materials Testing and Research (EMPA), Überlandstrasse 129, CH-8600, Dübendorf, Switzerland

Abstract

A method for 3D quantitative analysis of the swell behaviour in wood structure at the micro scale is presented. The full three-dimensional wood structures of a Scots pine wood sample, approximate Ø 1.0 mm and length 0.6 mm, was reconstructed at cellular level, with a spatial resolution of 0.7 \(\mu\)m – first in dry state and then after 5.5 h, 9.0 h and 12 h water immersion. The structural swelling, due to the water exposure, was analysed with digital volume correlation, in a region that measures 0.7 x 0.7 x 0.4 mm\(^3\). The results are presented as 3D displacement fields along the tangential, radial and longitudinal direction within the wood structure. The displacement fields are also presented as vector plots in 2D in order to give a better visual interpretation of the wood structure movements. The results describe a structural behaviour that in many respects reflects what is expected, based on macro scale analysis, and the magnitude of the measured swelling correlates well with what is found in literature. However, also unexpected phenomena can be found within the results. The radial swelling is, for example, strongly influenced by the presence of a resin channel, while the tangential swelling seems unaffected.

Key words:
Wood; Scots pine; Swell; X-ray tomography; Synchrotron; Digital volume correlation; Digital image correlation; 3D deformation, Micro structure; Ultra structure

Introduction

Wood is a porous material that changes properties both with temperature and moisture content (MC) changes. The latter has a significant influence on properties such as density, dimension, hardness, stiffness and strength. Water is bound in the cell walls, so called bound or hygroscopic water, when the MC is below the so called fibre saturation point, which is approximately 30% MC for most wood species at room temperature. The water in the cell walls is interacting with the wood constituents, as cellulose, hemicellulose and lignin.

In this study the studied wood has been Scots Pine (\textit{Pinus Sylvestris}), which is one of two dominating softwood species in Scandinavia. Fig. 1 shows a 3D rendering of the Scots pine wood structure, based on image data acquired during these experiments. Also, an attempt to mark the most significant features, according to reference [1], in the wood anatomy has been done in Fig. 1. The wood cells are hollow and the cavities (lumen) form a porous network system that is connected via smaller pores, so called bordered pits, denoted (d) in Fig. 1. In softwood there are two types of cells, tracheid and parenchyma cells (Fig. 1). Tracheid cells, denoted (a) in Fig. 1, are orientated mainly in the longitudinal direction of the wood and represents almost 95% of the wood volume [1]. The parenchyma cells are divided into ray parenchyma, in radial direction, and longitudinal parenchyma. In Fig. 1 a ray (c) that consists of several ray parenchyma and tracheid cells (e) that propagates in the radial direction (c1-c2),
Figure 1. The Scots pine wood microstructure (as imaged in this work), where the most significant features have been marked out. (a) Lumen of a tracheid, (b) tracheid cell wall, (c) uniseriate heterogeneous ray, (c1-c2) radial direction, (d) radial bordered pit, (e) ray parenchyma and/or tracheid cell. The full visualized volume is 112 x 56 x 112 μm³.

is shown. One form of the parenchyma is the epithelial cells that surround the resin canals. Energy storage of carbohydrates in the living tree is stored in the parenchyma cells. During the growth of the tree annual rings are formed, consisting of earlywood and latewood tracheids. The latter have smaller lumen and thicker cell wall and thereby higher density. The cell wall of a tracheid (denoted (b) in Fig.1) is built of different layers, from lumen and outwards; warty membrane, S3, S2, S1, primary wall and the middle lamella that separates two adjacent cells. The S2 layer is the thickest part of the cell wall and has strong crystalline cellulose microfibrils in a matrix of lignin and hemicellulose. The microfibrils are orientated in a helical form around lumen. Microfibrils are of great importance for the shrinkage/swelling behaviour and mechanical properties of wood fibres and solid wood. When the microfibrils in the S2 layer are aligned with the longitudinal axis of the tracheid, then the so called microfibril angle is 0°. When the angle increases the microfibrils form a more helical shape in the cell wall. In normal softwood the angle is small and the microfibrils strengthen the tracheid in its main direction and also reduces the longitudinal shrinkage/swelling to small movements, since crystalline cellulose has poor interaction with water. In reaction wood (compression wood in softwoods) the microfibril angle increases which also results in a larger longitudinal shrinkage and lower tangential shrinkage, both on the micro- and macro level of wood. The term “ultra structure” is often used when looking into the cell walls and the cell wall configuration while “micro structure” is used on the level
of annual rings and early- and latewood distribution. However, in this report this classification has not been made and both are referred to as micro structure level. This means that wood is a multi scale material with characteristic features at several hierarchical levels – from annual rings at the cm-level, and visible for the bare eye, down to micro fibrils at the nanoscale and cellobiose molecules at the Å-scale. At each scale there exists a number of imaging techniques that can be used for investigation of the structure. Which technique to use depends on what kind of features or processes that is to be studied. The versatility of the material yields that imaging modalities used for investigation of the wood structure need to meet great demands and cover as large spatial scale as possible.

Traditionally, optical microscopy has been the most frequent technique used for high resolution characterisation of wood. It is straightforward to use and a flexible tool as the spatial scale can be adjusted rather easily. However it has its limitations as it only describes two-dimensional structures. An alternative method for structural investigations at the nano- and micro-scale is scanning electron microscopy (SEM) which also manages to describe the topography of the cellular structure, and therefore resolve the 3D cellular structure close to the surface. A drawback with all surface imaging methods is that the investigated specimen needs a lot of preparation in advance in order to give trustworthy information. It is very hard to produce a surface usable for investigations without also introducing distortions from the micro-machining tools. An exposed surface is also susceptible to humidity changes in the surrounding air, which may influence the studied properties of the wood.

From this perspective it is easy to see that non-destructive imaging techniques are getting more popular. They can be used to investigate a series of cross-sections of the specimen without the need of making perfect physical cuts through it. There are a number of different non-destructive imaging techniques that can be used to investigate wood structure, each with a characteristic range of spatial resolution. These methods and their suitability for different applications are described thoroughly in reference [2].

X-ray Computed tomography (CT) has been used for cross-sectional examination of large wood structures such as for example logs and timber ever since it was proposed for these kind of applications in the mid-eighties [3]. It has been an efficient tool for quality assurance as long as the structures have not been too small. X-ray CT is theoretically the best candidate for high resolution imaging due to the extremely short wave length of the x-rays. X-rays used in conventional CT systems occupies a relatively broad spectrum, which in combination with the fact that the beams in most cases are divergent makes the use of magnification optics almost impossible. Therefore, CT like the rest of the non-destructive imaging methods has been confined to the upper regions of the spatial scale – with resolution in the mm-range and upwards. However, recently Trtik et al. [4] proposed the use of a synchrotron based CT for 3D imaging of wood structure, which can be performed at the cellular scale. This technique, called synchrotron radiation micro-computed tomography (SRμCT), is the same technique as used here. It is based on the use of synchrotron x-rays, i.e. a parallel beam of x-rays with a very narrow bandwidth. These semi-coherent x-rays allow magnification optics to be used and hence the spatial resolution is increased drastically – to values below 1 μm.

Trtik and co-workers [4] made the SRμCT-imaging in phase contrast mode, i.e. the images are based on information about phase differences that occur in the x-rays as they cross the wood specimen. Here, on the other hand, the images are conventional absorption contrast images, based on the differential absorption of x-rays as they cross the wood material, which in its turn is determined by the density distribution in the wood.

In this work the analysis of the wood structure at cellular level, in dry and wet state, is complemented with a 3D digital image processing method called digital volume correlation (DVC)[5]. This method is used to get a quantitative measure of the swelling. The result is
presented as a 3D displacement field that describes how much the structure in each point within the investigated volume has moved due to the swelling.

The use of correlation based methods for analysis of deformation within wood structure was, to the authors’ knowledge, first proposed by Choi et al. in 1991 [6]. In their work video sequences of the deforming wood surface was captured and analysed with digital image correlation (DIC). The resulting two-dimensional displacement fields were then used to calculate the strain at the surface. Furthermore, Murata [7, 8] has used DIC to analyse the swelling of wood at the cellular level based on confocal scanning laser microscopy images of the surface. The rather homogeneous structure of the wood imaged with conventional CT systems, due to the limitation of spatial resolution mentioned above, has not allowed the use of methods like DIC for analysis of internal structures. An approach to move the correlation based deformation analysis to regions beyond the surface was however proposed by Danvind and Synnergren [9]. Here, an experiment where two cross-sections of drying wood were imaged sequentially with optical digital imaging and CT at the same time, complementary. The optical images with higher spatial resolution contained features that could be used for image analysis with DIC. The results from the surface were then assumed to also hold for the adjacent inner regions imaged with CT. The problem with the method was that the lacquer used to seal the end surface was not fully watertight, which caused problems when studying wood-moisture relations.

The use of three-dimensional deformation analysis by use of DVC was first proposed for measurements of 3D strain fields in trabecular bone tissue exposed to uni-axial compression loads [5]. It has become a popular method for measurements of strength in different bone types and similar research around osteoporoses, which also has been the driving force for the development and improvement of accuracy [10-12]. The technique was proposed after the introduction of micro-computed tomography (μCT) – a small scale CT technique with improved spatial resolution compared to conventional CT systems. DVC is continuously under development and has during the last decade been applied to a number of other materials as well, such as alumina foam and argillaceous rock [13, 14]. Recently it also has been used for deformation analysis in wood, exposed to three-point-bending [15] and based on SRμCT image data.

There are many measuring applications were the described combination of SRμCT and DVC may be motivated to use, some examples will be given in the text below.

Drying and wetting of wood causes shrinkage and swelling on all structural levels of wood. As described earlier, the complex micro structure of wood causes shrinkage anisotropy inside single cells (tracheids, parenchyma) and in the 3D matrix of cells. Often the macro structure shrinkage/swelling is studied, which is a reflection of the micro structure properties. However, there may be local differences on the micro level that is not reflected in the higher levels due to the averaging effect. These differences might cause stress concentration that influences the macro behaviour of wood. In studies of shrinkage on the micro structure level it is often 2D measurements in the radial and tangential plane. One example is Badel and Perré 2007 [16] who present an approach for predicting macro shrinkage/swelling in the radial and tangential plane for any random piece of oak by acquiring parts of the micro structure by ESEM and base a 2D finite element model, FE model, on the sampled micro structure. The FE model is then used for calculation of the macro shrinkage/swelling properties. Their approach has better prediction ability than more statistical models. The combined SRμCT and DVC methodology could be used in order to put forward a similar 3D approach for the relations between micro- and macro shrinkage/swelling. Using SRμCT it is possible to observe cell wall buckling, collapse and possibly tension or compression wood, that all influence the shrinkage/swelling behaviour.
Perré 2007 [17] presents an equipment for dynamic moisture-shrinkage measurements in 2D, based on a weighing balance, laser-scan micrometers and a climate chamber. If the method used here is complemented with a climate controlled chamber around the sample and methods for determining moisture content, the equipment could be used for static, or semi-dynamic, 3D non-destructive investigation of moisture-shrinkage relations, simultaneously as information of the micro morphology is achieved.

The 3D morphology of wood has also connection to the mechanical strength and fracture of wood. A typical drying crack in wood, on the macro scale, appears on the tangential surface and propagates in longitudinal- and radial direction. In radial direction the crack may follow the ray cells, since these are orientated in radial direction. [18] gives an example of how the crack propagates through the cell walls in the early wood and follow the ray cells in the late wood. Also on the micro scale, cracks in and across the cell walls appear during drying from the green state [19, 20]. These micro cracks are believed to reduce the strength and strength losses from 10 to more than 50% have been reported for spruce [21] and pine [19], respectively. In this study, the 3D shrinkage anisotropy that is causing internal stresses was studied on the micro level.

The combination of morphology, moisture, shrinkage, mechanical strength and fracture propagations are often not studied [22], the focus is laid on some of the parameters, not all. By combining SRμCT, DVC, mechanical loading and climate control it is possible to perform measurements that will contribute with valuable information to the field of hygromechanical properties of wood. In [22] more methods for measurements of hygromechanical characteristics of wood are presented.

Another field for SRμCT is to further study the water flow in the wood structure. For example when studying capillary water flow in the capillary network, where the validity of, for example, the percolation approach [23] could be investigated. However, water flow studies are not in the objective of this study.

The scope for this work is to introduce a new method for 3D quantitative hygromechanical analysis, by exemplifying a 3D non-destructive method for structural investigations and displacement measurements of wood on the micro scale. The structure of a Scots pine (Pinus Sylvestris) sample has been imaged in 3D in both dry and wet state, with a spatial resolution of 0.7x0.7x0.7 μm³, by use of SRμCT. The 3D cellular structures in dry state has been compared with the corresponding structures in three different wet states – captured after 5.5 h, 9.0h and 12 h exposure to water, respectively. Finally DVC is used to measure the displacement field due to volumetric swelling of the cellular structure, based on image data in dry state and in wet state after 12 h.

**Experimental procedure**

A small Scots pine (Pinus Sylvestris) wood sample, approximate Ø 1 mm and length 10 mm, was cut from a larger sample which had been dried at max 60°C dry bulb temperature from the green state. The wood specimen was glued to an Ø 3.15 mm metallic pin that was used to attach the specimen to the SRμCT controller stage at the beamline. The specimen and pin adapter configuration is shown in comparison with a standard sized match in Fig. 2(a). Furthermore, Fig.2(b) shows a 3D layout of the specimen with a close up on the investigated region, denoted R. The cross-section in the investigated region is, in dry state, approximately elliptic with minor and major axis 0.9 mm and 1.3 mm, respectively. The specimen was processed with fine sandpaper with the aim to get it as cylindrical as possible and with maximum dimension 1.3 mm in diameter. The specimen is scanned four times, first in dry state and then again after being placed 5.5h, 9.0h and 12 h in water, respectively. The measurements have been carried out at the TOMCAT beamline at the Swiss Light Source (SLS), Paul Scherrer Institute, Villegen, Switzerland. The x-ray photon energy was 9.4 keV,
Figure 2. (a) The specimen and the Ø 3.15 mm pin adapter used to mount the specimen to the SRμCT controller stage is compared with a standard sized match. (b) A 3D layout of the specimen and pin configuration with a close up on the investigated region, R.

Correlation procedure

The shape and position of the region $R_c$ that is analysed by DVC is specified in Fig. 3 (a)-(d). Fig. 3 (a) and (b) shows the extension of $R_c$ in the centre $z$-slice of the reconstructed wood structure in dry state and after 12 h in water (wet state), respectively. In 3D, the region measures $0.715 \times 0.715 \times 0.447 \text{ mm}^3$ and is described by $512 \times 512 \times 160$ voxels, which yield...
that the voxel dimension during the correlation analysis is $1.40 \times 1.40 \times 2.79 \, \mu m^3$. Fig 3. (c) and (d) show volume renderings of the wood structure in $R_c$ in dry- and wet state, respectively. In the correlation analysis these two data sets are referred to as the reference volume (dry wood structure) and the deformed volume (wood structure in wet state), respectively.

The volume correlation method requires that the analysed volume data contain random structures, of roughly the same size, in all three directions. At the macro-scale, the cellular structure seems to have a certain periodicity in the radial-tangential plane. However at the micro-scale this structure can be considered as reasonably random, as shown in Fig. 1.

In order to analyze as large region as possible in the xy-plane (RT-plane), using limited computational resources, the spatial resolution of the original image data was reduced a factor two along these directions. Furthermore, the physical extension of the tracheids in the longitudinal direction is about 100 times greater than in the radial and tangential direction. In

Figure 3. The correlated region $R_c$ is positioned in the investigated wood specimen according to (a) and (b), for the dry and wet state, respectively. (c) and (d) shows renderings of the reconstructed 3D structure of entire $R_c$, for these two states. The region $R_c$ has the dimensions $0.715 \times 0.715 \times 0.447 \, mm^3$, described by $512 \times 512 \times 160$ voxels.
In order to compensate for the elongated tracheids and make the features more isotropic and suitable for correlation analysis the spatial resolution along the L-direction was reduced by a factor of four.

The two volumes, shown in Fig 3 (b) and (c), are initially divided into smaller volumetric regions called subvolumes (or correlation windows), in which the correlation analysis is carried out. In this experiment the analysis is based on subvolumes of the size 64³ voxels or 89.4 x 89.4 x 179 μm³.

In the reference volume, the correlation procedure contains two subroutines. The reason for this is to gain time efficiency and robustness in combination with high spatial resolution and accuracy. The first routine makes an initial analysis at a rather coarse scale and estimates the integer displacement of each subvolume. The integer displacements are determined by cross-covariance estimation between each subvolume in the reference volume and the corresponding ones in the deformed volume. This is carried out in the spectral domain by use of a fast Fourier transform. Next, a new set of modified subvolumes are made from the deformed volume, based on the results from the initial correlation routine. The subsequent second correlation routine is carried out based on a continuous Chebyshev spline representation of the displacements.

The distance between the centres of two neighbouring subvolumes along each direction, is half the side of a subvolume (32 voxels). This yields that every subvolume is entirely overlapped by its neighbouring subvolumes. The final displacement in each point is in the end determined as a weighted average of the correlation results obtained for that particular point. This results in one continuous displacement field that cover the full correlated region $R_c$.

For a thorough description of the DVC algorithm, refer to [25].

### Results and discussion

Fig.4 (a)-(f) shows the displacement fields $u$, $v$ and $w$, corresponding to x-, y- and z-directions respectively, due to the swelling of the pine wood after 12 h exposure to water. Fig. 4(a), (c) and (e) shows the three displacement fields (only) whilst Fig. 4(b), (d) and (f) show them overlaid of the wood structure in wet state. After this exposure time, the wood is assumed to have reached fibre saturation point, i.e. the cell walls were water saturated and the most swollen state was reached. The $u$-displacement field, Fig.4 (a) and (b), describes the displacement in the tangential direction of the wood structure, and ranges between -22.4 μm and 24.5 μm. Furthermore, the $u$-displacement seems to have a linear variation along the x-direction while being rather constant with respect to the y- and z-direction. It was expected that the tangential swelling, $u$, would change in the radial direction also because of the different properties of early- and latewood. Since this was not found, it is assumed that the studied region, $R_c$, only contain one of the cell types.

The $v$-displacement field, in Fig. 4(c) and (d), describes the radial swelling of the wood structure, in the y-direction, and ranges between -8.63 μm and 14.1 μm. The spatial relationship of $v$ seems to be mostly linear in the tangential direction, which is expected. However the displacement field also reveal large structural movements in the radial direction that occur very locally near a resin canal, which is visible at the border of $R_c$ (at y=0 mm) in Fig. 4(d). The resin canal seems to have a rather strong influence on the structural behaviour in the radial direction while it does not seem to have any significant effect on the tangential behaviour. Moreover, the $u$- and $v$-displacements has been used to estimate the maximum strain in the tangential and radial direction, respectively. The maximum tangential strain is 6.56 % which is close to the 7.7 % that is found in literature [26]. In the radial direction the estimated maximum strain value is 3.18 % to compare with the 4 % found in literature for maximum shrinkage from fibre saturation point to 0 % MC. In this study the true change in
Figure 4. The three-dimensional $u$, $v$ and $w$-displacement fields. In (a), (c) and (e) the pure displacement fields are shown whilst in (b), (d) and (f) the displacement fields, overlaid on the wood structure in wet state are shown. The $u$-displacement field, shown in (a) and (b), describes the structural swelling of the wood in the $x$- (tangential) direction and range between $-22.4 \, \mu m$ and $24.5 \, \mu m$. The $v$-displacement field, shown in (c) and (d), describes the radial swelling of the wood structure. $v$-values range between $-8.63 \, \mu m$ and $14.1 \, \mu m$. The $w$-displacement field, shown in (e) and (f), describes the structural motion in the longitudinal direction. It is mostly close to zero. However, in some regions one finds radical movements that occur very locally. These range from $-12.8 \, \mu m$ to $15.5 \, \mu m$. 
MC was not known and therefore it was expected that the measured strains should be lower than maximum shrinkage found in literature. The \(w\)-displacement field, shown in Fig. 4(e) and (f), describes the structural motion in the longitudinal direction. It is mostly close to zero. However, in a few regions we find large \(w\)-displacements that range from -12.8 \(\mu\)m to 15.5 \(\mu\)m and occur very locally. Most of these extreme \(w\)-fluctuations are found near the borders of \(R_c\), where the structural movement has its maximum, in both the x- and y-direction. This in combination with the reduced number of features along the z-(longitudinal-) direction could result in a partial de-correlation in certain regions during the DVC analysis which would explain the extreme \(w\)-displacements. Therefore these \(w\)-displacements can most probably be explained as de-correlation effects caused by the large structural movements in combination with the reduced number of features along the z-(longitudinal-) direction.

Fig. 5 (a)-(d) describes the structural motion in selected regions of the correlated region, \(R_c\). Fig. 5(a) shows the correlated region together with the three orthogonal planes in which the displacement fields are presented in (b)-(d). Fig. 5 (b) shows a vector field describing the structural swelling of the wood in the xy-plane at \(z=0.223\) mm. The displacement field shows that the \(u\)- and \(v\)-displacements both are smooth and seems to have a linear variation with the spatial coordinates. Furthermore, one can see that the swelling in the x-direction (tangential

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**Figure 5.** Two-dimensional deformation patterns in three orthogonal planes, through the correlated region. (a) shows the correlated region, \(R_c\), together with the three planes that corresponds to the deformation patterns presented in (b), (c) and (d). The vector field in (b) describes the swelling of the cellular structure in the plane \(z=0.223\) mm. (c) shows the 2D displacement field that describes the structural motion in the yz-plane \(x=0.358\) mm. (d), finally, describes the structural motion in the xz-plane at \(y=0.358\) mm.
direction) is the dominating one. Fig. 5 (c) and (d) show the displacement fields in the yz-plane at $x=0.358$ mm, and in the xz-plane at $y=0.358$ mm, respectively. Note that Fig. 4 (a) and (c) and Fig. 5 (c)-(d) indicate that the $u$- and $v$-displacement are nearly constant in the $z$-direction, which is a behaviour that is expected on the macro structure level.

The resulting strong variations in the longitudinal direction of the cells are interesting. This behaviour is not reflected on the macro-level of wood. However, these variations might influence the macro behaviour due to local stress concentrations that are likely to appear in these areas. Stress concentrations might in turn be of importance for the macro mechanical properties and fracture mechanics.

A cause of the local longitudinal swelling might be influence of radial rays that interact with the tracheids. Also, longitudinal shrinkage is known to be influenced by the microfibril angle in the S2 layer and hence it would be interesting to further study that angle in cells in the regions of interest. Another cause could be strong local differences in MC, caused by for example water transport ways and local resin concentration. But this is not likely, since the discontinuities in longitudinal swelling caused by MC differences also would have been reflected in the other directions and this was not the case. A more likely cause is that this could be a measuring artefact of the DVC when calculating longitudinal displacements. The correlation region, $R_c$, is 0.447 mm in the longitudinal direction and hence 7-9 times shorter than the tracheids, which might be 3-4 mm in length. Tracheids are very uniform in the longitudinal direction and since the correlation algorithm is based on recognisable random patterns, it might be difficult to get good DVC correlation for longitudinal swelling of all tracheids. Though, apart from the tracheids, there are also far smaller features contributing to a recognisable pattern along the longitudinal direction, like for example the bordered pits and the radial oriented wood rays – as shown in Fig. 1. Also, during the correlation procedure, the significance estimates from the DVC-algorithm gave no clear indications of the analysed structure being de-correlated.

If the great longitudinal displacements are not a coincident but frequently appearing, then it is likely that high local stresses can appear on micro level, stresses that result in fracture of the wood structure in the vicinity of the rays. If this is the case, then it is likely that the first drying from green state will cause stresses and fractures on micro level, which is in accordance with the findings of Kifetew et al. [19]. The method used here was not able to detect internal checking in the S2 layers of the cell walls for the studied wood. However, when doing a preliminary visual inspection of not reported data from dry Scots pine, heat treated at maximum 170$^\circ$C, micro cracks in the cell walls were found, Fig. 6. It is also interesting to note that the green MC of heartwood in Scots pine is not below fibre saturation point, but slightly above it. Maybe the reason for this is that the wood structure has the most advantageous mechanical properties for the living tree when cell walls are full of water. If this discussion is valid, then there is a reason to avoid cell wall drying prior to cell wall bulking modification of wood, in order to achieve a wood structure that is as intact as possible. This work then strengthens the findings of Thuvander et al. [20] were the tensile strength and the energy to failure of glycerol modified wood was increased by a factor two, if cell wall bulking was done in green state.

Tschegg et al. [27] study crack propagation in larch and beech, showing that larger number and cross-section area of rays in beech makes the wood reinforced in the longitudinal-radial, LR, plane. In addition, it would be interesting to study local shrinkage in the vicinity of the rays, since it might have a correlation to the fracture behaviour. Maybe there are drying-induced local shrinkage that causes stresses and micro-failures, i.e. it is not only the rays that
reinforce in one direction, the rays might also weaken the structure in the longitudinal-tangential direction.

If the shrinkage/swelling properties of wood on micro level cause lower mechanical strength on the macro level it may be of economical relevance for the wood industry. Mechanical strength of wood for construction has great variations and the relations between predicted and actual mechanical strength are quite poor. Only a small increase of the mechanical strength and a decrease of the variations in properties may be of economical importance. However, this is contradicted in [21]. The method presented here, possibly in combination with SEM, may be of use for studying micro structure properties related to mechanical strength.

Shear strain in the tangential and radial planes can be denoted as $\partial u/\partial y$ and $\partial v/\partial x$. If the sample to be measured is rotated in the $xy$-plane it will be reflected in the $u$ and $v$ displacements, and both of them will get a linear variation along both $x$ and $y$ axes. When the $u$ displacements have a variation in the $y$ direction, then $\partial u/\partial y \neq 0$, which means that there are shear strains present.

Water inside the lumen caused problems for the DVC algorithm, since the presence of capillary water altered the random recognisable pattern which is needed for correlation. This is the reason why the results from measurements after 5.5 and 9.0 h water immersion were not presented here, since no correlation was found in $R_c$.

Fig. 7(a)-(h) shows image data of the cellular wood structure captured in dry state and in wet state after 5.5h, 9h and 12h water immersion. Fig. 7(a)-(d) shows the centre tangential-radial slice (xy-slice) trough the correlated region $R_c$, whilst Fig. 7(e)-(h) shows the 3D structure in a region that measures $83.8 \times 126 \times 126 \ \mu m^3$.

Here the capillary water inside the lumen is visible, for the structures collected after 5.5 and 9.0 h in water, (b), (f) and (c), (g), respectively. After 12 h immersion, Fig. 7(d) and (h), the sample shows no capillary water, which was not expected. The reason for this is not known,
Figure 7. The cellular structure in two different regions of the wood specimen in four different hygro-states. Both regions have the dimensions 83.8 x 126 x 126 μm³. One can see that the structures exposed to water for 5.5 h and 9 h, shown in (b), (f) and (c), (g), respectively contain water in the lumen which decomposes the recognisable structure required for correlation analysis. However, the wood structure exposed to water for 12 h, (d) and (h), shows no visible signs of water filled lumens and has a structure comparable with that from the dry state, (a) and (e).

but it is likely that a partial drying of the sample occurred prior to x-ray scanning, due to the experimental procedure. However, this gave usable data for swelling correlation, since the structure now is comparable with that in the dry state (Fig. 7(a) and (e)). It is probable that the MC was near the fibre saturation of the cell walls, which is believed to be the most swollen state of wood. To avoid this problem with low correlation it is suggested to develop methods for filtering away the capillary water from 3D data prior to applying the DVC algorithm. Maybe such a filtering is possible by slicing out data points with a density value near to the density of liquid water. Another way is to use phase contrast information for the SRμCT imaging of wood structure, as proposed by Trtik et al.[4], instead of absorption imaging as used here. In general, phase contrast imaging is carried out with higher photon energy and extended exposure time compared to conventional absorption contrast imaging. In pure phase contrast mode no information about the absorption (and hence density) is gathered from the material. Instead the information obtained describes the interfaces between the different (low absorption) materials within the specimen. Therefore phase-contrast images of a specimen with the constituents wood/air and wood/water/air are fully comparable with each other, from a structural point of view. However, since the scanning time, in general, is longer (due to the increased exposure time) this imaging method will be more sensitive to motion blur artefacts of the reconstructed structure, which occur as the wood structure dries and starts to shrink during scanning. These effects could hopefully be reduced through use of a climate chamber.
where the specimen is placed in an environment with constant humidity and temperature during scanning. Recently it has also been reported that mixed phase and absorption imaging can be performed in parallel, based on the same projection data [28, 29]. Hence it would be possible to measure the swelling (or drying) through use of DVC on either of the data sets and through subtraction of the absorption and phase image data isolate the water inside lumen. Capillary water movement in wood can proceed relatively fast if pressure differences and capillary communication network exist. That makes it a drawback that the total scanning time is approximately 12 min, since a redistribution of moisture may occur during the on-going scanning sequence and hence influence the accuracy of the measurements. Also moisture changes within the cell walls may occur during the scanning, since the scanning volume is very small. Further investigation of the influence of scanning time on the measuring accuracy would be possible, but not simple, by a combined simulation of material structure, moisture flow and scanning.

In this study density and displacements are achieved in 3D. In future work it is of interest to also derive strains and moisture content. A method used by Danvind [30] to derive MC in 3D data is based on identification of the same points, or small regions, in the material at different time steps or shrinkage/swelling states. This method will be difficult to apply here where there are many sharp changes in density in the data and the displacement data is smoothened over more than one cell. This means that it will be difficult to identify positions in for example a cell wall in the deformed state. How to do the MC estimation is therefore to be developed. One way forward might be to control the RH in a testing climate chamber and always wait for the sample to stabilise on equilibrium MC throughout the sample before scanning. However, that will not be a solution for more dynamical studies.

Due to the resolution and averaging of the DVC calculations it may be difficult to fully resolve the sharp displacements caused by local material variations. Hence, there are possibilities of improving the displacement measurement, for example by adopting the sizes and positions of the sub volumes (correlation windows) to the micro structure and avoid averaging of calculated displacements in some regions of interest. In the SRμCT data it seems to be possible to measure cell wall shrinkage and swelling, since there are recognisable individual structural irregularities of the cell walls, especially where there are bordered pits. However, not by using the sub-volume size of the DVC algorithm used in this study. Instead more manual image analysis can be done or the size of the sub-volumes might be decreased. Also, it would be possible to increase the magnification while performing the SRμCT imaging by a factor two and thus increase the (theoretical) spatial resolution a factor two, which would result in the field of view 0.715 mm² and voxels with the dimension 0.349 μm³. With the higher spatial resolution it might be possible to perform DVC analysis on individual cells and entirely rely on the smaller features present in the cell walls. This is to be further investigated.

It would be interesting to measure the micro swelling of the cell walls and the macro swelling over some annual rings in order to evaluate modelling approaches, for example Nakano [31], of the relationship between them. Pang [32] shows a micro structure modelling approach where this method may be useful for providing good data on both the geometrical structure and the shrinkage/swelling properties. Also when working with modification through compression of wood, for example Blomberg and Persson [33], this method is believed to be useful.

The sorption isotherm of wood does not follow the same pattern in desorption and sorption [34]. Also, the first desorption from the green state does not follow the same pattern as desorption after re-wetting [35]. This also changes the shrinkage behaviour to some extent after re-wetting. This might be caused by the reduction of water bonding sites to the wood molecules, which in turn may be influenced by fractures on the micro structure level that appear during drying from the green state. This might also be the cause of different sorption
behaviour of wood pulp and gross wood as found by Stamm [36]. The SRμCT method may be usable for simultaneous studies of MC, shrinkage/swelling and changes on the micro structure level, to further understand the hysteresis of the sorption isotherm.

The SRμCT and DVC have some limitations (or disadvantages) that were noted for the study done in this work. The sample size is restricted to a field of view that ranges from 0.715 mm² to 11.4 mm², depending on the magnifying optics used in the detector unit (the magnification ranges from 1.25x to 20x). This restricts the studies that can be done. The relatively long scanning times make it difficult to study dynamical phenomena. No climate controlled chamber for placing the samples in, during scanning, was available. The DVC algorithm did not measure displacements of individual cell walls, or within cell walls. Capillary water in lumen made it impossible to calculate displacements using DVC. The resolution was not good enough to directly resolve for example micro fibril angle and whether pits were opened or closed. Trtik et. al [[4]] proposes a promising water-shed methodology for studying the state of the pits.

**Conclusions**

By bringing non-destructive 3D measurements for structure and algorithms for displacements calculations to the micro-scale level of wood, better understanding of wood will be achieved. It can be concluded that the methods presented here, SRμCT in combination with DVC, are very useful for studying shrinkage and swelling in 3D on micro level of wood below fibre saturation point.

The swelling in radial and tangential direction, found in this study, resembled the expected swelling on macro level. However, also unexpected phenomena was found within the results. The radial swelling was, for example, strongly influenced by the presence of a resin channel, while the tangential swelling seemed unaffected.

The DVC had too low correlation for calculating displacements when comparing wood without water in cell lumen and deformed wood with capillary water.

The longitudinal swelling showed local variations, which was not reflected on the macro level. The variations were suggested to be caused by material variations and/or weaknesses in the displacement measurement. The latter was assumed to be caused by the homogenous structure of the tracheids and their long elongation in longitudinal direction.

**Future work**

During the work the computational speed and RAM requirements of the DVC algorithm has been improved. Hence the correlation volume, $R_c$, may be enlarged in future work, or alternatively, the analysis may be performed with increased spatial resolution, without loss of speed. For future work, it is of great importance that methods for handling the capillary water in lumen are developed. A possible solution is to perform the 3D imaging based on phase-contras information, as discussed in the text. Furthermore, there are several examples for future work based on the methodology used in this study. It would be of interest to carry out similar experiments with improved spatial resolution, and determine the structural response within individual cells. Also, it would be of interest to carry out calculation of moisture content and study moisture flow. A further possibility is to also introduce fracture mechanics in the study, for investigation of crack propagation through loading to failure, as a function of moisture content.
Acknowledgements
The authors would like to thank Samuel McDonald at the Swiss Light Source (SLS) at the Paul Scherrer Institute (PSI), Villigen, Switzerland, for helping us at late hours while performing the synchrotron experiments at the TOMCAT beamline.

F. Forsberg would like to thank The Swedish Research Council (VR) for the funding of his research project. Furthermore, he would like to acknowledge the foundations The Royal Swedish Academy of Science, Nordeas Norrlandstiftelse and The Wallenberg foundation for their financial support of his three month guest research at EMPA in Dübendorf, Switzerland, through scholarships.

References


Paper E

3D deformation and strain analysis in compacted granular sugar using x-ray microtomography and digital volume correlation
3D deformation and strain analysis in compacted granular sugar using x-ray microtomography and digital volume correlation

F. Forsberg a, C.R. Siviour b

a Division of Experimental Mechanics, Department of Applied Physics and Mechanical Engineering, Luleå University of Technology, SE-971 87 Luleå, Sweden
b Department of Engineering Science, University of Oxford, Parks Road, Oxford OX1 3PJ, UK

Abstract

Understanding the displacement of granular beds under compaction is important for a range of industrial, geological and civil engineering applications. Such materials exhibit inhomogeneous internal displacements, including strain localisation, which mean that a method for in-situ evaluation of internal 3D displacement fields at high spatial resolutions would be a major development. This paper presents results from the compaction of a cylindrical bed of sugar, with the dimensions Ø7.0 mm and height 8.2 mm, using X-ray microtomography to evaluate the internal structure, and Digital Volume Correlation to calculate 3D displacement information from these data. In contrast to previous studies, which generally track a small number of marker particles, the research here uses the natural structure of the sugar to provide a random pattern for image correlation, thus allowing full-field information to be captured. The maximum displacement parallel with the compressive force is 240 μm and obtained in the top region of the bed, and induces approximately 7 % compressive strain. The results show good agreement when compared with a well established 2D image correlation technique.

1. Introduction

The behaviour of granular materials and powders under compaction is an important area of study with relevance to a range of industrial, civil engineering and geophysical applications including production of ceramics, metals, pharmaceuticals and explosives [1-3]. The behaviour of granular beds is also important for explosives safety, where suitable compaction laws, which relate intergranular stress to porosity, are required to understand processes such as the deflagration to detonation transition [4, 5]. The behaviour of granular materials is often characterised by the formation of shear bands. As well as affecting density homogeneity of the bed, and thus the mechanical integrity of a finished product, shear bands, which are regions of large relative flow, act to dissipate mechanical work as heat in a localised region, which may have important consequences for explosive safety [6]. For these reasons, it is important to develop experimental capability for better investigating the behaviour of granular beds under compaction. A key aspect of this development is the ability to measure local displacements fields within a specimen. Whilst the use of optically transparent confinements allows a range of important quantitative measurements to be performed [3], edge effects mean that representative results cannot be obtained for many processes. Full-field three dimensional displacement measurements, such as can be obtained from a combination of X-ray microtomography and 3D Image Correlation, will provide an important extension to the range of obtainable data and the processes that can be investigated. X-ray microtomography or micro computed tomography (μCT) is a non-destructive (non-contact) imaging method where the 3D density distribution of an object is obtained from a sequence of 2D X-ray projections, captured from different angles during a rotation of the
object. The reconstructed volumetric data gives a full representation of the micro-architecture of the investigated material and has become a popular tool for structural analysis of materials [7].

There exist a number of publications where structure or motion within granular materials has been analysed by x-ray computed tomography (CT) or microtomography in combination with various image processing techniques [1, 8-11]. In particular, the nature of CT is that free and occupied volumes within a compacting material, and thus the density of the bed, can be calculated from the acquired data [9, 12]; reconstruction and characterisation of particle-particle interactions has also been performed [1]. A number of authors have used tracer particles to obtain information about the movement of the bed [8, 10, 11, 13]; however, such measurements only give pointwise information, even though, in principle, full field measurements should be achievable from CT data.

Image correlation methods are a special type of image processing techniques where the displacement field, describing the structural deformation of an object, is determined from cross-correlation of two images, taken before and after the change of state. In two-dimensions, based on planar image data, this technique is called Digital Image Correlation (DIC) [14] or Electronic Speckle Photography (ESP) [15]. The analysis is typically carried out in a number of sub-regimes (correlation windows), within the object, which together give the full representation of the displacement field. It is of great importance that the imaged structure contains random features – either natural or synthetic – that the correlation analysis is based upon. The smaller features there are, the smaller the correlation windows can be made, and hence the better resolution in the image correlation procedure.

There are a number of previous studies in which DIC has been used that are relevant to the current paper. Using optical image data, researchers have performed analysis of shear band formation in sand due to compressive loads [16], and for granular flow analysis, with strong resemblance to particle image velocimetry (PIV) used in fluid mechanics [17]. An important development is to combine DIC with X-ray imaging to analyse internal deformation processes. This may be done for impact processes, using flash X-ray to capture the high speed deformation [18]. In another paper, granular flow analysis in a hopper was performed; velocity fields and strain fields were obtained in the centre plane of the granular material, defined by a single sheet of seeded tungsten particles [19].

More recently, McDonald et al. combined DIC and X-ray microtomography for analysis of internal deformation and particle movements during loading by a punch into a die containing a mixture of aluminium and tin powder. Here, the tin (volume fraction 15%) was used as a high contrast ‘tracer’. However, unlike the studies discussed above, individual particles were not tracked, rather, the random distribution of the tin particles gave the required pattern for image correlation to be performed, based on both 2D slices and 3D volumes of the reconstructed image data, thus giving much improved data density and spatial resolution [2]. The emphasis of this study was the flow field of particles around the indenter, rather than the compaction behaviour of the powder, which will be the focus of the current paper.

Correlation based analysis in three dimensions, based on volumetric data, is called Digital Volume Correlation (DVC) and was first proposed by Bay et al. for strain analysis in bone tissue exposed to compression loads [20]. The use of DVC combined with x-ray microtomography enables the full 3D displacement field and strain field to be determined throughout the material. DVC is continuously under development and has during the last decade been applied to a number of different materials such as argillaceous rock [21], alumina foam [22] and wood [23].

In the current paper, Digital Volume Correlation is applied to X-ray microtomography data from a compacted bed of sugar. The random spatial distribution of particles is ideally suited
to the application of correlation techniques, without the use of tracer particles that may affect
the overall behaviour of the bed. Apart from the 3D displacement fields that are the direct
outcome of the correlation algorithm, the experimental results also cover calculations of strain
and identification of a shear band.

2. Experimental procedure

The experimental procedure is here divided into two separate sub-sections. The first describes
the pure experimental work and methods. The second gives a brief description of the
correlation methodology together with specific details about the analysis performed.

2.1 Experimental set-up and imaging procedure

Experiments were performed on sugar, which is commonly used as a simulant of explosive
crystals in mechanical experiments [6]. Fig. 1 shows the loading device used in the
experiment, where the sugar (c) is encapsulated in a hollow Perspex cylinder (b). The inner-
and outer diameter of the cylinder are 7.00 mm and 10.1 mm, respectively. The compaction of
the material is achieved by applying a compressive load in the axial (z) direction with a 6.95
mm diameter solid cylinder, made out of brass (a).

The main components of the X-ray microtomography system are a microfocus X-ray source
from Hamamatsu Photonics (L7901-01), a motorized rotation stage from Linos Photonics (RT
120 ST) and an X-ray detector unit from Hamamatsu Photonics (C7876-10). The X-ray tube
has a voltage range of 20 – 100 kV and the tube current ranges from 0 – 250 μA. In these
experiments the voltage and current were held at 25 kV and 100 μA, respectively. With these
settings the effective source spot size is 5μm. The rotation stage has a microstep resolution of
0.002° and an absolute positioning accuracy of 0.05°. The detector consists of an image
intensifier and a 17 mm CCD of 600 x 800 pixels. The effective field of view of the detector
is 72 x 54 mm and the spatial resolution is 4.5 Lp/mm. The source-to-specimen and source-to-
detector distances were 72 mm and 320 mm, respectively, resulting in a 4.44 times
magnification of the specimen.

During the experiment 545 projections were captured, distributed at equal angles over 360
degrees. The exposure time for each acquired projection was 2.56 s.

Figure 1. Photo showing the loading device used for the compaction experiment, the most significant parts are
marked: (a) Brass rod, (b) Perspex tube, (c) Sugar crystals, (d) Strip with tungsten markers.
The reconstructed cylindrical bed of granular sugar. Here, one quarter of the volume data has been removed for better visualisation of the features. The reconstructed data reveals that the spatial range of the features is rather large. The material is dominated by large sugar crystals.

The reconstruction was carried out with a Feldkamp cone-beam reconstruction algorithm [24], on a standard PC with dual CPUs (Pentium4 XEON 2.2 GHz processors) and 2 GB of RAM. Prior to reconstruction a set of correction schemes is applied to the projection data. These are used to reduce image artefacts in the reconstructed data, such as ring artefacts and double structures. More information about the image quality assurance methods used here is given in [25]. The reconstructed data has dimensions 7.98 x 7.98 x 8.87 mm³, described by 342 x 342 x 380 voxels and the region includes the granular sugar as well as the innermost parts of the Perspex cylinder. Fig. 2 shows a volume rendering of the reconstructed granular sugar.

2.2 Volume correlation procedure
It is of great importance that the investigated material contains an isotropic distribution of random features – a unique 3D microstructure on which the correlation analysis can be based. Here, the sugar grains are stochastically shaped and distributed and meet these structural requirements. If, however, the grains had been perfect spheres this would have caused problems since the individual features would have been unique.

During the correlation procedure the correlated volumes are divided into smaller regions - sub-volumes or correlation windows, each containing a small number of features. The size of these sub-volumes is here 0.75³ mm³, corresponding to 32³ voxels. The displacements within each of these sub-volumes are found through minimization of the cross-covariance function between the reference- and the deformed sub-volume data. The result gives the displacement field that deforms the original volume to fit into the grid of the deformed structure. The displacements are here represented continuously as Chebyshev polynomials. The correlation windows (sub-volumes) overlap each other by 16 pixels (half the side of the correlation window) in each direction. Thus, each point in the full correlated region, except those near the borders, is a member of eight individual correlation windows. The Chebyshev-approximation of the displacements is very effective and accurate if the displacements are small. However it is only valid for a limited range of displacements. Therefore, to ensure successful correlation,
when the deformations are large, an initial correlation routine is used. The initial routine makes a coarse estimation of the displacements and supplies the main routine with adequate initial values that ensure convergence of the cross-covariance minimization. The full set of results from all sub-volumes is finally mapped together to one continuous displacement field, covering the whole investigated volume. A more thorough description of the methodology is given in [26].

The correlation analysis is carried out in a region $R_c$, with dimensions 6.72 x 6.72 x 8.21 mm$^3$, described by 288 x 288 x 352 voxels. The theoretical spatial resolution, determined by the dimension of the cubic voxels, is thus 23.3 $\mu$m. The shape of the region has been optimised to describe as much of the cylindrical specimen as possible, with the given sub-volume size, as shown in Fig. 3 (a)-(c). Here, the reconstructed cylindrical specimen is shown (a) together with a schematic representation of $R_c$, describing the shape and location of the region within the reconstructed cylinder. Fig. 3 (c), finally, shows a volume rendering of the reconstructed granular sugar inside this region. The homogeneous structure of the perspex cylinder that holds the sugar is avoided in order to prevent de-correlation, which apart from degrading the results also leads to longer calculation times.

3. Results

The $w$-displacement field, describing the motion of the sugar along the $z$-axis, parallel to the applied load, is shown in Fig. 4 (a)-(d). (a) to (c) describe subsets of the $w$-displacement field, obtained by cuts in the yz-plane through the region $R_e$ at $x$-positions 1.68 mm, 3.36 mm and 5.04 mm, respectively. In (d) the full $w$-displacement field is shown. The displacements range from 55 to 240 $\mu$m in the negative $z$-direction. As expected, the largest displacements occur in the top region, where the force is applied. This region is rather well defined and the large movements in the sugar decline rapidly with distance from the top face. The smaller sugar crystals can move more freely than the bigger ones. This figure also shows how edge effects can affect the material behaviour. As expected, there is a variation of displacement across the top face; regions near the walls of the tube move less due to the friction between the sugar crystals and the walls. At the bottom region, the sugar crystal displacement reaches its
3.1 Comparison to DIC

As well as performing DVC analysis of the 3D reconstruction, it is also possible to perform 2D DIC on planes cut through the cylinder. Here, two-dimensional correlation analysis is carried for comparison reasons by use of a well established DIC-algorithm [15, 27, 28]. In Fig. 5 (a)-(c) a comparison is given of the results obtained from the two techniques. Fig. 5 (a) shows a vector plot describing the v- and w-displacements in the yz-plane at x=3.36 mm, obtained from DVC. The vector plot is overlaid onto the continuous w-displacement field, which is the same as that shown in Fig. 4 (b). In Fig. 5 (b) the results from DIC analysis in the same yz-plane is shown. The DIC analysis has been performed with correlation parameters comparable with those used for DVC; the correlation windows were 32 x 32 pixels in size and there was a 16 pixels overlap between them in both directions. More details about the DIC algorithm used can be found in ref [15]. It is important to note that the vectors in Fig. 5 (a) represent a subset of the full result, which has been chosen so that they corresponds the ones from (b). The full representation of the w-displacement is given by the coloured field. Comparison of the vectors in Fig. 5 (a) and (b) shows that the w-displacements are fully comparable with each other. This is more apparent in Fig. 5 (c), which shows the results for the w-displacement along the z-direction at two different y-positions. The first is taken at y=3.36 mm, which is the centre column of the vector field in (a) and (b). Globally, this represents the absolute centre of the cylinder. In (c) these results are denoted I. The second set of data is taken at y=6.35 mm, which correspond to the border region at the right hand side in (a) and (b). These results are in (c) denoted II.

These graphs show that the results from DVC and DIC give highly comparable representations of the w-displacement in both these regions. The fluctuations of w that observed in the results obtained by DVC also show up in the DIC results. Generally, the DVC method presents slightly lower displacement values, especially in the upper regimes of the
Figure 5. The DVC-results compared with results from DIC. The analysis have been carried out in the yz-plane at x=3.36 mm (the centre plane). (a) and (b) shows vector representations of the v- and w-displacements for DVC and DIC, respectively. In (a) the vector plot have been overlaid on the continuous w-displacement field, obtained by DVC. In (c) the results from the two methods are compared for two columns along the z-direction - at y=3.36 mm (the centre) and at y=6.35 mm (right hand side border).

cylinder. However, these discrepancies are small, and in several regions the two methods give close to identical results. Fig. 5(c) also shows how differently the granular material behaves in the centre region and at the border (near the cylinder walls). From these data, the sugar crystals in the border region have a displacement approximately 20 μm less than those in the centre. Towards the top of the cylinder, however, this difference increases to approximately 40 μm. By comparison of the vector fields in (a) and (b) it is also observed that the results from the DIC-analysis report a slightly larger displacement in the y-direction. The lack of DIC-results in the uppermost region in Fig. 5 (b) and (c) is due to the large displacements, which resulted in de-correlation.

3.2 Strain calculation

The DVC procedure gives three full field displacement maps, representing the displacement of each voxel in the x, y and z directions. This allows all nine components of strain to be calculated at the voxel scale which should enable principal strains, effective strain and shear strains to be derived.

However, an effect of the procedure used to find 3D displacement maps does cause discontinuities in displacement at the borders of the subvolumes used for the correlation procedure. When differentiating to find the gradient, these discontinuities cause a ‘grid’ effect on the data.

Therefore, in order to perform strain calculations, the displacement data were first smoothed to remove the underlying grid. Another approach would be to calculate strains for the data corresponding to each subvolume individually, and then stitch the results together for the whole specimen. The effect of smoothing is shown in Fig. 6(a), a smoothed version of figure 5(a) using the csaps cubic spline in Matlab with a smoothing parameter of 0.000125. The strains described by $\frac{\partial w}{\partial y}$ and $\frac{\partial w}{\partial z}$ are shown in Fig. 6(b) and 6(c) respectively, and are discussed further below.
4. Discussion

4.1 Results from this experiment

Fig. 4 (a)-(d) and Fig. 5 (a)-(c) clearly shows that the greatest response to the applied force is found in the top region of the cylinder, directly beneath the tip of the brass cylinder. The applied load leads to a compaction of the sugar with greatest displacement at the top of the tube. The gradient of the curves in Fig. 5(c) also decreases with increasing distance along the negative z-direction. This is expected, since the frictional effect of the sides becomes more important further down the tube, and is reflected in the strain fields shown in Fig. 6(c).

With reference to the inhomogeneous deformation of granular materials, a further feature of the deformation is highlighted in the plots in Fig. 7. The contours of constant displacement are at approximately constant values of z in the centre of the plot, whilst they slope upwards on the left and right hand sides. As expected, these correspond to regions of increased (more negative) strains in Fig. 6(b) and 6(c). Of particular interest are those contours for displacements between -100 and -146 μm; the advantage of full field displacement measurements is the ability to confirm the relevance of such features with reference to the whole of the specimen, as indicated in 7(b). In this case this behaviour is repeated in the isosurface (in 3D) throughout the specimen.

These angled contours represent regions of shear in the specimen; however, sensible calculations of effective and shear strain at a voxel level have not been made. This is due to noise in the displacement data, which it amplified on taking gradients and again on rotating to principal axes. Small amounts of noise in the displacement field can have a large effect on
the principal strain calculation. An approach to deal with this would be calculation of strain fields over volumes consisting of a number of voxels: selecting a suitable length scale for strain measurements. Whilst this would reduce the spatial resolution of the strain measurements, it should improve the calculation of both the magnitude and direction of the principal strains. From Fig. 7, it is seen that the length scale over which displacement inhomogeneity is observed, which is determined by the size of the sugar crystals, is much larger than the voxel size, so the reduction in spatial resolution would not prevent useful data from being obtained.

In addition to this problem, the issue of displacement discontinuities at the subvolume boundaries must also be addressed. Therefore, two areas of development that will be addressed in a future paper are calculation of strains over a suitable length scale, and stitching together of strain fields from different subvolumes.

4.2 Digital Volume Correlation

In correlation based analysis, the spatial resolution is normally described by the size of the correlation windows - the smaller the windows, the higher the resolution. Furthermore, the sizes of the correlation windows are limited by the features used in the correlation. Here, the sugar crystals are rather large and the size of the correlation windows has been set to 32³. However, with the continuous representation of the displacements with Chebyshev-polynomials the resolution is no longer strictly dependent on the size of the correlation windows.

An advantage of image correlation methods is that they rely on the distribution of features, rather than tracking individual grains. In the experiment reported here, the size distribution of the features was far from uniform, as shown in Fig 2; however the correlation procedure was performed without any noticeable degradation of the results. Although there is a preferable
size of the features, for a given size of the correlation window, the correlation method has a good tolerance. In the current set-up, it would be possible to investigate materials with significantly smaller grains, which would also allow the size of the correlation windows to be reduced. This would allow the region of interest, \( R_c \), to be better adapted to the cylindrical shape of the specimen. Alternatively, for the current grain size, a larger specimen could be used, with correlation windows of the same size. Thus, the technique is very flexible, and adaptable to the application or material to be investigated.

If the sugar crystals break during the compaction process, entirely new features are created and the microstructure is partially changed. This will make the correlation process more difficult in these regions and partially degrade the results. The correlation procedure successfully handles continuous deformations of the structure, such as for example translation, shear and rotation. However, problems arise when large discontinuities occur, or when there are large micro-structural changes. Therefore in order to have a successful correlation analysis it is important that the applied load does not reach the limit where it crushes the features beyond recognition between successive image acquisitions.

The DVC analysis is carried out based on additional information from a third dimension, compared with DIC. The increased amount of data about the structure and the displaced features yield a better statistical representation of the deformation process. This might be an explanation to why the specimen was analysed successfully by the DVC method while the DIC suffer from de-correlation effects in the uppermost region, where the largest displacements occur.

5. Conclusions

Digital Volume Correlation has been used successfully to calculate the full field displacements of a tube of compacting sugar, using the structure of the granular material to provide the texture required for the correlation procedure. The procedure therefore avoids the addition of marker particles to the specimen. The displacements obtained from the volume correlation procedure have been compared to those from 2D Digital Image Correlation on a slice through the same specimen. The expected features of granular compaction were observed in the data; displacements and strains are larger at the top of the cylinder, and in the centre, where edge effects are less. Further, a continuous region of shear deformation is observed in the displacement data. Future research will concentrate on more robust strain calculations, in particular the calculation of principal and maximum shear strains, as well as application to a series of experiments on the force-displacement response of sugar as a modal granular system.

Acknowledgements

F. Forsberg thanks the Swedish Research Council (VR) for the financial support of his research project.

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