Spindle vibration and sound field measurement using optical vibrometry

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By
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Dedicated to the memory of my baby sister
Afsaneh Tatar
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ABSTRACT

Mechanical systems often produce a considerable amount of vibration and noise. To be able to obtain a complete picture of the dynamic behaviour of these systems, vibration and sound measurements are of significant importance. Optical metrology is well-suited for non-intrusive measurements on complex objects.

The development and the use of remote non-contact vibration measurement methods for spindles are described and vibration measurements on thin-walled structures and sound field measurements are made.

It was shown that by making the surface of the spindle optically smooth, both harmonic speckle noise and crosstalk between vibration components could be avoided in laser vibrometry measurements. The radial misalignment and the out-of-roundness of the spindle could also be determined from the signal. Furthermore, the technique was also used for measuring the vibrations of a tool during milling of an aluminium workpiece. The cutting vibrations were determined from the laser vibrometry signal and were compared to the measured cutting force and to the spindle head vibration.

Measurement of radial vibrations along a line on a rotating polished shaft was made using digital holographic interferometry. This technique enables full field vibration measurements in two or more directions simultaneously. This method also provides mode shapes directly and may be helpful in vibration testing.

Modal analysis of a thin-walled workpiece fixed in the milling machine table has been carried out for different stages of machining using scanning laser vibrometry. The result has been used for obtaining the correct boundary conditions of a finite element model of the workpiece. The finite element model together with the measured tool response obtained by laser vibrometry has been used as input parameters for predicting machining stability.

Laser vibrometry measurements on a violin excited by a rotating disc were performed. The chain of interacting parts of the played violin was studied: the string, the bridge and the plates as well as the generated sound field. The measurements on the string showed stick-slip behaviour and the bridge measurements showed that the string vibrations were transmitted to the bridge both in the horizontal and the vertical direction. Measurements on the plates showed complex operational deflection shapes. The sound fields were measured and visualized for different harmonic partials of the played tone. However, the measured sound field is a two-dimensional projection of a three-dimensional sound field. This projection effect is illustrated by measurements of a sound field emitted from several ultrasound transducers from different projection angles. It was shown that by making a sufficient number of laser vibrometry measurements, the three-dimensional sound field could be reconstructed using a tomography algorithm. The idea is to apply the measurement method in rotating machines, where near-field acoustic measurements may provide additional information about a rotating machine part.

The measurement methods that are developed and used provide increased understanding of the dynamics of complex thin-walled structures and rotating spindles. This may be utilized in the optimization of the machines currently available and in the development of machine parts.
This thesis consists of a summary and the following seven papers:


**Paper D**  Kourosh Tatar, Per Gren and Henrik Lycksam, Digital holographic interferometry for radial vibration measurements along rotating shafts, accepted after minor revisions for publication in Applied Optics.


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Part I

Summary
1 INTRODUCTION

1.1 Background

Today’s manufacturing industry demands higher productivity with maintained or even smaller tolerances. The demand of higher productivity in machining implies higher rates of material removal and reduced production time and cost. Using high-speed milling, increased material removal rates are achieved through higher spindle speeds, increased feed rate, and greater depth of cut. High-speed milling offers a gain in productivity and successful machining is thus highly dependent on the proper selection of these cutting parameters. Improper selection of cutting parameters not only reduces productivity, but may also cause instabilities in the system which shows up as vibrations. Apart from a reduced or damaged workpiece, tool, or spindle, machining vibrations can produce unpleasant noise with levels so high that they may be dangerous for the operators. Since vibration in machining is caused by the relative movement between the cutter and the workpiece, the dynamic behaviour of both the spindle and the workpiece must be considered.

Generally, milling machine tool vibration can be divided into three different types: the first type is free vibrations and may originate from a suddenly applied impulse force from, for example, the impact from the initial engagement of cutting tools. The tool and the workpiece are excited and oscillate freely in their natural vibration modes until the damping present causes the vibration amplitudes to eventually die out. Forced vibrations resulting from periodic cutting forces is the second type. Forced vibrations are a great concern in milling, causing tolerance violations [1, 2], especially if the tooth passing frequency or one of its harmonics is close to one of the natural modes of the machine. The bending of the cutting tool may be amplified so that the tool exits the cut. The exit and re-entering of the tool in the workpiece excite the system further which may lead to greater instability. And finally, self-excited vibrations, a well-known phenomenon commonly referred to as “chatter” in machining literature, see for example Ref. [2]. Chatter can be caused by various physical mechanisms. Frictional chatter as a result of friction between the tool and the workpiece, mode-coupling chatter as a result of the relative phase between the vibrations in two directions in the plane of cut, and thermo-mechanical chatter caused by the thermodynamics of the cutting process are often called primary chatter. Secondary chatter is due to the regeneration of waviness of the workpiece surface finish originally caused by forced vibrations. Secondary chatter is considered to be one of the most important causes of vibration instabilities in milling. During milling, as the tool enters the workpiece, the impact causes the system to oscillate at one of its natural frequencies. This oscillation produces in turn waviness on the machined surface finish. This waviness causes alternation in the chip thickness and consequently cutting forces and in the worst case the cutting forces grow to levels so high that they lead to spindle failure.

Often in manufacturing processes, especially when manufacturing components for aerospace application, the workpiece is a monolith in the initial stage and material removal is up to 90% of the original volume. Normally, in different stages of milling, the cutting parameters are changed by the operator since the workpiece rigidity and consequently the dynamic behaviour of the system changes continuously. The critical stage is the finishing
stage, or when machining thin-walled structures. In order to optimize the cutting parameters, so-called stability diagrams are used which identify stable and unstable depth of cuts for specific spindle speeds. These diagrams are based on the tool vibration response (the frequency response function of the system) and furthermore require specific knowledge of depth of cut, number of teeth on the cutter, and the cutting coefficients (a constant parameter of tool geometry-workpiece material that must be established from experimental measurements). The tool vibration response is typically obtained using impact testing. An instrumented hammer is usually used to excite the tip of the tool and the response is measured with the aid of a low mass accelerometer. The measured response together with the cutting coefficients is then exported to additional software for computing the stability diagrams. Although stability diagrams can be helpful to understand the process, many times they are insufficient. The uncertainty associated with stability diagrams depends surely on the technique used to create them and the input parameters that are experimentally obtained. The predicted stability diagram is however, specific for the machine structure, the spindle, the tool holder, the tool and the workpiece. Any changes in the workpiece or other cutting parameters will result in a different response. The approach mentioned to predict the milling condition lacks two important issues; the rotation of the cutting tool and the change of workpiece stiffness while the material is removed.

To be able to remove large amounts of material while maintaining high quality vibration behaviour of milling machines under actual operating conditions must be able to be determined and high precision measurements are thus required. Contacting vibration transducers such as accelerometers can not easily be applied on the rotating spindle itself during the milling process. The vibrations transmitted from the spindle into a non-rotating part, usually the spindle head are therefore measured, and the experimental work on the workpiece is performed in discrete steps, where the milling is stopped just before the tool enters a problem area. However, two problems can occur using this approach: low vibration transmission and/or magnetic disturbance from the motor during high spindle speeds makes the spindle head measurements unreliable. Non-contact position transducers such as capacitive and inductive displacement sensors are possible measurement transducers, but these kinds of sensors must be positioned very close to the rotating part and are sensitive to shaft run-out. Moreover, when performing measurements at very high spindle speed, the air flow that arise in the gap between the spindle and the sensor produces noise in the signal [3]. In order to avoid such limitations, remote, non-disturbing measurement techniques are desired that do not interact with the object to be measured.

1.2 Objective and scope of work

The objective of this thesis is to develop and use remote non-contact optical techniques to measure vibrations on a rotating milling machine spindle. Laser vibrometry and digital holographic interferometry are two techniques that possibly can do this and these techniques have been investigated. The laser vibrometer is a well-established instrument normally used on stationary vibrating objects. This instrument complements contact transducers such as accelerometers and provides vibration measurements in situations where a non-contact measurement technique is required as in vibration measurement of light or rotating objects. However, the spectra of laser vibrometry measurements on rotating objects contain additional surface motion information. In paragraph 2.1, the principles of laser vibrometry are introduced and an approach to determine the translational vibration component in rotating object measurements is
presented in paragraph 3.3. This technique is used for spindles vibration measurements. Laser vibrometry is a point measuring technique and in order to obtain addition spatial information about the vibration behaviour of the rotating spindle, the use of high-speed digital holographic interferometry has been proposed. Moreover, modal analysis of a thin-walled workpiece fixed in the milling machine table has been carried out for different stages of machining using scanning laser vibrometry. The result has been used for obtaining the correct boundary conditions of a finite element model. The finite element model together with the measured tool response obtained by laser vibrometry has been used as input parameters for commercially available software for predicting stability.

The stability of milling may also be evaluated using audio signals during machining, see for example Refs. [2, 4]. Laser vibrometry may be used in an unconventional way to measure sound fields without disturbing the measurement field. It is plausible that sound measurements using laser vibrometry can be performed close to a cutting tool. The objective is that non-intrusive measurement methodology will provide useful information about the milling process. The use of scanning laser vibrometry for sound field measurements has been studied on well-known sound sources; namely a bowed violin and ultrasound transducers. Since the measured sound is a line integral through the measurement volume between the laser vibrometer head and a rigid object to reflect the laser beam, a two-dimensional projection of a three-dimensional sound field is obtained. The projection effect always requires attention since pressure variations along the probing laser beam can result in low integrated pressure values. Multiple projections around the sound source provide a better picture of the actual acoustic field. Tomographic reconstruction of the sound field of ultrasound transducers was conducted which provided the three-dimensional sound field.
2 LASER VIBROMETRY

2.1 Principle of laser vibrometry

Basically the laser vibrometer is an instrument that uses the Doppler shift of the backscattered light from a vibrating surface. The Doppler frequency, \( f_D \), is directly proportional to the velocity, \( v \), of the surface in the incident direction and is given by [5]

\[
f_D = \frac{2v}{\lambda},
\]

where \( \lambda \) is the laser wavelength. Normally an He-Ne laser with a wavelength of 632.8 nm is used as a light source. Since photodetectors are not quick enough to respond to the light frequency in air (\( f = 4.7 \times 10^{14} \) Hz) an interferometric approach is used where the back scattered light from the vibrating object is mixed with a reference light. A common configuration is the Mach-Zehnder interferometer [6]. A schematic sketch of the laser vibrometer system is shown in Figure 2-1. A laser beam is divided by a beam splitter, BS, into a reference and an object beam. The reference beam is then frequency shifted by a known amount, \( f_B \), by a modulator, which is needed for obtaining the direction of the vibration velocity. The object beam reflects from the object and hence is Doppler shifted, \( f_D \), due to the object velocity and mixes with the pre-shifted reference beam on the photodetector. Depending on the optical path difference between these two beams, they will interfere constructively or destructively. The photodetector measures the intensity of the mixed light, which will be time dependent. When the object is vibrating, an optical beat of frequency \( |f_B - f_D| \) is recorded by the photodetector. A displacement of the object towards the instrument produces an optical beat with a frequency lower than the pre-shifted reference beam, while a displacement away from the instrument produces an optical beat with a frequency higher than the pre-shifted reference beam. Frequency demodulation of the photodetector signal by a Doppler signal processor produces a time resolved velocity component of the vibrating object.

The measurement bandwidth is depending on the electronics of the instrument; if a instrument using a laser source with a wavelength of 632.8 nm is capable of measuring velocities between -10 m/s and +10 m/s, the bandwidth would according to Equation (2.1) be 64 MHz.
Figure 2-1 Schematic illustration of the laser vibrometer. The laser beam is divided into a reference and an object beam. These two interfere on the photodetector and the velocity of the object is obtained after demodulation of the signal. Beam splitter (BS), mirror (M), laser frequency ($f$), Bragg frequency shift ($f_B$) and Doppler frequency caused by the object velocity ($f_D$).

2.2 Description of the laser vibrometry system

The Polytec PSV 300 scanning laser vibrometer system was used in the experiments. It uses an He-Ne laser with a wavelength of 632.8 nm, and an acousto-optic modulator (Bragg cell) [5] driven at 40 MHz. Two configurations are available; a four channel system, capable of measuring vibrations with frequencies up to 40 kHz and a two channel system, capable of measuring vibrations with frequencies up to 1 MHz. With velocity sensitivities 2, 5, 10, 25 and 1000 mm/s /V, velocities up to 10 m/s can be measured. The system has a resolution of about 0.3 - 10 $\mu$m/s, depending on the measurement range.

The scanning head has two small servo-controlled mirrors, which make it possible to deflect the beam both in the horizontal and vertical direction by about ±20°. Moreover, it contains a live colour video-camera that together with sophisticated software allows the operator to produce a custom-defined grid of measurement points on the area of interest. The focused laser beam moves quickly and precisely and according to the manufacturer of the scanning laser vibrometry system the settling time is less than 10 ms and the angular resolution is about 0.002°. The system can automatically collect vibration data from 512 x 512 individual points on a defined measurement area resulting in high spatial resolution measurements in a relatively short time.
3 LASER VIBROMETRY FOR ROTATING OBJECTS

When investigating vibrations in spindles, it is of value to know how the spindle responds to a certain excitation. In this way the effect can be related to the cause and any relationship between them that can be modelled mathematically can be found. In vibration testing, the excitation method is important and this must be performed in a controlled and repeatable manner. When studying spindle dynamics, non-contact excitation is preferable to contact excitation. One solution to this requirement is the use of adaptive magnetic bearings, AMB. This technique has been used in paper B.

Analysis of mechanical vibrations is often performed in the frequency domain. If the laser beam from the laser vibrometer is aimed at the rotating optically rough surface of the spindle the spectrum of the laser vibrometer output will be of no use due to harmonic speckle noise and crosstalk between the velocity components of the moving surface. These signal distortions are the two most important limitations of laser vibrometry measuring on rotating objects. The principles behind these two effects are discussed and an approach to resolve the vibration component along the measurement direction is presented.

3.1 Speckle effects in laser vibrometry measurements on rotating spindles

Consider Figure 3-1(a). When polarized coherent laser light of wavelength \( \lambda \) illuminates a surface that is optically rough, which most surfaces are, i.e. the surface roughness \( \sigma \) is large on the scale of the laser wavelength, each scattering surface element acts like a point source of coherent light. Generally the laser spot covers many such uncorrelated source elements which produces a sum of uncorrelated wavelets. As a result of superposition between these uncorrelated wavelets a granular pattern called speckle will be formed on the detector. These dark and bright spots are unique for every different point (subvolume) in space. Figure 3-1(b) shows as an example a typical speckle pattern generated from an optically rough surface. Speckles have random amplitude and phase, where the phase values are uniformly distributed between 0 and \( 2\pi \). If the speckle pattern changes during the laser vibrometry measurement, the rate of change in the resulting phase will be nonzero, which will result in noise in the laser vibrometry signal where the amount of noise depends on the rate of change in the speckle pattern caused by tilt or in-plane motion of the object. The presence of speckle noise in a laser vibrometry signal is characterised by large signal jumps and if the raw signal is converted to a sound wave, it will sound as a scratching sound. Speckle noise caused by random disturbances can be averaged out by measuring a great number of measurements. However, noise induced by systematic speckle fluctuations due to non-normal target motions, such as tilt or in-plane motions can not be averaged out. A rotational motion is a combination of tilt and in-plane motion, and the speckle noise from a rotating object, such as a rotating spindle is generated by the moving speckle pattern on the laser vibrometer photodetector. Although the scattering for one revolution is totally random, the pattern will repeat itself
for every revolution and a beat is introduced with the same fundamental frequency as the rotation frequency in the frequency spectrum of the signal, a signal known as pseudo vibrations [8]. The spectrum of the pseudo vibration signal consists of peaks at the fundamental frequency and the subsequent harmonics. These speckle harmonics are difficult to distinguish from the true vibrations and since they contain amplitude and phase, they can increase or decrease the true vibration level.

Figure 3-1: (a) Generation of speckles on the photodetector when coherent light of wavelength $\lambda$, scatters back from an optically rough surface. (b) A typical speckle pattern.

An experiment was arranged to study the speckle effects in laser vibrometry measurement on a milling machine spindle, see Figure 3-2. A dummy tool of ferromagnetic material without cutter was manufactured and mounted in a Dynamite milling machine. An adaptive magnetic bearing (AMB) was used to apply non-contact electromagnetic force to the dummy tool while embedded inductive displacement sensors (DS) measured the tool response simultaneously. Comparisons between laser vibrometry and inductive displacement sensors were made.

Figure 3-2: Above: Schematic sketch of the experimental arrangement, Adaptive magnetic bearing (AMB). Below: Photo and schematic representation of the AMB, Collet holder (CH), Dummy tool (DT), Laser measurement point (LMP), Electromagnet (EM), displacement sensors (DS), Ferro magnetic material (FM) and spindle (S).
In Figure 3-3 a measurement on the non-rotating spindle with an optically rough surface is shown. The dummy tool is harmonically excited at 400 Hz in the y-direction along the direction of the probing laser beam. The appearance of the laser vibrometer time history in (a) is as expected for a harmonically excited object. In (b) the spectrum of the laser vibrometer signal is shown: beside the peak at the excitation frequency the harmonics are excited. The spectrum also contains a high-frequency noise floor, which is at an acceptable level. This spectrum is typical for laser vibrometry measurements on harmonically excited objects. In (c) the displacement sensor time history is presented, differentiated once for obtaining velocity. The differentiated signal is then smoothed using a moving average window with a span of seven points. The differentiation of the displacement signal is for making the comparison easier and the smoothing is the same as lowpass filtering the signal. The result is very similar to the laser vibrometer signal in (a). The spectrum of the displacement sensor signal is shown in (d). The noise floor is higher and some additional peaks at 940 Hz, 1400 Hz and 1900 Hz are observed. This is, however, explained by the fact that the laser vibrometer and the displacement sensors measured at different positions on the rotating dummy tool along the z-axis causing a small difference in the output amplitude.

In Figure 3-4, the same measurement is performed with the spindle rotating at 700 rpm corresponding to 12 Hz. In Figure 3-4(a) and (c) the time response for one rotation and 34 cycles of translational vibration are shown obtained from the laser vibrometer and the displacement sensors, respectively. The time history of the laser vibrometer signal is
no longer smooth like the response measured with the displacement sensors, but contains amplitude jumps. In (b), the spectrum of the full length (1 s) of the laser vibrometer signal contains speckle peaks at every multiple of the rotation harmonics. The level of the speckle harmonics is about 0.1 mm/s with some random fluctuation. This is typical for laser vibrometer measurements on rotating objects, where the speckle noise completely masks other low amplitude vibration components. In (d) the spectrum of the full length (1 s) of the displacement sensor signal is shown for comparison. The spectrum contains the rotation frequency, the excitation frequency with the harmonics and some other frequencies caused by the rotation. The second and the third harmonics are broader compared to the non-rotating spindle. The additional peaks found in Figure 3-3(d) also occur here.

![Figure 3-4](image_url)

(a) (b) (c) (d)

Figure 3-4: Measurements on the spindle rotating at 700 rpm (12 Hz) harmonically excited at 400 Hz. The laser measurement surface is optically rough. (a) Laser vibrometer time history, (b) laser vibrometer spectrum, (c) displacement sensor time history, and (d) displacement sensor spectrum.

The speckle noise in laser vibrometry measurements on rotating objects has been studied since the late 1980s and is now well known as a major problem, especially if you are studying vibrations in a frequency range close to the rotation harmonics. Different methods have been proposed and investigated, for example by optimizing the detector size and the position relative to a rotating target, the speckle noise level can be reduced in experiments by up to 10 dB [9]. However, this method does not remove the speckle noise completely. Speckle harmonics can be removed by breaking the repetitively of the measurement path between each revolution either by randomising the laser measurement position along the axis of rotation [10], or by changing the surface structure. The latter has been achieved by continuously applying oil to the surface during the measurement [11].
The second and the third method have only been verified experimentally by laser torsional vibrometers and no radial measurements have been presented using “standard” laser vibrometry.

Studying structural vibrations in the frequency domain, normally gives a spectrum containing a series of peaks where the damping of the structure is related to the sharpness of the peaks. On highly damped structures the frequency components are broad and relatively easy to distinguish from harmonic speckle noise. On lightly damped structures on the other hand, the natural frequencies are narrow and may more easily be misinterpreted as harmonic speckle noise. One way to investigate if any speckle harmonics are overlie structural vibrations is to measure at several number of different spindle speeds. However, some frequency components are spindle speed dependent and some uncertainty still may be present.

The vibration behaviour of a cutting tool is typically a function of the spindle rotation speed and the number of cutter teeth. Since the speeds of high-speed milling machine spindles are almost constant and the fact that a measurement sequence normally contains many revolutions, the frequency components of cutting vibrations are very narrow. As an example a force measurement during shoulder milling an aluminium workpiece is shown in Figure 3-5, where the frequency axis is normalized with the spindle rotation frequency. Components up to the 12th order are present. It is easy then to realize that it is difficult to identify the true cutting vibrations in laser vibrometry measurements since they coincide with the harmonic speckle noise. An approach to avoid speckle harmonics is described in 3.3.

![Figure 3-5: Typical cutting force measurement in shoulder milling.](image)

3.2 Crosstalk in laser vibrometry measurements on rotating spindles

The laser vibrometer is sensitive to target velocity in the direction of the laser beam and therefore when measuring on rotating targets the total surface velocity in the laser direction will be recorded. Resolving the different velocity components will not be trivial. If the desired vibration component is the axial velocity component, the crosstalk problem can be avoided by moving the laser beam synchronously with the rotation; such tracking systems have been applied in e.g., tyres and propellers [12, 13]. In this way measurements may be made at different positions on the rotating surface, or a grid of points even may be measured. However, for radial vibration measurements the tracking of the measurement surface is not possible since the laser beam has to be stationary in space and the measured vibration velocity will be a mix of vibration components. A detailed and thorough velocity sensitivity model is described in Refs. [14-17].
Consider Figure 3-6 where the profile of a rotating and vibrating shaft is shown. The shaft undergoes a translation vibration displacement, \( \mathbf{A} = x \hat{x} + y \hat{y} \), and a total rotation, \( \boldsymbol{\Omega} = \Omega_2 \hat{z} \) (including torsional vibrations, neglecting pitch and yaw). The total velocity at the laser measurement point, LMP, is the sum of the translational velocity and the velocity relative to the instantaneous rotation axis
\[
V_{\text{LMP}} = \frac{d}{dt} \mathbf{A} + \boldsymbol{\Omega} \times \mathbf{r}_{\text{LMP}},
\]
where \( \mathbf{r}_{\text{LMP}} \) is the instantaneous position vector of the laser measurement point relative to the rotation axis. The position vector, \( \mathbf{r}_{\text{LMP}} \), can be written with respect to the initial undeflected position vector \( \mathbf{r}_0 = r_0 \hat{y} \), the translational vector \( \mathbf{A} \), and the displacement vector \( \mathbf{p} = (p - r_0) \hat{y} \)
\[
\mathbf{r}_{\text{LMP}} = (r_0 - \mathbf{A}) + \mathbf{p}.
\]
Or
\[
\mathbf{r}_{\text{LMP}} = \begin{bmatrix}
-x \\
-y + p \\
0
\end{bmatrix}
\]
By inserting expression (3.3) into Eq. (3.1) we obtain the total surface velocity at the LMP
\[
V_{\text{LMP}} = \begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{0}
\end{bmatrix} + \begin{bmatrix}
\Omega_2 (y - p) \\
-\Omega_2 x \\
0
\end{bmatrix}.
\]
The measured velocity along the y-axis will therefore be
\[
V_{\text{LMP},y} = \dot{y} - \Omega_2 x,
\]
where the first term, \( \dot{y} \) is the desired translational velocity and the second term, \( \Omega_2 x \) is the error-term that can be of sufficient magnitude to make the laser vibrometry measurements on rotating shafts ambiguous.
Figure 3-6: Measured velocity component of a vibrating rotating shaft. Translation vibration displacement vector (A), angular velocity of the shaft (Ω), laser measurement point (LMP), displacement vector p, and position vectors r₀ and r_LMP.

The crosstalk can influence the vibration measurement producing a vibration level increase or decrease with respect to the correct level depending on the relative phase between the x- and y-vibration components and in the worse case the laser vibrometer output can be zero despite large translational vibrations. This trend can be explained by the following example.

Suppose that the displacements are

\[ x = A_x \sin(\omega t), \]

and

\[ y = A_y \sin(\omega t + \phi), \]

where \( \omega \) is the vibration frequency and \( A_x \) and \( A_y \) are the displacements amplitudes. Differentiating expression (3.7) gives the translational velocity in the y-direction

\[ \dot{y} = A_y \omega \cos(\omega t + \phi). \]

Inserting expression (3.6) and (3.8) into (3.5) gives the measured velocity

\[ V_{LMP,y} = A_y \omega \cos(\omega t + \phi) - \Omega A_x \sin(\omega t). \]

If the vibration frequency is equal to the rotation frequency, \( \omega = \Omega \) and if the phase difference between the displacements in the x- and the y-direction is 90°, the measured velocity (3.9) becomes
and in the worst case: when the amplitudes $A_x$ and $A_y$ are equal the laser vibrometer output becomes zero.

The experimental arrangement described in Figure 3-2 is again taken under consideration. The set-up allowed laser vibrometry measurements on the dummy tool along the y-direction and simultaneous independent measurements with inductive displacement sensors in two orthogonal, x- and y-directions. New series of measurements were performed where the dummy tool was harmonically excited by the AMB at 400 Hz perpendicular to the laser vibrometer measurement direction with a vibration displacement amplitude of about 6 μm (15000 μm/s). Figure 3-7 (from paper B) shows the crosstalk phenomenon in the laser vibrometry measurements for five different spindle speeds. The measurement shows a clear spindle speed dependent crosstalk although the external excitation is almost constant for all the spindle speeds. To verify if the crosstalk effect follows equation (3.5), calculations were performed using the displacement sensor signals $x$ and $y$. The velocity \( \dot{y} \) is obtained by numerical differentiation of the displacement signal in the Fourier domain. Numerically there is good agreement between the calculated velocity (circle) and the measured velocity (triangle up). The small deviation will be discussed later.

Figure 3-7: Crosstalk in laser vibrometry measurements on a rotating spindle for five different spindle speeds. Non-contact electromagnetic excitation at 400 Hz in the cross direction. Excitation amplitude 6 μm (15000 μm/s). Laser vibrometry measurement (triangle up). Calculated velocity using the displacement signals $x$ and $y$ (circle).

A method for resolving the translational vibrations using a setup of two simultaneously measuring laser vibrometers in both orthogonal directions and an accurate measurement of the rotational angular velocity using laser torsional vibrometry has been developed by Halkon and Rothberg [17]. A different approach, described in 3.3, below, has been taken in these studies.
3.3 An approach to avoid the speckle and the crosstalk problem

Since speckles are generated when the backscattered light is randomly dephased due to the surface microstructure, a simple approach to solve the problem is by making the measurement surface optically smooth, i.e. the surface variation is much smaller than the laser wavelength. In this way the backscattered light will retain its phase. When the measurement surface is optically smooth, the laser vibrometer should be insensitive to the rotating motion as long as the cross section of the rotating object is circular and significantly larger than the vibration amplitude. In order to investigate this approach the surface of the dummy tool in the measurement region seen in Figure 3-2 was polished optically smooth. In Figure 3-8 a measurement on the optically smooth spindle rotating at 700 rpm and excited at 400 Hz is presented. In (a) the laser vibrometer time history is shown. The appearance of this time history is very similar to that shown in Figure 3-4(c), which was obtained by the displacement sensors. The small difference in amplitude and phase is due to different measurement positions and time. The spectrum of the laser vibrometer signal is shown in (b) and resembles the spectrum obtained by the displacement sensor shown in Figure 3-4(d). The main difference is in the lower frequency part of the spectrum, where a number of harmonics to the rotation frequency is observed, which can be explained as follows:

![Figure 3-8: Measurements on the optically smooth spindle, rotating at 700 rpm (12 Hz), harmonically excited at 400 Hz. The laser measurement surface is optically smooth. (a) LDV time history, and (b) LDV spectrum.](image)

The profile of the polished measurement surface is not perfectly round. Since the laser vibrometer is a relative measurement device, the actual deviation from a perfect circle will also be present in the measurement signal. The out-of-roundness information will be seen in the spectrum as peaks at some lower rotation harmonics. The main difference between harmonic speckle peaks and out-of-roundness components in the frequency domain is that the amplitudes of the out-of-roundness components have a decaying appearance with increasing frequency while the amplitude of the harmonic speckle noise is almost constant.

In order to investigate the out-of-roundness of the tool further, laser vibrometry measurements were conducted during free run, which means that no external forces were applied. The displacement amplitudes at the rotational harmonics showed to be spindle speed independent and rapidly decaying. The third harmonic had the highest value, which indicated that the triangular component is the dominant one. By band-pass filtering the displacement signal it is possible to reconstruct the profile of the surface. Figure 3-9(a)
shows a polar plot of the filtered time signal for one revolution. Actually, the out-of-roundness of the profile can be presented in different ways depending on the choice of the reference circle; least square reference circle, minimum zone reference circle, minimum circumscribed reference circle or maximum inscribed reference circle are the most common reference circles. In our case, the measured deviation is simply linearly plotted with respect to the dashed circle. An independent measurement (at a later time) of the surface using a mechanical roundness tester (C E Johansson) confirms the reconstructed profile.

![Image](image.png)

Figure 3-9: (a) full-curve: Reconstructed roundness profile from the LDV measurement. Broken curve: reference circle. The distance between two circles is 2 μm. (b) Photo of the result obtained from the roundness test of the surface. The distance between the grids in the radial direction is 0.6 μm.

It has therefore, so far been established that by making the measurement surface optically smooth, the speckle harmonics can be avoided and that the out-of-roundness of the measured surface can be determined. Figure 3-10 shows laser vibrometry measurements on the optically smooth dummy tool where the spindle was excited at 400Hz in the cross direction. No spindle speed dependence can be observed; hence the crosstalk is also avoided when measuring on an optically smooth surface.

![Image](image.png)

Figure 3-10: Laser vibrometry measurements on the optically smooth rotating dummy tool. The spindle is excited at 400 Hz in the cross direction. Excitation amplitude 6 μm (15000 μm/s).

The laser beam alignment is critical in laser vibrometry measurements on rotating objects. In order to draw meaningful conclusions based on the comparison between the velocities obtained by the displacement sensors and the laser vibrometer the two
instruments must be perfectly aligned. Since it was difficult to align the laser vibrometer with the displacement sensors their outputs differed slightly. This implies that the displacement sensors that were intended to measure the displacements in the cross direction were also not perfectly orthogonal to the laser vibrometry measurement direction. As a consequence, the calculated velocities obtained from the displacement sensors also differed slightly from the laser vibrometer in Figure 3-7.

The laser beam is assumed to be perpendicular to the rotation axis and a misalignment angle can result in further inaccuracies. In our measurements the alignment of the laser beam is performed on an optically smooth surface and since the stand-off distance is about 1 m, even very small misalignments with the rotation axis results in signal drop-outs. It has been difficult to calibrate the displacement sensors with the laser vibrometer and the angular misalignment between the two instruments has been shown to affect the outputs. On the other hand, the misalignment effect does not interfere with the interpretation of the crosstalk effect and the conclusion is that it is possible to determine the true translational vibrations in the laser vibrometer measurement direction if the measurement surface is optically smooth.

3.4 Measurement of milling tool vibrations during cutting

In this subsection, the use of laser vibrometry for milling tool vibration measurements during cutting is presented. The experimental set-up is schematically shown in Figure 3-11. A highly polished cylindrical casing was mounted on the cutting tool, a 16 mm R390 mill with two teeth. The tool was in turn mounted in a five-axis high-speed milling machine centre, Liechti Turbomill ST1200. Shoulder cutting tests of an aluminium workpiece were performed with a spindle speed of 19,000 rpm. The spindle head was held stationary in space while the feed of the workpiece was achieved by the machine table. The axial and the radial depth of cut were set to 5 and 1 mm respectively. The laser vibrometer measured the tool vibrations from a safe operating distance of 1800 mm. A table dynamometer and two accelerometers served as comparative signals.

![Figure 3-11: Experimental set-up for milling tool vibration measurement during cutting. Left: a schematic sketch. Right: a photo showing the cutting tool with the polished cylindrical casing, the workpiece, the table dynamometer and the clamps.](image)

In Figure 3-12, the measured tool displacement together with the measured cutting force is shown as a function of spindle rotation angle. In order to obtain the desired displacements, the radial misalignment and the out-of-roundness information were filtered out in the Fourier domain and the filtered signal was then numerically integrated. In order to check the repeatability, ten revolutions of the middle part of the signals are plotted.
Cutter one is engaged between rotation angles 80° and 110°, and after the first cutter disengagement the tool continues to vibrate freely until the next cutter edge engages between rotation angles 260° and 290°. The amplitude of the first cutter is higher in both tool displacement measurements and cutting force measurements. The difference can be explained by the geometry and the positions of the cutters inserted in the tool. Actually, the present asymmetry in the tool not only affect the amplitude but also there is a small difference in the angular range of the teeth since the first cutter enters the cut at a smaller entry angle and later leaves the cut at a larger exit angle. To summarise, the figure shows the dynamic response of both the tool and the workpiece during cutting. This is discussed in further detail in paper C.

![Graph showing tool displacement and cutting force](image.png)

**Figure 3-12:** Tool displacement, $D_y$, and cutting force, $F_y$, in the feed direction as a function of spindle rotation angle. Ten revolutions overlaid.
Holography is a non-contacting whole field method for recording and reconstructing optical wavefronts. With this technique, both the amplitude and the phase of the original wavefronts are reconstructed. The technique has been used for recording static and dynamic events in solids and transparent media. Holographic interferometry is a method of comparing holographic reconstructed wave fields with a reference wave field interferometrically. Detailed presentation of the subject can be found in e.g. Refs., [18, 19]. The method is suitable for non-intrusive vibration measurements. In classic holographic interferometry photographic film was usually used as the recording medium and the reconstruction was performed by illuminating the film with a reference wave. The use of wet chemical processing of the film and the reconstruction process was time-consuming and a drawback of the classic technique. Today solid state detectors are often used as the recording device and the analysis is performed using a computer. The most common method of determining the phase change between digital holograms is the Fourier transform method [20]. A review of the principles of digital hologram recording, numerical reconstruction and phase evaluation for interferometry application can be found in Ref., [21].

4.1 Radial vibration measurements along a rotating shaft

The laser speckle problem discussed in previous sections also is valid here. Holographic interferometry measurements on a rotating and vibrating highly polished shaft are presented in paper D. Figure 4-1 shows the experimental set-up. Laser light is divided by a polarizing beam splitter PBS into two beams, where the intensity ratio between them can be changed by rotating the λ/2-plate. One beam is directed at the rotating shaft along the x-direction i1. The light reflected from the object o1 is then directed towards a diffuser D. The other beam is expanded and then collimated by the cylindrical lenses L1, and L2, respectively. The light that now has the shape of a sharp line with a height of 45 mm in the z-direction illuminates the shaft along i2. The reflected light, o2, falls onto a different part of the diffuser. The diffuser is then imaged onto the camera detector by a 100 mm lens. A rectangular aperture A, with a size of 1.34 × 5.0 mm² in front of the imaging lens serves as a lowpass filter. A small portion of the light is reflected at the flat surface of L1 and is used as reference beam R. In order to get interference between the reference beam and the two object beams, the reference beam passes a polarization rotator (45°). A circular aperture, about 1 mm in diameter and an expanding lens system ensure that the reference beam is sufficiently smooth and spherical. The reference beam is reflected by the cube beam splitter and interferes with the object beam on the detector and a digital image-plane hologram is recorded. The reference beam is slightly off-axis to ensure the separation of the interference terms in the Fourier domain.

With this arrangement, a 1 rad phase change will be equivalent to λ/(4πR) displacement.
The detector of the high-speed digital camera (REDLAKE MotionPro X3) has a resolution of 1280 × 1024 pixels and a pixel size of 12 × 12 μm². The maximum framing rate at full resolution is 1,000 frames per second (fps). The number of the pixels was reduced to 1280 × 100, and the framing rate was increased to 10,000 fps. The exposure time was set to 96 μs.

The change in phase between two exposures is limited to π radians and the maximum allowed displacement between those frames is λ/4, which corresponds to the maximum vibration velocity of

\[ v_{\text{max}} < \frac{\lambda}{4} f_{\text{camera}}, \]  

where \( f_{\text{camera}} \) is the camera frame rate in Hz. Thus the maximum allowed vibration velocity is 1.3 mm/s for a framing rate of 10,000 fps.

Figure 4-1: Digital holographic interferometry arrangement for radial vibration measurements. Above: Schematic sketch. Below: Photo of the set-up. Beam splitter (BS), polarizing beam splitter (PBS), cylindrical lenses (L1 and L2), electromagnet (EM) and diffuser (D).
A highly polished cylindrical steel shaft with a diameter of 15 mm and a height of 210 mm was positioned vertically on a motorized rotation stage with a rotation speed of 1.15 revolutions per minute. The shaft was then excited at 20 Hz in the y-direction by an electromagnetic coil close to the free end of the shaft without contacting the shaft surface.

A laser vibrometer was positioned 90 cm from the shaft and measured the vibrations in the y-direction from the opposite side of the shaft at a single point. Both the high-speed camera and the laser vibrometer were triggered at a certain phase of the rotation via a delay unit. The recorded image sequence was 2 seconds.

Figure 4-2 shows an example of obtained vibration displacements at t = 25.1 ms, after subtraction of the radial misalignment and the out-of-roundness components. In Figure 4-3 displacements measured both by holographic interferometry and laser vibrometry are shown. Beside the fundamental excitation frequency, a small modulation at half the excitation frequency (10 Hz) due to remanence and a 100 Hz modulation due to free vibration of the shaft can be observed. The curve obtained by laser vibrometry is shifted up 0.1 μm for easier comparison. The results (y-curves) in Figure 4-3 are very close.

To summarise, the measurement is very promising and may be applied on e.g., high-speed spindles. The method provides operational vibration mode in different direction of a rotating spindle which gives a more complete picture of the vibrating spindle.

![Figure 4-2: Obtained radial displacements, y, measured along the shaft at t = 25.1 ms.](image1)

![Figure 4-3: Displacement in both x- and y-directions as a functions of time. The displacement obtained by laser vibrometry (LDV) is shifted up 0.1 μm for easier comparison.](image2)
5 SCANNING LASER VIBROMETRY
MEASUREMENTS

5.1 Measurements on a thin-walled aluminium workpiece

Machine tool vibrations are the result of relative movement between the cutter and
the workpiece [2]. In most situations the workpiece is considered as a solid part fixed to
the machine table with no significant modal properties of its own. This assumption tends
not to apply when machining components with low rigidity. In manufacturing of
components for aerospace applications, for example, the material removal can be up to
90% of the original volume [22]. Hence the modal parameters of the workpiece, such as
natural frequencies, natural mode shapes and stiffness changes, and consequently so does
the vibration behaviour of the whole system as machining progress. A detailed
presentation of the theory of modal analysis can be found in e.g., a book by Ewins [23].

In order to be able to predict reliable stable machining parameters, such as spindle
speed and depth of cut, knowledge of the workpiece response is essential throughout the
whole milling process. The modal parameters of the workpiece may be obtained for the
continuous milling process by the use of theoretical models and mathematical
computations. However, these models must be verified by experiments since on many
occasions they often prove to be inaccurate due to the complex geometry and boundary
conditions.

The mass loading of contacting vibration transducers such as accelerometers induce
local stiffness variations on the object and produce uncertainties in vibration
measurements on light structures, especially at higher frequencies. There is therefore a
need for non-contact measurement techniques, and laser vibrometry is here an alternative.
The laser vibrometer is a point measuring instrument and for obtaining field
measurements, a sequence of single point measurements is what is required. Scanning
laser vibrometers offer this possibility as long as the event being studied is repetitive.

As mentioned earlier, the laser vibrometer measures the velocity component along
the laser beam direction. Misalignment and in-plane vibration velocities thus produce
errors in the measurements and must always be considered. Direction uncertainty and
error due to in-plane vibrations are outlined in e.g., the review article by Castellini et. al.
[24].

The laser vibrometer was used to study an L-shaped aluminium (7010T7451) detail
for aerospace application. The thickness of the detail was reduced from 40 mm to 1.0 mm.
The detail has shown vibration instability during machining especially at the final stage.
The objective of the experimental test was to compare the measured vibration modes with
the data produced by a finite element model (FE-model) and to calibrate the boundary
conditions of the FE-model of the detail clamped in the machine table. Further, the FE-
model is used in a model for predicting stability diagrams for milling. These diagrams are
presented in paper E. The outputs from the modal tests were the natural mode frequencies
and mode shapes. A photo of the experimental setup is shown in Figure 5-1.
Figure 5-1: Photo of the experimental set-up for modal testing of the aluminium detail clamped in the machine table.

An electromechanical shaker was used to excite the detail. The input force was measured with a force transducer. Synchronization between the scan points was obtained using the force signal as the reference.

In order to ensure that minimum in-plane vibrations were induced, careful alignment of the shaker-stinger-force-transducer-object was conducted. The stand-off distance was about 250 cm, and the length of the detail was 30 cm. The laser vibrometer was aligned so that the laser beam at the centre of the measurement grid was orthogonal to the plane of the detail. This arrangement produces a maximum misalignment angle of below 5° at the boundaries of the measurement grid. An angular misalignment of 5° to the optical axis corresponds to a velocity uncertainty of 0.38 % [24]. Even more important is the measurement error due to in-plane motion of the object. The error made is below 2 % for a misalignment of 5° and a ratio between in-plane and out-of-plane components of 0.1, which was acceptable in this case study. Vibration mode shapes and frequencies were then extracted from the measurement data by the vibrometer system software.

Figure 5-2 shows the first mode of the initial geometry. In (a), the laser vibrometry measurement is shown at 2835 Hz. The colour coding represents quantitative displacements, where the blue and the red areas are vibrating out of phase relative to each other. A sequence of calculations was run to iterate to the correct boundary conditions. The result is presented in Figure 5-2(b). The calculated natural frequency was at 2940 Hz, which is close to the measured natural frequency. The first mode of the final geometry is shown in Figure 5-3. The measured mode shape is shown in (a). The natural frequency is decreased to 685 Hz and the position where the maximum vibration amplitude occurs is moved about 12 cm. The FE-calculation shown in (b) is at a frequency of 800 Hz.
5.2 Measurements on a bowed violin

The mechanical response of a bowed violin was studied. Scanning laser vibrometry together with a specially constructed continuous bowing device, can measure different parts of an assembled violin under conditions very close to real life. The investigation shows how vibrations from the excited string transmit to the violin body via the bridge and produce the characteristic sound of the violin. The measured operational deflection shapes give additional understanding of how vibrating plates emit sound. Figure 5.4(a) shows a photo of the experimental set-up. For a detailed description of the experimental arrangement, see appended paper F. The operational deflection shape of the front of the violin at 1130 Hz, which is the fourth partial of the played tone, is shown in Figure 5.4(b). The blue and the red areas are vibrating out of phase relative to each other. Colour coding is set within the measuring range so that the results can be visualized as clearly as possible; ± 4 μm. The vibration pattern of the violin is complex and the illustrations become even more interesting when the sound fields produced are also observed, see paper F. The sound measurement will be described in chapter 6.
Figure 5-4: (a) Photo of the experimental set-up. (b) Operational deflection shape of the violin at 1130 Hz. The blue and the red areas are vibrating out of phase relative to each other. Colour coding: ± 4 μm.
6 SOUND FIELD MEASUREMENTS USING LASER VIBROMETRY

The use of laser vibrometry for sound measurements was proposed by Zipser et al [25, 26] in the year 2000. Since then several studies in different applications have been carried out [27-33]. In this chapter the principles of sound measurements using laser vibrometry and sound field measurements of a bowed violin and also sound field measurements of ultrasound transducers are presented.

6.1 Sound field projection recordings

In order to use the laser vibrometer for sound measurements the vibrating object in Figure 2-1 is replaced by a rigid reflector and the distance, \( L \) is kept constant. The arrangement can be seen in Figure 6-1. The sound field propagating through the constant measurement volume, the distance the laser beam travels between the laser vibrometer head and the rigid reflector, will disturb the density distribution in air and consequently cause pressure fluctuations [25, 26]. The spatial and temporal pressure fluctuations cause in turn changes in the refractive index, \( n(x, y, z, t) \). More general than described in paragraph, 2.1, the laser vibrometer measures the rate of change in optical path length, which is described by

\[
s = \int n(x, y, z, t) dl,
\]

where \( dl \) is the differential distance for the travelled light.

\[\text{Figure 6-1: Arrangement for sound field measurements using scanning laser vibrometry.}\]

Since the geometrical distance for the laser beam is constant, and the fact that the laser beam passes through the measurement volume twice, the rate of change in optical path is

\[
\dot{s} = 2 \int_0^L n(x, y, z, t) dl,
\]

\[\text{(6.2)}\]
where \( \dot{n} \) is the time derivative of refractive index \( n \). Thus, the laser vibrometer output is a virtual velocity of the non-moving reflector. So then, the phase distribution of the sound field is given directly. In order to obtain the amplitude of the sound pressure the following calculations are needed. The density of a gas is coupled to the refractive index by the Gladstone-Dale equation [18]
\[
\frac{n - 1}{n} = K \rho,
\]
where \( K \) is the Gladstone-Dale constant. The Gladstone-Dale constant is a property of the gas and is a function of the wavelength of light, the temperature and the pressure. However, under moderate physical conditions, \( K \) is only wavelength dependent and is about \( 0.2256 \times 10^{-3} \text{ m}^3/\text{kg} \text{ in air (}\lambda = 632.8 \text{ nm}) \). Further, under adiabatic conditions the pressure is related to the density as
\[
\left( \frac{p}{p_0} \right)^{\gamma} = \left( \frac{\rho}{\rho_0} \right)^{\gamma},
\]
where \( p_0 \) and \( \rho_0 \) are the undisturbed pressure and density respectively. The variables \( p \) and \( \rho \) represent the acoustic contribution to the overall pressure and density fields and \( \gamma \) is the specific-heat ratio which is about 1.4 in air. Assuming that the sound pressure fluctuations \( p \) are small compared to the undisturbed atmospheric pressure, the time derivative of the refractive index is given by
\[
\dot{n} = \frac{n_0 - 1}{\gamma} \frac{\dot{p}}{p_0},
\]
where \( n_0 \) is the undisturbed refractive index. Combining Equation (6.2) and Equation (6.5) yields
\[
\dot{s} = 2L \frac{n_0 - 1}{\gamma} \frac{\dot{p}}{p_0} dl.
\]
This means that the pressure variation along the light path can not be resolved directly.

For the special case where the pressure variations are constant along the path of integration, the Equation (6.6) becomes
\[
\dot{s} = 2L \frac{n_0 - 1}{\gamma} \frac{\dot{p}}{p_0}.
\]
Solving for pressure, we get
\[
\dot{p} = \frac{\dot{s}}{2L} \frac{\gamma}{n_0 - 1} p_0.
\]
However, in most cases the pressure variations will not be constant along the path of integration and the probing beam will traverse different regions with different pressure variations. By scanning the laser beam across the reflector we will obtain a two-dimensional projection of the actual three-dimensional sound field propagating through
the measurement volume. Since sound fields are not necessarily symmetrical, different projection angles result in different laser vibrometry signals and account must always be taken of this projection effect. Some of the projection effects were investigated in Ref. [34]. In Figure 6-2 the measurement direction dependence is shown. The same sound field that was emitted from three 40 kHz ultrasound transducers positioned in a line on an xy-plane (see Figure 6-3) were measured along two different directions (for description of the experimental arrangement, see paper G). The projected sound field is presented in different graphical ways; the modulus of the measured data, shown in the first column, gives the amplitude distribution of the disturbed air which is a measure of magnitude at a certain line. The argument of the measured data, shown in the second column, gives the phase distribution which is important to gain more understanding of the sound field, and the real part of the measured data, shown in the third column, shows the real physical distribution of the wave. By creating a sequence of images of the real part of the wave for various phase shifts, we get a cyclic animation illustrating the propagation of the wave through the measured field. The first row presents the yz-plane. Here the shape of the sound field resembles the sound field from a single sound source. The second row presents the xz-plane. Here the interference pattern from the three sources is seen. These results clearly visualize the projection effect and hence the sound fields from laser vibrometry measurements have to be interpreted with caution. The colour coding of the images are tabulated in Table 6-1.

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>Phase</th>
<th>Real part</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Image]</td>
<td>[Image]</td>
<td>[Image]</td>
</tr>
</tbody>
</table>

Figure 6-2: Projection effect in laser vibrometry measurement of sound field. The same sound field is measured at two different angles. Top row: yz-plane. Bottom row: xz-plane.

Figure 6-3: Photo of the ultrasound transducers and the transducer holder.
Table 6-1: Colour coding for the ultrasound field measurements.

<table>
<thead>
<tr>
<th>Plane</th>
<th>Magnitude</th>
<th>Phase</th>
<th>Real wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>yz</td>
<td>0-5 mm/s</td>
<td>±π</td>
<td>± 5 mm/s</td>
</tr>
<tr>
<td>xz</td>
<td>0-2 mm/s</td>
<td>±π</td>
<td>± 2 mm/s</td>
</tr>
</tbody>
</table>

Transient events can also be recorded in single point measurements, but for field measurements the sound field has to be in a stationary condition where the relative phase between each measurement point has to be known. This is obtained by, for example, using a reference usually obtained from a microphone at a fixed position.

Let the example of the violin be once again regarded. The ODS for the fourth partial played tone of the front plate of the violin was shown in Figure 5-4. In Figure 6-4, the ODSs of both front and back plate together with the generated sound field are illustrated. The top plate is divided in antinodes with two horizontal and one vertical nodal line. The vertical nodal line should make the measured radiated sound less efficient since the optical path difference is integrated in a line along the width of the violin and two nearby vibrations in anti-phase cancel out each other. The upper part of the top plate vibrates in phase with the lower part and in anti-phase with the middle part, which probably cause the lobe-shaped radiation. However, the radiated sound field from the top plate is also affected from the two f-shaped holes, so-called f-holes and the sound from the violin is a combination of plate vibrations and air modes. It is easier to relate the emitted sound from the back plate to the measured ODS since no vertical nodal line can be seen. The two horizontal nodal lines make the back plate resemble three sources aligned in a row where the middle one is emitting in anti-phase with the two others. For a more thorough discussion about the relation between the body ODSs and the emitted sound field refer to paper F and Ref. [34].

Figure 6-4: LDV measurements on a bowed violin and the generated sound at 1130 Hz, the fourth harmonic of the played tone. Colour coding: top plate = ± 4 μm, sound field = ±0.8 μm and back plate = ±1.6 μm.
6.2 Tomographic reconstruction of sound fields

The word "tomography" is derived from the Greek *tomos* (slice) and *graphein* (to write) and tomography refers therefore to the cross-sectional imaging of an object. The mathematical basis for the reconstruction from projections was suggested by the Austrian mathematician Johann Radon. It is outside the scope of this thesis to give a detailed description of the tomographic principle, the interested reader should consult e.g., Ref. [35]. The purpose of this subsection is to describe briefly the basic steps of the subject.

The first step is to collect all the projections. Although tomography is most commonly carried out in medical applications using x-rays, it can also be applied to other types of projection data from multiple directions. Tomographic reconstructions of sound fields using TV-Holography recordings has for example been reported by Lökberg et al. [36] and Gren et al. [37]. Ultrasound field reconstruction using laser vibrometry was proposed by Matar et al. [38, 39] where projections of a single ultrasound transducer were measured distributed over 180° and the results obtained showed the pressure variations in a plane perpendicular to the transducer. Here we will use scanning laser vibrometry projections of a non-symmetric sound source constructed by ultrasound transducers.

The reconstruction algorithm that is mostly used in practice is the filtered backprojection algorithm. This tomographic reconstruction algorithm can in turn be divided into two steps. The first step is to filter the full set of projection data. This filtering has shown to be necessary to avoid distortions and misrepresentations. Due to computer efficiency the filtration is carried out in the Fourier domain, i.e. the Fourier transform of the projection data is multiplied with a suitable weight function and then is inverse Fourier transformed back. The ramp-filter has been shown to be the optimal filter and is that most used in backprojection algorithms. The last step is to backproject all the filtered data. This backprojection operation takes each point on the filtered projection and "smears" it back over a line. When this is carried out for all the points for every projection the three-dimensional complex amplitude of the measured sound field will be obtained.

Figure 6-5 is a schematic representation of the experimental set-up for the laser vibrometry projection recordings of the sound field emitted by ultrasound transducers presented in paper G. The laser head and the rigid reflector are held stationary and by manually rotating the transducer holder between each projection recording, various projection angles are obtained.

![Figure 6-5: Experimental set-up for laser vibrometry projection recordings.](image-url)
The transducers were arranged in an L-shape formation (Figure 6-6) driven at 40 kHz by a signal generator. The lower left transducer was set to operate out of phase relative to the other three.

Figure 6-7 shows the real part of a reconstructed sound field. This reconstruction is based on 72 different laser vibrometry projections, evenly distributed over 360°. About 7000 points were measured for each projection recording, which gives a spatial resolution limit of about 2 mm in each measured grid plane. In the Figure on the left, the sound field is plotted for xy-planes at different distances from the source plane and in the Figure on the right, the sound field is plotted for different xz-planes. It can also be seen that one of the positive connected transducers at x = 100 mm is shifted in phase relative to the other two. This phase shift is due to the fact that it was positioned a little lower than the other transducers. Since the wavelength in air is only about 8.5 mm, a shift in position of 2 mm corresponds to a phase shift of 85°. Since the density of the measured radial projections becomes sparser as one gets farther away from the rotation axis, the spatial resolution decreases towards the edges of the reconstructed volume. This can be improved by increasing the number of projection recordings per 360°. To summarise, it has been shown that laser vibrometry measurements combined with tomographic reconstruction is suitable for investigation of ultrasound fields in air.

Figure 6-7: The real part of the reconstructed sound field emitted by four ultrasound transducers arranged in an L-shape formation.
This research has developed the use of a single laser vibrometer for measuring the vibration of a high-speed spindle. This method requires that the spindle head is stationary in space, that the laser beam has a clear line of sight and is aligned with the spindle rotation axis, and that the measurement surface is optically smooth and kept clean during the measurement.

In paper A, we demonstrated that by making the measurement surface optically smooth the speckle noise harmonics in laser vibrometry measurements on a rotating spindle can be avoided. By bandpass filtering the spectrum of the laser vibrometry signal, the radial misalignment and the out-of-roundness of the surface could be determined. These components should be excluded in spindle vibration testing.

In paper B, we demonstrated that the orthogonal displacements introduced by a magnetic bearing do not leak in the laser vibrometry signal using our approach. This automatically implies that the torsional vibrations are also excluded from the measurement signal. This leads to the conclusion that the translational velocity component in the measurement direction can be determined using the proposed approach. The method requires neither simultaneous orthogonal measurements, nor independent rotational speed measurement and therefore also can be applied on rotating objects where only one radial direction can be accessed. It is possible to apply the measurement method on any rotating shaft with a circular cross section that can be polished and kept clean.

In paper C, the measurement method was applied on a real cutting test in a milling machine at a spindle speed of 19,000 rpm. Cutting tool vibrations were determined by subtraction of the radial misalignment and the out-of-roundness components. The tool displacements obtained followed the corresponding cutting force signal that was simultaneously measured using a dynamometer. Also, transient vibrations could be measured.

In paper D, full field measurements were performed on an optically smooth rotating shaft. The measurement provided radial vibrations along the shaft. Moreover, the experimental arrangement allowed simultaneous orthogonal measurements. It is possible to obtain a three-dimensional vibration description of the shaft, by a small modification of the experimental set-up. In recent years solid state detectors and increasing computer capacity have enabled high-speed digital holographic interferometry more useful for industrial applications. Hence, the technique may be applied to rotating spindles.

Stability predictions of milling processes are usually based on two types of milling experiments, either a rigid workpiece and a flexible tool, or a flexible tool and a rigid workpiece. Predicting stability for a milling process where the rigidity of the tool is close to the rigidity of the workpiece is far more complicated. However, a reliable tool-workpiece transfer function is needed. This is hard to achieve in practice, since the rigidity of the workpiece is continuously changed as the material is removed during machining. In paper E, a model for predicting stability limits in high-speed machining of a thin-walled structure is presented. The model is based on laser vibrometry measurements on the spindle and finite element calculations on the workpiece for different stages in the milling process. The laser vibrometry measurements of the workpiece clamped in the machine
table have been crucial for improvement of the boundary conditions of the finite element model.

In paper $F$, the vibrations of the different parts of the violin as well as the generated sound fields were studied. The measurements on the string shows stick-slip behaviour and the bridge measurements show that the string vibrations transmit to the bridge both in the horizontal and the vertical direction. Measurements on the plates show complex deflection shapes which are combinations of different Eigenmodes. The sound fields emitted from the violin were measured and visualized for different harmonic partials of the played tone. However, the visualized sound field obtained by the laser vibrometer is a projection of the sound field along the laser light and the image obtained is a two-dimensional map of the real three-dimensional sound field, which must be considered. In the worst case the probing laser beam may pass through two equally strong sound pressures with opposite phase resulting in zero laser vibrometer output.

In paper $G$, it was shown that sound field projections, measured using laser vibrometry can be used in computed tomography to obtain the three-dimensional distribution of the sound field. This additional information can be essential in cases where the vibration behaviour of the structure that radiates the sound is unknown.

The measurement methods provide non-contact high-stand off distance vibration and sound measurement on complex structures. The methods can be applied on high-speed spindles, for vibration measurements, runout measurements, roundness measurements or sound measurements. The new sound measurement technique has big potentials for automatic measurements, on-line monitoring and quality control in mechanical systems with thin-walled structures and rotating parts.
High-speed digital holographic interferometry has been used to measure the two orthogonal radial vibration components along a rotating polished shaft. The measurement method is yet to be applied on a high-speed spindle. Such measurement would provide the operational deflection modes of the rotating spindle and a better picture of the dynamics of the rotating spindle. Moreover, it is well known that rotating shafts tend to bow out at certain speeds and whirl in a complicated manner [40, 41]. This may be the case when long and slender tools are used in machining [42]. The whirling of the tool may be in the same direction or in the opposite direction of the spindle rotation and the velocity may or may not be equal to the spindle speed. In order to be able to determine the direction of the whirling Yamamoto et al. [41] has proposed a complex-FFT-method where the whirling plane of the rotor is mapped to the complex plane. For this purpose two orthogonal simultaneous time histories of the tool are needed, which can be obtained, for example, from the high-speed digital holographic interferometry recordings, or from two orthogonal laser vibrometry signals. The digital holographic interferometry technique in particular may provide whirling along the tool axis. It is worth investigating the whirling phenomenon in an actual cutting test. The whirling vibrations are most likely related to the relative radial misalignment between the tool and the spindle shaft.

The milling of metal produces noise. The sound of stable milling is calm while instabilities usually produce unpleasant noise. Laser vibrometry can possibly perform remote non-intrusive sound measurements during machining. It is likely that laser vibrometry measurements of the sound field near the tool can give addition information regarding the cutting process. This has yet to be investigated. Nevertheless, the spectrum of the generated sound can be used for setting thresholds for stable machining regions. Successful monitoring though requires real-time post processing of the signal where the tooth passing frequencies are filtered out. Such automated data processing requires an evaluation program that has access to the spindle speed.
9 SUMMARY OF APPENDED PAPERS

The appended papers are listed according to their content in Table 9-1 below, thereafter follows a short summary and conclusion to each paper.

Table 9-1: Contents of appended papers

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<th>Paper</th>
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<th>Thin-walled structures</th>
<th>Machining/Production</th>
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Paper A: Laser doppler vibrometry measurements of a rotating milling machine spindle
By: M. Rantatalo, K. Tatar and P. Norman
Summary: The harmonic speckle noise in the laser vibrometer signal of a rotating spindle was studied. For avoiding the harmonic speckle noise, it was proposed to make the measurement surface optically smooth. The spectra of two different measurement surfaces were compared.
Conclusions: By making the measurement surface optically smooth the harmonic speckle noise in the laser vibrometry signal was avoided and the radial misalignment as well as the out-of-roundness of the surface could be determined.

Paper B: Laser vibrometry measurements of an optically smooth rotating spindle
By: K. Tatar, M. Rantatalo and P. Gren
Summary: The crosstalk between the vibration velocity components in laser vibrometry measurement of a rotating spindle was studied. The spindle was excited by an adaptive magnetic bearing and the response was measured by both laser vibrometry and inductive displacement sensors. Two different measurement surfaces for five spindle speeds were compared.
Conclusions: Both harmonic speckle noise and crosstalk in laser vibrometry measurement of a rotating spindle could be avoided. The non-contact excitation and measurement method may be applied in spindle vibration analysis.
**Paper C:** Measurement of milling tool vibrations during cutting using laser vibrometry  
**By:** Kourosh Tatar and Per Gren  
**Summary:** The use of laser vibrometry for milling tool vibration measurement was demonstrated.  
**Conclusions:** A remote non-contact machine tool vibration measurement method was presented. It was shown that cutting vibrations could be determined from the laser vibrometer signal in the Fourier domain. The method also enabled transient vibration measurements of the tool during cutting, which might contribute to the understanding of the dynamics of milling systems at the start and end of a cutting path. The method may be used for analysis and also as a tool in the development of spindles and machine tools.

**Paper D:** Digital holographic interferometry for radial vibration measurements along rotating shafts  
**By:** Kourosh Tatar, Per Gren and Henrik Lycksam.  
**Summary:** A method to measure radial vibrations along a highly polished rotating shaft was developed using a high-speed camera and a digital holographic interferometry set-up. Simultaneous measurements with a laser vibrometer were made.  
**Conclusions:** The two orthogonal radial vibrations and also the deflection mode shapes of a rotating shaft could be determined. The technique may be applied for spindle vibration analysis, spindle rotation error measurements, out-of-roundness measurements and also for investigating the whirling phenomenon in long and slender spindles.

**Paper E:** Integrated Approach for Prediction of Stability Limits for Machining with Large Volumes of Material Removal  
**By:** A. Svoboda, K. Tatar, P. Norman and M. Bäckström  
**Summary:** A model for the prediction of stability limits as a function of process parameters for machining of a thin-walled aluminium detail was presented. The model was based on finite element calculation of the workpiece and laser vibrometry measurements of the spindle. In order to improve the boundary conditions of the finite element model, experimental modal analysis of the detail clamped in the machine table was performed using laser vibrometry.  
**Conclusions:** The method predicts stability limits in high speed machining of a thin-walled workpiece.
**Paper F:** Laser vibrometry measurements of vibration and sound fields of a bowed violin

**By:** P. Gren, K. Tatar, J. Granström, N-E. Molin and E. V. Jansson.

**Summary:** The vibration and the sound field of a violin was studied using laser vibrometry. The string was excited using a rotating bow apparatus and the vibrations from the string transmitted to the violin body via the bridge and produced the sound. The measurements on the string showed stick-slip behaviour and the bridge measurements showed that the string vibrations transmitted to the bridge both in the horizontal and the vertical direction. Measurements on the plates showed complex deflection shapes which were combinations of different eigenmodes. The sound fields emitted from the violin was measured and visualized for different harmonic partials of the played tone.

**Conclusions:** The designed bowing apparatus was an effective devise to excite the violin in a long and controlled manner. Vibrations of the different parts of the bowed violin as well as the emitted sound were measured. The visualized sound field obtained by the laser vibrometer is a projection of the sound field along the laser light; the image obtained is a two-dimensional map of the real three-dimensional sound field. The projection effect must be considered in sound field measurements using scanning laser vibrometry.

**Paper G:** Tomographic reconstruction of ultrasound fields measured using laser vibrometry

**By:** Kourosh Tatar, Erik Olsson and Fredrik Forsberg.

**Summary:** The possibility to obtain the three-dimensional amplitude and phase distribution of a sound field using laser vibrometry recordings and computed tomography was investigated. The sound field of both a symmetric and a non-symmetric ultrasound transducer configuration was measured and reconstructed.

**Conclusions:** Scanning laser vibrometry has shown to be a technique that provides remote non-disturbing two-dimensional projection of a three-dimensional sound field. By recording a sufficient number of projections at evenly distributed angles around a rotation axis perpendicular to the plane of the projections, the full three-dimensional complex amplitude of the sound field was reconstructed using a cone-beam tomography algorithm.


Part II

Papers
Paper A
ABSTRACT

Finding an optimum process window to avoid vibrations during machining is of great importance; especially when manufacturing parts with high accuracy and/or high productivity demands. In order to make more accurate predictions of the dynamic modal properties of a machining system in use, a non-contact method of measuring vibrations in the rotating spindle is required. Laser Doppler Vibrometry (LDV) is a non-contact method, which is commonly used for vibration measurements. The work presented consists of an investigation into the use of LDV to measure vibrations of a rotating tool in a milling machine, and the effects of speckle noise on measurement quality. The work demonstrates how the axial misalignment and the roundness of a polished shaft can be evaluated from LDV measurements.

1 INTRODUCTION

Manufacturers of modern machine tools are increasingly implementing advanced process monitoring and supervisory process control (1) to complement the basic functionality of the machine tool control system. At there simplest, process monitoring systems are used to help prevent or limit the effects of catastrophic events such as tool breakage (2) or spindle failure. Such events can be detected by monitoring the current drawn by axis drives and spindle motor (3, 4), or by more advanced techniques such as cutting force monitoring or measurements of vibrations using accelerometers or acoustic emission using sensitive transducers and signal conditioning software (5-9). By setting safe limits for the monitored parameter(s) based on experience or trials, unusual or unexpected events which may indicate a catastrophic failure can be used as a trigger to stop the machine.
Since vibrations are the result of relative movement between the cutter and work piece, the dynamic behaviour of both the machine structure and rotating spindle/cutter together with the behaviour of the component being machined has to be considered. In most situations, the work piece can be considered a solid part fixed to the machine table with no significant modal properties of its own. This assumption tends to weaken, however, when machining components with relatively thin walls (10).

Regenerative machine tool chatter is a fundamental type of vibration that can occur during milling. These vibrations have their origin in the closed loop nature of the cutting process and are dependent on the structural vibration modes, described by the frequency response function (FRF) of the machine tool. The FRF is normally measured on a non rotating/static system from which the limits for chatter free machining can be calculated (11). In modern machine tools, spindle speeds of 20,000 rpm and upwards are not uncommon, since the dynamic characteristics of the spindle such as damping change, this causes the FRF of the system as a whole to change.

To be able to fully investigate the behaviour of a high-speed rotating system, such as a machine tool spindle, it is necessary to use non-contact measurement methods. Several approaches to the non-contact measurement of rotating objects have been developed. These include optical techniques such as Pulsed Laser TV-Holography (12) and Laser Doppler Vibrometer techniques (LDV) (13).

LDV is a well-established technique for measuring the velocity of a moving object. It is based on the Doppler effect, which explains the fact that light changes its frequency when detected by a stationary observer after being reflected from a moving object. The vibrating object scatters or reflects light from the laser beam and the Doppler frequency shift is used to measure the component of velocity which lies along the axis of the laser beam. As the laser light has a very high frequency, direct demodulation of the light is not possible and optical interferometry is therefore used.

When a coherent light source illuminates a surface that is optically rough, i.e. the surface roughness is large on the scale of the laser wavelength, a granular pattern called speckle which has random amplitude and phase is seen. This is due to interference between the components of backscattered light. The intensity of a speckle pattern obeys negative exponential statistics and their phases are uniformly distributed over all values between \(-\pi\) and \(\pi\) (14). If the speckle pattern changes during LDV measurement the rate of change in the resulting phase will be nonzero, and the frequency spectrum will contain peaks. These kinds of speckle fluctuations are induced by non-normal target motions, such as tilt, in-plane motions or rotation (15). Speckle fluctuations due to target rotation are periodic and will repeat for each revolution. This leads to peaks in the spectrum at the fundamental rotation frequency and higher order harmonics. These modulations are difficult to distinguish from the true vibrations and in the worst case, can almost completely mask the vibration pattern. It is therefore important that the target to be measured has a surface smooth enough so that the speckle noise is avoided.
2 EXPERIMENTAL SET-UP AND PROCEDURE

2.1 Preparation of the dummy tool
A dummy tool with a radius of 10 mm and a length of 100 mm was manufactured from a solid stainless steel tool blank. The shaft was mounted in a lathe and polished using emery paper with grades ranging from 400 (grains/mm) to 1200. The shaft was finally polished using diamond paste with particles ranging from 9 μm to 0.25 μm and with a chemical polishing fluid. Quality control of the polished surface was performed using non-contact optical surface profile measurement (www.veeco.com). In the actual experiment a spray which is normally used for crack detection was used to create a removable diffuse (optically rough) surface on the polished dummy tool. Both the polished tool surface and the sprayed surface were measured by the optical profiler, and a representative area of the tool of 304 x 199 μm was sampled in steps of 414 nm. The measurements showed a normally distributed surface structure with Ra = 11.29 nm, implying that the polished surface is optically smooth compared to the laser wavelength of 633 nm. The sprayed surface showed substantially higher values, Ra = 21.02 μm, giving an optically raw surface.

2.2 The milling machine
The LDV measurements were made on a Liechti Turbomill ST1200 ‘state-of-the-art’ machining centre offering multiple (5-axis) movement and a spindle capable of speeds of up to 24,000 rpm. The polished dummy tool was mounted in a Corogrip holder with an HSK shank which was in turn mounted in the machine and was not removed until all the measurements had been made.

2.3 Setting up the LDV
For the measurements, a PSV 300 LDV system from Polytec GmbH (www.polytec.com) including a displacement decoder was used. The LDV scanning head was mounted on a sturdy tripod and placed approximately 2 m from the tip of the dummy tool on a soft damped material to reduce the influence of structural floor vibrations. Care was taken to align the laser beam so that it’s centre line passed through the centre line of the shaft and was perpendicular to the shaft’s axis of rotation. This was necessary to ensure that the true velocity vector associated with the vibrations was along the incident direction of the laser beam. The LDV system was set up to perform sampling with a frequency of 40.96 kHz. The maximum detectable frequency was set by the system to 16 kHz. The LDV system produced frequency spectra with a standard FFT algorithm using a complex averaging method with 100 averages of 800 ms each giving a frequency resolution of 1.25 Hz and a total measuring time of 12.8 s.

2.4 LDV measurements
A series of experiments were carried out to establish whether vibrations of a rotating tool could be measured using the LDV system. Four different spindle speeds 2700, 4200, 6000 and 7200 rpm were studied. The LDV was used to measure the vibrations at the tip of the polished dummy tool in the radial direction at these speeds. The same set of measurements was carried out on the tool after being sprayed to give an optically raw finish.
Logged data was exported from the LDV as ASCII files and then imported into Matlab 6.0 where more detailed analysis and filtering of the data was carried out. The large-scale profile around the circumference of the dummy tool was measured using a mechanical roundness tester from C E Johansson (www.cej.se), with an accuracy of $\pm 0.3\mu m$. This was performed after that the LDV measurements were carried out.

3 RESULTS

In this section the results from the measurements at 6000 rpm are presented. The velocity spectrum of the polished and rough dummy tool measurements are displayed in the same chart for different frequency bands, Figure 2-5. The spectrum of the rough surface has been flipped down to the negative side in the charts to simplify comparison of the two spectra. In the charts it can be seen that the spectrum of the rough dummy tool contains peaks at $f \times n$ Hz where $f$ is the rotational speed of 100 Hz (6000 rpm) and $n = 1, 2, 3, \ldots$ These peaks are expected due to the presence of a speckle noise repeated for each dummy tool revolution in the sampled data.

A zoomed part of the spectrum covering the frequency band 8.8-10 kHz shows clearly the speckle noise in the form of peaks at integer multiples of the rotational speed of 100 Hz. These are marked with circles along the frequency axis, Figure 3. These peaks could not be seen in the graph of the polished tool. Between 1.1-1.5 kHz contains both speckle noise peaks...
and ordinary vibrations, Figure 4. Note that the vibrations are present in both curves but the peaks are only present in the spectrum of the rough surface.

The frequency band covering 0-1 kHz shows harmonic peaks in both FFT graphs, see Figure 5. However, the first peak at 100 Hz in the polished measurement spectrum was detected as the dummy tool axial misalignment and the other six harmonics as the roundness profile. Figure 6 shows the signal from the displacement decoder. This signal is band-pass filtered between 0.15-0.75 kHz, thus filtering out the roundness profile. The result is shown in Figure 7, where the filtered time signal for one revolution is presented in a polar plot (dashed line), together with an independent mechanical measurement of the roundness made by the roundness tester (solid line). The difference between the curves is less than the error given by the manufacturer of the roundness tester ($\pm 0.3 \mu m$). For the sprayed dummy tool the roundness could not be measured properly due to speckle noise caused by the rough surface. Similar results where achieved for measurements made at spindle speeds of 2700, 4200, and 7200 rpm.

Figure 2. Spectra of the polished and the sprayed surface at a spindle speed of 6000 rpm. The spectrum of the rough dummy tool has been mirrored along the frequency axis down to the negative side to simplify comparison between the two. Multiple harmonics of $n\times 100$ Hz where $n= 1,2,3,...$ can be seen in the spectrum of the sprayed surface.
Figure 3. Zoomed part of the spectrum, 8.8-10 kHz. Frequencies where peaks are expected due to speckle noise are marked with a ring on the frequency axis.

Figure 4. Zoomed part of the spectrum, 1.1-1.5 kHz. Frequencies where peaks are expected due to speckle noise are marked with a ring on the frequency axis. It can clearly be seen that no peaks is present in the spectrum of the polished surface at the marked positions. Note that the vibration signal is present in both measurements.
Figure 5. Zoomed part of the spectrum, 0-1 kHz. Frequencies where peaks are expected due to speckle noise are marked with a ring on the frequency axis. In this graph peaks in both spectra are present at the marked frequency positions. For the polished case the peaks are identified as tool axial misalignment and roundness.

Figure 6. Displacement measurement at 6000 rpm.
DISCUSSION AND CONCLUDING REMARKS

Speckle noise interference was avoided by polishing the surface of the dummy tool (in effect a rotating shaft) until an optically smooth surface was achieved. The optically smooth surface of the rotating tool generated no repeated speckle noise and hence no unwanted peaks at integer multiples of the rotational frequency. This allowed the axial misalignment and roundness of the dummy tool to be measured at speeds of up to at least 7200 rpm using an LDV. This implies that radial vibration measurements of the tool can also be conducted; for example, when investigating dynamics of the cutting process.

The possibility of extracting roundness and alignment information is based on the fact that at a rotational speed of 6000 rpm (100Hz) any misalignment would be seen as a 100Hz signal. Since the tool is not perfectly round, harmonics of 100Hz will be present. The first component of the out of roundness is an elliptical form and results in a frequency peak at 2*100Hz (200 Hz). The second roundness component, a tri-lobed form, would be seen as a peak at 3*100Hz (300Hz) and so on. In the experiments, no significant peaks at integer multiples of the
rotational speed could be detected above the 6:th component at 7*100Hz. To eliminate the possibility of structural vibrations being misinterpreted as misalignment or out of roundness, measurements were made at a number of rotational speeds (2700, 4200, 6000 and 7200 rpm). At each of these speeds, the misalignment data would be seen as a peak at a different frequency; namely 45Hz, 70Hz, 100Hz and 120Hz. Out of roundness data would be at multiples of the primary frequency. When measuring at speeds other than 6000rpm, any structural vibration around 100Hz would become clear, and the peak due to misalignment shifted. This makes it possible to analyse the presence of structural vibrations overlaying axial misalignment and out of roundness. No significant structural vibrations overlaying the roundness and misalignment data were detected in the experimental data.

However, since some frequency components of structural vibrations are spindle speed dependant it is not possible to draw firm conclusions about misalignment and roundness based solely on the LDV measurements. This effect must be investigated further. Good correspondence was however seen between LDV measurements and direct mechanical measurements of misalignment and out of roundness made using a dial test indicator and roundness tester. This indicates that no significant structural vibrations are overlaying the pitch and roundness data at the spindle speeds investigated.

Several problems must be overcome if LDV measurements are to be made of a rotating tool when it is cutting. Firstly, the laser beam must have a clear line of sight to the target surface on the tool without interference from cooling fluid or metal chips generated by the cutting process. The target surface must also be kept free of particles such as dust or process fluids. In milling machines where the tool moves relative to the machine base it must be possible to track the moving tool either by physically mounting the LDV on the moving axis of the machine or by some other tracking system. Finally, the alignment of the laser beam relative to the optically smooth target surface on the tool is also an issue that has to be considered. Axial misalignment, roundness, in-plane vibrations or applied cutting forces can affect the direction in which the laser beam is reflected from the target surface which could lead to a poor signal level or drop outs. In plane vibrations and deflection due to cutting forces together with the geometry of the shaft can also lead to misinterpretation of vibration data due to cross sensitivity. This has not been investigated in this work.

Different spindle / cutting speeds, cutting forces and changing component geometry affect the dynamics of a machining system. Measurement techniques based on physically mounting sensors, such as accelerometers, on the machine or workpiece can also affect the system dynamics. The ability to perform non-contact measurements of vibrations will allow measurement of changes in machine dynamics to be made during the cutting process without affecting the process itself. This is the subject of ongoing work.

5 ACKNOWLEDGEMENTS

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REFERENCES


Paper B
Laser vibrometry measurements of an optically smooth rotating spindle

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Abstract

Laser doppler vibrometry (LDV) is a well-established non-contact method, commonly used for vibration measurements on static objects. However, the method has limitations when applied to rotating objects. The LDV signal will contain periodically repeated speckle noise and a mix of vibration velocity components.

In this paper, the crosstalk between vibration velocity components in laser vibrometry measurements of a rotating dummy tool in a milling machine spindle is studied. The spindle is excited by an active magnetic bearing (AMB) and the response is measured by LDV in one direction and inductive displacement sensors in two orthogonal directions simultaneously. The work shows how the LDV crosstalk problem can be avoided if the measurement surface is optically smooth, hence the LDV technique can be used when measuring spindle dynamics.

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Keywords: Laser vibrometry; Crosstalk; Speckle noise; Spindle dynamics

1. Introduction

To be able to fully investigate the behaviour of a rotating system, such as a milling machine spindle, it is necessary to make measurements directly on the spindle during rotation. This can be done either by electronically or optically based non-contact measurement methods such as capacitive displacement sensors (DS) [1], laser distance sensors [2] or laser doppler vibrometry (LDV) [3]. The laser vibrometer is a powerful tool for measuring vibration velocities. The nature of the LDV system renders measurements without additional mass loading and allows a wide range of distances between the sensor head and the object (from millimetres up to several metres and scanning angles of about $\pm 20^\circ$). However, two major problems occur when performing LDV measurements on rotating objects; the presence of speckle noise and crosstalk between vibration velocity components. In some cases, a tracking system can be used, where the laser beam follows the rotating surface. Measurements on propellers and tiers have been demonstrated [4,5].
Speckles are a random pattern of dark and bright spots formed in space when a diffusely reflective surface is illuminated by coherent light (laser light). This is a result of superimposing wavelets of light with different travelled path length due to the surface structure. The speckle noise from a rotating shaft is generated by the moving speckle pattern on the LDV detector. This pattern is repeated for each revolution and will create a repeated noise in the measurement signal, called pseudo vibrations [6]. Larger surface structures than half the laser wavelength will result in fully developed speckles. It has been shown by prior authors that the speckle noise level can be reduced or removed with different methods. By optimizing the target-detector separation within a laser vibrometer the noise level can be reduced but not completely removed [7]. The speckle noise in laser torsional vibrometry measurements can be removed by randomising the path that the laser light is undertaking during the revolutions, either by moving the laser along the shaft [8] or simply by adding a new surface structure. The latter can be achieved by continuously applying e.g. oil or some other substances to the surface during the measurement [9]. In theory this technique should also work in laser doppler measurements.

The crosstalk problem can be described as an error-term in the measurement caused by a velocity component due to the rotation [10–13]. The measured velocities of a rotating optically rough shaft in the two orthogonal directions, $v_x$, $v_y$, can be expressed as [13]

\[
\begin{align*}
    v_y &= \dot{y} + \Omega(x - x_0) \\
    v_x &= \dot{x} - \Omega(y - y_0)
\end{align*}
\]

where $\dot{x}$ and $\dot{y}$ are vibration velocities, $x$ and $y$ are the vibration displacements, $x_0$ and $y_0$ are the distances to the spin axis due to alignment errors and $\Omega$ is the total angular velocity including torsional vibrations. $\dot{x}$ and $\dot{y}$ are the desired velocities to measure.

The methods for speckle noise reduction/removal described above cope with the specific speckle noise problem but are not able to neutralise the effect of crosstalk in a single beam LDV measurement. Consequently; the signal obtained during measurements under these circumstances will be a mix of the velocities in both directions.

A method for resolving the true vibrations in the two $x$- and $y$-direction using a set-up of two simultaneously measuring lasers in both directions and an accurate measurement of the rotational angular velocity has been developed by Halkon and Rothberg [13].

In [3], it is shown that the speckle noise in laser vibrometry can be avoided by polishing the surface optically smooth, i.e. the surface roughness is much smaller than the laser wavelength. The out of roundness was measured and showed a good agreement with a mechanical roundness measurement. However, the crosstalk was not investigated.

In this work, we investigate experimentally the crosstalk between the two directions ($x$ and $y$), in laser vibrometry measurements of a rotating spindle with a polished surface. The spindle is excited by an active magnetic bearing (AMB) manufactured by SKF Revolve, and the response in the cross direction is measured by LDV and inductive DS.

2. Experimental set-up and procedure

Fig. 1(a) is a photo showing the AMB mounted in the milling machine table and Fig. 1(b) is a sketch of the AMB. The measurements were made in a Dynamite; 3-axis vertical table-top milling machine. The spindle (S) is capable of speeds of up to 7000 rpm. The dummy tool (DT) was mounted in a Collet holder (CH) with a Morse taper which was in turn mounted in the machine. The surface of the LDV measurement position (LMP) was polished optically smooth. The surface roughness was measured to $R_s = 0.021 \mu m$, using a Wyko NT1100 optical profiler (www.veeco.com). The surface could be made temporary rough by spraying it with a developer for crack testing (paint). This paint could easily be removed without scratching the surface. The spindle was harmonically excited by the AMB in the $x$- or $y$-direction with electromagnets (EM) at a cylindrical segmented part of the dummy tool consisting of a ferromagnetic material (FM). The EM are arranged in two pairs opposite to each other. The vibrations of the spindle in the $y$-direction were measured by the vibrometer at LMP. Inductive DS within the AMB measured the displacements in the $x$- and $y$-direction simultaneously.
The inductive DS are arranged in pairs, one pair measure the displacement in the \(x\)-direction and the other pair in the \(y\)-direction. The sensitivity of the inductive DS are 110 \(\mu\text{m}/\text{V}\). The measurement range is 0–5 kHz and the limiting gap between the rotating dummy tool and the DS is 150 \(\mu\text{m}\).

A PSV 300 LDV system from Polytec GmbH including a displacement decoder was used. The LDV scanning head was mounted on a sturdy tripod and placed 1 m from the polished dummy tool. Specular surfaces obey the law of reflection; angle of incidence = angle of reflection. For successful measurements on a specular surface the laser vibrometer must be aligned properly with the target rotation axis. In practice, the arrangement was aligned by looking at the reflected laser light. A paper sheet with a hole for passing the laser beam was mounted on the scanning head. The laser beam was focused on the polished surface behaving like a cylindrical mirror. The reflected light sheet was adjusted so that the central part passed the aperture of the scanning head. The signal quality indicator of the LDV system showed a very high value at all spindle speeds so no dropout errors were present at the measurements. The LDV system was set up to perform sampling with a frequency of 32 kHz. The LDV sensitivity was set to 5 mm s\(^{-1}\) V\(^{-1}\) during the measurements on the smooth surface. The sensitivity was then decreased to 25 mm s\(^{-1}\) V\(^{-1}\) when measuring on the rough surface to avoid overloads due to the increased signal energy caused by the speckle noise. The measurement ranges were 10 and 31.6 V, respectively.

Five different spindle speeds, 700, 1400, 2800, 5600 and 7000 rpm were studied.

The presence of speckle noise was examined in the frequency domain of the LDV output during free run, which means that no forces were applied by the AMB.

The eigenfrequencies of the dummy tool/spindle were extracted from the frequency response functions shown in Fig. 2, using the AMB controlling system software. Three different excitation cases were examined; excitation close to the first eigenfrequency of the tool spindle system (400 Hz), excitation above the first eigenfrequency (700 Hz), and excitation at the rotation frequencies.

The crosstalk in the LDV measurements of a rough and polished rotating shaft was studied. For each case, the shaft was excited with different frequencies in the \(x\)-direction for the complete set of spindle speeds.
3. Results

Measurements were first performed at free run. Fig. 3 shows the displacement amplitudes of the 2–19th rotational harmonics of the polished dummy tool. The amplitudes are independent of spindle speed and are rapidly decaying, which indicates that they are not caused by random speckle noise. The profile of the polished measurement surface is not perfectly round. The actual deviation from a perfect circle will be recorded by the LDV. These roundness components are seen in the FFT as rotation harmonics. The third harmonic has the highest value, which means that the triangular component is the dominant one. By band-pass filtering the displacement signal it is possible to reconstruct the profile of the surface [3]. An independent measurement (at a later time) of the surface using a mechanical roundness tester (C E Johansson) confirms these peaks and some scratches. The scratches are also confirmed by the surface profile measurements performed by the Wyko equipment. Otherwise, the surface was optically smooth ($R_a = 21$ nm).

To examine the crosstalk, the dummy tool was excited in the $x$-direction (cross direction to the LDV) at 400, 700 Hz and at the rotational frequencies. In each case, two sets of measurements were made; firstly on the polished dummy tool with an optically smooth surface and secondly on the dummy tool after being sprayed with paint to give an optically rough surface. Fig. 4 illustrates the effect of crosstalk in LDV measurements of a rotating rough surface for different spindle speeds. The vibration velocity measured on the dummy tool after being sprayed with paint (triangle up) shows a spindle speed dependant crosstalk as expected from Eq. (1), while the same measurements on the smooth surface (triangles down) does not. The outputs from the displacement sensor (DS) in the $y$-direction for both sets of measurements (smooth and rough measurement surface) are also presented in the graph (square and pentagram). The signal to noise ratio was checked to be 43 dB in a typical case for the inductive DS. The differentiation of the signal was performed in the frequency domain at the excitation frequency. Inserting the signals from the DS $\dot{y}$ and $x$ into Eq. (1a) results in the expected velocity from the vibrometer when the surface is rough (circle). Numerically there is a good agreement between the calculated and the measured velocity. In (a), the excitation frequency is at 400 Hz and

![Fig. 2. Frequency and phase response functions of the spindle in the x- and y-directions.](image-url)
Fig. 3. The displacement amplitudes of the second to 19th rotational order of the polished dummy tool.

Fig. 4. Crosstalk in LDV measurements for different spindle speeds. Excitation at (a) 400 Hz, (b) 700 Hz and (c) at rotation frequencies.
the displacement and velocity amplitudes in the $x$-direction are about $6 \mu m$ and $15,100 \mu m \, s^{-1}$ at the LMP, respectively. In (b), the excitation frequency is at $700 \, Hz$ and the displacement and velocity amplitudes in the $x$-direction are about $4.5 \, \mu m$ and $19,800 \, \mu m \, s^{-1}$ at the LMP, respectively. Despite the different excitation level and frequency the results in (a) and (b) are consistent. There are some differences between the LDV output from the measurements on the optically smooth surface and the velocities obtained by the displacement sensor in the $y$-direction (triangle down and square). These differences are due to a small misalignment (few degrees) between the LDV and the displacement sensor in the $y$-direction. The sensitivity to misalignment has been checked by calculations. Since the excitation in the $x$-direction is comparatively large, even small misalignment angles do change the level of the output considerably. For example, the difference in (a) can be compensated by an angle of about $4.5^\circ$. In (c), the dummy tool was excited at the rotation frequencies, $11.7, 23.4, 46.7, 93.3$ and $116.7 \, Hz$. The vibrometer output from the measurements on the smooth surface shows no crosstalk and follows the differentiated displacement sensor output. The crosstalk effect lowers the vibration amplitude level in this case contrary to the previous two excitation frequencies. This shows that the crosstalk can result in either a higher vibration level or a lower one than the correct one.

4. Discussion

The laser beam alignment is one of the most important and critical step in rotating components measurement. In this study, the laser vibrometer must also be aligned with the inductive DS for comparison. Further the amplitude of the orthogonal excitation displacement is even more important when studying the crosstalk problem, which means that the inductive DS measuring the displacements in the cross direction must be perfectly orthogonal to the laser vibrometer. This task showed to be difficult to overcome. The inevitable instrument angular misalignment was though minimised by try-and-error which was time consuming. The actual value of the angular misalignment is unknown and difficult to control now afterwards. Backward calculations estimate the misalignment to be about $4.5^\circ$ in Fig. 4(a) which is not unreasonable. However, the objective of this paper is to investigate the crosstalk in laser vibrometry measurements on an optically smooth rotating spindle. The crosstalk is angular velocity dependence and even small displacements in the orthogonal direction result in amplitude differences for different but still high spindle speeds. Despite the systematic misalignment error the laser vibrometry measurements show no such spindle speed dependence. The differences between the level of the laser vibrometry measurements and the level of the displacement sensor outputs associated with the instrument misalignment, noise and measurement error should not interfere with the crosstalk investigation.

It is shown that the crosstalk can influence the vibration measurement producing a vibration level increase or decrease with respect to the correct level. The latter trend can be explained by the following example.

Suppose that the displacements are

$$x = A_x \sin(o t)$$  \hspace{1cm} (2)

and

$$y = A_y \sin(o t + \phi)$$  \hspace{1cm} (3)

where $o$ is the vibration frequency and $A_x$ and $A_y$ are the displacements amplitudes. Differentiating (3) gives the translational velocity in the $y$-direction

$$\dot{y} = A_y o \cos(o t + \phi).$$  \hspace{1cm} (4)

Inserting expressions (2) and (4) into (1a) and neglecting the alignment error $x_0$ gives the measured velocity

$$\dot{v}_y = A_y o \cos(o t + \phi) + \Omega A_x \sin(o t).$$  \hspace{1cm} (5)

If the vibration frequency is equal to the rotation frequency, $o = \Omega$ and if the phase difference between the displacements in the $x$- and the $y$-direction is $90^\circ$, the measured velocity (5) becomes

$$\dot{v}_y = \Omega (A_x - A_y) \sin(\Omega t)$$  \hspace{1cm} (6)

and in the worst case; when the amplitudes $A_x$ and $A_y$ are equal the LDV output becomes zero.
5. Conclusions

Laser vibrometry is normally used on stationary vibrating objects, although synchronous tracking of the laser beam with the moving surface has been tried in number of cases by others. But to measure vibrations of a rotating spindle necessitates that the laser beam is stationary in space. If the rotating surface is optically rough, a moving speckle pattern will occur on the detector, which gives a repeatable speckle noise in the measurement signal and also crosstalk from other velocity components.

By using a polished surface, the crosstalk in LDV measurements is avoided and the desired vibration can be measured. The fact that the cross vibrations applied by the AMB is removed from the LDV measurements automatically imply the removal of torsional vibration contributions.

The frequency content in the signal due to roundness components did not indicate in any detectable crosstalk.

The scanning LDV also provides the possibility to measure vibrations on different parts of a rotating machine, i.e. the milling machine spindle housing and other significant components during machining, which will give an overall picture of the system.

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References

Paper C
Measurement of milling tool vibrations during cutting using laser vibrometry

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Abstract

Spindle and tool vibration measurements are of great importance in both the development and monitoring of high-speed milling. Measurements of cutting forces and vibrations on the stationary spindle head is the most used technique today. But since the milling results depend on the relative movement between the workpiece and the tool, it is desirable to measure on the rotating tool as close to the cutters as possible. In this paper the use of laser vibrometry (LDV) for milling tool vibration measurements during cutting is demonstrated. However, laser vibrometry measurements on rotating surfaces are not in general straightforward. Crosstalk between vibration velocity components and harmonic speckle noise generated from the repeating revolution of the surface topography are problems that must be considered. In order to overcome the mentioned issues, a cylindrical casing with a highly optically smooth surface was manufactured and mounted on the tool to be measured. The spindle vibrations, radial tool misalignment, and out-of-roundness of the measured surface were filtered out from the signal; hence, the vibrations of the cutting tool were resolved. Simultaneous measurements of cutting forces and spindle head vibrations were performed and comparisons between the signals were conducted. The results showed that vibration velocities or displacements of the tool can be obtained with high temporal resolution during cutting load and therefore the approach is proven to be feasible for analysing high-frequency milling tool vibrations.

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Keywords: Laser vibrometry; Tool vibrations; Milling

1. Introduction

Today’s manufacturing industry demands higher productivity with preserved or even smaller tolerances. The demand on high productivity leads to increased material removal per unit time and higher spindle speeds, increased feed rate, and greater depth of cut. However, at certain combinations of machining parameters; process instabilities and vibrations can occur which result in decreased accuracy, poorer surface finish, reduced tool life time and in the worst case spindle failure. Vibrations in milling have been investigated by many researchers using cutting force sensors, microphones and accelerometers. Although cutting force measurements may be addressed as the key information needed to be monitored, today’s available force measuring platforms, dynamometers, are limited to relative small workpieces. Microphones are best suited for setting up thresholds based on experience or trials. The sound of a stable cutting process is usually calm and contains only frequencies originating from the spindle speed and the cutting teeth. However, microphones cannot give any information about deformations and forced vibrations. Since accelerometers cannot easily be applied on the cutting tool itself, the first option is to measure the vibrations transmitted from the tool into a closest non-rotating part, the workpiece and or the spindle head. Unfortunately, two major problems may occur during vibration measurements on the spindle head: (1) low vibration transmission and disturbances from the bearings making the measurements unreliable, and (2) magnetic disturbances from the motor. Non-contact sensors close to the tool are therefore desirable to use. Electronically non-contact measuring techniques like inductive and capacitive displacement sensors [1–3] and also optical techniques like laser displacement sensors [4] based on triangulation offer...
the possibility to non-contact measurements. They are however limited to close range measurements and are sensitive to thermal dilation and must therefore be arranged in two pairs. The arrangements using this kind of proximity sensors normally get lumbering and difficult to mount close to the cutters of the tool.

Laser vibrometry (LDV) [5] is a well-established non-contact and remote measurement method normally used for measurements on stationary vibrating surfaces. The nature of the laser vibrometers renders measurements with high temporal and spatial resolution. However, two major problems occur when measuring on rotating spindles, namely harmonic speckle noise and crosstalk between the velocity components. This paper discusses how these problems can be handled and presents results from a real milling process where cutting forces, tool vibrations and spindle head vibrations were recorded.

2. Speckle noise and crosstalk in laser vibrometry measurements of a rotating spindle

When polarized coherent laser light illuminates a surface that is optically rough, which most surfaces are, i.e. the surface roughness is large on the scale of the laser wavelength, each scattering surface element acts like a point source of coherent light. As a result of superposition between these uncorrelated wavelets a granular pattern of dark and bright spots called speckles will be formed on the photodetector. Speckles are unique for every different point in space, they have random amplitude and phase and if the speckle pattern changes during the LDV measurement, the rate of change in the resulting phase will be non-zero which will result in noise in the frequency spectrum of the signal. Speckle noise caused by random disturbances can be averaged out by measuring a great number of measurements. However, noise induced by systematic speckle fluctuations due to non-normal target motions, such as rotational motions cannot be averaged out, thus when measuring on rotating spindles the speckle pattern move across the LDV photodetector with a tremendous velocity. Although the scattering for one revolution is totally random, the pattern repeats itself for every revolution and a beat is introduced with the same fundamental frequency as the rotation frequency, a signal known as pseudo-vibrations [6]. A spectrum of a typical vibrometry output from a spindle vibration measurement during free run (no cutting, rough surface) is shown in Fig. 1. The frequency axis is normalized with the spindle rotational frequency and components up to the 15th order are seen. These harmonics contain amplitude and phase; hence they can increase or decrease the correct vibration level. The vibration behaviour of a cutting tool is typically a function of the spindle rotation speed and number of cutter inserts. It is easy then to realize that speckle harmonics in LDV measurements are difficult to distinguish from the true cutting vibrations since they coincide with the tooth passing frequency and its harmonics. An example of a dynamometer measurement of cutting forces during shoulder milling an aluminium profile with two cutter inserts is shown in Fig. 2, where frequency components from the 1st up to the 15th order of the spindle rotation can be seen. Here, the fundamental tooth passing frequency is at the second spindle rotation order. Further, it must be understood that the laser vibrometer is sensitive to the target velocity in the direction of the laser beam and therefore when measuring on rotating spindles the total surface velocity, projected in the laser beam direction is recorded. A detailed and thorough description of the velocity sensitivity in LDV measurements on rotating objects is given in Refs. [7–10]. In summary, the laser vibrometer output is approximately a sum of the desired translational velocity and products of the spindle rotation speed and displacements in perpendicular directions to the intended measurement. If $\dot{x}$ and $\dot{y}$ are vibration velocities intended to be measured, $x$ and $y$ are the vibration displacements, and $\Omega_s$ is the total angular velocity, the measured velocities of a rotating optically rough surface in the two orthogonal directions, $v_x$, $v_y$, can be expressed as

$$v_x = \dot{x} - \Omega_s y$$
(1)

and

$$v_y = \dot{y} + \Omega_s x.$$  
(2)

Consequently, even small displacements in the cross-direction cause large measurement errors if the spindle speed is high, which is the case in high-speed milling. By making the measurement surface optically smooth, i.e. the surface roughness is much smaller than the laser wavelength, both speckle harmonics and crosstalk are avoided in the measurements. A more thorough examination of the crosstalk and speckle noise removal is presented in Refs. [11,12].
Now, the prime interest in this work is to measure the milling tool vibrations in a real cutting trial.

3. Velocity in favour of direct displacement measurements

Basically the laser vibrometer is an interferometer using the Doppler shift of the backscattered light from a vibrating surface. The Doppler frequency, \( f_D \), is directly proportional to the velocity, \( v \), of the surface in the incident direction \[ f_D = \frac{2v}{\lambda}, \] where \( \lambda \) is the laser wavelength. By demodulating the Doppler frequency signal, the velocity will be obtained. Many times the vibration displacement of the cutting tool tip is desired. The commercial laser vibrometer used in the experiments has a built-in displacement decoder which offers the possibility to perform direct displacement measurements without transforming the Doppler frequency into vibration velocity; instead, the laser vibrometer counts the bright–dark fringes on the detector. However, displacement demodulation is better suited for low-frequency measurements. In the cutting experiments, the displacement signal dropped out occasionally, especially in the transient phase of the cutting cycle. By lowering the sensitivity and setting the range to full-scale, dropouts could be avoided. On the other hand, the resolution of the signal was decreased below an acceptable level. Therefore, the tool vibration velocities were measured and numerically integrated to obtain the desired displacements. In order to ensure that the numerical integration of the velocity signal does not cause information loss and produces similar results as direct displacement measurement, the displacement obtained by a direct displacement measurement of the tool tip was compared to the integrated velocity signal that was simultaneously recorded. The integrated velocity and the direct displacement measurement showed to be compatible and contained almost identical information. The difference was at lower frequencies, far below the fundamental spindle rotation frequency. This error was minimized by subtracting the DC-bias in the raw measurement signal before the integration procedure.

4. Experimental setup and procedure

Fig. 3 is a schematic representation of the experimental arrangement. In order to avoid the speckle noise and the crosstalk problem in the measurements, a highly polished stainless cylindrical casing with an outer diameter of 20 mm was mounted on the cutting tool, a 16 mm R390 mill with two inserts, see Fig. 4. The tool was mounted in an HMD tool holder, which in turn, was mounted in the machine, a five-axis high-speed vertical milling machine centre, Liechti Turbomill ST1200. The cutting forces were measured by a 9257A Kistler table dynamometer with a 5 kN range. An AA7010 aluminium workpiece was faced, drilled to fit the table dynamometer holes and firmly bolted to ensure rigid mounting. The dynamometer was, in turn, fixed to the machine table with clamps. Two Bruel & Kjaer, type 4507 accelerometers, with a sensitivity of 100 mV/(m/s²) and a frequency range between 2 Hz and 5 kHz were mounted on the spindle head at closest possible positions to the rotating part of the spindle in the feed and cross-feed directions. The accelerometers measure the spindle vibrations and serve as another comparative signal. The tool vibration was measured by a Polytec PSV 300 scanning laser vibrometer (LDV), standing steadily on a support located at about 1800 mm from the measurement point. The machine centre foundation was vibration isolated from the rest of the floor, so the vibrations transmitted to the laser head were minimized. The cutting forces, spindle head vibrations and the laser vibrometer signal from the rotating tool were acquired simultaneously. The sensitivity of the vibrometer was set to 125 mm/s/V with a maximum range of 31.62 V and the sampling frequency was 102.4 kHz.

Shoulder cutting tests were performed with a constant spindle speed of 19,000 rpm. The axial and the radial depth of cut were set to 5 and 1 mm, respectively. The feed rate was set to 6200 mm/min resulting in chip loads of 0.1632 mm/tooth. All cutting tests were conducted under dry conditions. The cross-feed and the feed directions are aligned with the Cartesian \( x \) and \( y \) axes, respectively.

5. Results and discussion

5.1. Radial misalignment and out-of-roundness

Radial misalignment is defined as the deviation of the geometrical tool axis from the spindle rotation axis. The typical misalignment test involves a dial indicator
positioned against a gage pin, with the spindle manually rotated. The laser vibrometer measures the rate of change in optical path between the laser head and the measured surface. Hence, displacements due to the radial misalignment and also the out-of-roundness of the measured surface are recorded in the signal. The magnitudes of the radial misalignment and the out-of-roundness components are seen in the spectrum of the LDV measurement during free run at the spindle frequency and its higher rotational harmonics, respectively. The amplitudes of the out-of-roundness components are independent of spindle speed and easy to distinguish [12]. In order to resolve the cutting vibrations, the radial misalignment error and the out-of-roundness errors must be subtracted from the measured signals. This means that free run measurements at the operation spindle speed must be conducted and synchronized with the actual cutting measurement. The synchronization can be done using a reference signal that tracks the angular spindle position. In this work, the pre-triggered part of the laser vibrometer signal is used for this purpose and is subtracted from the signal in the Fourier domain. The pre-triggered part of the signal is stationary and an exact integer number of spindle revolutions is selected for a trustworthy Fourier spectrum. In Fig. 5, the LDV signal in the feed direction from the cutting experiment is shown. In (a) the whole raw signal and in (b) a zoomed part of the pre-triggered region (free run, no cutting) are shown. The magnitude of the radial misalignment is marked in the time history plot as two dashed lines. The high-frequency content in the signal is due to out-of-roundness of the surface and self-excited spindle vibrations. In (c) a zoomed part of the raw signal in the cutting region is shown, where the velocity is formed by superposition of the radial misalignment, out-of-roundness, spindle vibrations and forced vibrations. Finally, in (d) the filtered signal, i.e. after subtraction by the free run signal, is shown in the early stage of cutting where the radial depth of cut is increasing with time. Such time history illustrates also the damping of the machine tool, which comes not only from the structure of the tool itself, but also from the joints between the tool, the tool holder and the spindle nose.

The radial misalignment and the out-of-roundness of the tool in measurements in the cross-feed direction are subtracted from the raw signal in the same manner.

5.2. Milling tool vibrations during cutting

In Fig. 6, the displacement of the tool together with the measured cutting force measured in the feed direction (y-direction) is shown as a function of spindle rotation angle. Cutter one is engaged between 80° and 110°, and after the first cutter disengagement the tool continues to vibrate freely until the next cutter edge engages between 260° and 290°. In Fig. 7, the tool displacement together with the cutting force in the cross-feed direction (x-direction) is shown. It must be remembered that measurements in the feed and the cross-feed directions are made at different cutting tests but otherwise under equal conditions. In Figs. 6 and 7, the amplitude of the first cutter is higher than for the second cutter in both tool displacement measurements and cutting force measurements. The difference can be explained, for instance, by that the axial and radial positions and also the angle of the two individual cutters inserted in the tool body differ slightly, see Fig. 8. The small variations in the cutter location, \( R_1 \neq R_2 \), \( z_1 \neq z_2 \), and also different angle of the cutters, \( \theta_1 \neq \theta_2 \), are common in commercial cutting tools, especially in worn tools. These asymmetries may have significant influence on
the cutting forces since the individual tooth removes different amount of material and consequently causes different tool displacements and surface finish [13–15]. In this case, the radial and axial differences are measured roughly to about 10 and 25 μm, respectively, using a Toolmaster3, which is an optical imaging and magnifying measuring apparatus. Even though the radial misalignment of the tool body at the polished surface is compensated in the LDV measurement, still in reality the dynamic cutting forces are influenced due to the variation of the chip load caused by the misalignment at the cutter edge. The actual total misalignment at the cutters is a combination of cutter axis offset, cutter axis tilt, individual cutter size, shape and position. In Fig. 9, a photo of the milled surface is shown, where the finished workpiece surface normal vector is perpendicular to the direction of the feed. The field of view is about 4.5 x 5 mm², where the horizontal axis is the feed direction and the vertical axis is the axial depth of cut profile. The photo of the surface topography may be divided into three regions. The upper and the lower regions are due to different cutters and the intermediate region is caused by milling by both cutters. The spacing between the cutter paths in the upper and the lower region is about 0.33 mm where the pattern in the lower region is shifted half a cutter path distance relative to the pattern in the upper region. Although the generation of the milled surface is far complex, the cutter paths in the photo may indicate...
that the true trajectories of the individual cutters are different.

In order to check the robustness and repeatability of the measurement method, 10 revolutions of the middle part of the LDV signals are plotted in Fig. 10. As one can see the curves are very close to each other in terms of both amplitude and spindle rotation angle.

5.3. Vibration monitoring in milling

As mentioned in Section 1, vibration monitoring of the tool close to the cutting region is desired. At the same time, the method described in this paper may not be the most appropriate way of monitoring today since the sensors required for vibration monitoring in industry should be insensitive to machining environment and also be cost effective. An ideal on-line system should include permanently installed, relatively cheap sensors that provide vibrations with high accuracy. Although, signals by accelerometers mounted on the spindle head do not provide direct vibration measures of the cutting tool, they may fulfill the requirements needed for process monitoring sensors since they are low-cost and easy to use sensors. Therefore, in hopes of being able to connect vibrations measured on the spindle head to the vibrations of the cutting tool, simultaneous vibration measurements of the spindle head were conducted. Figs. 11–16 compare the tool vibrations and spindle head vibrations together with the cutting force measurements.

Fig. 11 shows the spectrum of the cutting force measured with the table dynamometer in the feed direction. The measurement demonstrates peaks up to 6TPF. TPF is an abbreviation for tooth passing frequency which is defined as

\[
TPF = \frac{\Omega \times N}{60},
\]

where \( \Omega \) is the spindle speed, revolution per minute, and \( N \) is the number of cutters. The presence of cutter misalignment is revealed as a cutting force component at the spindle frequency (SF). Fig. 12 presents the spectrum of the measured vibration displacement of the tool in the feed direction. SF: spindle frequency; TPF: tooth passing frequency.
frequencies from SF up to 6TPF, in good agreement with the cutting force. The spectrum of the accelerometer signal measured on the spindle head in the feed direction is shown in Fig. 13. The accelerometer signal contains peaks, besides the one from the spindle speed harmonics and the tooth passing frequency, probably originating from the bearings. A high peak at the 8TPF is observed, which may indicate that the bearings are easily excited around 5 kHz. Fig. 14 shows the spectrum of the measured cutting force in the cross-feed direction where no peak at the spindle frequency can be seen. The magnitude of the third and the fourth TPF are almost equal to the ones from the force measurement at the feed direction. The spectrum of the measured vibration displacement of the cutting tool in the cross-feed direction is shown in Fig. 15. Note that the magnitudes of the vibrations are much lower than those in the feed direction since the radial depth of cut is only 1 mm. The relation between the cutting force and the displacement of the tool at the fundamental tooth passing frequency is the same for measurements in both feed and cross-feed directions. The spectrum of the accelerometer signal measured on the spindle head in the cross-feed direction is shown in Fig. 16. It can be observed that the spectrum contains many peaks above the 14th spindle rotation order. Some of these can be originating from the bearings. Note that the accelerometer signal has a vibration amplitude of 73 m/s² at 8TPF, which reveals that the spindle head is far more sensitive around 5 kHz in the cross-feed direction. This frequency has shown to be also activated at free run measurements, and the big amplitude difference in the x- and y-directions may be due to how the spindle is supported by the milling machine. However, neither the cutting force measurements nor the LDV measurements showed such vibrations at 8TPF.

More data and investigations are needed to be able to couple the measured spindle head vibrations to the measured cutting tool vibrations. However, finding a suitable transfer function was not the main objective of this paper.

6. Conclusions

A milling tool vibration measurement method using LDV is presented. The method requires that the spindle head is held stationary in space, that the laser beam has a clear line of sight and that the measurement surface is optically smooth and kept clean. The laser vibrometer signal contains information on tool vibrations, radial misalignment and out-of-roundness. It is shown that the cutting vibration information can be extracted by properly subtraction of the free-run (no cutting) part of the laser vibrometer signal from the cutting part of the same recording. Cutting tests in a milling machine at a spindle speed of 19,000 rpm showed that the tool displacement peaks follow the corresponding peaks in the force signals during the time each cutter is engaged in the workpiece. Asymmetries in the tool geometry could be seen in the laser vibrometer and the force signals as amplitude alteration between the individual cutting teeth and also by studying the machined surface topography.

The main advantage of the method is that it is possible to perform remote non-contact measurements on a rotating milling tool very close to the cutters. The method also enables transient vibration measurements of the tool during cutting, which can contribute to the understanding of the dynamics of milling systems at the start and end of a cutting path. The method can be used for high-frequency machine tool vibration analysis and also as a tool in the development of spindles and tools.

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References


Paper D
Digital holographic interferometry for radial vibration measurements along rotating shafts

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A digital holographic interferometry set-up is used to measure radial vibrations along a rotating shaft. A continuous Nd:YAG laser and a high-speed digital camera are used for recording the holograms. In order to avoid speckle noise from the rotating surface, the shaft was polished optically smooth. Simultaneous measurements with a laser vibrometer at one point and comparisons between the signals showed good agreement. It is shown that different vibration components of a rotating shaft can be simultaneously measured with this technique. © 2007 Optical Society of America

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Introduction

Vibration measurements are made for a variety of reasons; product designs and development, condition monitoring, fault detection for maintenance, verification of an analytical model proposed for a specific mechanical system etc. Rotating parts of a mechanical system often become a source of vibration of that system. Radial vibrations of rotating shafts have been studied by many researchers and engineers and vibration measurements have often brought practical difficulties and challenges. Vibrations have been traditionally measured by accelerometers; and for rotating shafts the vibrations transmitted to a non-rotating part have been the most common solution. However, in order to be able to fully investigate the behavior of a rotating system, it is necessary to make measurements directly on the rotating part under the actual operational condition. Therefore non-contact measurements are desirable. Capacitive and inductive displacement sensors [1-5], laser distance sensors based on triangulation [6, 7] and laser vibrometry (LDV) [4, 8-12] are such vibration transducers that make non-contact measurements possible on rotating parts. In many applications field information of the vibration pattern is required. The above mentioned techniques are all point measuring methods and for obtaining field information a sequence of measurements over the area of interest is needed. A two-dimensional measurement set-up using several vibration transducers can be lumbering, and although scanning laser vibrometers are commercially available where the laser beam moves automatically from point to point over a pre-defined grid of points, spatially dense measurements can be time-consuming and also require repetitive events. Holographic interferometry however is a non-contact optical technique that provides full field measurements with high spatial resolution [13]. Different vibration measurements on static objects have been reported e.g. Refs [14-16]. This work presents a technique to measure vibrations of a rotating shaft using digital holographic interferometry. In order to validate the result, simultaneous measurements were performed by a laser vibrometer.
Digital holographic interferometry

In classic holographic interferometry photographic film were usually used as the recording medium. The reconstruction of the object wave was then made by illuminating the hologram with the reference wave, also called the reconstruction wave. A detailed presentation of holographic interferometry and some important methods for analyze of the interference pattern is given in e.g. Refs [13, 17]. Today solid state detectors are often used as the recording device and the Fourier transform method [18] is often used to determine the phase change between digital holograms. If a small angular offset is introduced between the object and the reference beam the two-dimensional Fourier spectrum of the hologram will contain three distinct parts. By filtering out the interference term and inverse transform it back, a complex valued amplitude will be obtained where the phase (or the argument), \( \Phi \) can be calculated with the arctangent operation [18, 19]. Next the desired phase difference, \( \Delta \Phi \), between two digital holograms is calculated where the phase values are wrapped between \(-\pi\) and \(\pi\). In order to obtain a smooth and continuous phase map a spatial unwrapping operation can be performed. Finally the desired relative displacement vector, \( \mathbf{d} \), between two exposures is proportional to the phase difference [13]:

\[
\Delta \Phi = \frac{2\pi}{\lambda} \mathbf{d} \cdot \mathbf{S}
\]

where \( \lambda \) is the laser wavelength and \( \mathbf{S} \) is the sensitivity vector given by the geometry of the holographic set-up; defined as an approximation as the vector difference between the observation vector \( \mathbf{o} \) and the illumination vector \( \mathbf{i} \):

\[
\mathbf{S} = \mathbf{o} - \mathbf{i}.
\]

A review of the principles of digital hologram recording, numerical reconstruction and phase evaluation for interferometry application can be found in e.g. [20].

When laser light is scattered by an optically rough surface speckles are produced. If the speckle pattern undergoes spatial changes between two recordings the calculated phase difference will deteriorate. If the speckle change is too large it is no longer possible to extract the wanted phase change of the object. Tilt, in-plane motion or rotation of the object cause such speckle movements on the detector. If the in-plane motion is a rotation, the unwanted effect can be compensated by an optical derotator [21] or by combining digital holographic interferometry with digital speckle photography (DSP) technique [22, 23]. In some cases, it is possible to move the measuring system synchronously with the in-plane translation or rotation of the object [24]. However, in order to measure radial vibrations of a rotating shaft, building such tracking system is not trivial. Also laser vibrometry is sensitive to speckle variations caused by tilt, in-plane motion and rotation of the object [25, 26]. In previous works [8-10] it has been shown that by making the measurement surface optically smooth the speckle noise problem in laser vibrometry measurements can be avoided. This approach is also applied in this work.
**Experimental set-up and procedure**

Fig. 1 shows a schematic sketch of the experimental set-up. Light from a 400 mW laser of wavelength 532 nm is divided by a polarizing beam splitter PBS into two beams of different polarization. The intensity ratio between the two beams can be changed by rotating the λ/2-plate. One beam is directed to illuminate the rotating shaft along the x-direction \( \mathbf{i}_1 \). The light reflected from the object \( \mathbf{o}_1 \) is then directed towards a diffuser \( D \). The other beam is expanded and then collimated by the cylindrical lenses \( L_1 \) and \( L_2 \), respectively. The light that now has a shape of a sharp line with a height of 45 mm in the z-direction illuminates the shaft along \( \mathbf{i}_2 \). The reflected light, \( \mathbf{o}_2 \), falls onto a different part of the diffuser. The diffuser is then imaged onto the camera detector by a 100 mm lens. A rectangular aperture \( A \), with a size of 1.34 × 5.0 mm\(^2\) in front of the imaging lens serves as a lowpass filter preventing aliasing. A small portion of the light is reflected at the flat surface of \( L_1 \) and is used as reference beam \( R \). In order to get interference between the reference beam and the two object beams, the reference beam passes a polarization rotator (45°). A circular aperture (about 1 mm in diameter) and an expanding lens system ensure that the reference beam is smooth and spherical enough. The reference beam is reflected by the cube beam splitter and interferes with the object beam on the detector and a digital image-plane hologram is recorded. The reference beam is slightly off-axis to ensure the separation of the interference terms in the Fourier domain.

![Fig. 1. Experimental set-up. Beam splitter (BS), polarizing beam splitter (PBS), cylindrical lenses (L1 and L2), and diffuser (D).](image)

The set-up is arranged such that the sensitivity vectors coincide with the intended displacement measurement directions. The illumination vector \( \mathbf{i}_1 \) is parallel to the object light vector \( \mathbf{o}_1 \). With this arrangement the absolute value of the sensitivity vector will be 2 and a 1 rad phase change will be equivalent to \( \lambda/(4\pi) \) displacement. The angle between the illumination vector \( \mathbf{i}_2 \) and the object vector \( \mathbf{o}_2 \) is exaggerated in the sketch and is in reality about 5 degrees which makes the absolute value of the sensitivity vector close to 2.

The detector of the high-speed digital camera (REDLAKE MotionPro X3) has a resolution of 1280 × 1024 pixels and a pixel size of 12 × 12 μm\(^2\). The maximum framing rate at full resolution is 1,000 frames-per-second (fps). By reducing the field of view the speed can be increased. Since in this case the useful information is along a line, the number of the pixels was reduced to 1280 × 100, and the framing rate was increased to 10,000 fps. The
exposure time was set to 96 μs. The high-speed camera was connected to a PC via an USB2 interface.

The object; a highly polished cylindrical shaft with a diameter of 15 mm and a height of 210 mm was positioned vertically on a motorized rotation stage with a rotation speed of 1.15 revolutions per minute. The object was then excited at 20 Hz in the y-direction by an electromagnetic coil close to the free end of the shaft without contacting the shaft surface (see Fig. 2). The upper edge of the illumination light for the holographic interferometry measurement in the y-direction was at $z = 45$ mm ($132$ mm above the base of the rotation stage). The measurement point in the x-direction was at $z = 29$ mm ($115$ mm above the base of the rotation stage).

Fig. 2. Schematic representation of the polished shaft mounted in the rotation stage. The LDV measurement point (LMP) is $115$ mm above the base of the rotation stage ($z = 29$ mm). The digital holographic interferometry measurement line (DHI) starts $87$ mm above the base of the rotation stage ($z = 0$).

A Polytec PSV300 scanning laser vibrometer was positioned $90$ cm from the shaft and measured the vibrations in the y-direction from the opposite side of the shaft at $z = 29$ mm ($115$ mm above the base of the rotation stage, Figs. 1-2). The laser vibrometer has a built in displacement decoder which offers the possibility to perform direct displacement measurements. Both the high-speed camera and the laser vibrometer were triggered at a certain phase of the rotation via a delay unit. The recorded image sequence was $2$ seconds, i.e. $20001$ images.
Results and discussion

Fig. 3 shows the calculated phase map at $t = 25.1$ ms in the sequence indicating the relative displacement of the shaft. In order to reduce the noise, the raw data was smoothed using a convolution kernel of $10 \times 10$ pixels prior to the phase calculation. The smoothing procedure reduces the spatial resolution to about 1 mm which is enough in this case. The first 1140 pixels correspond to the $y$ displacement and the rest (within the black border) correspond to the $x$ displacement. The phase values are wrapped between $-\pi$ and $\pi$. By performing temporal unwrapping to all the wrapped phase maps in the recorded sequence, continuous phase maps can be obtained at any instant of time. Fig. 4 shows the unwrapped phase at $t = 25.1$ ms. Some of the rows for the first 20 columns are noisy due to low-light. Physically the phase can be assumed constant along the columns and therefore the row(s) that contain minimum noise can be selected for further analysis. These continuous phase maps are then converted to relative displacements.

Fig. 3. Wrapped phase map at $t = 25.1$ ms. The first 1140 pixels correspond to the $y$ displacement and the last 100 pixels within the black border correspond to the $x$ displacement.

Fig. 4. Continuous phase at 25.1 ms. The first 1140 pixels correspond to the $y$ displacement between $z = 0$ mm and $z = 45$ mm and the last 100 pixels correspond to the $x$ displacement at $z = 29$ mm.

Displacements due to inevitable radial misalignment of the shaft and also the out-of-roundness of the measured surface are also present in the signal and need to be subtracted from the measurements. For this purpose, free run measurements (rotation, no external excitation) were conducted where the angular position was tracked and synchronized using an independent reference signal that triggered the measurements. The converted displacement in the $y$-direction after subtracting the out-of-roundness and radial misalignment at $t = 25.1$ ms together with the end positions are shown in Fig. 5. Asymmetry in the end positions of the shaft movement can be explained by following; the magnitude of the non-contact electromagnetic excitation force applied at the free end of the shaft is varying with the modulus of the sine-wave signal of the signal generator. As the excitation force approaches its maximum value the shaft deflects towards the positive $y$-direction and as the excitation force approaches zero the shaft springs back. Consequently the magnitude of the displacement in the positive $y$-direction is larger then in the opposite direction. A time animation of how the shaft actually vibrates can be presented as a video if a sequence of calculated displacements...
for different time is created. Such an animation is available in the on-line version of this article, where Fig. 5 is a representative diagram from that movie.

In Fig. 6 displacement curves measured both by holographic interferometry (in x and y-direction) and laser vibrometry (in y-direction) are shown as a function of time at \( z = 29 \) mm. The window is chosen so that four periods of the excitation can be seen. Beside the fundamental excitation frequency at 20 Hz, the following frequency contents can be observed: a small modulation at half the excitation frequency (10 Hz) due to the remanence (the magnetization left behind in the shaft when the external magnetic field is zero), a 100 Hz modulation due to free vibration of the shaft. The curve obtained by laser vibrometry is shifted up 0.1 \( \mu m \) for easier comparison. Although the measurement techniques for the two systems differ, the results (y-curves) in Fig. 6 are very close.

The change in phase between two exposures is limited to \( \pi \) radians and since the sensitivity vectors are almost parallel to the measurement directions, from Eq. 1 we get that the maximum allowed displacement between two exposures is 133 nm. Thus the maximum allowed vibration velocity is 1.3 mm/s for a framing rate of 10,000 fps. In our case, the measured phase changes between two successive holograms were below 2 radians.

![Fig. 5. (Color online) Displacement measured in the y-direction at \( t = 25.1 \) ms together with the end positions. File size: 3 MB.](image)

![Fig. 6. (Color online) Displacement in both x- and y-directions measured at \( z = 29 \) mm as functions of time. The displacement obtained by laser vibrometry (LDV) is shifted up 0.1 \( \mu m \) for easier comparison.](image)
Conclusions

The presented work shows that it is possible to simultaneously measure the two orthogonal radial vibrations (x,y) of a rotating shaft using digital holographic interferometry if the surface of the shaft is optically smooth. Simultaneous measurements by laser vibrometry verify the results. In order to obtain a three-dimensional vibration description of the shaft it is possible to modify the set-up so that a third light vector illuminates the shaft such that the third sensitivity vector coincides with the z-axis. By spatially separating different object lights over the diffuser surface any vibration component of the shaft may be simultaneously achieved using one single high-speed camera.

The technique is non-contacting and non-disturbing. Remote and non-contact measurements are particularly desirable in machine tool applications. The technique may be applied for spindle or tool vibration analysis, spindle or tool rotation error (radial misalignment) measurements, out-of-roundness measurements and also for investigating the whirling phenomenon in long and slender shafts.

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References


Paper E
Integrated approach for prediction of stability limits for machining with large volumes of material removal

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High-speed machining of thin-walled structures is widely used in the aeronautical industry. Higher spindle speed and machining feed rate, combined with a greater depth of cut, increases the removal rate and with it, productivity. The combination of higher spindle speed and depth of cut makes instabilities (chatter) a far more significant concern. Chatter causes reduced surface quality and accelerated tool wear. Since chatter is so prevalent, traditional cutting parameters and processes are frequently rendered ineffective and inaccurate. For the machine tool to reach its full utility, the chatter vibrations must be identified and avoided. In order to avoid chatter and implement optimum cutting parameters, the machine tool including all components and the work piece must be dynamically mapped to identify vibration characteristics. The aim of the presented work is to develop a model for the prediction of stability limits as a function of process parameters. The model consists of experimentally measured vibration properties of the spindle-tool, and finite element calculations of the work piece in (three) different stages of the process. Commercial software packages used for integration into the model prove to accomplish demands for functionality and performance. A reference geometry that is typical for an aircraft detail is used for evaluation of the prediction methodology. In order to validate the model, the stability limits predicted by the use of numerical simulation are compared with the results based on the experimental work.

Keywords: High-speed machining; Regenerative chatter; Stability lobes; Finite element analysis; Laser Doppler Vibrometry

1. Introduction

An increasing trend in the modern manufacturing industry points towards larger ranges of variation in production systems. This is a direct consequence of the effort to satisfy customer demands for fast deliveries of new products. The present situation in the manufacturing industry is that they have to adjust much of their cutting processes as a consequence of the uncertainty that exists in the influencing machining conditions. In order to do this, techniques must be used to handle variations in the manufacturing conditions as the frequent reconfigurations and setups generate.

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In manufacturing of components for aerospace applications, aluminium is one of the most frequently used materials. In machining aluminium, higher spindle speed and machining feed rate combined with a greater depth of cut, increase the removal rate and with it, productivity. The combination of higher spindle speed and depth of cut makes instabilities (chatter) a far more significant concern. In machining, chatter is perceived as an unwanted excessive vibration between the tool and the work piece, resulting in a poor surface finish with possible initiation of micro-cracks. Chatter vibrations accelerate also tool wear which has a deteriorating effect on the tool life, and the reliability of the machining operations.

Especially for the cases where long slender end mills or highly flexible thin-wall parts are involved, chatter is almost unavoidable unless special suppression techniques are used or the removal rate of material is substantially reduced.

Currently working with the preparation machining process, the natural frequencies of the work piece are measured and analysed when there is a problem with the quality of the surface or cutting process. The manufacturing process is stopped just before the cutting starts in the problem area and sensors are put in the same area. The extensive experimental work is a limiting factor in application of this approach to manufacturing of aluminium aerospace components. The removal of material in aerospace applications is vast and figures up to 90% of the original volume which results in non-linear behaviour due to time variant geometry and stiffness. Due to time and costs consuming experimental work, the long-term goal is to develop a tool for the prediction of chatter vibrations that is based on a digital prototype.

The first attempt to describe chatter was made by Arnold 1946. Tobias and Fishwick 1958 presented a comprehensive mathematical model and analysis results of chatter vibrations. The importance of predicting stability in milling has increased due to advances in high-speed milling technology. At high speeds, the stabilizing effect of process damping diminishes, making the process more prone to chatter. On the other hand, stability limits usually referred to as stability lobes exist at certain high spindle speeds which can be used to substantially increase the chatter-free material removal rate, provided that they are predicted accurately, see e.g. Smith and Tlusty 1993. Wiercigroh and Budak 2001 presented a critical review of the modelling and experimental investigations. In this work, sources of nonlinearities, chatter generation and suppression in metal cutting are studied. In the papers by Moon and Kalmar-Nagy 2001 and Balachandran 2001, the prediction of complex, unsteady and chaotic dynamics associated with cutting processes through nonlinear dynamical models is reviewed. A mathematical model of mechanics and dynamics of general milling cutters is presented by Engin and Altintas 2001.

In general, several physical mechanisms causing chatter can be distinguished. Frictional chatter due to friction between the tool and work piece, mode-coupling chatter and thermo-mechanical chatter caused by the thermodynamics of the cutting process are often called primary chatter.

Secondary chatter is caused by the regeneration of waviness on the surface of the work piece. This phenomenon is called regenerative chatter and is considered to be one of the most important causes of instability in the cutting process.

During machining as the cutter tooth enters into the cut, the cutting system (tool-holder, tool) deforms in bending due to applied cutting forces. Forces released by the tooth exiting the cut cause the cutting system to vibrate with its
natural frequency. The vibration leaves small waviness on a surface of the workpiece, as illustrated in figure 1. If the following tooth impact does not match the natural frequency of the cutting system, the chip thickness will increase as well as the applied cutting forces that result in a larger deformation of the system. The worst condition is when the vibration from cutting edges moves and the mirror image of the surface waviness is 180° out of phase, see figure 1. In order to avoid chatter in machining means that the tooth impact frequency has to match the natural frequency of the cutting system. The ideal condition is when the surface waviness and cutting vibration are in phase.

Various models to predict the stability boundaries related to chatter have been suggested by e.g. Altintas 2000, Li and Li 2000, Tlusty 2000, Peigne et al. 2004, Solis et al. 2004 and Toh 2004. These analytical-experimental models describe the cutting process dynamics using cutting parameters that are constant for the spindle speed range under consideration. This means that the dependencies of the dynamic behaviour of the milling machine or the cutting process on the spindle speed are not modelled.

In order to investigate such dependencies, dedicated experiments have been performed and reported by Faassen et al. 2003. In this paper, the method for prediction of the chatter boundaries is proposed and applied to predict the chatter stability as a function of process parameters. A method for calculation of stability limits considering the flexibility of work piece is presented by Bravo et al. 2005.

The approach that follows the methodology developed by Altintas 2000 is illustrated in figure 2. Here, the prediction of a stable cutting process is based on experimental modal analysis. For the evaluation of experiments and calculation of

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**Figure 1.** Chip thickness variation (dashed area) between the tool and the work piece. In phase thickness generation is shown in upper half of the figure and out of phase generation in lower half.
stability lobes, the commercial software CutPro was used. CutPro is an analytic simulation software package, developed for off-line milling process optimisation.

In models discussed previously, the vibration properties of the work piece are predicted from the constant material parameters and the geometry. Alternatively, the vibrations properties of the work piece are calculated from experimental modal analysis.

In manufacturing thin-walled structures, the material removal is vast which results in significant changes of vibration properties. Hence, it is unfeasible to use constant cutting parameters for prediction of the complete machining process.

In order to obtain reliable prediction for machining thin-walled components, an extensive trial and error experimental work was needed. The main advantage of the proposed model (approach) is using a digital prototype for the preparation of the manufacturing process. This means that all trial-error experiments are run numerically instead of using physical prototypes which is absolutely necessary in the case of machining structures with vibration properties that change during machining. Saving time and money is the most desirable result of the application.

2. Method

2.1 Digital model for prediction of chatter vibrations

The majority of critical frequencies are the natural frequencies of each component in the machining system. The machine tool itself and the work piece are influencing the stability and have to be handled.

In order to avoid time-consuming and expensive experiments, a supportive integrated tool based on existing commercial software was introduced (see figure 3). Application of a digital model facilitates the analysis of vibration and the prediction of a chatter-free cutting process. The proposed model integrates the analytical
calculations of the tool (Altintas 2000) and the dynamic finite element analysis of the work piece in several stages of the process.

The vibration properties of the spindle are investigated experimentally using Laser Doppler Vibrometry (LDV). The software CutPro was used for the evaluation of a stable cutting process just as any knowledge database.

In the presented digital model, the continuous machining process is discretized in time to obtain models of current stages in the machining process. Stages of the process when the cutting tool is changed are chosen for the analysis.

2.2 Definition of tool path and geometry for the analysis

A reference geometry that is typical for an aircraft detail (see figure 4) is used for evaluation of the prediction methodology. Material removal in manufacturing this detail is large. For the illustration, the volume of the initial stock was $1.896 \times 10^6 \text{ mm}^3$ and the volume after final milling operation was $0.218 \times 10^6 \text{ mm}^3$.

Figure 3. The model for the prediction of the chatter-free cutting process that integrates numerical simulation with experimental procedures based on LDV measurements.

Figure 4. Sequence of in-process stock (as cut) geometries: Left – initial stock; right – final geometry.
The thickness of the initial monolith was 40 mm, the final thickness of the bottom was 1.0 mm and the flange thickness was 1.5 mm.

The structure has shown significant sensibility to vibrations during the machining. Before manufacturing, it must be first decided which operations are required to go from a stock geometry to the finished final component. A part programme for machining of the geometry is generated in a Computer Aided Manufacturing (CAM) system. The solid model of the structure that is created with nominal finished dimensions in a Computer Aided Design (CAD) system is transferred to the CAM module by selecting from the system database. In the presented work, the I-DEAS CAD/CAM system was used.

Modern CAM software offers high levels of support for creating tool paths which is necessary for the definition of the component geometry from a piece of stock material. The CAM software automatically creates suitable paths for the tool to follow by the use of built-in algorithms. It is generally possible to specify how critical parts of the tool path such as entry and exit from the part or significant changes in the direction of the tool path such as machining in a corner are to be handled.

In order to perform high-speed roughing in volume milling, the scan type constant load is to be used. The milling is based on principles of constant cutting conditions, constant chip load, continuous tool engagement and also minimizing of sudden changes of tool direction. For the optimization of the mentioned strategies, a smooth tool path is used starting from rough milling down to the finishing. In order to achieve a stable as possible tool path, the machining is started from the middle of the working part with the direction outwards.

The analysis of natural frequencies and response analysis are carried out for the sequence of cut geometries (in-process stock geometries) that are identified to be significant for the manufacturing process. The in-process stock geometry is generated from the current state of the process at the end of each operation or even at discrete time intervals.

I-DEAS/CAM software allows one to calculate and to export a solid model geometry for each step (in-process stock) in the machining of the part. In-process stock is a representation of the stock after the tool has cut away excess material for each operation. After a tool path is created, the software updates the shape of the in-process stock geometry. For the first operation in the first setup, the tool path removes material from the initial stock in the assembly.

The updated in-process stock is saved and used for the next cutting operation. The geometry for two significant stages of the process is shown in figure 4. The generation of in-process stock sequences supports the definition of the time variant geometry for every step in the machining process and preparation of simulation models for the stability analysis.

2.3 Modelling of stable milling processes

As mentioned previously, instabilities that can lead to chatter during milling operations are of interest. Roughing operations such as volume clear are often sensitive to chatter. Vibrations significant for finishing operations with a small radial depth of cut are a combination of regenerative chatter vibrations and forced vibrations.
A schematic work piece tool model of the milling process is represented by an equivalent two-degrees-of-freedom spring-mass-damper system and shown in figure 5. Details of modelling can be found in e.g. (Altintas 2000).

In figure 5, the feed direction and spindle rotation are shown for an up milling operation. The initial surface of the work piece is smooth without waves during the first revolution, but the tool starts leaving a wavy surface behind because of the structural modes of the machine tool-work piece system in the feed direction. When the second revolution starts, the surface has waves both inside the cut where the tool is cutting (inner modulation \(t\)) and on the outside surface of the cut owing to vibrations during the previous revolution of cut (outer modulation \(t - T\)). Depending on the phase shift between the two successive waves (see also figure 1), the maximum chip thickness may exponentially grow while oscillating at a chatter frequency that is close but not equal to a dominant structural mode in the system. Hence, the resulting dynamic chip thickness is no longer constant but varies as a function of vibration frequency and the speed of the spindle. Generally, the dynamic thickness can be expressed as follows:

\[
h(t) = h_0 \sin \Omega(t) + [x(t) - x(t - T)] \sin \Omega(t) + [y(t) - y(t - T)] \cos \Omega(t),
\]

where \(h_0\) is the intended chip thickness, which is equal to the feed rate of the machine, and \(x, y\) are components of the dynamic chip thickness produced owing to vibrations at the present time \(t\) and one spindle revolution period \(T\) before. Assuming that the work piece is approximated by a two-degree-of-freedom system in two uncoupled and orthogonal directions, the following equation of motion is obtained:

\[
\begin{bmatrix}
m_x & 0 \\
0 & m_y
\end{bmatrix}
\begin{bmatrix}
\ddot{x}(t) \\
\ddot{y}(t)
\end{bmatrix} +
\begin{bmatrix}
c_x & 0 \\
0 & c_y
\end{bmatrix}
\begin{bmatrix}
\dot{x}(t) \\
\dot{y}(t)
\end{bmatrix} +
\begin{bmatrix}
k_x & 0 \\
0 & k_y
\end{bmatrix}
\begin{bmatrix}
x(t) \\
y(t)
\end{bmatrix} =
\begin{bmatrix}
F_x(t) \\
F_y(t)
\end{bmatrix}.
\]

Here, the terms \(m_{x,y}\), \(c_{x,y}\) and \(k_{x,y}\) are the modal mass, damping and stiffness, respectively, and \(F_x\) and \(F_y\) are the tangential and radial cutting forces components resolved in \(x, y\) directions. The tangential and radial components of cutting forces

Figure 5. Regenerative effect due to chip-thickness variation.
are proportional to the axial depth of cut $a$, the dynamic chip thickness $h(t)$ and can be expressed as follows:

$$F_t(t) = K_f a h(t)$$
$$F_n(t) = K_r F_t(t).$$

The coefficients $K_f$ and $K_r$ are cutting constants in the feed and normal directions. Equation (2) can be rewritten in a more compact matrix form as follows:

$$M \ddot{u}(t) + C \dot{u}(t) + K u(t) = F(u(t) - u(t - T)).$$

Equation (4) is a time-delay differential equation and is solved numerically. For the solution of the dynamic response problem, the mode superposition method can be used. The mode superposition method for time-history analysis is based on normal mode dynamics. Normal mode dynamics calculates the natural frequencies and natural modes of vibrations by means of the finite element method (FEM). Mode shapes and natural frequencies are used for identification of structural resonances that may produce an undesirably large structural response to the dynamic input. Further, the response of structures to dynamic inputs can often be assumed to be a combination of the mode shapes corresponding to each mode. This lets the mode shapes construct a numerically efficient representation of the structure (modal representation) for use in further analyses.

The Lanczos method is applied to the present analysis as the most effective method for the solution of large-scale problems.

2.4 Calibration of the simulation model

For the machine tool to reach its full utility, the vibrations that contribute to chatter must be identified. It is usually the mismatch between the frequency response of the spindle and the frequency behaviour at the cutting point (contact between cutter and work piece) which contributes to uncertainties in the machine modelling. In order to obtain reliable results, a simulation model that is a model of the real behaviour has to be used. The more the process is accurately simulated; the closer to reality results will be obtained. This is a way to decrease the possibility of modelling errors.

In order to make a more accurate prediction of the dynamic behaviour of the cutting process, LDV was used for calibrating the simulation model. LDV is a non-intrusive optical method that is fast becoming common for vibration measurements. The non-contact nature of LDV means that the structure can be analysed without introducing additional mass loading which is important in the cases of light weighted, thin-walled parts. Basically, the device is a heterodyne interferometer based on the Doppler effect of backscattered light, as schematically presented in figure 6. A laser beam is divided by a beam splitter (BS) into a reference beam and an object beam. The reference beam reflects on a mirror (M) and is redirected to a modulator where the beam is then shifted by a known amount of frequency ($f_B$). This frequency shift is needed for resolving the direction of the measured vibration velocity. Systems differ by the method used for obtaining this frequency shift; our Polytec laser vibrometer uses an acousto-optic modulator (Bragg cell) with a frequency shift of about 40 MHz. The object beam reflects from the target and, hence, is Doppler shifted ($f_D$) due to the target velocity and mixes with the shifted
reference beam on the photo detector. Depending on the optical path difference between these two beams, they will interfere constructively or destructively. When the target is moving, the intensity measured by the photo detector will be time dependent. Frequency demodulation of the photo detector signal by a Doppler signal processor produces a time-resolved velocity component of the moving target.

In the application of laser vibrometry to experimental modal analysis, the surface of the vibrating object is scanned. A scan is a sequence of single point measurements. Two small servo-controlled mirrors in the scanning head make it possible to deflect the beam both in the horizontal and the vertical direction. The measured data in each scan point are then compared to a reference signal. In this case, the measured object is excited by an electrodynamic shaker. A force transducer at the driving point acts as a reference. Mode shapes and frequencies are then extracted from the measurements data by the LDV system software.

The LDV was used to calibrate the boundary conditions in the FEM model and also to experimentally determine the vibration properties of the setup cutting tool-spindle. Details about measurements on the spindle are presented in Rantatalo et al. 2004.

The performance of the finite elements is evaluated using the stock part that is fixed in the measuring frame by means of flexible suspensions (rubber band). The flexible fixture allows the part to reproduce all rigid body modes. A shaker is used to excite the stock by a random pulse. The first non-singular mode and the corresponding natural frequency of 1947 Hz were chosen to illustrate the result of the experiment (see figure 7).
The finite element model with the same type of flexible constraint allowing all rigid body modes is analysed using an eigen problem solver. For the FEM discretization, 10-node tetrahedron elements with quadratic interpolation were used. Material parameters for the reference part were as follows: Elasticity modulus $7.1 \times 10^4$ MPa, Poisson’s ratio 0.315, Density $2.8 \times 10^{-6}$ kg mm$^{-3}$. The material type was: Zn-Mg-Cu-Zr-Al – aircraft grade.

The simulation result – first non-singular mode with a frequency of 1970 Hz is also presented in figure 7. The simulation shows good agreement with the experiment.

The next step in the calibration of the finite element model was an accurate definition of the boundary conditions for the analysis. The work piece was mounted on the machine table in a Liechti Turbomill machining centre by the use of stiff clamps. The setup containing the stock part and the shaker is shown in figure 8.

The frequency response to a random signal was measured by means of a LDV. The first mode with corresponding frequency is shown in figure 9. A sequence of numerical experiments was run to iterate to the correct stiffness of the fixing clamps. The result of the analysis is presented in figure 9. Again, the first mode and frequency of the fixed stock were chosen to compare the numerical result with the experiment. The simulation shows again good agreement with the experimental results.

3. Modelling results and experimental verification

3.1 Evaluation of a stable cutting process

The digital model, constructed and calibrated in previous sections was used to pursue an analysis of stability boundaries. The sequence of as cut geometries shown in figure 4 was used for generation of FEM models. During the machining, the initial stock mass of 5.31 kg is reduced to 0.61 kg after finishing the operations. This factor and a lack of symmetry in geometry which is very common in this type of manufacturing had a large impact on the vibration properties. In order to show the significant changes of the vibration properties, the first natural frequency and the modes of the initial stock and final geometry were chosen for the comparison. The simulated first frequency 2940 Hz for the stock decreases after the final milling
operation to 800 Hz. Simulated and LDV experimental results for the final geometry are shown in figure 10.

The FE model used for solving the eigenvalue problem was also used for transient response analysis by including appropriate initial and boundary conditions. Transient analysis compares the dynamic response of a structure to a set of simultaneous transient excitations. The response at each time instant was calculated by combining the modal response obtained using time integration. The work piece was loaded by a short force pulse in two orthogonal directions in the plane $x - y$ which was parallel to the bottom plane of the work piece.

Figure 8. Photo of the setup with the stock and the shaker mounted in the Liechi machine.

Figure 9. First mode and natural frequency are shown for the comparison of LDV measurements with FEM analysis. Left – FEM modelling. The colour scale represents the first normalized mode corresponding to the first natural frequency of 2940 Hz. Right – Experimental result. The colour scale represents quantitative displacements corresponding to the first frequency of 2835 Hz.
The frequency response functions (FRF) in mm \( N^{-1} \) obtained from FE analysis for a sequence of cut geometries and also obtained from LDV measurements on the spindle were exported into the CutPro software and were used for processing and stability lobes calculations. Features implemented in the CutPro software for the generation of data describing a machining process. In the preparatory stage, the cutter data and cutting coefficients for the current work piece material and a cutting force model were defined. After all the preparatory steps were accomplished, the necessary input data to continue with the stability lobes calculation were available.

The stability lobe defines region of stable and unstable cutting zones as a function of maximum depth of cut and spindle speed and is used to select appropriate machining parameters. Stability lobes that are predicted for the initial stock geometry and for a \( \phi 40 \)-mm CoroMill cutter are shown in figure 11. Finish milling of the final geometry is done using a \( \phi 16 \)-mm CoroMill R216 cutter. The predicted stability lobes for final operations are presented in figure 12.
3.2 Experimental verification

In order to study the accuracy of the prediction, the stability boundaries in terms of depth of cut and spindle speed obtained by FE modelling had to be validated with the experimental procedures.

A comparison of stability lobes predicted by the experimental modal analysis using CutPro with stability lobes calculated by the use of FEM analysis is presented in figure 13. The prediction based on FEM analysis yields lower values for the critical depth of cut than the prediction based only on experimental procedures. The difference in critical depth of cut above all for higher spindle speeds can be explained by the fact that the FEM-based prediction involves analysis in 3D that is not the case in the experimental approach. The prediction based on experimental modal analysis involves only excitations and response measurements in the same direction (testing in 1D space).

A series of milling tests in the Liechti machine were performed for the verification purpose on the initial monolithic stock with a R390, Ø40-mm tool. The spindle speed and feed rate were kept constant at 9500 rpm and 3000 mm min$^{-1}$, respectively, while the axial depth of cut was varied. Three different axial depths of cut, 1.5 mm, 2 mm and 3 mm were studied. The cuts were placed sequentially along the y-direction. The radial depth of cut was equal to the radius of the tool. The length of each cut was 58 mm and for the next cut, the tool was moved to the right by a distance of 42 mm leaving 2 mm between the cut paths. The range of depth of cut was sufficient to move the system across different regions of stability. These cutting points are marked in figure 13. At a 1.5-mm depth of cut, the system was clearly within the stable region of the curve predicted by the experimental work, and falls just within the stable region of the stability curve predicted by the FEM analysis. At a 2-mm depth of cut, the system was inside the stable region of the experimental predicted curve but outside the FEM based one. Finally, at a 3-mm depth of cut, the system fell outside the stable region of the two curves. A Brüel & Kjaer Type 4393 accelerometer was mounted on the work piece, about 10 cm from the tool path, in the feed direction. Also, a Brüel & Kjaer Type 4189 microphone was used for sound recordings.
The microphone was placed about 70 cm from the tool path, directed towards the tool. The sensitivity of the accelerometer and the microphone was 0.314 pC m$^{-1}$ s$^{-1}$ and 45.5 mV Pa$^{-1}$, respectively. The data were sampled at 32.8 kHz and a low-pass filter of 10 kHz was used. After machining, the generated surface quality was examined in order to determine the process stability. Figure 14 shows a photo of the milled surface.

At a 1.5-mm depth of cut, the sound pressure level (SPL) at the spindle rotation frequency was 99 dB, and the surface finish was very good (see figure 14(a)). The system was in a stable state. At a 2-mm depth of cut, the SPL was 103 dB and the surface finish was acceptable but not perfect (see figure 14(b)). Removed chips were smeary and sticking on the tool, and hence, a coolant was needed. The system was in a transition state between the stable and unstable positions. At a 3-mm depth of cut, the SPL was 106 dB and the accelerometer level was higher, especially at

![Figure 14. Photo of the machined surface. (a) 1.5-mm axial depth of cut; (b) 2-mm axial depth of cut; and (c) 3-mm axial depth of cut.](image-url)
about the 10th, 22nd, and 51st spindle harmonics. The surface finish was bad (see figure 14(c)) and the system was unstable.

4. Conclusions

Chatter is a dynamic phenomenon that can appear in machining at almost any spindle speed. Stable machining cannot be achieved without investigating the influence of chatter on the cutting process. A standard way to obtain reliable prediction of a stable machining process is the application of the methodology based on the experimental modal analysis. This approach requires, in the case of thin-walled structures, extensive trial and error experimental work.

In this work, an integrated tool for prediction stability boundaries was presented. A digital model for the milling process based on integration of CAD, CAM, and FEM was developed. Commercial software packages used for integration into the model prove to accomplish demands for functionality and performance. Not only does the suggested approach give safer machining, it also carries promises of an overall higher productivity.

In order to avoid the usual mismatch between the frequency response of the spindle and the frequency behaviour at the cutting point (contact between cutter and work piece) which contributes to uncertainties in the machine modelling, dedicated experimental procedures which were based on LDV measurements were used.

The non-contact nature of the LDV makes accurate and fast measurements possible with an easy setup without any mass loads. This is of crucial importance when measuring on thin-wall structures with low mass and rigidity since the eigen mass of the accelerometers disturbs the measurements. Due to the complex geometry and fixture of the detail the dynamic FEM models needed to be validated and improved, especially regarding boundary conditions where even small changes in the fixture could result in considerable differences in the natural frequencies.

The LDV was also able to make measurements on the rotating spindle, spindle housing, tool, tool holder, work piece, clamps and machine table in one setup and gave an overall picture of the machining.

The performed milling tests showed good agreement with the predicted stability lobes (at 9500 rpm). Machining just within the stable region predicted using the FEM analysis resulted in the best surface finish.

Acknowledgement

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Paper F
Laser vibrometry measurements of vibration and sound fields of a bowed violin

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Abstract
Laser vibrometry measurements on a bowed violin are performed. A rotating disc apparatus, acting as a violin bow, is developed. It produces a continuous, long, repeatable, multi-frequency sound from the instrument that imitates the real bow–string interaction for a ‘very long bow’. What mainly differs is that the back and forward motion of the real bow is replaced by the rotating motion with constant velocity of the disc and constant bowing force (bowing pressure). This procedure is repeatable. It is long lasting and allows laser vibrometry techniques to be used, which measure forced vibrations by bowing at all excited frequencies simultaneously. A chain of interacting parts of the played violin is studied: the string, the bridge and the plates as well as the emitted sound field. A description of the mechanics and the sound production of the bowed violin is given, i.e. the production chain from the bowed string to the produced tone.

Keywords: laser vibrometry, violin, sound fields

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(Some figures in this article are in colour only in the electronic version)

1. Introduction
The physics of the violin as known up to 1984 was thoroughly described by Cremer [1]. The construction, properties of separate parts, etc are presented in a more applied way in [2]. However, as in many earlier investigations of the violin, the violin parts are investigated separately and not so much as a played, assembled instrument. The holographic interferometry investigations from the early 1970s [3, 4] used electromagnetic single frequency excitation, and not bowing. Successful experiments at that time demanded high stability. Today’s optical measuring techniques such as the laser vibrometer are much less disturbed by rigid body motions and have a much higher sensitivity.

The violin body is made of three main parts: the top plate, the ribs and the back plate, see figure 1. The top plate is made of spruce and made arched (a shell), has two f-holes, is strengthened by the bassbar and supported by the soundpost. The edges of the top plate are glued to the maple ribs with six blocks: the upper block, the lower block and the four corner blocks. To the other side of the ribs, the back plate of maple (also made arched, a shell) is glued. The soundpost is a small wooden cylinder placed close to the treble foot of the bridge and is squeezed in between the top and back plates. Its position can be adjusted if necessary.

Laser Doppler vibrometry (LDV) [5] is a well-established non-contact method normally used for measurements on vibrating surfaces. By scanning the laser beam across the surface of the object, the resonance frequencies and the corresponding modes can be measured and visualized. Since LDV is an interferometric technique, changes in an optical path in general can be measured. Zipser et al proposed to use
LDV to visualize and measure sound and flow fields [6, 7]. In such measurements the background should be stable and the medium between the laser head and the background ‘vibrates’ due to pressure changes.

The present investigation differs from earlier optical investigations [3, 4, 8–11] in that the violin is now excited by a rotating bow and measured with LDV when it is bowed instead of exciting it at a single frequency at a time. The prime interest is now not to measure the eigennodes but the ODSs (operating deflection shapes) [12] and the sound field that is excited by the bowing of the real assembled instrument. It also differs from investigations where pulsed lasers have been used to study the violin, often excited with impacts or transients. Molin et al [13, 14] showed the bending wave propagation in the body of the violin for impulses at the bridge top when the two bridge feet moved in phase or out of phase. One observation from these experiments was the high speed with which the back plate of the violin was set in vibration by the waves transmitted to the back plate by the soundpost. Electro-mechanical measurement for the modal analysis of the violin was also performed [15–17] but such investigations were performed either with instrumented, impact hammers or from white noise excitation, and not by bowing. In this investigation, however, it is not the start or the end of a tone that is of prime interest but rather the more continuous and constant middle part of a played tone.

The sound radiations of three violins were measured and compared by Wang and Buroughs [18]. They used near-field acoustic holography for the measurements and a haired rosined infinite belt for the bowing.

Linaza et al used numerical FEM analysis to study the violin [19]. This method has many advantages since it allows numerical studies of the influence of thickness, curvature, material, etc, that is many numerical experiments which are hard or impossible to perform physically. On the other hand, there are a number of unknown or less known factors in such often quite simplified numerical FEM models.

2. Experimental set-up and technique

In his study of the mechanical actions of violins, Saunders [20] constructed and used a mechanical, rosined rotating bow to obtain repeatable conditions for his experiments. Meinel [21] also constructed a bowing machine. He used an infinite rosined stripe. Our reason for developing the rotating bow was that the LDV measurements need a repeatable, long and well-defined vibration signal to allow the modal analysis of the signal. Since the measuring time sometimes exceeds 10 min, ordinary bowing by hand was not possible, and it is not stable enough. A rotating bow was therefore constructed from which the violin string can be excited continuously and repeatedly for more than 15 min, see figure 2. It consists of a dc-engine (24 V) driven disc (D) of PMMA and is hinged like a pendulum in a frame supported by two bearings (B). The normal bowing force on the string can easily be changed by moving a small mass along the lever (L), and the rotational speed is controlled by a dc-voltage supply. The diameter and the thickness of the disc are 110 mm and 6 mm, respectively. The edge of the rotating disc was slightly rounded. It was rosined in the same way prior to all measuring series. Any change in the sound from the bowed violin was not detected by the measuring microphone during 20 min bowing.

The violin and the rotating bow were mounted in such a way that they can be rotated together around a vertical axis, see figure 3. This makes it easy to measure on different sides of the violin and also different projections of the sound field without changing bowing conditions or moving the scanning head of the LDV. The rotating device is firmly bolted to the white-painted heavy concrete block (1.3 × 1.2 × 0.4 m³), seen as the background in figure 3. This rigid non-vibrating background reflector is necessary for measuring the small changes in the optical path length for the LDV laser beam probing through the sound field [6, 22].
The scanning head of the LDV system (Polytec PSV-300) can be seen at the lower-left corner of figure 3. It was mounted on a sturdy tripod and was placed 3 m from the concrete block when measuring the sound fields and about 1 m from the violin when measuring the top and back plate vibrations. Our Polytec LDV can be operated in two different modes: FFT and time mode. In the FFT mode, after scanning across the field of interest, frequency bands corresponding to the resonances or sound frequencies can be selected and animations of the modes can be presented. An Endevco, 1-TEDS accelerometer (A) and a Bruel & Kjaer type 4189 microphone (M) were used as reference signals and their position can be seen in figure 3.

By recording in the time domain, time animations of how the object actually vibrates (including all modes) can be presented. To make this possible, a repeatable trigger signal is necessary so that the recording of each scanning point can start at exactly the same phase of the vibration. The slip portion of the bow–string interaction occurs once every fundamental period and induces a sharp well-defined slope in the microphone signal, which is used as the trigger.

The relative phase of the vibration fields of the top plate and the back plate was determined by sending the probing laser ray through the f-holes to the inside of the back plate at the same time as the top plate was measured. One obstacle was that the violin label (a piece of paper) inside the violin glued to the back plate, with the violin makers name on it, did not always work well as a reflector of laser light. It was not firmly enough glued to the back plate.

The measurements were performed in a quite big optics laboratory. It would have been better to put all the measuring equipment in a room with better sound absorbing walls. At certain frequencies, we could detect standing waves generated by wall reflections. We tried to improve our room using curtains and other sound absorbing materials. On the other hand, in sound measuring experiments, the flat, heavy concrete block does reflect both light and sound quite well, and it is necessary to use it in the sound measurements. So even if we had been in an acoustic ‘silent’ room, the heavy concrete block would have been used. The so-called reverberation radius of the room could be estimated from room dimensions and wall, floor and ceiling materials to 0.7 m, i.e. making observations over the full concrete block meaningful. Within the reverberation radius, the directly radiated sound is expected to dominate over the reverberant sound of the room.

The violin used in the experiment is the same violin that was built and investigated during all steps in making [3]. Harry Sundin made the violin in 1971 and it was named HS71. The violin has since then been used in several other investigations [8, 13, 14].

3. String vibrations

The bowed string has been studied extensively by many authors [23–25], and its fundamental behaviour is quite well known. The reason for our measurements of the bowed string was twofold: to see if it was at all possible to measure on a bowed violin string with LDV and if so, then to determine the shape of the travelling waves that were excited. The same conditions (i.e. bowing speed, position and bowing force) in the bow–string interaction should be used as in the rest of the measurements: on the bridge, the violin body and the sound fields. In this way, a more complete picture of the mechanics of the violin could be obtained.

In all experiments the open D-string is bowed, somewhat low tuned at 285 Hz (period time \( T = 3.5 \text{ ms} \)). By illuminating the string with a stroboscope, it was easy to observe that the amplitude of vibration in the middle of the string was several millimetres (as high as 5 mm), both parallel and transversal to the bowing direction. Since large motions in the transverse direction to the laser beam would deteriorate the signal, it was only possible to measure the string vibrations close to the disc. This problem would be possible to solve if the laser beam could follow the motion of the string. For this purpose, a sophisticated tracking system is needed. In figure 4, the displacement of the string in the bowing direction at a point close to the rotating bow is shown as a function of time. The peak-to-peak displacement is rather large, about 1.8 mm. The displacement curve in figure 4 has the same shape as observed by Helmholtz, see figure 24(B) in [23]. The graph starts at the left with a maximum negative slope of about 0.7 m s\(^{-1}\) which is quite close to the estimated bow speed of 0.75 m s\(^{-1}\). This corresponds to the sticking part of the period between...
the bow and the string. It is much longer than the slip part, which is even more transient, where the string and the bow move relative to each other. The radius of the rotating disc is 5.5 cm and it rotates with about 2.1 rev s$^{-1}$. The bowing force was measured to be about 1 N. These numbers are in the playable range as given by Askenfelt [24] for the violin as being between 0.5 and 1.5 N at a bow velocity of 1–0.1 m s$^{-1}$.

The almost triangular wave in figure 4 propagates in both directions along (up and down) the string and is reflected at the ends of the string. At the bridge, it is partly reflected and it transmits part of its energy and momentum to the bridge as a series of pulses. Observe that this transmission takes place in both directions, parallel and at right angles to the top plate. The pulses are traceable as a complicated sum of allowed eigenmodes of the string. The bow–string interaction thus results in forced vibrations of the bridge at the eigenfrequencies of the string in mainly two directions. This results in a bridge motion as illustrated in the following section. A very thorough description of the bowed string is also given by Woodhouse [25].

4. Bridge vibrations

Time sequences of the bridge motion were measured with the vibrometer of both horizontal and vertical components, when the violin was bowed. The LDV system was set in time mode with five averages using a sampling frequency of 102.4 kHz. The LDV system was set to integrate the vibrometer velocity signal giving a displacement output. The measurement positions and directions are shown in figure 5.

Figures 6 and 7 show the vibration displacement as a function of time. It can be noted that the horizontal amplitude of vibration (point 3 in figure 5) is about ten times larger on the side of the bridge than the vertical component on the bassbar side (point 1 in figure 5). At the side of the bridge, the amplitude at the top is about 3.5 times larger than that at the bottom (points 3 and 4 in figure 5), and they are vibrating in phase. The vertical motion at the top of the bridge is different not only in that it is smaller, but also in that the treble side motion is mostly negative when the bassbar side is positive and vice versa, i.e., they move in anti-phase. The vertical amplitude on the soundpost side is smaller than that on the bassbar side. The shapes of the horizontal and vertical components (points 1 and 3) remind us very much of each other despite the large difference in amplitudes. This might be explained by the rocking motion of the bridge [1, 26].

Actually, several points at the top and at the side of the bridge were measured though only two of them from each side are shown in figures 6 and 7. These measurements indicated that the bridge mainly moves as a ‘rigid’ body with a rocking and stamping motion. Eigenmodes of the bridge itself were not detected in the signals.

5. Sound spectrum of the bowed violin

In the experiments, the violin HS71 was played on the open D-string by the rosinated rotating bow at approximately one-tenth of the open string length. The violin was mounted 35 cm from the concrete block, hung upside down and held by a special clamp at the tailpiece button and a clamp covered by foam plastic at the scroll. The reason for hanging it upside down was to minimize the covered parts of the violin front plate as seen from the LDV measuring head when bowed. Sound reflections at the concrete block and the disc of the bow give minor disturbance to the recorded and presented sound fields. Figure 8 shows the sound spectrum obtained with a microphone. It was placed as shown in figure 3. Frequency components from the 1st up to the 17th harmonic of variable strength starting with the fundamental one at 285 Hz can be seen. Between 2 and 3 kHz (7th to 9th harmonics) and at
a ray through a transparent medium is defined as

\[ l = \int_0^L \Delta n(x, y, z, t) \, dx, \]  

where \( \Delta n(x, y, z, t) \) is the refractive index variation and \( dL \) is a differential distance along the light path. The rate of change in the optical path length is

\[ L = \frac{dl}{dt} = \int_0^L \Delta n(x, y, z, t) \, dx, \]  

where \( \Delta n(x, y, z, t) \) is the refractive index variation along the light path and \( dl \) is a differential distance along the light path. The rate of change in the optical path length is given by

\[ \Delta L = \int_0^L \Delta n(x, y, z, t) \, dx. \]  

If the refractive index variations along the light path are small, and the fact that \( n \approx 1.00027 \) is very close to 1, then

\[ \Delta L = \frac{v}{c}, \]  

where \( v \) is the velocity of the reflecting surface in the direction of the laser beam. This is the normal situation for vibration measurements. If, on the other hand, the reflecting surface is rigid, then

\[ \Delta L = 2 \int_0^L \Delta n(x, y, z, t) \, dx, \]  

Figure 8. Sound pressure level for the bowed violin measured by the microphone.

3.5 kHz (12th harmonics) some strong components show up. This frequency range is often referred to as the bridge hill at about 3 kHz of the violin bridge. It must be remembered however that this was for a rigidly supported bridge and not a bridge placed on the sprung top plate of a violin. For an assembled violin, the motion of the top plate is also important for the ‘bridge hill’. It is noticeable that the violin sound has many strong harmonics, many of them in the frequency range where human beings have highest hearing sensitivity. The minima at the 10th to 11th partials are set by the frequency range where human beings have highest hearing sensitivity. The minima at the 10th to 11th partials are set by the bowing at approximately one-tenth of the string length.

In the following section, we have selected four harmonics from the spectrum for presentation of plate vibrations and sound fields.

6. Plate vibrations and sound fields

6.1. Vibration field and sound field measurements

As mentioned in the introduction, LDV is an interferometric method that measures the rate of change in the optical path length for the probing laser beam. The optical path length for a ray through a transparent medium is defined as

\[ \Delta L = \int n(x, y, z, t) \, dx, \]  

where \( n(x, y, z, t) \) is the refractive index of the medium. The rate of change in the optical path length is

\[ \frac{dl}{dt} = \frac{d}{dx} \int n(x, y, z, t) \, dx, \]  

where \( \frac{d}{dx} \) is the derivative with respect to the spatial coordinate \( x \). The rate of change in the optical path length is given by

\[ \Delta L = \frac{d}{dx} \int n(x, y, z, t) \, dx, \]  

where \( \frac{d}{dx} \) is the derivative with respect to the spatial coordinate \( x \). The rate of change in the optical path length is given by

\[ \Delta L = \frac{d}{dt} \int_0^L n(x, y, z, t) \, dx, \]  

where \( \frac{d}{dt} \) is the time derivative.

Combining equations (7) and (4) yields

\[ \Delta L = 2 \int_0^L \frac{n_x - 1}{\gamma} \, dx. \]  

Assuming the pressure variations to be constant along a distance \( x \) and zero elsewhere gives

\[ \Delta L = 2 \mu \left( \frac{n_x - 1}{\gamma} \right). \]  

For each frequency component of the sound, the amplitude of \( \tilde{p} \) is the pressure amplitude multiplied by the angular frequency \( \omega = 2\pi f \). The pressure amplitude can thus be expressed as

\[ \tilde{p} = \frac{\Delta L}{2x} \left( \frac{n_x - 1}{\gamma} \right) p_0, \]  

where \( p_0 \) is the unattenuated sound pressure amplitude. The pressure amplitude can thus be expressed as

\[ \tilde{p} = \frac{\Delta L}{2x} \left( \frac{n_x - 1}{\gamma} \right) p_0. \]  

Table 1. Vibration displacement ranges in figures 9–12.

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where \( L \) is the distance from the scanning head to the reflecting surface. The density of a gas is coupled to the refractive index by the Gladstone–Dale equation [27]

\[ \gamma = n - 1 = K \rho. \]  

Under adiabatic conditions, the pressure is related to the density as

\[ p = \frac{\gamma}{\gamma - 1} p_0 \]  

where \( p_0 \) and \( \rho_0 \) are the undisturbed pressure and density, respectively. \( p \) and \( \rho \) represent the acoustic contribution to the overall pressure and density fields, respectively, and \( \gamma \) is the specific-heat ratio. Assuming that the sound pressure fluctuations \( p \) are small compared to the undisturbed atmospheric pressure, the time derivative of the refractive index is given by

\[ \frac{d}{dt} n = \frac{n_0 - 1}{\gamma} \frac{\rho}{\rho_0}. \]  

Combining equations (7) and (4) yields

\[ \Delta L = 2 \int_0^L \frac{n_x - 1}{\gamma} \, dx. \]  

Assuming the pressure variations to be constant along a distance \( x \) and zero elsewhere gives

\[ \Delta L = 2n_x \left( \frac{n_x - 1}{\gamma} \right). \]  

The refractive index changes in our case are caused by the sound pressure variations from the violin. Of course, the sound pressure field is present even when we measure the plate vibrations, but with negligible influence since the refractive index changes are very small compared with the changes caused by the plate vibrations, see table 1.

Table 1. Vibration displacement ranges in figures 9–12.

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along the whole light path. By scanning the laser beam across the reflector around the violin, we will obtain a 2D map of the integrated 3D sound field [6]. The results of the plate vibration fields and the sound fields are best presented as animations.

A comparison between microphone and LDV measurements was performed. The microphone was directed towards and quite close (a few millimetres) to the back plate of the bowed violin. The phase difference between the microphone and the reference accelerometer signals of the fourth harmonic (1130 Hz) was observed in situ on the computer screen. Knowing the mode shape at this frequency from preliminary LDV measurements, the microphone was moved across the back plate through areas vibrating in different phases. Moving from two nearby positions vibrating in anti-phase resulted in a phase shift of 180° in the microphone signal. From this measurement, it was clear that the phase of the emitted near-field sound varied in the same way as the plate vibrations recorded by the LDV system. This qualitative check thus verifies the expected basic behaviour of the LDV system.

In the experiments presented below, the LDV system operated in the FFT mode with a sampling frequency of 20.48 kHz. The LDV system produced frequency spectra using a complex averaging method with three averages of 200 ms, each giving a frequency resolution of 5 Hz.

For the top and back plate measurements, the scanning points were spaced approximately 2 cm giving a spatial resolution of about 9 cm.

6.2. Results

Figures 9–12 show the top and back plate ODSs (a) and (c)) and the integrated sound field (b) at different frequencies (285 Hz, 1130 Hz, 1415 Hz and 2265 Hz). These figures are ‘snapshots’ of the attached video animations. It must be understood that the LDV software automatically scales the colour coding (blue–red) within the measuring range, unless the operator sets the scales manually. Table 1 summarizes the displacement ranges for figures 9–12 and the corresponding animations. Observe the large differences between the plate vibration amplitudes and the integrated sound field amplitudes (amplitude of the optical path length change caused by refractive index changes).

The operating deflection shapes and sound field at 285 Hz (fundamental of the played tone) are shown in figure 9. The corresponding animations to these images are shown in animated figures 9(a)–(c). Red areas vibrate in anti-phase to the blue areas in the top and back plate images.

This frequency is close to the A0 or f-hole mode (the ‘main air mode’ or ‘Helmholtz mode’) resonance. A nodal line runs into six antinodes with one vertical and two horizontal nodal lines. The anti-phase motions of the left and right parts should make the radiation measured with the vibrometer less efficient, but still represent a symmetrical motion along the body. The lower and upper parts of the front plate are in phase but in anti-phase with the middle part. It is plausible that this generates a somewhat deformed radiation lobe pointing to the left as is found in figure 10(b). The back plate shows four areas vibrating in anti-phase relative to its neighbours. No vertical nodal line is seen; rather the phase of the antinodes is more or less constant across the violin back. The radiation close to the back plate, figure 10(b), mainly shows one part on the top and one at the bottom emitting sound in phase with one part between them in anti-phase. This radiation pattern reminds us somewhat of three sound sources on a line in anti-phase as in Cremer [1], p 418. The directivity [28–31] of this mode is quite strong compared to the more spherical sound distribution found in figure 9(b). The wavelength of the sound in air is now slightly larger than the wavelength of the bending waves of the plates.

The operating deflection shapes and sound field at 1415 Hz (fifth partial of the played tone) are shown in figure 11. The corresponding animations to these images are shown in animated figures 11(a)–(c). The ODS of the top plate shows a similar split-up as in the previous figure 10(a). The two feet of the bridge again vibrate in anti-phase. The vibrations at the soundpost are noticeable, also in the back plate. The ODS of the back plate now shows four areas of anti-phase motion in the lower part. In the top part, the antinodes cover almost the whole width of the violin. The sound field in figure 11(b)
Laser vibrometry measurements of vibration and sound fields of a bowed violin

(remember that it is the projected sound field that is visualized) is harder to interpret and weaker than before. It however shows radiation lobes of varying strength as can be expected. One lobe is radiating from the upper part and one from the lower part of the violin. Again the wavelengths of the sound and the longitudinal bending waves are of the same magnitudes.

The operating deflection shapes and sound field at 2265 Hz (eighth partial of the played tone) are shown in figure 12. The corresponding animations to these images are shown in animated figures 12(a)–(c). This partial is of almost the same strength as the fifth partial in figure 11. The ODSs are now divided into a larger number of antinodes. As many as six antinodes can be seen in the lower part of the top plate and four in the lowest part of the back plate. Several horizontal nodal lines, four to five, can be seen in both plates. The radiation from such complicated vibration patterns will of course also be quite complicated, compared to the near-field distribution and the more distant radiation pattern. Radiation wavefronts from the top plate, the upper part and another set from the middle part can however be seen. The two sets of wavefronts are of anti-phase. Still, it must be noticed that such modes are important to the total picture of the violin sound.

7. Discussion

The sound of a single note from a melodic musical instrument can be divided into a fundamental frequency tone and a number of harmonic partials. The string has this ability; it has a fundamental frequency plus a number of higher partials, depending upon the way it has been excited. The string itself is however a very weak emitter of sound, and its diameter is too thin compared to the aerial wavelength of the sound to be produced. Ordinary wooden plates and shells normally do not generate harmonic partials when moderately excited, but, on the other hand, they are efficient sound emitters. The violin uses a combination of string vibrations and plate/body modes. It operates in such a way that forced vibrations at the harmonic string partials set the violin body into vibration to produce the sound. In this way, both harmonic partials and an efficient sound production are achieved.

Bowing the string is a very efficient way to excite the string both vigorously and continuously. It vibrates in a fundamental and a high number of partials and produces a long sustained tone. The rosin on the rotating bow in our experiments models the bow–string interaction in an idealized way.
bowing velocity, bowing force and direction of bowing are kept constant. The string is excited both horizontally and vertically as well by rotation. Neither the start nor the ends of a real played note are considered in our experiments. The set-up models the middle, constant portion of a real played note, repeated every fundamental period. In real life, a skilled musician can vary the bowing speed, the bowing pressure (i.e. the force by which the bow is pressed against the string) and bowing position to change the relative frequency content of the sound in a wide range as well as the radiated sound power. Measurements have been performed on the bowed open D-string vibrations, close to the bowing point. The bow–string interaction produces a triangular shaped wave, see figure 4, which is reflected at the bridge and transmits energy and momentum into forced vibrations to the rest of the instrument. These forced vibrations set the instrument into vibration at the fundamental string frequency and all partial frequencies simultaneously. The bridge is constructed in an ingenious way so that it efficiently transmits the vibration, mainly as a rocking and a stamping vibration, to both its feet which sets the rest of the violin body into vibration, see figures 6 and 7.

The violin body may look symmetric from the outside but its construction is not. Close to and beneath the treble bridge foot, the soundpost is situated. It transmits vibrations from the bridge and the top plate to the back plate in a very direct way. Beneath the bass bridge foot, there is a bar glued to the inside of the top plate that efficiently transmits bending waves in the top plate along the instrument. The top plate is made out of spruce and the back plate of maple, wooden plates with quite different mechanical behaviour. Spruce is both a very stiff and very light anisotropic material that allows a very high bending wave speed along the grains and a somewhat slower speed across the grains. The top plate, back plate and the ribs form an almost closed air volume open only at the two f-holes in the top plate. Measurements of the top and back plate vibration fields of the bowed violin at the open D-string show that a high number of strong harmonic vibration components are generated. In the frequency range between 2 and 5 kHz, where the sensitivity of human hearing is high, the violin has a number of quite strong components. It is clear that the mode density (number of eigenmodes/frequency interval) of the violin body is quite low below, say, 1 kHz. This means that it is more likely that the fifth partial of the string is close to an eigenmode of the violin body than the first partial. In the lower frequency range, the violin should be treated as a body, for instance, as an elliptical tube closed at the ends [11].

Figure 11. Vibration and sound fields at the fifth harmonic (1415 Hz). (a) Top plate ODS, (b) sound field and (c) back plate ODS.

An MPEG movie of this figure is available from stacks.iop.org/MST/17/635

Figure 12. Vibration and sound fields at the eighth harmonic (2265 Hz). (a) Top plate ODS, (b) sound field and (c) back plate ODS.

An MPEG movie of this figure is available from stacks.iop.org/MST/17/635
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so that both the top and the back plate and the ribs are parts of a number of the first lower vibration modes below, say, 800 Hz. For higher modes, the vibrating areas get smaller and they tend to localize inside the parts of the violin plates. Forced vibrations of the violin body at a certain frequency will set the different vibration modes into vibration, both those below and above the forced vibration frequency. This often results in quite complicated sums of nearby vibration modes (ODSs) that operate together but not in simple phase relations any longer.

The dimensions of the violin are small compared with the aeral wavelength at frequencies below, say, 1 kHz. Above, say, 2 kHz, the bending wavelength of the wooden violin plates is more of the same order as the aerial wavelength and consequently the violin is an efficient radiator of sound at higher frequencies.

There are two main sound sources of the violin body: the air modes and the body (plate) modes. Both are important for the total performance of the violin. They are both set into forced vibrations by the bow-string interaction, mainly transmitted through the bridge. In figure 9, the ‘main air mode, $A_0$', or ‘Helmholtz mode’ is seen. It is acting as an efficient sound source, almost as a mono-source. Note also that the violin is small compared to the aerial wavelength at this frequency. The violin body, here, is vibrating as a ‘breathing’ mode at this frequency so that the total sound of both the air mode and the co-vibrating walls is generated together. The asymmetry of the violin is also noticeable. At frequencies below, say, 600 Hz, the relative frequency difference between sound generating modes is quite large (the mode density is low) and it is very crucial to the performance of the instrument that these modes are distributed evenly.

Measurements of the sound fields generated by the bowed violin were performed. Interesting animated movies of the fundamental component as well as higher harmonics are visualized. When analysing these fields it must always be remembered that it is the projected, radiated sound field across the violin which is shown. If the probing laser is passing through both a high sound pressure and an equally strong sound pressure in opposite phase, the projected result may look as if no sound was produced at all. If the measurements had been performed so that these two fields were beside and not behind each other, two strong radiation centres would have been seen. Below 800 Hz, the violin acts mainly as a mono-source with almost equal sound intensity in all directions. But already at frequencies above, say, 800 Hz, the violin directivity starts to get important.

When looking at the fourth and fifth harmonics, figures 10 and 11, the aerial wavelength is getting comparable in size to the bending waves in the violin body. These harmonics start to show directivity. In the eighth harmonics, figure 12, the mode density of the violin body is quite high, and many smaller vibrating areas show up. The radiation from the top plate side is strong and is divided into directional lobes. Good violins often show a set of strong radiation modes at frequencies from 2 to 5 kHz.

8. Conclusions

Laser vibrometry has been shown to be a versatile technique to study vibration fields not only of most parts of the violin but also of the sound fields in air, generated by the vibration fields of the violin. The technique is non-contacting and non-disturbing. It is the middle, sustained, part of a bowed tone that is studied in this investigation. A rotating bow was therefore developed to generate the long lasting sound that allows scanning laser vibrometry to be used. A critical point in the measurements is to obtain a triggering signal from a microphone or an accelerometer that allows the measuring point to be moved (scanned) and still keep track of the relative phase of the signal at the different measuring points. This is crucial for a successful modal analysis of the measurements.

Measurements of the bridge motion confirm that the string vibrations are transferred to the bridge both in the horizontal and vertical directions. The bridge efficiently transfers the string vibrations to the body of the instrument at the bridge feet, mainly by a combination of a rocking and a stamping motion of the bridge.

Both air modes of the enclosed air and body and plate modes contribute to the total generated sound. It is also apparent that the back plate is an equally important sound radiator as the top plate. In spite of all investigations, much is lacking for a complete fundamental understanding of the physics of the violin, especially at higher frequencies. The measured ODSs verify previous results obtained with the HS71 violin [3, 8] and add understanding of how eigenmodes may combine.

One disadvantage of using the LDV sound field visualization is the fact that the sound field is integrated over a line. This means nearby vibrations in anti-phase, for example, could result in low integrated pressure values. On the other hand, this gives a 2D image of the sound field and opens possibilities for tomographic 3D reconstructions of the sound fields if several projections can be obtained.

Another possibility is opened by the measurements; the sound field generated by the vibrating violin panels can be calculated from the measured ODSs (phase and amplitude). These calculations can be compared to measured LDV sound fields. This work is in progress. To examine the parts that originate from the vibrating panels and the enclosed air, one could try to measure with or without cotton pads in the f-holes. Indeed the projected sound fields measured with LDV open several possibilities which are difficult to perform using microphones.

In a planned subsequent paper, three violins of varying quality—all bowed in the same way at the open E-string—will be compared, firstly, to verify if the main findings of the HS71 violin are valid for violins in general and, secondly, to test if differences can be proved between the best violins (Old Italian violins) and quite different but still good violins (French violins).

Will new acoustical factors be introduced or are the above presented factors differently used?

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Paper G
A method for obtaining the three-dimensional amplitude and phase distribution of an ultrasound field in air, using laser vibrometry and computed tomography, is described. Radiating ultrasound transducers causes pressure changes in the air, which lead to refractive index changes. This change in refractive index can be measured using optical methods like laser vibrometry. If the geometry is kept constant during a measurement with a laser vibrometer, the measured rate of change in optical path is strictly due to the rate of change in refractive index. The measured complex amplitude is an integral of the refractive index changes along the path of the probing beam of the laser vibrometer through the measurement volume, which means that a measurement on a three-dimensional sound field will result in a two-dimensional projection. By rotating the transducer configuration 360 degrees and making measurements at evenly distributed angles the full three-dimensional complex amplitude is reconstructed using a cone-beam tomography algorithm.

**Introduction**

Ultrasound in the frequency from 20 kHz up to megahertz is applied in many industrial and medical applications. The conventional ultrasound sensors, microphones; transducer arrays or mechanical scanners, have in the application of mapping a 2D sound field one major drawback; they obviously disturb the measurement field due to the sensor body, especially at close region of the transducer. Optical measurement methods are non contact, and non intrusive and therefore suitable for measurement and visualization of sound fields. In 1994 Lökberg [1] proposed TV-holography, a whole field measurement method for harmonic sound field measurements in air. Since then numerous papers have been published on measurements of acoustic fields using TV holography. Lökberg et al. [2] and Gren et al. [3] showed that by recording several projections of the sound field and using tomographic principles, the amplitude and phase of the sound field can be reconstructed in every part of the measured volume. Another optical measuring technique is the light diffraction method which is based on the Schlieren set-up system [4]. Almqvist et al. [5-7] have for example used the light diffraction tomography on ultrasound fields for the characterization of the ultrasonic transducers.

Zipser et al. [8, 9] proposed a point measuring method, scanning laser vibrometry, for the measurement and visualization of sound fields, which is the method used in this work. With this technique the absolute phase of the field is obtained directly from the measurements while the amplitude needs some calculations. The nature of the laser vibrometer opens the possibility to measure the sound field without influencing the acoustic field from frequencies of hundreds of hertz up to megahertz with higher sensitivity compared to TV holography [2, 3, 10]. Several measurements and investigations in different applications have been conducted during the years. Molin et al. [10] and Gren et al. [11] have presented visualizations of 2D projections of sound fields in air from music instruments. Harland et al. [12] and Wang et al. [13] have measured underwater acoustic fields, the former investigated how the angle of incidence of the laser light and the acoustic axis affects the laser vibrometer output. In [14] by Nakamura et al. the air inside a aluminium ring was excited by a vibrating transducer and the resulting standing wave inside the ring, also the radiated field outside the ring were measured and visualized using a scanning laser vibrometer. Šegrová et al. [15] showed the possibility and benefits of the laser interferometry technique for measurements in closed chambers with transparent walls i.e. an air field glass tube. However, the laser vibrometry technique discussed in the mentioned studies provides a two-dimensional projection of a three-dimensional sound field and one must always pay attention to this effect if there is a pressure variation along the laser probe. Some of this projection effects are investigated thoroughly by Olsson and Tatar in [16].
Tomographic reconstruction of ultrasound fields using 2D laser vibrometry projections was for the first time proposed by Matar et al. [17, 18]. However, they used only a single ultrasound transducer, assumed symmetric sound source and recorded projections distributed over only 180°. The obtained results showed the ultrasonic pressure variations in a plane, perpendicular to the transducer.

In this paper we reconstruct ultrasound filed from three to four transducers arranged in a 1) symmetric and 2) non-symmetric way emitting at different phases. Using Laser Projections distributed over 360° the interaction between the waves from the different transducers are resolved. Before the actual sound measurements, the vibrating membrane of the transducers were measured by the laser vibrometer which showed that they were vibrating at their fundamental mode. Hence, we assume that the transducers are ideal plane-piston acoustic sources.

**Sound measurements using the Laser Doppler Vibrometer.**

Basically, the laser vibrometer is an interferometer which measures the rate of change in the optical path length [19], normally used on vibrating surfaces. In order to use the laser vibrometer for sound measurements, a special arrangement must be used, see Figure 1. The sound field propagating through the measurement volume (the distance the laser beam undertakes between the laser vibrometer head and the rigid reflector) will disturb the density distribution in air and consequently cause pressure fluctuations [8, 9]. The spatial and temporal pressure fluctuations cause in turn changes in the refractive index, \( n(x,y,z,t) \). The optical path length for the ray through the air is described by

\[
\delta l = \int n(x,y,z)dl, \quad (1)
\]

where \( dl \) is the differential distance for the travelled light. Since the geometrical distance for the laser beam is constant, and the fact that the laser beam passes through the measurement volume twice, the rate of change in optical path is

\[
\delta l = 2 \int n(x,y,z)dl, \quad (2)
\]

where \( L \) is the distance from the laser vibrometer head to the reflector. Thus, the laser vibrometer output is a virtual velocity of the non-moving reflector and gives both qualitative and quantitative information of the sound field, i.e. the phase of the sound field. In order to obtain the amplitude of the sound pressure the following calculations are needed:

The density distribution of the gas (air) is coupled to the refractive index by the Gladstone-Dale relation [20]

\[
n = 1 - \frac{K}{\rho}, \quad (3)
\]

where \( K \), the Gladstone-Dale constant, is a property of the air which is a function of the wavelength and temperature. Under adiabatic conditions the pressure is related to the density as

\[
\frac{p_0 + \rho}{\rho} = \left[ \frac{p_0 + \rho}{\rho} \right]^{\gamma}, \quad (4)
\]

where \( p_0 \) and \( \rho_0 \) are the undisturbed pressure and density respectively, \( \rho \) and \( \rho \) represent the acoustic contribution to the overall pressure and density fields and \( y \) is the specific-heat ratio. If the pressure fluctuations \( p \), are small in comparasion to the undisturbed atmospheric pressure, \( p_0 \), the time derivative of the refractive index is given by

\[
n = \frac{n_0 - 1}{\gamma} \frac{\rho}{\rho}, \quad (5)
\]

Combining Eq. (5) and Eq. (2) yield

\[
\delta l = 2 \int \frac{n_0 - 1}{\gamma} \frac{\rho}{\rho} dl, \quad (6)
\]
For the special case where the pressure variations are constant along the path of integration, the relation between the pressure variations and the rate of change in the optical path is linear

\[ s = 2\pi \frac{n - 1}{\gamma} \frac{\theta}{p_i} \tag{7} \]

However, in most cases the pressure variations will not be constant along the path of integration. When measuring a sound field the probing beam from the LDV will traverse regions with different pressure variations and the resulting measurement will be a two-dimensional projection of the actual three-dimensional field. The projection effects were investigated in previous work [16] and are in this paper used to obtain a tomographic reconstruction of the sound field.

**Reconstruction procedure**

The methodology for reconstructing spatial distributions in a cross-section of an object with computed tomography (CT) is most commonly carried out, in medical applications, using x-rays. This methodology can however be abduced to other regimes of the electromagnetic spectrum, provided that there is an interaction between the light and matter. The interaction implies that the complex amplitude of the light can be related to the propagation distance through the object.

Here we make use of the CT methodology to reconstruct sound pressure variations in free space (air) based on data acquired using LDV. The imaging geometry using a point scanning LDV is principally similar to the geometry of an x-ray CT system. The most noticeably difference is that the projection data here is acquired using interferometry. The sound pressure variations yield a phase shift, \( \phi \), that stand in direct relation to the change in optical path. Thus the phase of the probing beam can be written as

\[ \phi(x) = ks + \delta(x) \tag{8} \]

where \( \phi \) is the phase, \( k \) is the wave number, \( s \) is the optical path length and \( \delta \) is the phase shift. Another difference between these measurements and the conventional CT-setup is that the source and detector is here coinciding geometrically, as seen in Figure 2. This means that the projected ray is propagating along a path two times through the object before reaching the detector. The return travel of the laser beam yields that the phase shift is doubled. When calculating the sound pressure distribution one has to compensate for this extra phase shift.

Finally, the sound pressure variation at each point in the investigated region is assumed to be stationary and free from external perturbations during acquisition of projection data.
The theory described below gives a brief insight into the reconstruction method. A much more detailed and thorough description of the algorithms and implementations in computed tomography is given in the book by Kak and Slaney [21].

We will here look at the simplest imaging case, where we have parallel rays propagating through the object, see Figure 3. In CT most imaging set-ups use fan or cone shaped beams which require a little more complex formulation than the one for parallel beams. The basic principle is however the same.

The expression for a projection generated by a bundle of parallel rays propagating through an object \( f(x,y) \), at a given angle \( \theta \) is given by,

\[
P(\theta, t) = \int_{0}^{2\pi} f(x, y) ds
\]

where \( t \) is the radial distance from the origin to the rays along a line orthogonal to the propagation direction, see Figure 3. This is a two-dimensional generalisation of equation (1) that only holds for a cross-section (slice) through the object. The object function, \( f(x,y) \), is generally also dependent of the wavelength of the photons but here we assume monochromatic illumination.

The Fourier slice theorem states that the radial line \( k \) in the two-dimensional Fourier transform of the object, \( \mathcal{F} \{ f \} \), can be found from the one-dimensional Fourier transform of a parallel projection, \( \mathcal{F} \{ P \} \). The statement can be written as

\[
\mathcal{F} \{ P \}(\theta, k) = \mathcal{F} \{ f \}(k \cos \theta, k \sin \theta).
\]

This relationship is visualised in Figure 3. Therefore, by capturing projection data at all angles during a 180° rotation of the object (or imaging system) we get information at all points in the two dimensional spectrum of the object. By inverse transformation we can then determine the object function in the imaged plane

\[
f(x, y) = \int \mathcal{F}^{-1} \{ f \}(k \cos \theta, k \sin \theta) dk, dk.
\]

The results from using the Fourier slice theorem is a neat conceptual model for retrieving the object function in the imaged plane. However, when it comes to solving this problem practically there is another method that is more suitable for handling discrete data and implementation – this method is called Filtered Backprojection (FBP). The basic steps of the filtered backprojection algorithm are given by the name. First the projection data is filtered - using a ramp filter - and then back projected along the scanning direction. A simple expression for the FBP-algorithm in the parallel beam case can be established from equation (10) and (11) by a change of variables and rearrangement of terms [21, 22]

\[
f(x, y) = \int P * g_\theta 0, x \cos \theta + y \sin \theta d\theta.
\]
The Fourier slice theorem relates the one dimensional Fourier transform of a parallel projection with the two dimensional Fourier transform of the object along a radial line. The filter $g_\nu$ can in the time domain be written as,

$$g_\nu(t) = \frac{1}{2} \int h^{\nu,\omega}(\omega) \, d\omega$$

which in the frequency domain represent a ramp shaped high pass filter that sharpens the reconstructed slice. Most commonly, however, the beam is not parallel but fan shaped or cone shaped, as Figure 1 shows. In these cases equation (12) needs to be modified with case specific weights in order to compensate the fact that all rays are passing through the object along paths of different length. A more intense description of the complete algorithm is given in reference [21].

Here, the sound pressure variation is reconstructed from two dimensional projection data using a cone-beam algorithm. This sort of algorithm is normally used in x-ray micro-computed tomography ($\mu$CT) to reconstruct the three dimensional distribution of the linear attenuation coefficient. The cone beam algorithm was first proposed by Feldkamp et al. [23] and is based upon a two dimensional filtered back projection algorithm.

The great advantage with cone-beam CT is that all projection data needed for the reconstruction of an entire object is captured during one single rotation, instead of reconstructing each slice subsequently as in fan-beam CT.

Experiments

Figure 4 is a schematic representation of the experimental arrangement. The ultrasound transducers used in the experiment have a diameter of 1 cm and radiate sound with the centre frequency of 40 kHz. They were arranged in two different configurations; 1) in a line, and 2) in an L-shape, in a special home made transducer holder (Figure 5). The holder was in turn placed on a rotation stage position between a white painted heavy concrete block of measures 1.3\times1.2\times0.4 \, m^3, and the laser head of a PSV 300 vibrometer system from Polytec GmbH. The concrete block acts as a rigid reflector which is necessary for measuring the small changes in the optical path length for the vibrometer laser beam probing through the sound field in the measurement volume [8, 9, 24, 25]. The transducer-holder-rotation stage arrangement is in such a way that the transducers can rotate around a vertical axis. This makes it possible to record different projections of the sound field keeping the laser head stationary. The transducers are driven at 40 kHz by a signal generator. The excitation signal is also used as a reference signal and trigs the LDV measurements in time.

A grid of 140\times50 \, mm^2 with 1 mm increments in both directions was established on a plane above the transducers, which gives a spatial resolution limit (in the grid plane) of 2 mm which corresponds to an upper frequency resolution of 170 kHz in air, for each projection recordings. The distance between the laser head and the reflector is 3.2 m and the rotation axis is 0.82 m in front of the reflector. The sensitivity of the laser vibrometer was set to 5 mms^{-1}V^{-1} and the system was set up to perform sampling with a frequency of 128 kHz with a maximum detectable frequency of 50 kHz. The first experiment was made with a symmetric configuration of the sound transducers. Three transducers were fixed in a straight line in the transducer holder mounted on the rotation stage; see Figure 5(a). The transducers were operating in phase from the same signal during the measurements. In order to save time not all projections were measured, 72 projections were obtained, 19 from measurements and the rest using the symmetry in the setup. Consider Figure 6, the angle $\theta$ is defined as the angle between the normal to the rigid reflector surface and the line on which the transducers are situated. The angle difference between each measurement...
was chosen to be 5 degrees resulting in 72 necessary projections for the three-dimensional reconstruction of the sound field. Since the sound source configuration consisted of three sound sources positioned in a straight line, and if the sound sources can be assumed to be identical, only the measurements at angles between 0 to 90 degrees produce unique projections. Let a point in a projection be denoted by \( P_n(x,y) \), where \( n \) is for the projection number varying from 0 to 71. \( x \) and \( y \) are the number of pixels confined to \([1,N_x]\) and \([1,N_y]\) where \( N_x \) and \( N_y \) are the number of pixels along \( x \) and \( y \) respectively. If the first 19 projections are measured according to Figure 6(a), then the next 18 projections can be obtained through a mirror operation defined as

\[
P^*_i(x,y) = P_{19+i}(N_x - x, y) \quad n = 19 \ldots 36 .
\]
As seen in Figure 6(b) the source configuration of projection \( n \) is a flip around the y-axis of the source configuration of projection 36-\( n \) when \( n \) varies from 19 to 36. This means that also the corresponding measurement result also should be flipped around the y-axis, which is described by the operation in equation (15). The last 35 projections are then obtained by copying the first according to

\[
P_{n}(x,y) = P_{37-n}(x,y) \quad n = 37...71.
\]  

see Figure 6(c). This way all necessary 72 projections are obtained from only 19 sound field measurements.

Results

Both the real and the imaginary part were then reconstructed, separately, and afterwards recombined in order to obtain the full 3D complex amplitude of the sound field. In Figure 7 the real part of the sound field is plotted for different planes. The transducers are situated in the xy-plane at \( z=0 \) mm. In (a) the sound field is plotted for xy-planes at different distances from the source plane. In (b) the sound field is plotted for different xz-planes. In (a) it can be seen that the sound source in middle seem to be somewhat out of phase compared to the other two. This is due to the fact that it is positioned a little bit lower relative to the others. Since the wavelength is only about 9 mm a shift in position of only 2 mm corresponds to a phase shift of 80 degrees. In Figure 7(b) the propagation of sound wave is visualized more clearly. In the plane plotted at \( y=70 \) mm, it can be seen that the sound is strongest in the z-direction, and directions of minimum sound radiation can also be observed.

![Figure 7](image-url)  

Figure 7. Results from the symmetric measurement. The real part of the measured sound field plotted for different xy- and xz-planes are plotted in (a) and (b) respectively.

In Figure 8, the results for the unsymmetrical sound source configuration (Figure 5(b)) is plotted. This time all 72 projections were obtained through measurements. Now, one of the transducers, the lower left one in the L, was set to operate out of phase relative to the other three. This also shows in the reconstruction plotted in Figure 8. In the same figure it can also be seen that the transducer in the middle once again is positioned a little bit lower than the others, resulting in a phase shift. A common source to image artefacts in tomographic reconstruction is due to angular under sampling of projection data. Theoretically the number of acquired projections over 360° should equal the maximum object dimension in voxels times the factor \( n/2 \) [21], in order to avoid such effects. In our case, it would take 228 equiangular sampled projections to fulfil this requirement. Achieving this high sampling frequency, in practice, would have been a very time consuming process. Therefore we have allowed a certain degree of aliasing effects in the reconstructed data, and reduced the number of acquired projections to 72. The resulting under sampling artefacts is visible as streaks in the reconstructed data. These streaks can be seen in the z-slices in Figure (8) and are more pronounced peripherally. However, they do not have any vital importance to the results and conclusions of these investigations.
Laser vibrometry which is normally used for surface vibration measurements have shown to be also a method for measuring the refractive index variation of the air caused by a sound field. The remote, and non-contact method is non-perturbing and if an area of interest is scanned a two dimensional projection of the sound field can be obtained rapidly and with high spatial resolution. These projections can be visualized in illustrative ways; as images or movies which often give an idea of the whole field if the sound source is well known [11]. However, when analyzing sound fields using laser vibrometry, one must always have in mind that the output is an integral of the refractive index variation along the laser path, which in the worst case scenario; positive and negative variation of the refractive index with equal amplitude cancel each other. In order to obtain the refractive index variation for all points in the measurement volume, tomographic reconstruction using several projections of the sound field is conducted. The cone-beam algorithm for computed tomography is used for the reconstruction of the sound fields in the experiments and the symmetry of the transducer configuration is taken into advantage when possible to reduce the measurement time and amount of data to be storage. The spatial resolution of the tomographic reconstructed sound field is higher in the centre and decreases towards the edges of the measurement volume. However, the resolution can be improved further by increasing the density of the measurement grid and by increasing the number of projection recordings per revolution.

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References


